

Harvard Math 129: Algebraic Number Theory

Homework Assignment 8

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Due: THURSDAY, April 21, 2005

The problems have equal point value, and multi-part problems are of the same value. In any problem where you use a computer, include in your solution the exact commands you type and their output. You may use any software, including (but not limited to) MAGMA and PARI.

1. Look up the Riemann-Roch theorem in a book on algebraic curves.
 - (a) Write it down, somewhat in your own words.
 - (b) Let E be an elliptic curve over a field K . Use the Riemann-Roch theorem to deduce that the natural map

$$E(K) \rightarrow \text{Pic}^0(E/K)$$

is an isomorphism. (If you can't figure out the argument because you aren't very familiar with algebraic curves, try to find it in a book and put it in your own words—this is not cheating.)

2. Write down a “proposal” or summary of your final project. Do something reasonable here for full credit.