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INTRODUCTION

Introduction to the First (1992) Edition

This book is in three sections. First, we describe in detail an algorithm based on modular symbols for computing modular elliptic curves: that is, one-dimensional factors of the Jacobian of the modular curve $X_0(N)$, which are attached to certain cusp forms for the congruence subgroup $\Gamma_0(N)$. In the second section, various algorithms for studying the arithmetic of elliptic curves (defined over the rationals) are described. These are for the most part not new, but they have not all appeared in book form, and it seemed appropriate to include them here. Lastly, we report on the results obtained when the modular symbols algorithm was carried out for all $N \leq 1000$. In a comprehensive set of tables we give details of the curves found, together with all isogenous curves (5113 curves in all, in 2463 isogeny classes¹). Specifically, we give for each curve the rank and generators for the points of infinite order, the number of torsion points, the regulator, the traces of Frobenius for primes less than 100, and the leading coefficient of the L -series at $s = 1$; we also give the reduction data (Kodaira symbols, and local constants) for all primes of bad reduction, and information about isogenies.

For $N \leq 200$ these curves can be found in the well-known tables usually referred to as “Antwerp IV” [2], as computed by Tingley [67], who in turn extended earlier tables of curves found by systematic search; our calculations agree with that list in all 281 cases. For values of N in the range $200 < N \leq 320$ Tingley computed the modular curves attached to newforms for $\Gamma_0(N)$ only when there was no known curve of conductor N corresponding to the newform: these appear in his thesis [67] but are unpublished. As in [2], the curves E we list for each N have the following properties.

- (1) They have conductor N , as determined by Tate’s algorithm [65].
- (2) The coefficients given are those of a global minimal model for E , and these coefficients (or, more precisely, the c_4 and c_6 invariants) agree with the numerical values obtained from the modular calculation to several decimal places: in most cases, depending on the accuracy obtained—see below—differing by no more than 10^{-30} .
- (3) Their traces of Frobenius agree with those of the modular curves for all primes $p < 1000$.

We have also investigated, for each curve, certain numbers related to the Birch–Swinnerton-Dyer conjecture. Let $f(z)$ be a newform for $\Gamma_0(N)$ with rational Fourier coefficients, and E the elliptic curve defined over \mathbb{Q} attached to f . The value of $L(f, 1)$ is a rational multiple of a period of f , and may be computed easily using modular symbols (see [37] and Section 2.8 below). We have computed this rational number in each case, and find that it is always consistent with the Birch–Swinnerton-Dyer conjecture for E . More specifically, let $\Omega_0(f)$ be the least positive real period of F and $\Omega(f) = 2\Omega_0(f)$ or $\Omega_0(f)$ according as the period lattice of f is or is not rectangular. Then we find that $L(f, 1)/\Omega(f) = 0$ if (and only if) the Mordell-Weil group $E(\mathbb{Q})$

¹In the first edition, these numbers were given as 5089 and 2447 respectively, as the curves of conductor 702 were inadvertently omitted.

has positive rank, and when $E(\mathbb{Q})$ is finite we find in each case that

$$L(f, 1)/\Omega(f) = \prod_{p|N} c_p \cdot |E(\mathbb{Q})|^{-2} \cdot S$$

with $S \in \mathbb{N}$ (in fact $S = 1$ in all but four cases: $S = 4$ in three cases and $S = 9$ in one case). This is consistent with the Birch–Swinnerton-Dyer conjecture if the Tate–Shafarevich group III is finite of order S . (Here c_p is the local index $[E(\mathbb{Q}_p) : E_0(\mathbb{Q}_p)]$; see [58, p.362].) When $L(f, 1) = 0$, we compute the sign w of the functional equation for $L(f, s)$, and verify that $w = +1$ if and only if the curve has even rank. More precisely, we also compute the value of $L^{(r)}(f, 1)$, where r is the rank, the regulator R , and the quotient

$$S = \frac{L^{(r)}(f, 1)}{r! \Omega(f)} \bigg/ \frac{(\prod c_p) R}{|E(\mathbb{Q})_{\text{tors}}|^2}.$$

In all but the four cases mentioned above we find that $S = 1$ to within the accuracy of the computation.

Our algorithm uses modular symbols to compute the 1-homology of $\Gamma_0(N) \backslash \mathcal{H}^*$ where \mathcal{H}^* is the extended upper half-plane $\{z \in \mathbb{C} : \text{Im}(z) > 0\} \cup \{\infty\} \cup \mathbb{Q}$. While similar in some respects to Tingley’s original algorithm described in [67], it also uses ideas from [37] together with some new ideas which will be described in detail below. One important advantage of our method, compared with Tingley’s, is that we do not need to consider explicitly the exact geometric shape of a fundamental region for the action of $\Gamma_0(N)$ on \mathcal{H}^* : this means that highly composite N can be dealt with in exactly the same way as, say, prime N . Of course, for prime N there are other methods, such as that of Mestre [43], which are probably faster in that case, though not apparently yielding the values of the “Birch–Swinnerton-Dyer numbers” $L(f, 1)/\Omega(f)$. There is also a strong similarity between the algorithms described here and those developed by the author in his investigation of cusp forms of weight two over imaginary quadratic fields [12], [13], [15]. A variant of this algorithm has also been used successfully to study modular forms for $\Gamma_0(N)$ with quadratic character, thus answering some questions raised by Pinch (see [48] or [49]) concerning elliptic curves of everywhere good reduction over real quadratic fields. See [14] for details of this, and for a generalization to $\Gamma_1(N)$: one could find cusp forms of weight two with arbitrary character using this extension of the modular symbol method, though at present it has only been implemented for quadratic characters, as described in [14].

It is not our intention in this book to discuss the theory of modular forms in any detail, though we will summarize the facts that we need, and give references to suitable texts. The theoretical construction and properties of the modular elliptic curves will also be excluded, except for a brief summary. Likewise, we will assume that the reader has some knowledge of the theory of elliptic curves, such as can be obtained from one of the growing number of excellent books on the subject. Instead we will be concentrating on computational aspects, and hope thus to complement other, more theoretical, treatments.

In Chapter 2 we describe the various steps in the modular symbol algorithm in detail. At each step we give the theoretical foundations of the method used, with proofs or references to the literature. Included here are some remarks on our implementation of the algorithms, which might be useful to those wishing to write their own programs. At the end of this stage we have equations for the curves, together with certain other data for the associated cusp form: Hecke eigenvalues, sign of the functional equation, and the ratio $L(f, 1)/\Omega(f)$.

Following Chapter 2, we give some worked examples to illustrate the various methods.

In Chapter 3 we describe the algorithms we used to study the elliptic curves we found using modular symbols, including the finding of all curves isogenous to those in the original list.

These algorithms are more generally applicable to arbitrary elliptic curves over \mathbb{Q} , although we do not consider questions which might arise with curves having bad reduction at very large primes. (For example, we do not consider how to factorize the discriminant in order to find the bad primes, as in all cases in the tables this is trivially achieved by trial division). Here we compute minimal equations, local reduction types, rank and torsion, generators for the Mordell–Weil group, the regulator, and traces of Frobenius. This includes all the information published in the earlier Antwerp IV tables. The final calculations, relating to the Birch–Swinnerton-Dyer conjecture, are also described here; these combine values obtained from the cusp forms (specifically, the leading coefficient of the expansion of the L -series at $s = 1$, and the real period) with the regulator and local factors obtained directly from the curves. Thus we can compute in each case the conjectural value S of the order of III, the Tate–Shafarevich group.

Finally, in Chapter 4 we discuss the results of the computations for $N \leq 1000$, and introduce the tables which follow.

All the computer programs used were written in Algol68 (amounting to over 10000 lines of code in all) and run on the ICL3980 computer at the South West Universities Regional Computing Centre at Bath, U.K.. The author would like to express his thanks to the staff of SWURCC for their friendly help and cooperation, and also to Richard Pinch for the use of his Algol68 multiple-length arithmetic package. At present, our programs are not easily portable, mainly because of the choice of Algol68 as programming language, which is not very generally available. However we are currently working on a new version of the programs, written in a standard version of the object-oriented language C++, which would be easily portable. The elliptic curve algorithms themselves are currently (1991) available more readily, in a number of computer packages.² In particular, the package `apecs`, written in Maple and available free via anonymous file transfer from Ian Connell of McGill University, will compute all the data we have included for each curve. (A slightly limited version of `apecs`, known as `upecs`, runs under UBASIC on MS-DOS machines). There are also elliptic curve functions available for Mathematica (Silverman’s Elliptic Curve Calculator) and in the PARI/GP package. These packages are all in the process of rapid development.

An earlier version of Chapter 2 of this book, with the tables, has been fairly widely circulated, and several people have pointed out errors which somehow crept in to the original tables. We have made every effort to eliminate typographical errors in the tables, which were typeset directly from data files produced by the programs which did the calculations. Where possible, the data for each curve has been checked independently using other programs. Amongst those who have spotted earlier errors or have helped with checking, I would like to mention Richard Pinch, Harvey Rose, Ian Connell, Noam Elkies, and Wah Keung Chan; obviously there may still be some incorrect entries, but these remain solely my responsibility.

Introduction to the Second (1996) Edition

Since the first edition of this book appeared in 1992, some significant advances have been made in the algorithms described and in their implementation. The second edition contains an account of these advances, as well as correcting many errors and omissions in the original text and tables. We give here a summary of the more substantial changes to the text and tables.

Of course, the most significant theoretical advance of the last four years is the proof by Wiles, Taylor–Wiles and others of most cases of the Shimura–Taniyama–Weil conjecture, which almost makes the word “modular” in the title of this book redundant. However, the only effect the new results have on this work are to guarantee that every elliptic curve defined over the rationals and of conductor less than 1000 is isomorphic to one of those in our Table 1.

²See the end of the introduction for more on obtaining these packages.

Chapter 2. Section 2.1 has been completely rewritten and expanded to give a much more coherent, self-contained, and (we hope) correct account of the theoretical background to the modular symbol method. The text here is based closely on some unpublished lecture notes of the author for a series of lectures he gave in Bordeaux in 1995 at the meeting “État de la Recherche en Algorithmique Arithmétique” organized by the Société Mathématique de France.

In Section 2.4, we give a self-contained treatment of the method of Heilbronn matrices for computing Hecke operators, similar to the treatment by Merel in [42], as this now forms part of our implementation.

In Section 2.10, we give a new method of computing periods of cusp forms, as described in [18], which is as efficient as the “indirect” method; this largely makes the indirect method redundant, but we still include it in Section 2.11. Also in Section 2.11, we include some tricks and shortcuts which we have developed as we pushed the computations to higher levels, which can greatly reduce the computation time needed to find equations for the curves of conductor N , at the expense of not necessarily knowing which is the so-called “strong Weil” curve in its isogeny class.

Section 2.14 has been rewritten to take into account the results of Edixhoven on the Manin constant (see [21]), which imply that the values of c_4 and c_6 which we compute for each curve are known *a priori* to be integral. This means that the values we compute are guaranteed to be correct, and eliminates the uncertainty previously existing as to whether the curves we obtain by rounding the computed values are the modular elliptic curves they are supposed to be.

Section 2.15 is entirely new: we show how to compute the degree of the modular parametrization map $\varphi: X_0(N) \rightarrow E$ for a modular elliptic curve of conductor N , using our version (see [17]) of a method of Zagier [69]. This method is easy to implement within the modular symbol framework, and we have added it to our programs, so that we now compute the degree automatically for each curve we find.

The appendix to Chapter 2, containing worked examples, now includes the Heilbronn matrix method, and also illustrates some of the tricks mentioned in Section 2.11.

Implementation changes. The implementations of all the algorithms described here have been completely rewritten in C++, to be easily portable. We use the GNU compiler `gcc` for this. For multiprecision arithmetic we use either the GNU package `libg++` or the package `LiDIA`. For solving the systems of linear equations giving the relations between M-symbols, we use sparse matrix routines which not only reduce memory requirements, but also speed up that part of the computations considerably. These routines were written by L. Figueiredo specifically for his work on imaginary quadratic fields (see [24]) which in turn built on the author’s work in [12] and [13].

Chapter 3. In Section 3.1 we give simpler formulae for recovering the Weierstrass coefficients of a curve from the invariants c_4 and c_6 ; this enables us to simplify the Kraus–Laska–Connell algorithm slightly. In Section 3.4 we give a slightly improved formula for the global canonical height, and include this as a separate algorithm. Section 3.5 now contains references to other bounds between the naive and canonical heights, and other methods for the infinite descent step, but without details.

The main changes in this chapter are to Section 3.6 on two-descent algorithms. On the one hand, we give a better explanation of the theoretical basis for these algorithms, making the account more self-contained (though we do not include all proofs). We have also moved the discussion on testing homogeneous spaces for local and global solubility forward, as this is common to the two main algorithms (general two-descent and two-descent via 2-isogeny). On the other hand, several parts of the algorithm have been subject to major improvements over the last few years, thanks to collaboration with P. Serf, S. Siksek and N. P. Smart, and these

are now included. Notable here are the syzygy sieve in the search for quartics, the systematic use of group structure in the 2-isogeny case, and the use of quadratic sieving in searching for rational points on homogeneous spaces. We also simplify the test for equivalence of quartics and the process of recovering rational points on the curve from points on the homogeneous spaces. Many of these improvements are from the author's paper [20], which contains some proofs omitted here.

Implementation changes. As with the modular symbol algorithms, we have rewritten all the elliptic curve algorithms in C++. In the case of the program to find isogenies, which is very sensitive to the precision used, we have written an independent implementation in PARI/GP; using this we have a check on the isogeny computations which gave the isogenous curves listed in Table 1. (The standard precision version of this program, while much faster, does miss several of the isogenies, for reasons given in Section 3.8.)

Versions of our algorithms will shortly become generally available in two forms. First, the package LiDIA (a library of C++ classes for computational number theory, developed by the LiDIA group at the Universität des Saarlandes in Germany) will include them in a coming release. Secondly, the package MAGMA is also in the process of implementing the algorithms.

In addition to these packages and those mentioned in the original Introduction, we should also mention the package SIMATH, developed by H. G. Zimmer's research group in Saarbrücken, which also has a large collection of very efficient elliptic curve algorithms.

See the end of this Introduction for how to obtain more information on these packages.

Chapter 4 and Tables. The two main changes in the tables are to include all the data for $N = 702$ in Tables 1–4 and include the new Table 5 giving the degree of the modular parametrization for each strong Weil curve. The omission of level 702 in the first edition is hard to explain; in our original implementation and file structure, it was not possible to distinguish between a level which had run successfully, but with no rational newforms found, and a level which had not yet run. The original runs were done as batch jobs on a remote mainframe computer, with manual record-keeping to keep track of which levels had run successfully. Our current implementation is much more robust in this respect. We are grateful to Henri Cohen who first discovered this error on comparing our data with his own tables (of modular forms of varying weight and level, computed by him together with Skoruppa and Zagier). The omission was also noted by Jacques Basmaji of Essen, who recomputed Table 3 independently.

The new implementation finds the newforms at each level in a consistent order. In the original runs, the order in which the newforms were found changed as the program developed. Unfortunately, we did not recompute the earlier levels with the final version of the program before publishing the first edition of the tables, and the identifying letter for each newform given in the tables has now become standard. Hence our current implementation reorders the newforms during output to agree with the order as originally published (this is necessary for 147 levels in all, the largest being 450).

Also concerning the order and naming of the curves: the convention we normally use is that in each isogeny class the first curve is the strong Weil curve whose period lattice is exactly that of the corresponding newform for $\Gamma_0(N)$, such as 11A1 for example. In precisely one case, an error caused the first curve listed in class 990H to be not the strong Weil curve but a curve isogenous to it. The strong Weil curve in this class is in fact 990H3 and not 990H1. In the notation of Section 2.11, the correct values of l^+ and m^+ to obtain the strong Weil curve 990H3 are 13 and 8, but for some reason we had used the value $m^+ = 24$ which leads to the 3-isogenous curve 990H1.

In Table 1, the other corrections are: $N = 160$ has the Antwerp codes corrected, and 916B1 has a spurious indication of a non-existent 3-isogeny removed.

In Table 2, we include the generators for the curves of conductor 702 and positive rank, and

again correct the Antwerp code for curve 160A1. We also give the generator of 427C1 correctly as $(-3, 1)$ rather than $(-3, 0)$ as previously, and for 990H we give the generator $(-35, 97)$ of the strong Weil curve 990H3 rather than a generator of 990H1 as before.

In Table 3, as well as inserting the data for $N = 702$, we correct the eigenvalues for $N = 100$ which had been given incorrectly.

In Table 4, we insert the data for $N = 702$ and also for 600E–600I which had been omitted by mistake. Moreover, for $N = 990$ we give the data for 990H3 instead of 990H1 as before, as this is the strong Weil curve (the only difference being that Ω has been multiplied by 3 and R divided by 3).

Extension of the Tables. Using our new implementation of the algorithms of Chapter 2, we have extended the computations of all modular elliptic curves up to conductor 5077 (chosen as the smallest conductor of a curve of rank 3). We have also computed in each case the other data tabulated here for conductors up to 1000. For reasons of space, we cannot print extended versions of the tables: as there are 17598 newforms (or isogeny classes) and a total of 31586 curves up to 5077, this would have made this book approximately six times as thick as it is at present!

Instead, the data for curves whose conductors lie in the range from 1001 to 5077 (and beyond, as they become available) may be obtained by anonymous file transfer from the author's ftp site at <ftp://euclid.ex.ac.uk/pub/cremona/data>.

Finally, many thanks to those who have told me of misprints and other errors in the First Edition, including J. Basmaji, G. Bailey, B. Brock, F. Calegari, J. W. S. Cassels, T. Kagawa, B. Kaskel, P. Serf, S. Siksek, and N. Smart. Apologies to any whose names have been omitted. Extra thanks are also due to Nigel Smart, who read a draft of Chapter 3 of the Second Edition, and made useful suggestions.

web and ftp sites

More information on the packages mentioned above, and in most cases the packages themselves, can be obtained from the following web and ftp sites. Apart from MAGMA they are all free.

apecs (for Maple):	ftp://math.mcgill.ca/pub/apecs
Elliptic Curve Calculator (for Mathematica):	ftp://gauss.math.brown.edu/dist/EllipticCurve
LiDIA:	http://www-jb.cs.uni-sb.de/LiDIA
MAGMA:	http://www.maths.usyd.edu.au:8000/comp/magma
mwrnk:	ftp://euclid.ex.ac.uk/pub/cremona/progs
PARI/GP:	ftp://megrez.math.u-bordeaux.fr/pub/pari
SIMATH:	http://emmy.math.uni-sb.de/~simath
upecs (for UBASIC):	ftp://math.mcgill.ca/pub/upecs

Links to all of these can be found at

<http://www.maths.ex.ac.uk/~cremona/packages.html>.

MODULAR SYMBOL ALGORITHMS

In this chapter we describe the modular symbol method in detail. First, in Sections 2.1 to 2.5, we describe the use of modular symbols and M-symbols to compute the homology space $H_1(X_0(N), \mathbb{Q})$ and the action of the Hecke algebra, for an arbitrary positive integer N . At this stage it is already possible to identify rational newforms f , and obtain some information about the modular elliptic curves E_f attached to them: these are introduced in Section 2.6. To obtain equations for the curves E_f we compute their period lattices: the methods used for this stage occupy most of the remaining sections of the chapter. The final section 2.15 shows how to compute the degree of the associated map $\varphi : X_0(N) \rightarrow E_f$.

To illustrate the methods, we also give some worked examples in an Appendix to the chapter.

2.1 Modular Symbols and Homology

2.1.1. The upper half-plane, the modular group and cusp forms.

Let \mathcal{H} denote the upper half-plane

$$\mathcal{H} = \{z = x + iy \in \mathbb{C} \mid y > 0\},$$

and $\mathcal{H}^* = \mathcal{H} \cup \mathbb{Q} \cup \{\infty\}$ the extended upper half-plane, obtained by including the cusps $\mathbb{Q} \cup \{\infty\}$. The group $\mathrm{PSL}_2(\mathbb{R})$ acts on \mathcal{H}^* via linear fractional transformations:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}: z \mapsto \frac{az + b}{cz + d};$$

these are the isometries of the hyperbolic geometry on \mathcal{H} , for which geodesics are either half-lines perpendicular to the real axis \mathbb{R} , or semicircles perpendicular to \mathbb{R} .

The modular group $\Gamma = \mathrm{PSL}_2(\mathbb{Z})$ is a discrete subgroup of $\mathrm{PSL}_2(\mathbb{R})$ (in the topology induced from $\mathrm{SL}(2, \mathbb{R}) \subset M_2(\mathbb{R}) \cong \mathbb{R}^4$), and acts discontinuously on \mathcal{H} , in the sense that for each $z \in \mathcal{H}$ the orbit $\Gamma.z$ is discrete. Note that the cusps $\mathbb{Q} \cup \{\infty\}$ form a complete Γ -orbit.

The elements $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}: z \mapsto -1/z$ (of order 2) and $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}: z \mapsto z + 1$ (of infinite order) generate Γ . This fact, and the related fact that a fundamental region for the action of Γ on \mathcal{H} is given by the set \mathcal{F} defined by

$$(2.1.1) \quad \mathcal{F} = \{z = x + iy \in \mathcal{H} \mid |x| \leq \frac{1}{2}, |z| \geq 1\},$$

are standard and will not be proved here. Both results depend essentially on the fact that \mathbb{Z} is Euclidean.

Let G be a subgroup of Γ of finite index e . Then G also acts discretely on \mathcal{H} . A fundamental region for G on \mathcal{H} is given by $\cup M_i.\mathcal{F}$, where the M_i (for $1 \leq i \leq e$) are right coset representatives for G in Γ .

Let $X_G = G \backslash \mathcal{H}^*$ denote the quotient space; this may be given the structure of a compact Riemann surface. Around most points the local parameter is just z , but more care is needed about the “parabolic points” or cusps, and the “elliptic points” which have non-trivial stabilizers in the Γ -action. These elliptic points for G (if any) are in the Γ -orbits of i (stabilized by S of order 2), and of $\rho = (1 + \sqrt{-3})/2$ (stabilized by TS of order 3). See the books of Lang [32], Shimura [55] or Knapp [28] for details of the Riemann surface construction.

Let g denote the genus of the surface X_G ; as a real manifold¹, X_G is a g -holed torus. We will be concerned with the explicit computation of the 1-homology $H_1(X_G, \mathbb{Z})$, which is a free \mathbb{Z} -module of rank $2g$. (See Subsection 2.1.2 below for a brief review of homology). This homology will be expressed in terms of “modular symbols”, defined below. We must also explain the connection between homology, modular forms, and elliptic curves. First we review the definition of cusp forms.

The space of *cusp forms of weight 2* for G will be denoted by $S_2(G)$. These cusp forms are holomorphic functions $f(z)$ for $z \in \mathcal{H}$ which satisfy

(1) $f|_M = f$ for all $M \in G$, where

$$\left(f \left| \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right. \right) (z) = (cz + d)^{-2} f \left(\frac{az + b}{cz + d} \right).$$

Thus, since $(cz + d)^{-2} = (d/dz)(M(z))$ for $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, we have, for all $M \in G$,

$$f(M(z))d(M(z)) = f(z)dz.$$

(2) $f(z)$ behaves nicely at the cusps. The significance of this condition is that, by (1), a cusp form of weight 2 for G is the pull-back of a (holomorphic) differential on the Riemann surface $G \backslash \mathcal{H}$, of which X_G is the compactification after adding the (finitely many) G -inequivalent cusps, and we want this differential to be holomorphic on the whole of X_G . In future we will identify cusp forms of weight 2 for G with holomorphic differentials on X_G . From standard Riemann surface theory, we then know that $S_2(G)$ is a complex vector space of dimension g .

We can make explicit the condition that $f(z)dz$ is holomorphic at the cusp ∞ (for the other cusps, see one of the references on the theory of modular forms). The stabilizer of ∞ in Γ is the infinite cyclic subgroup generated by T ; if h is the least positive integer such that $T^h \in G$, then clearly we have

$$\text{Stab}(\infty) \cap G = \langle T^h \rangle,$$

and every $f \in S_2(G)$ has a Fourier expansion of the form

$$(2.1.2) \quad f(z) = \sum_{n=1}^{\infty} a_n e^{2\pi i n z / h}$$

with coefficients $a_n \in \mathbb{C}$. The integer h is called the *width* of the cusp ∞ ; for $G = \Gamma_0(N)$, the case we will be most interested in, we have $h = 1$, since $T \in \Gamma_0(N)$.

¹Strictly speaking, X_G is not a manifold unless G has no elements of finite order, because of the branching over the elliptic points. However this will make no difference in practice and we may safely ignore it.

2.1.2. The duality between cusp forms and homology.

The basis for our method is the explicit computation of the homology (specifically, the rational 1-homology) of the Riemann surface X_G . This is useful for various reasons. On the one hand, this gives us a very explicit vector space on which Hecke operators act, which is isomorphic (or more strictly, dual) to the space of cusp forms. Thus by computing homology and the Hecke action on it, we are indirectly also able to obtain information about the space of cusp forms. The Fourier coefficients of the cusp forms are determined by their Hecke eigenvalues (see Section 2.6), so we obtain these indirectly as eigenvalues of Hecke operators acting on homology. Secondly, in order to actually compute the elliptic curves attached to these cusp forms, we need to know their periods, which are obtained by integrating the corresponding differentials around closed paths on the surface X_G ; since two paths give the same period (for all forms) if and only if they are homologous (essentially by Cauchy's Theorem on X_G), it is clear that to determine the whole period lattice we will also require an explicit knowledge of the homology of X_G .

The integral homology $H_1(X_G, \mathbb{Z})$ is most easily defined geometrically: it is the abelian group obtained by taking as generators all closed paths on X_G , and factoring out by the relation that two closed paths are equivalent (or *homologous*) if one can be continuously deformed into the other. If the genus of the surface X_G is g , this gives a free abelian group of rank $2g$: roughly speaking, the surface is a g -holed torus, and there are two generating loops around each hole. To determine this homology group in practice, one triangulates the surface, so that every path is homologous to a path along the edges of the triangulation. Now the generators are the directed edges of the triangulation, modulo relations given by the sum of the edges around each triangle being homologous to zero. A typical element of $H_1(X_G, \mathbb{Z})$ will then be given as a \mathbb{Z} -linear combination of these directed edges. In Subsection 2.1.6 below, we will make this very explicit: there will be one edge of the triangulation for each coset of G in Γ , and the triangle relations will be expressed algebraically in terms of the coset action of Γ . This description will entirely algebraicize the situation, in a way which is then easy to implement on a computer.

For any other ring R , the homology with coefficients in R is obtained simply by tensoring with R :

$$H_1(X_G, R) = H_1(X_G, \mathbb{Z}) \otimes_{\mathbb{Z}} R.$$

Explicitly, one just takes R -linear combinations of the $2g$ generators of the \mathbb{Z} -module $H_1(X_G, \mathbb{Z})$ ("extension of scalars"); the result is then an R -module. In what follows we will only need to take $R = \mathbb{Q}$, $R = \mathbb{R}$, and $R = \mathbb{C}$.

Let $H_1(X_G, \mathbb{R}) = H_1(X_G, \mathbb{Z}) \otimes_{\mathbb{Z}} \mathbb{R}$, which is a real vector space of dimension $2g$. Abstractly, this space is obtained by formal extension of scalars from $H_1(X_G, \mathbb{Z})$; but we can be more concrete, if we introduce the notion of modular symbols.

First let $\alpha, \beta \in \mathcal{H}^*$ be points equivalent under the action of G , so that $\beta = M(\alpha)$ for some $M \in G$. Any smooth path (for instance, a geodesic path) from α to β in \mathcal{H}^* projects to a closed path in the quotient space X_G , and hence determines an integral homology class in $H_1(X_G, \mathbb{Z})$, which depends only on α and β and not on the path chosen, because \mathcal{H}^* is simply connected. (In fact, the class depends only on M : see (5) in Proposition 2.1.1 below). We denote this homology class by the *modular symbol* $\{\alpha, \beta\}_G$, or simply $\{\alpha, \beta\}$ when the group G is clear from the context.

Conversely, every integral homology class $\gamma \in H_1(X_G, \mathbb{Z})$ can be represented by such a modular symbol $\{\alpha, \beta\}$. Also, if $f \in S_2(G)$ then the integral

$$\int_{\gamma} 2\pi i f(z) dz = 2\pi i \int_{\alpha}^{\beta} f(z) dz$$

is well-defined, since $f(z)$ is holomorphic, and will be denoted either as $\langle \gamma, f \rangle$ or as $I_f(\alpha, \beta)$. The (complex) value of such an integral is called a *period* of the cusp form f , or of the associated differential $2\pi i f(z) dz$.

Let f_1, f_2, \dots, f_g be a fixed basis for $S_2(G)$, so that the differentials $2\pi i f_j(z) dz$ are a basis for the holomorphic differentials on X_G . Also let $\gamma_1, \gamma_2, \dots, \gamma_{2g}$ be a fixed \mathbb{Z} -basis for the integral homology $H_1(X_G, \mathbb{Z})$. Then we may form the $2g \times g$ complex *period matrix*

$$\Omega = (\omega_{jk}) = (\langle \gamma_j, f_k \rangle).$$

By standard Riemann surface theory, the $2g$ rows of Ω are linearly independent over \mathbb{R} , and so their \mathbb{Z} -span is a lattice (discrete subgroup) Λ of rank $2g$ in \mathbb{C}^g . The quotient $J(G) = \mathbb{C}^g / \Lambda$ is the Jacobian of X_G ; it is an abelian variety of dimension g .

The symbols $\{\alpha, \beta\}$ give \mathbb{C} -linear functionals $S_2(G) \rightarrow \mathbb{C}$ via $f \mapsto I_f(\alpha, \beta)$. We may identify $H_1(X_G, \mathbb{R})$ with the space of all \mathbb{C} -linear functionals on $S_2(G)$ as follows: given an element $\gamma \in H_1(X_G, \mathbb{R})$, we can write γ uniquely in the form

$$\gamma = \sum_{j=1}^{2g} c_j \gamma_j$$

with coefficients $c_j \in \mathbb{R}$. Define $\langle \gamma, f \rangle = \sum_{j=1}^{2g} c_j \langle \gamma_j, f \rangle$. Then the corresponding functional is $f \mapsto \langle \gamma, f \rangle$. Conversely, given a functional $\omega: S_2(G) \rightarrow \mathbb{C}$, the vector $(\omega(f_1), \omega(f_2), \dots, \omega(f_g)) \in \mathbb{C}^g$ may be expressed uniquely as an \mathbb{R} -linear combination of the rows of Ω , so there exist real scalars c_j ($1 \leq j \leq 2g$) such that

$$\omega(f) = \sum_{j=1}^{2g} c_j \langle \gamma_j, f \rangle$$

for all $f \in S_2(G)$; then $\omega(f) = \langle \gamma, f \rangle$ where $\gamma = \sum_{j=1}^{2g} c_j \gamma_j \in H_1(X_G, \mathbb{R})$.

In particular, let $\alpha, \beta \in \mathcal{H}^*$ be arbitrary (not necessarily in the same G -orbit); then the functional $f \mapsto I_f(\alpha, \beta)$ corresponds to a unique element $\gamma = \sum_{j=1}^{2g} c_j \gamma_j \in H_1(X_G, \mathbb{R})$, and we *define* the modular symbol $\{\alpha, \beta\}_G \in H_1(X_G, \mathbb{R})$ to be this element. Clearly this definition agrees with the earlier one in the special case where $\beta = M(\alpha)$ for some $M \in G$; indeed, this case holds if and only if all $c_j \in \mathbb{Z}$.

By the *field of definition* of an element $\gamma \in H_1(X_G, \mathbb{R})$ we mean the field generated over \mathbb{Q} by its coefficients c_j (with respect to the \mathbb{Z} -basis for the integral homology, as above). For example, γ is rational (has field of definition \mathbb{Q}) if and only if $\gamma \in H_1(X_G, \mathbb{Q})$.

We now have an \mathbb{R} -bilinear pairing

$$(2.1.3) \quad S_2(G) \times H_1(X_G, \mathbb{R}) \longrightarrow \mathbb{C}$$

given by

$$(f, \gamma) \mapsto \langle \gamma, f \rangle = \int_{\gamma} 2\pi i f(z) dz$$

which gives an exact duality between the two spaces on the left if we view $S_2(G)$ as a real vector space of dimension $2g$ by restriction of scalars from \mathbb{C} to \mathbb{R} .

To interpret this as a duality over \mathbb{C} , we can give $H_1(X_G, \mathbb{R})$ the structure of a vector space over \mathbb{C} (of dimension g) as follows. Given $\gamma \in H_1(X_G, \mathbb{R})$ and $c \in \mathbb{C}$, we define $c\gamma$ to be that element of $H_1(X_G, \mathbb{R})$ which satisfies $\langle c\gamma, f \rangle = \langle \gamma, cf \rangle$ for all $f \in S_2(G)$; in other words, $c\gamma$ is the element corresponding to the functional $f \mapsto c \langle \gamma, f \rangle$. Now the map $(f, \gamma) \mapsto \langle \gamma, f \rangle$ is \mathbb{C} -bilinear, and the dual pairing (2.1.3) between homology and cusp forms is an exact duality over \mathbb{C} .

2.1.3. Real structure.

For suitable groups G we can restrict the duality described above to a duality between real vector spaces of dimension g . This has important implications for explicit computations, where a halving of the dimension (from $2g$ to g) gives a significant saving of effort.

Let $J = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$. We say that a subgroup G of Γ is *of real type* if J normalizes G .

Explicitly, let $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in G$; then $J^{-1}MJ = \begin{pmatrix} a & -b \\ -c & d \end{pmatrix} = M^*$, say, and G is of real type when $M \in G \iff M^* \in G$. This will be true, in particular, for the congruence subgroups $\Gamma_0(N)$ and $\Gamma_1(N)$ of most interest to us.

For $z \in \mathcal{H}$ set $z^* = -\bar{z}$. Then a trivial calculation shows that $w = M(z) \iff w^* = M^*(z^*)$; it follows that, for G of real type, the map $z \mapsto z^*$ induces a well-defined map on the quotient X_G , and hence also on homology, via $\{\alpha, \beta\} \mapsto \{\alpha^*, \beta^*\}$. Clearly this is an \mathbb{R} -linear involution on $H_1(X_G, \mathbb{R})$. Hence we obtain a decomposition into $+1$ and -1 eigenspaces for $*$:

$$H_1(X_G, \mathbb{R}) = H_1^+(X_G, \mathbb{R}) \oplus H_1^-(X_G, \mathbb{R}).$$

REMARK. The involution $*$ also acts on the integral homology $H_1(X_G, \mathbb{Z})$, and we may set $H_1^\pm(X_G, \mathbb{Z}) = H_1^\pm(X_G, \mathbb{R}) \cap H_1(X_G, \mathbb{Z})$. However the direct sum $H_1^+(X_G, \mathbb{Z}) \oplus H_1^-(X_G, \mathbb{Z})$ will in general have finite index in $H_1(X_G, \mathbb{Z})$.

We now define dually an involution, also denoted $*$, on the space $S_2(G)$ where G is of real type. For a holomorphic function f on \mathcal{H} , we set $f^*(z) = \overline{f(z^*)}$. Then f^* is also holomorphic on \mathcal{H} , and the following facts are easily verified:

- (1) If f has Fourier expansion $f(z) = \sum a_n q^n$ (where $q = \exp(2\pi iz/h)$ as in (2.1.2) above), then $f^*(z) = \sum \overline{a_n} q^n$. In other words, the Fourier coefficients of f^* are the conjugates of those of f .
- (2) For $M \in \Gamma$, we have $f^* | M = (f | M^*)^*$.
- (3) $\langle \gamma^*, f^* \rangle = \overline{\langle \gamma, f \rangle}$ for all f, γ .

As a formal consequence of fact (2), we immediately see that, for G of real type, the map $f \mapsto f^*$ is an \mathbb{R} -linear map from $S_2(G)$ to itself, which is an involution. Denote by $S_2(G)_\mathbb{R}$ the \mathbb{R} -subspace of $S_2(G)$ fixed by this involution, which by fact (1) consists of those cusp forms with real Fourier coefficients. Then $\dim_{\mathbb{R}}(S_2(G)_\mathbb{R}) = g$, and $S_2(G)_\mathbb{R}$ spans $S_2(G)$ over \mathbb{C} .

For nonzero $f \in S_2(G)_\mathbb{R}$ we have (from fact (3)):

$$\langle \gamma, f \rangle \in \mathbb{R} \iff \gamma \in H_1^+(X_G, \mathbb{R}),$$

and also

$$\langle \gamma, f \rangle \in i\mathbb{R} \iff \gamma \in H_1^-(X_G, \mathbb{R}).$$

Moreover, multiplication by i on $H_1(X_G, \mathbb{R})$ interchanges the “real” and “pure imaginary” eigenspaces $H_1^\pm(X_G, \mathbb{R})$ since

$$\begin{aligned} \gamma \in H_1^+ &\iff \langle \gamma, f \rangle \in \mathbb{R} && \forall f \in S_2(G)_\mathbb{R} \\ &\iff \langle i\gamma, f \rangle \in i\mathbb{R} && \forall f \in S_2(G)_\mathbb{R} \\ &\iff i\gamma \in H_1^-. \end{aligned}$$

It follows that $\dim H_1^+(X_G, \mathbb{R}) = \dim H_1^-(X_G, \mathbb{R}) = g$.

Also, since the period $\langle \gamma, f \rangle$ is real for $\gamma \in H_1^+$ and $f \in S_2(G)_\mathbb{R}$, the duality over \mathbb{C} we had earlier now restricts to a duality over \mathbb{R} :

$$(2.1.4) \quad S_2(G)_\mathbb{R} \times H_1^+(X_G, \mathbb{R}) \longrightarrow \mathbb{R}.$$

It follows that the real vector spaces $S_2(G)_\mathbb{R}$ and $H_1^+(X_G, \mathbb{R})$ of dimension g are dual to each other. We will exploit this duality (which also respects the action of Hecke and other operators, see below), as we will compute $H_1(X_G, \mathbb{R})$ explicitly in order to gain information about the cusp forms in $S_2(G)$. Also, this duality is crucial in the definition of modular elliptic curves.

2.1.4. Modular symbol formalism.

We will need the following simple properties of the modular symbols $\{\alpha, \beta\}$.

PROPOSITION 2.1.1. *Let $\alpha, \beta, \gamma \in \mathcal{H}^*$, and let $M, M_1, M_2 \in G$. Then*

- (1) $\{\alpha, \alpha\} = 0$;
- (2) $\{\alpha, \beta\} + \{\beta, \alpha\} = 0$;
- (3) $\{\alpha, \beta\} + \{\beta, \gamma\} + \{\gamma, \alpha\} = 0$;
- (4) $\{M\alpha, M\beta\}_G = \{\alpha, \beta\}_G$;
- (5) $\{\alpha, M\alpha\}_G = \{\beta, M\beta\}_G$;
- (6) $\{\alpha, M_1M_2\alpha\}_G = \{\alpha, M_1\alpha\}_G + \{\alpha, M_2\alpha\}_G$;
- (7) $\{\alpha, M\alpha\}_G \in H_1(X_G, \mathbb{Z})$.

PROOF. Only (5) and (6) are not quite obvious. For (5), write $\{\alpha, M\alpha\} = \{\alpha, \beta\} + \{\beta, M\beta\} + \{M\beta, M\alpha\}$, using (2) and (3); now the first and third terms cancel by (4). For (6), we have $\{\alpha, M_1M_2\alpha\} = \{\alpha, M_1\alpha\} + \{M_1\alpha, M_1M_2\alpha\} = \{\alpha, M_1\alpha\} + \{\alpha, M_2\alpha\}$ using (4). \square

COROLLARY 2.1.2. *The map $M \mapsto \{\alpha, M\alpha\}_G$ is a surjective group homomorphism $G \rightarrow H_1(X_G, \mathbb{Z})$, which is independent of $\alpha \in \mathcal{H}^*$.*

The kernel of this homomorphism contains all commutators and elliptic elements (since the latter have finite order, and the image is a torsion-free abelian group), and also all parabolic elements: for if $M \in G$ is parabolic, it is a conjugate of T and hence fixes some $\alpha \in \mathbb{Q} \cup \{\infty\}$, so $M \mapsto \{\alpha, M\alpha\} = 0$. In fact, the kernel is generated by these elements, but we will not prove that here.

2.1.5. Rational structure and the Manin-Drinfeld Theorem.

We have seen that every element γ of $H_1(X_G, \mathbb{Z})$ has the form $\{\alpha, M\alpha\}$ with $M \in G$ and $\alpha \in \mathcal{H}^*$ arbitrary; usually we take α to be a cusp, so that γ is a path between G -equivalent cusps. It is not clear in general what is the field of definition of a modular symbol $\{\alpha, \beta\}_G$ for which α and β are both cusps. However, when G is a congruence subgroup, this is answered by the Manin-Drinfeld Theorem.

A *congruence subgroup* of Γ is a subgroup G such that membership of G is determined by means of congruence conditions on the entries of a matrix in Γ . A moment's thought shows that this is equivalent to the condition that for some positive integer N , G contains the *principal congruence subgroup* $\Gamma(N)$, which is defined to be the subgroup of Γ consisting of matrices congruent to the identity modulo N . The least such N is called the *level* of G .

The most important congruence subgroups are $\Gamma(N)$ itself;

$$\Gamma_0(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \mid c \equiv 0 \pmod{N} \right\};$$

and

$$\Gamma_1(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \mid c \equiv 0, a \equiv 1 \pmod{N} \right\}.$$

We can now state the Manin-Drinfeld Theorem.

THEOREM 2.1.3. (*Manin, Drinfeld*) *Let G be a congruence subgroup of the modular group Γ . Then for all pairs of cusps $\alpha, \beta \in \mathcal{H}^*$ we have*

$$\{\alpha, \beta\}_G \in H_1(X_G, \mathbb{Q}).$$

In particular, the modular symbol $\{0, \infty\}_G$ is rational; the denominator of this element is very important in many ways.

Thus the rational homology $H_1(X_G, \mathbb{Q})$ is generated by paths between cusps (since it is generated by the integral homology), and conversely every path between cusps is rational. We will see later how to use this fact to develop an algorithm for computing the rational homology.

The proof of the Manin-Drinfeld Theorem involves the use of Hecke operators: see the Remark in Section 2.9 for a sketch of this argument in the case of $\Gamma_0(N)$. Using Hecke operators, we can also prove that the space of cusp forms $S_2(G)$ has a \mathbb{Q} -structure, namely a basis consisting of forms with rational Fourier coefficients (when G is a congruence subgroup). This is related to the fact that the modular curve X_G , which as a Riemann surface is certainly an algebraic curve over \mathbb{C} , can in fact be given the structure of an algebraic curve over the field of algebraic numbers $\overline{\mathbb{Q}}$ (and even over the N th cyclotomic field, if G has level N). This rational structure is crucial to the construction of modular elliptic curves, to ensure that we obtain elliptic curves defined over \mathbb{Q} . For further details, see the books of Lang [32] and Knapp [28].

The duality between cusp forms and homology does not descend entirely to \mathbb{Q} , however, because even if $f \in S_2(G)$ has rational Fourier coefficients and $\gamma \in H_1(X_G, \mathbb{Q})$, the period $\langle \gamma, f \rangle$ will not be rational. Rationality questions for periods of modular forms have been studied extensively, notably by Manin, but we will not go into this further here.

2.1.6. Triangulations and homology.

From now on we will assume that G is a congruence subgroup, so that the rational homology of X_G is precisely the homology generated by paths between cusps.

We will compute the homology of X_G by first triangulating the upper half-plane \mathcal{H}^* , using a tessellation of hyperbolic triangles, and then passing to the quotient. This will give us a very explicit triangulation of the surface X_G , using which we can write down explicit generators and relations for its 1-homology.

For $M \in \Gamma$ let $\langle M \rangle$ denote $\{M(0), M(\infty)\}$, viewed as a path in \mathcal{H}^* ; this is the image under M of the imaginary axis $\{0, \infty\}$. These geodesic paths form the oriented edges of a triangulation of \mathcal{H}^* whose vertices are the cusps $\mathbb{Q} \cup \{\infty\}$. Explicitly, there is an edge from ∞ to n for all $n \in \mathbb{Z}$, and an edge between rational numbers a/c and b/d such that $ad - bc = 1$. The triangles of the triangulation are images under $M \in \Gamma$ of the basic triangle \mathcal{T} with vertices at 0, 1 and ∞ and edges (I) , (TS) , $((TS)^2)$. We denote by $\langle M \rangle$ the image of this triangle under M , which has vertices $M(0)$, $M(1)$ and $M(\infty)$ and edges (M) , (MTS) and $(M(TS)^2)$. This representation of the triangles is unique except for the relation

$$\langle M \rangle = \langle MTS \rangle = \langle M(TS)^2 \rangle.$$

Also, triangles $\langle M \rangle$ and $\langle MS \rangle$ meet along the edge (M) , since $(MS) = -(M)$ (the negative sign indicating reverse orientation).

We will use the symbol $(M)_G$ to denote the image of the path (M) in the quotient X_G , and also its image in the rational homology. The geometric observations of the previous paragraph now give us the following 2- and 3-term relations between these symbols:

$$(2.1.5) \quad \begin{aligned} (M)_G + (MTS)_G + (M(TS)^2)_G &= 0 \\ (M)_G + (MS)_G &= 0. \end{aligned}$$

We also clearly have the relations

$$(2.1.6) \quad (M'M)_G = (M)_G$$

for all $M' \in G$, so we may use as generators of the rational homology the finite set of symbols $(M_i)_G$, $(1 \leq i \leq e)$, where as before M_1, \dots, M_e are a set of right coset representatives for G in Γ .

Let $C(G)$ be the \mathbb{Q} -vector space with basis the formal symbols $(M)_G$ for each M in Γ , identified by the relations (2.1.6), so that $\dim(C(G)) = e = [\Gamma : G]$.

Let $B(G)$ be the subspace of $C(G)$ spanned by all elements of the form

$$(M)_G + (MS)_G, \\ (M)_G + (MTS)_G + (M(TS)^2)_G.$$

Let $C_0(G)$ be the \mathbb{Q} -vector space spanned by the G -cusps $[\alpha]_G$ for $\alpha \in \mathbb{Q} \cup \{\infty\}$ (so that $[\alpha]_G = [\beta]_G \iff \beta = M(\alpha)$ for some $M \in G$). Define the boundary map $\delta: C(G) \rightarrow C_0(G)$ by

$$\delta((M)_G) = [M(\infty)]_G - [M(0)]_G$$

and set $Z(G) = \ker(\delta)$. Note that $B(G) \subseteq Z(G)$, by a trivial calculation using the facts that S transposes 0 and ∞ while TS cycles 0, 1 and ∞ .

Finally we define $H(G) = Z(G)/B(G)$. The crucial result, due in this form to Manin [37, Theorem 1.9], is that this formal construction does in fact give us the rational homology of X_G :

THEOREM 2.1.4. *$H(G)$ is isomorphic to $H_1(X_G, \mathbb{Q})$, the isomorphism being induced by*

$$(M)_G \mapsto \{M(0), M(\infty)\}_G.$$

We may thus use the symbol $(M)_G$ either as an abstract symbol obeying certain relations, or to denote an element of $H_1(X_G, \mathbb{Q})$, without confusion. In future, as the subgroup G will be fixed, we will omit the subscript on these symbols and blur the distinction between (M) as a path in the upper half-plane and $(M)_G$ as representing an element of the rational 1-homology of X_G .

Note that the form of the relations between the generating symbols (M) does not depend at all on the specific group G . In particular we do not have to consider explicitly the shape of a fundamental region for the action of G on \mathcal{H}^* , or how the edges of such a region are identified. This represents a major simplification compared with earlier approaches, such as that used by Tingley [67]. In order to develop this result into an explicit algorithm for computing homology, we need to have a specific set of right coset representatives for the subgroup G of Γ , and also to have a test for G -equivalence of cusps. These are purely algebraic problems which can easily be solved for arithmetically defined subgroups G such as congruence subgroups. Observe that from this point on, we do not have to do any geometry at all.

One final remark before we specialize to the case $G = \Gamma_0(N)$: every path between cusps may be expressed as a finite sum of paths of the form (M) with $M \in \Gamma$. Writing $\{\alpha, \beta\} = \{0, \beta\} - \{0, \alpha\}$, it suffices to do this for modular symbols of the form $\{0, \alpha\}$. Let

$$(2.1.7) \quad \frac{p_{-2}}{q_{-2}} = \frac{0}{1}, \frac{p_{-1}}{q_{-1}} = \frac{1}{0}, \frac{p_0}{1} = \frac{p_0}{q_0}, \frac{p_1}{q_1}, \frac{p_2}{q_2}, \dots, \frac{p_k}{q_k} = \alpha$$

denote the continued fraction convergents of the rational number α . Then, as is well-known,

$$p_j q_{j-1} - p_{j-1} q_j = (-1)^{j-1} \quad \text{for } -1 \leq j \leq k.$$

Hence

$$(2.1.8) \quad \{0, \alpha\} = \sum_{j=-1}^k \left\{ \frac{p_{j-1}}{q_{j-1}}, \frac{p_j}{q_j} \right\} = \sum_{j=-1}^k \{M_j(0), M_j(\infty)\} = \sum_{j=-1}^k (M_j)$$

$$\text{where } M_j = \begin{pmatrix} (-1)^{j-1} p_j & p_{j-1} \\ (-1)^{j-1} q_j & q_{j-1} \end{pmatrix}.$$

2.2 M-symbols and $\Gamma_0(N)$

We now specialize to the case $G = \Gamma_0(N)$:

$$\Gamma_0(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \mid c \equiv 0 \pmod{N} \right\}.$$

The index of $\Gamma_0(N)$ in Γ is given (see [55, Proposition 1.43]) by

$$[\Gamma : \Gamma_0(N)] = N \prod_{p|N} (1 + p^{-1}).$$

Define $H(N) = H(\Gamma_0(N))$ and $X_0(N) = X_{\Gamma_0(N)}$. After Theorem 2.1.4, we will identify $H(N)$ with $H_1(X_0(N), \mathbb{Q})$ by identifying (M) with $\{M(0), M(\infty)\}$.

The next lemma is used to determine right coset representatives for $\Gamma_0(N)$ in Γ .

PROPOSITION 2.2.1. *For $j = 1, 2$ let $M_j = \begin{pmatrix} a_j & b_j \\ c_j & d_j \end{pmatrix} \in \Gamma$. The following are equivalent.*

- (1) *The right cosets $\Gamma_0(N)M_1$ and $\Gamma_0(N)M_2$ are equal;*
- (2) *$c_1 d_2 \equiv c_2 d_1 \pmod{N}$;*
- (3) *There exists u with $\gcd(u, N) = 1$ such that $c_1 \equiv u c_2$ and $d_1 \equiv u d_2 \pmod{N}$.*

PROOF. We have

$$M_1 M_2^{-1} = \begin{pmatrix} a_1 d_2 - b_1 c_2 & * \\ c_1 d_2 - d_1 c_2 & a_2 d_1 - b_2 c_1 \end{pmatrix},$$

which is in $\Gamma_0(N)$ if and only if $c_1 d_2 - d_1 c_2 \equiv 0 \pmod{N}$. Thus (1) and (2) are equivalent. Also, if (1) holds, then from $\det(M_1 M_2^{-1}) = 1$, we deduce also that $\gcd(u, N) = 1$, where $u = a_2 d_1 - b_2 c_1$. Now

$$\begin{aligned} u c_2 &= a_2 d_1 c_2 - b_2 c_1 c_2 \\ &\equiv a_2 d_2 c_1 - b_2 c_2 c_1 && \text{since } d_1 c_2 \equiv d_2 c_1 \pmod{N} \\ &= c_1 && \text{since } a_2 d_2 - b_2 c_2 = 1 \end{aligned}$$

and $u d_2 \equiv d_1$ similarly. Conversely, if $c_1 \equiv u c_2$ and $d_1 \equiv u d_2 \pmod{N}$ with $\gcd(u, N) = 1$, then the congruence in (2) follows easily. \square

On the set of ordered pairs $(c, d) \in \mathbb{Z}^2$ such that $\gcd(c, d, N) = 1$ we now define the relation \sim , where

$$(2.2.1) \quad (c_1, d_1) \sim (c_2, d_2) \iff c_1 d_2 \equiv c_2 d_1 \pmod{N}.$$

By Proposition 2.2.1, this is an equivalence relation. The equivalence class of (c, d) will be denoted $(c : d)$, and such symbols will be called M-symbols (after Manin, who introduced them in [37]). The set of these M-symbols modulo N is $P^1(N) = P^1(\mathbb{Z}/N\mathbb{Z})$, the projective line over the ring of integers modulo N .

Notice that in an M-symbol $(c : d)$, the integers c and d are only determined modulo N , and that we can always choose them such that $\gcd(c, d) = 1$.

Proposition 2.2.1 now implies the following.

PROPOSITION 2.2.2. *There exist bijections*

$$P^1(N) \longleftrightarrow [\Gamma : \Gamma_0(N)] \longleftrightarrow \{(M) : M \in [\Gamma : \Gamma_0(N)]\}$$

given by

$$(2.2.2) \quad (c : d) \leftrightarrow M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \leftrightarrow (M) = \{b/d, a/c\}$$

where $a, b \in \mathbb{Z}$ are chosen so that $ad - bc = 1$. \square

Note that a different choice of a, b in (2.2.2) has the effect of multiplying M on the left by a power of T which does not change the right coset of M , or the symbol (M) , since $T \in \Gamma_0(N)$ for all N .

The right coset action of Γ on $[\Gamma : \Gamma_0(N)]$ induces an action on $P^1(N)$:

$$(2.2.3) \quad (c : d) \begin{pmatrix} p & q \\ r & s \end{pmatrix} = (cp + dr : cq + ds).$$

In particular, we have

$$(2.2.4) \quad (c : d)S = (d : -c) \quad \text{and} \quad (c : d)T = (c : c + d).$$

The boundary map δ now takes the form

$$(2.2.5) \quad \delta: (c : d) \mapsto [a/c] - [b/d].$$

In order to compute $\ker(\delta)$, we must be able to determine when two cusps are $\Gamma_0(N)$ -equivalent. This is achieved by the following result.

PROPOSITION 2.2.3. *For $j = 1, 2$ let $\alpha_j = p_j/q_j$ be cusps written in lowest terms. The following are equivalent:*

- (1) $\alpha_2 = M(\alpha_1)$ for some $M \in \Gamma_0(N)$;
- (2) $q_2 \equiv uq_1 \pmod{N}$ and $up_2 \equiv p_1 \pmod{\gcd(q_1, N)}$, with $\gcd(u, N) = 1$.
- (3) $s_1q_2 \equiv s_2q_1 \pmod{\gcd(q_1q_2, N)}$, where s_j satisfies $p_js_j \equiv 1 \pmod{q_j}$.

PROOF. (1) \implies (2): Let $M = \begin{pmatrix} a & b \\ Nc & d \end{pmatrix} \in \Gamma_0(N)$; Then $p_2/q_2 = (ap_1 + bq_1)/(Ncp_1 + dq_1)$, with both fractions in lowest terms. Equating numerators and denominators (up to sign) gives (2), with $u = \pm d$, since $ad \equiv 1 \pmod{N}$.

(2) \implies (1): Here we use Proposition 2.2.1. Assume (2), and write $p_1s'_1 - q_1r'_1 = p_2s_2 - q_2r_2 = 1$ with $s'_1, r'_1, s_2, r_2 \in \mathbb{Z}$. Then $p_1s'_1 \equiv 1 \pmod{q_1}$ and $p_2s_2 \equiv 1 \pmod{q_2}$. Also $\gcd(q_1, N) = \gcd(q_2, N) = N_0$, say, since $q_2 \equiv uq_1 \pmod{N}$. Now $up_2 \equiv p_1 \pmod{N_0}$ implies $us'_1 \equiv s_2 \pmod{N_0}$, so we may find $x \in \mathbb{Z}$ such that $uxq_1 \equiv us'_1 - s_2 \pmod{N}$. Set $s_1 = s'_1 - xq_1$ and $r_1 = r'_1 - xp_1$. Then $p_1s_1 - q_1r_1 = 1$ and now $us_1 \equiv s_2 \pmod{N}$. By Proposition 2.2.1, there exists $M \in \Gamma_0(N)$ such that $\begin{pmatrix} p_2 & r_2 \\ q_2 & s_2 \end{pmatrix} = M \begin{pmatrix} p_1 & r_1 \\ q_1 & s_1 \end{pmatrix}$, and so $M(p_1/q_1) = p_2/q_2$ as required.

(1) \iff (3): As before, solve the equations $p_js_j - q_jr_j = 1$ for $j = 1, 2$. Set $M_j = \begin{pmatrix} p_j & r_j \\ q_j & s_j \end{pmatrix}$, so that $M_j(\infty) = \alpha_j$, and $M_2M_1^{-1}(\alpha_1) = \alpha_2$. This matrix is in $\Gamma_0(N)$ if and only if $q_2s_1 - q_1s_2 \equiv 0 \pmod{N}$. The most general such matrix is obtained by replacing s_1 by

$s'_1 = s_1 + xq_1$, and it follows that α_1 and α_2 are equivalent if and only if we can solve for $x \in \mathbb{Z}$ the congruence

$$0 \equiv q_2 s'_1 - q_1 s_2 \equiv q_2 s_1 - q_1 s_2 + x q_1 q_2 \pmod{N},$$

which is if and only if the congruence in (3) holds. \square

Henceforth, we can therefore assume that $H(N)$ is given explicitly in terms of M-symbols. Certain symbols will be generators, and each M-symbol $(c : d)$ will be expressed as a \mathbb{Q} -linear combination of these generating symbols, by means of the 2-term relations

$$(2.2.6) \quad (c : d) + (-d : c) = 0$$

and 3-term relations

$$(2.2.7) \quad (c : d) + (c + d : -c) + (d : -c - d) = 0.$$

These are just the relations (2.1.5) expressed in terms of M-symbols, using (2.2.4).

Implementation. We make a list of inequivalent M-symbols as follows: first, list the symbols $(c : 1)$ for $0 \leq c < N$; then the symbols $(1 : d)$ for $0 \leq d < N$ and $\gcd(d, N) > 1$; and finally a pairwise inequivalent set of symbols $(c : d)$ with $c|N$, $c \neq 1, N$, $\gcd(c, d) = 1$ and $\gcd(d, N) > 1$. (The latter symbols do not arise when N is a prime power.)

To speed up the looking up of M-symbols in the list, we have found it extremely worthwhile to prepare at the start of the program a collection of lookup tables, containing for example a table of inverses modulo N . We also used a simple “hashing” system, so that given any particular symbol $(c : d)$ we could quickly determine to which symbol in our standard list it is equivalent. While this preparation of look-up tables may seem rather trivial, in practice it has had a dramatic effect, speeding up the mass computation of Hecke eigenvalues a_p (see Section 2.9) by a factor of up to 50.

Using the 2-term relations (2.2.6) we may identify the M-symbols in pairs, up to sign. This immediately halves the number of generators needed. Then the 3-term relations (2.2.7) are computed, each M-symbol being replaced by plus or minus one of the current generators, and the resulting equations solved using integer Gaussian elimination. At the end of this stage we have a list of r (say) “free generators” from the list of M-symbols, and a table expressing each of the M-symbols in the list as a \mathbb{Q} -linear combination of the generators. In practice, we store \mathbb{Z} -linear combinations, keeping a common denominator d_1 separately; however, by judicious choice of the order of elimination of symbols, in practice this denominator is very frequently 1.

Next we compute the boundary map δ on each of the free generators, using (2.2.5). We have a procedure based on Proposition 2.2.3 to test cusp equivalence. Hence we do not have to compute in advance a complete list of inequivalent cusps. Instead, we keep a cumulative list: each cusp we come across is checked for equivalence with those in the list already, and is added to the list if it represents a new equivalence class. We found this simpler to implement than using a standard set of pairwise inequivalent cusps, as in [37, Corollary 2.6].

We thus compute a matrix with integer entries for the linear map δ , and by a second step of Gaussian elimination can compute a basis for its kernel, which by definition is $H(N)$. This basis is stored as a list of $2g$ integer vectors in \mathbb{Z}^r over a second common denominator d_2 . (Here g is the genus of $X_0(N)$, so that $\dim H(N) = 2g$.) We may arrange (by reducing the basis suitably) that whenever a linear combination of M-symbols (represented as a vector in \mathbb{Z}^r) is in $\ker(\delta)$, then its coefficients with respect to the basis are given by (a subset of) $2g$ components of these vectors, divided by the cumulative common denominator $d_1 d_2$.

From now on we will regard elements of $H(N)$ as being given by vectors in \mathbb{Z}^{2g} in this way.

2.3 Conversion between modular symbols and M-symbols

As noted above, each M-symbol $(c : d)$ has a representative with $\gcd(c, d) = 1$, and corresponds to the right coset representative $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ in Γ , where $ad - bc = 1$. The isomorphism of Theorem 2.1.4 thus becomes

$$(2.3.1) \quad (c : d) \mapsto \{b/d, a/c\}.$$

The modular symbol on the right of (2.3.1) is independent of the choice of a and b with $ad - bc = 1$, since $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \in \Gamma_0(N)$ for all N , and so

$$\{\alpha, \beta\}_{\Gamma_0(N)} = \{\alpha + k, \beta + l\}_{\Gamma_0(N)}$$

for all k, l in \mathbb{Z} and α, β in \mathcal{H}^* .

Conversely each modular symbol $\{\alpha, \beta\}$ with α and β in $\mathbb{Q} \cup \{\infty\}$ can be expressed, using continued fractions, as a sum of modular symbols of the special form $(M) = \{M(0), M(\infty)\}$ with $M \in \Gamma$, hence as a sum of M-symbols $(c : d)$, and finally as a linear combination of the generating M-symbols.

Using the notation introduced above in Subsection 2.1.6, if $q_0 = 1, q_1, \dots, q_k$ are the denominators of the continued fraction convergents to the rational number α as in (2.1.7), in terms of M-symbols we have

$$(2.3.2) \quad \{0, \alpha\} = \sum_{j=1}^k ((-1)^{j-1} q_j : q_{j-1})$$

since the first two terms in (2.1.8) cancel out. Note that it is only the denominators of the continued fraction convergents which are used.

2.4 Action of Hecke and other operators

For each prime p not dividing N , the Hecke operator T_p acts on modular symbols $\{\alpha, \beta\}$ via

$$(2.4.1) \quad \begin{aligned} T_p: \quad \{\alpha, \beta\} &\mapsto \left[\begin{pmatrix} p & 0 \\ 0 & 1 \end{pmatrix} + \sum_{r \pmod p} \begin{pmatrix} 1 & r \\ 0 & p \end{pmatrix} \right] \{\alpha, \beta\} \\ &= \{p\alpha, p\beta\} + \sum_{r \pmod p} \left\{ \frac{\alpha + r}{p}, \frac{\beta + r}{p} \right\}. \end{aligned}$$

This action induces a linear map from $H(N)$ to itself, provided that p does not divide N , which we again denote by T_p .

There are also Hecke operators, which we also denote T_p , acting on the space $S_2(N) = S_2(\Gamma_0(N))$ of cusp forms of weight 2 for $\Gamma_0(N)$. First recall that 2×2 matrices $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with $ad - bc > 0$ act on functions $f(z)$ on the right via

$$f \mapsto f|M \quad \text{where} \quad (f|M)(z) = \frac{ad - bc}{(cz + d)^2} f\left(\frac{az + b}{cz + d}\right).$$

A form of weight 2 for some group G will satisfy $f|M = f$ for all $M \in G$. This action extends by linearity to an action by formal linear combinations of matrices. The Hecke operator T_p is defined by

$$f|T_p = f \left| \left[\begin{pmatrix} p & 0 \\ 0 & 1 \end{pmatrix} + \sum_{r=0}^{p-1} \begin{pmatrix} 1 & r \\ 0 & p \end{pmatrix} \right] \right|,$$

so that

$$(f|T_p)(z) = p f(pz) + \frac{1}{p} \sum_{r=0}^{p-1} f\left(\frac{z+r}{p}\right).$$

A standard result is that T_p does act on $S_2(N)$, provided that $p \nmid N$. (There are similar operators U_p for primes p dividing N , but we will not need these).

These matrix actions on $S_2(N)$ and $H(N)$ are compatible, in the sense that they respect the duality between cusp forms and homology:

$$\langle \{\alpha, \beta\}, f|M \rangle = \langle \{M\alpha, M\beta\}, f \rangle,$$

since

$$\frac{d}{dz} \left(\frac{az+b}{cz+d} \right) = \frac{ad-bc}{(cz+d)^2},$$

and so

$$\int_{\alpha}^{\beta} (f|M)(z) dz = \int_{\alpha}^{\beta} \frac{ad-bc}{(cz+d)^2} f(M(z)) dz = \int_{M\alpha}^{M\beta} f(w) dw.$$

Thus, in particular,

$$\langle \{\alpha, \beta\}, f|T_p \rangle = \langle T_p\{\alpha, \beta\}, f \rangle.$$

Secondly, for each prime q dividing N there is an involution operator W_q acting on $H(N)$ and $S_2(N)$. We recall the definition. Let q^α be the exact power of q dividing N , and let x, y, z, w be any integers satisfying $q^\alpha xw - (N/q^\alpha)yz = 1$. Then the matrix $W_q = \begin{pmatrix} q^\alpha x & y \\ Nz & q^\alpha w \end{pmatrix}$ has determinant q^α and normalizes $\Gamma_0(N)$ (modular scalar matrices). Thus W_q induces an action on $H(N)$ and $S_2(N)$, which is an involution since $W_q^2 \in \Gamma_0(N)$ (modulo scalars), and is independent of the values x, y, z, w chosen. The product of all the W_q for q dividing N is the Fricke involution W_N , coming from the transformation $z \mapsto -1/Nz$, with matrix $\begin{pmatrix} 0 & -1 \\ N & 0 \end{pmatrix}$.

The operators T_p for primes p not dividing N and W_q for primes q dividing N together generate a commutative \mathbb{Q} -algebra, called the Hecke algebra and denoted \mathbb{T} . Moreover, each operator is self-adjoint with respect to the so-called Petersson inner product on $S_2(N)$, and so there exist bases for $S_2(N)$ consisting of simultaneous eigenforms for all the T_p and W_q , with real eigenvalues. (See [1, Theorem 2] or [32, Corollary 2 to Theorem 4.2].) Similarly, the action of \mathbb{T} on $H(N) \otimes \mathbb{R}$ can also be diagonalized.

Finally, recall from Subsection 2.1.3 that the transformation $z \mapsto z^* = -\bar{z}$ on \mathcal{H} commutes with the action of $\Gamma_0(N)$ and hence also induces an involution on $H(N)$ which we denote $*$. This operator commutes with all the T_p and W_q , which thus preserve the eigenspaces $H^+(N)$ and $H^-(N)$. Moreover, $H^+(N)$ and $H^-(N)$ are isomorphic as modules for the Hecke algebra \mathbb{T} . It follows that in order to compute eigenvalues of Hecke operators, we can restrict our attention to $H^+(N)$. This has some practical significance, as we elaborate in the next section.

Implementation. To compute the matrices giving the action of each of these operators on $H(N)$ we may proceed as follows. We convert each of the generating M-symbols to a modular symbol as in Section 2.3. To compute a T_p , we apply (2.4.1) to each, reconvert each term on the right of (2.4.1) to a sum of M-symbols using (2.3.2), and hence express it as a \mathbb{Z}^{2g} -vector giving it as a linear combination of the generating M-symbols. This gives one column of the $2g \times 2g$ matrix. Similarly with W_q and W_N . Computing the matrix of $*$ is easier, as we can work directly with the M-symbols, on which $*$ acts via $(c : d) \mapsto (-c : d)$. These integer matrices are in fact $d_1 d_2$ times the actual operator matrices (where d_1 and d_2 are the denominators which may have arisen earlier as a result of the Gaussian elimination steps). Obviously this must be taken into account when we look for eigenvalues later; however, for simplicity of exposition we will assume from now on that this denominator $d_1 d_2$ is 1. We use the convention that the space is represented by column vectors, with operator matrices acting on the left.

Heilbronn matrices. There is an alternative approach to computing the T_p , based on so-called *Heilbronn matrices* of level p . These were described by Mazur in [38], and their application to give an algorithm for computing the Hecke action on homology in terms of M-symbols was given by Merel in his paper [42]. We will describe our own version of this method, which is easy to implement; our approach differs slightly from, and is a little simpler than, that of Merel's paper [42].

Since with this method one acts directly on the M-symbols, one avoids the conversion to and from modular symbols. This makes the method faster in practice, particularly as we may precompute the Heilbronn matrices for all the small primes p (say $p < 30$) for which we need to compute the matrix of T_p in order to split off one-dimensional eigenspaces from $H(N)$.

From the definition in (2.4.1), the action of T_p is expressed as the sum of the actions on modular symbols, on the left, of $p+1$ matrices of determinant p , namely $\begin{pmatrix} p & 0 \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & r \\ 0 & p \end{pmatrix}$ for r modulo p . The following result shows how each of these acts on M-symbols directly, via an action on the right.

PROPOSITION 2.4.1. *Let p be a prime not dividing N and $(c : d)$ an M-symbol for N . The action on $(c : d)$ of the $p+1$ matrices appearing in (2.4.1) is as follows.*

(1)

$$\begin{pmatrix} p & 0 \\ 0 & 1 \end{pmatrix} (c : d) = (c : pd) = (c : d) \begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix}.$$

(2) *For $r \in \mathbb{Z}$, let $M_i \in \Gamma$ for $0 \leq i \leq k$ be the matrices constructed from the continued fraction convergents to r/p as in (2.1.8), so that*

$$M_0(0) = \infty, \quad M_1(0) = M_0(\infty), \quad \dots, \quad M_k(0) = M_{k-1}(\infty), \quad M_k(\infty) = \frac{r}{p}.$$

Set $M'_i = \begin{pmatrix} p & -r \\ 0 & 1 \end{pmatrix} M_i S$ for $0 \leq i \leq k$. Then

$$\begin{pmatrix} 1 & r \\ 0 & p \end{pmatrix} (c : d) = \sum_{i=0}^k (c : d) M'_i.$$

PROOF. In each case we first solve $ad - bc = 1$ for integers a, b and apply the appropriate matrix to the modular symbol $\{b/d, a/c\}$.

(1) Since $p \nmid N$ we may assume by the Chinese Remainder Theorem that c is a multiple of p , say $c = pc'$. Now $M = \begin{pmatrix} a & pb \\ c' & d \end{pmatrix} = \begin{pmatrix} p & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} p & 0 \\ 0 & 1 \end{pmatrix}^{-1} \in \Gamma$, and

$$(c : d) \begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix} = (c : pd) = (c' : d) = (M) = \left\{ \frac{pb}{d}, \frac{a}{c'} \right\} = \begin{pmatrix} p & 0 \\ 0 & 1 \end{pmatrix} \left\{ \frac{b}{d}, \frac{a}{c} \right\}.$$

(2) By construction of the M_i , we have

$$\sum_{i=0}^k (M_i) = \sum_{i=0}^k \{M_i(0), M_i(\infty)\} = \left\{ \infty, \frac{r}{p} \right\}.$$

Given an M-symbol $(c : d)$, we will show how to choose a and b which satisfy $ad - bc = 1$ and also

$$M = \begin{pmatrix} 1 & r \\ 0 & p \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & r \\ 0 & p \end{pmatrix}^{-1} \in \Gamma.$$

Then

$$\sum_{i=0}^k (MM_i S) = M \left\{ \frac{r}{p}, \infty \right\} = \left(M \begin{pmatrix} 1 & r \\ 0 & p \end{pmatrix} \right) = \left(\begin{pmatrix} 1 & r \\ 0 & p \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right) = \begin{pmatrix} 1 & r \\ 0 & p \end{pmatrix} \left\{ \frac{b}{d}, \frac{a}{c} \right\}.$$

Now we show that suitable values of a and b exist. Replacing d by $d + N$ if necessary, we may assume that $cr \not\equiv d \pmod{p}$. Given an arbitrary solution to $ad - bc = 1$, solve for t the congruence

$$(cr - d)t \equiv (b + dr) - r(a + cr) \pmod{p}.$$

Replacing (a, b) by $(a + ct, b + dt)$ we then still have $ad - bc = 1$ and now

$$(b + dr) - r(a + cr) = pb'$$

with $b' \in \mathbb{Z}$, and a simple calculation shows that $M = \begin{pmatrix} a + rc & b' \\ pc & d - rc \end{pmatrix}$ has the desired properties.

Since the bottom row of $MM_i S$ is $(pc : d - rc)M_i S = (c : d) \begin{pmatrix} p & -r \\ 0 & 1 \end{pmatrix} M_i S = (c : d)M'_i$, the result follows. \square

Hence for each prime p there exists a finite set R_p of matrices in $M_2(\mathbb{Z})$ with determinant p , called the Heilbronn matrices of level p , such that the Hecke operator T_p acts on M-symbols via

$$(c : d) \mapsto \sum_{M \in R_p} (c : d)M.$$

The usual definition of the set R_p (for an odd prime p not dividing N) is as follows: R_p is the set of matrices $\begin{pmatrix} x & -y \\ y' & x' \end{pmatrix} \in M_2(\mathbb{Z})$ with determinant $xx' + yy' = p$, and either (i) $x > |y| > 0$, $x' > |y'| > 0$, and $yy' > 0$; or (ii) $y = 0$, and $|y'| < x'/2$; or (iii) $y' = 0$, and $|y| < x/2$. This description, while closer to the original definition by Heilbronn and used by both Mazur and Merel, is not so easy to use in practice. One can show that the matrices in this definition may be constructed using the continued fraction expansions of r/p for r modulo p , and this leads to the presentation we have given here.

For example, for the first few primes we have

$$R_2 = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} \right\},$$

$$R_3 = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}, \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 3 \end{pmatrix}, \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 3 & -1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 1 & -3 \end{pmatrix} \right\},$$

and

$$R_5 = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}, \begin{pmatrix} 5 & 2 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 3 & 5 \end{pmatrix}, \begin{pmatrix} 5 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 5 \end{pmatrix}, \begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 5 & -1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 1 & -5 \end{pmatrix}, \begin{pmatrix} 5 & -2 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -2 & 1 \\ 1 & -3 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ -3 & 5 \end{pmatrix} \right\}.$$

To compute the complete set R_p for any prime p we may use the following algorithm. Effectively, we are computing the continued fraction expansions of each rational r/p with denominator p , and recording the matrices denoted M'_i in the preceding Proposition. In line 3 of the algorithm, the loop is over a complete set of residues r modulo p , such as $-p/2 \leq r < p/2$.

Algorithm for computing Heilbronn matrices

```

INPUT:      p (a prime).
OUTPUT:     the Heilbronn matrices of level p.

1.  BEGIN
2.  OUTPUT  $\begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix}$ ;
3.  FOR r MODULO p DO
4.  BEGIN
5.      x1=p; x2=-r; y1=0; y2=1; a=-p; b=r;
6.      OUTPUT  $\begin{pmatrix} x1 & x2 \\ y1 & y2 \end{pmatrix}$ ;
7.      WHILE b≠0 DO
8.      BEGIN
9.          q=nearest_integer(a/b);
10.         c=a-b*q; a=-b; b=c;
11.         x3=q*x2-x1; x1=x2; x2=x3;
12.         y3=q*y2-y1; y1=y2; y2=y3;
13.         OUTPUT  $\begin{pmatrix} x1 & x2 \\ y1 & y2 \end{pmatrix}$ 
14.     END
15. END
16. END

```

For example, take $p = 7$ and $r = 3$. The continued fraction convergents linking ∞ to $3/7$ are

$$\infty, 0, \frac{1}{2}, \frac{3}{7}$$

with associated unimodular matrices

$$M_0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad M_1 = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, \quad \text{and} \quad M_2 = \begin{pmatrix} -3 & 1 \\ -7 & 2 \end{pmatrix}.$$

The matrices M'_i constructed in Proposition 2.4.1 are then

$$M'_0 = \begin{pmatrix} 7 & -3 \\ 0 & 1 \end{pmatrix}, \quad M'_1 = \begin{pmatrix} -3 & -1 \\ 1 & -2 \end{pmatrix}, \quad \text{and} \quad M'_2 = \begin{pmatrix} 1 & 0 \\ 2 & 7 \end{pmatrix}.$$

These are (up to sign) the same as the matrices output by lines 3–14 of the algorithm when $p = 7$ and $r = 3$.

2.5 Working in $H^+(N)$

Recall that $H^+(N)$ is the +1 eigenspace for the operator $*$: $z \mapsto -\bar{z}$ acting on $H(N) = H_1(X_0(N), \mathbb{Q})$. We would like to work in $H^+(N)$ to compute the action of the Hecke algebra \mathbb{T} , since there are obvious savings in computation time and storage space achieved by working in a space with half the dimension of $H(N)$. To do this, note that $H^+(N) \cong H(N)/H^-(N)$ (as vector spaces). We can thus compute $H^+(N)$ in terms of M-symbols by including extra 2-term relations

$$(2.5.1) \quad (c : d) = (-c : d)$$

between the M-symbols. We must also identify the cusp equivalence classes $[\alpha]$ and $[\alpha^*] = [-\alpha]$ for $\alpha \in \mathbb{Q}$.

Effectively we are replacing $\Gamma_0(N)$ by the larger group

$$\widetilde{\Gamma_0(N)} = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbb{Z}, ad - bc = \pm 1, c \equiv 0 \pmod{N} \right\} = \langle \Gamma_0(N), J \rangle$$

which still acts discretely on \mathcal{H}^* via

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} : z \mapsto \begin{cases} \frac{az + b}{cz + d} & \text{if } ad - bc = +1, \\ \frac{a\bar{z} + b}{c\bar{z} + d} & \text{if } ad - bc = -1; \end{cases}$$

in particular, $J = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ sends z to $z^* = -\bar{z}$, giving the action of $*$. Hence, in effect, $\widetilde{\Gamma_0(N)} = \langle \Gamma_0(N), * \rangle$, and $H^+(N) \cong H_1(\widetilde{\Gamma_0(N)} \backslash \mathcal{H}^*, \mathbb{Q})$. (A similar procedure is possible for other subgroups G of Γ of real type.)

As a further saving, use of the extra relation (2.5.1) enables us to cut out half the 3-term relations (2.2.7), as follows. Using (2.5.1) on the second and third terms of (2.2.7) yields

$$(c : d) + (c + d : c) + (d : c + d) = 0.$$

Also, (2.5.1) and (2.2.6) together imply

$$(d : c) = -(c : d).$$

Hence relation (2.2.7) for $(d : c)$ now gives the same information as (2.2.7) for $(c : d)$, and can be omitted. Geometrically, the triangles which determine the 3-term relations have been identified in pairs by the action of the larger group, since the effect of the transformation J is to fold the upper half-plane in two along the imaginary half-axis.

Implementation. We modify the procedure of Section 2.2 in three ways: taking the 2-term relations (2.2.6) and (2.5.1) together we may identify M-symbols in sets of four (instead of two), up to sign, at the first stage of elimination. Then in the second stage we have only half the number of 3-term relations to consider, as noted above, and each can be expressed in terms of half the number of current generators: so we have half the number of equations in half the number of variables to solve, giving a four-fold saving in space and time. Finally, in computing $\ker(\delta)$ we must use a wider notion of cusp equivalence, since for $\alpha, \beta \in \mathbb{Q}$,

$$\alpha \equiv \beta \pmod{\widetilde{\Gamma_0(N)}} \iff \alpha \equiv \pm\beta \pmod{\Gamma_0(N)}.$$

Working in $H^+(N)$ is sufficient for the first stage of our algorithm, when we want to find certain cusp forms in $S_2(N)$, since $H^+(N) \otimes_{\mathbb{Q}} \mathbb{C} \cong S_2(N)$, both as vector spaces and as modules for the Hecke algebra \mathbb{T} . Hence eigenvectors in $H^+(N)$ correspond to eigenforms in $S_2(N)$. Since these eigenforms (or, more accurately, newforms—see the next section) have Fourier expansions in which the Fourier coefficients are determined by their Hecke eigenvalues, we can determine these coefficients indirectly by computing explicitly the action of the Hecke algebra \mathbb{T} on $H^+(N)$.

2.6 Modular forms and modular elliptic curves

Let $S_2(N)$ denote, as above, the space of cusp forms of weight 2 on $\Gamma_0(N)$. Forms $f(z) \in S_2(N)$ have Fourier expansions of the form

$$f(z) = \sum_{n=1}^{\infty} a(n, f) e^{2\pi i n z},$$

with coefficients $a(n, f) \in \mathbb{C}$. The corresponding differentials $2\pi i f(z) dz$ are (the pullbacks of) holomorphic differentials on the Riemann surface $X_0(N)$. Hence $S_2(N)$ is a complex vector space of dimension g , where g is the genus of $X_0(N)$, and $2g = \dim H(N)$. Moreover, $S_2(N) = S_2(N)_{\mathbb{Q}} \otimes_{\mathbb{Q}} \mathbb{C}$ where $S_2(N)_{\mathbb{Q}}$ is the subset of $S_2(N)$ consisting of forms $f(z)$ with *rational* Fourier coefficients $a(n, f)$. This rational structure on $S_2(N)$ is a consequence of the deep fact that $X_0(N)$ may be viewed as the complex points of an algebraic curve defined over \mathbb{Q} ; it may also be proved using Hecke operators and the duality with homology.

We are interested here in “rational newforms” f : that is, forms f which have rational Fourier coefficients $a(n, f)$, are simultaneous eigenforms for all the Hecke operators, and which are also “newforms” in the sense of Atkin and Lehner (see [1]). We briefly recall the definition.

For each proper divisor M of N and each $g \in S_2(M)$, the forms $g(Dz)$ for divisors D of N/M are in $S_2(N)$. The subspace $S_2^{\text{old}}(N)$ of $S_2(N)$ spanned by all such forms is called the space of *oldforms*. There is also an inner product on $S_2(N)$, called the Petersson inner product, with respect to which the Hecke operators are self-adjoint (Hermitian). Define $S_2^{\text{new}}(N)$ to be the orthogonal complement in $S_2(N)$ of $S_2^{\text{old}}(N)$ with respect to the Petersson inner product. The restriction of the Hecke algebra \mathbb{T} to $S_2^{\text{new}}(N)$ is semisimple; $S_2^{\text{new}}(N)$ has a basis consisting of simultaneous eigenforms, and these eigenforms are called *newforms*.

In general, newforms come in conjugate sets of $d \geq 1$ forms with eigenvalues generating an algebraic number field of degree d . The periods of such a set of conjugates $\{f\}$ form a lattice Λ of rank $2d$ in \mathbb{C}^d , and hence an abelian variety $A_f = \mathbb{C}^d / \Lambda$, which is defined over \mathbb{Q} . Here we will only be interested in the case $d = 1$, where the Hecke eigenvalues and hence Fourier coefficients of f are rational (in fact integers, being eigenvalues of integral matrices and hence algebraic integers). We will call such a form f a *rational newform*. Thus a rational newform f has an associated period lattice Λ_f :

$$\Lambda_f = \{ \langle \{\alpha, \beta\}, f \rangle \mid \alpha, \beta \in \mathcal{H}^*, \alpha \equiv \beta \pmod{\Gamma_0(N)} \}$$

which is a discrete rank 2 subgroup of \mathbb{C} . Then $E_f = \mathbb{C}/\Lambda_f$ is an elliptic curve, the modular elliptic curve attached to f . Moreover it is known that E_f is defined over \mathbb{Q} , has conductor N , and has L -series $L(E_f, s) = \sum a(n, f)n^{-s}$ where $f = \sum a(n, f) \exp(2\pi inz)$. (See [64], [55], and [7] for proofs of these statements, and [28] for a fuller discussion.)

The Fourier coefficients $a(n, f)$ of a newform $f(z) = \sum a(n, f) \exp(2\pi inz)$ are obtained from the Hecke eigenvalues of f as follows (see [1]). Firstly, for a newform f we always have $a(1, f) \neq 0$, and we normalize so that $a(1, f) = 1$. Then:

If p is a prime not dividing N , and $f|T_p = a_p f$, then $a(p, f) = a_p$.

If q is a prime dividing N , and $f|W_q = \varepsilon_q f$ with $\varepsilon_q = \pm 1$, then

$$(2.6.1) \quad a(q, f) = \begin{cases} -\varepsilon_q & \text{if } q^2 \nmid N, \\ 0 & \text{if } q^2 | N. \end{cases}$$

For prime powers, we have the recurrence relation

$$(2.6.2) \quad a(p^{r+1}, f) = a(p, f)a(p^r, f) - \delta_N(p)pa(p^{r-1}, f) \quad (r \geq 1)$$

where

$$\delta_N(p) = \begin{cases} 1 & \text{if } p \nmid N, \\ 0 & \text{if } p | N. \end{cases}$$

Finally, for composite indices we have multiplicativity: $a(mn, f) = a(m, f)a(n, f)$ when m and n are relatively prime.

With this background we may now make more precise what we mean by “computing the modular elliptic curves of conductor N ”. We do the following:

- (1) Compute the space $H^+(N)$ in terms of M-symbols and their relations.
- (2) Compute the action of sufficient Hecke operators W_q and T_p on $H^+(N)$ to determine the one-dimensional eigenspaces with rational eigenvalues; by duality, we now know the rational newforms in $S_2(N)$. Oldforms can be recognized, since in any systematic computation we will have already found them at some lower level M dividing N .
- (3) Find a \mathbb{Z} -basis for the period lattice Λ_f , for each rational newform f , computing the generating periods to high precision.
- (4) Given a \mathbb{Z} -basis for Λ_f , compute the coefficients of an equation for the attached elliptic curve E_f .
- (5) As well as the period lattice of the curves E_f , we can also compute the rational number $L(E_f, 1)/\Omega(E_f)$ (exactly) and the real value $L(E_f, 1)$ (approximately). Also, when $L(E_f, 1) = 0$ we can also determine the order of vanishing of $L(E_f, s)$ at $s = 1$, giving the analytic rank r of E_f , and the value of the derivative $L^{(r)}(E_f, 1)$, which is important in view of the Birch–Swinnerton-Dyer conjecture; we will discuss the latter computations in a later section.

This is the program which we wish to carry out, and have in fact carried out for all $N \leq 5077$. In sections 2.7–2.14 we discuss steps (2)–(5) in more detail.

2.7 Splitting off one-dimensional eigenspaces

Having computed a representation of $H^+(N)$ in terms of M-symbols, we now wish to identify the one-dimensional eigenspaces with rational integer eigenvalues for all the Hecke operators. For each eigenspace we will later need a *dual* basis vector in order to compute the projection of an arbitrary vector onto the eigenspace. Explicitly, we identify $H^+(N)$ with \mathbb{Q}^g via our M-symbol basis, representing each cycle as a column vector; each operator matrix acts on the left. Elements of the dual space will then be represented as row vectors. Projection onto a

one-dimensional eigenspace is then achieved by multiplying on the left by the appropriate row vector, which is defined up to scalar multiple by its being a simultaneous left eigenvector of each matrix. In our implementation, we do not distinguish between row and column vectors, and our linear algebra routines are designed to give right eigenvectors, so in practice all we do is find simultaneous eigenvectors for the transposes of the operator matrices. Projection (of a column vector) is then achieved by taking the dot product with the appropriate dual (row) vector. These remarks seem fairly trivial, but we need to be completely explicit if we are to implement these ideas successfully.

We wish to compute as few T_p as possible at this stage, to save time; we will have a much faster way of computing many Hecke eigenvalues later (see Section 2.9), once the eigenspaces have been found.

We also need to identify “oldclasses”: these are also common eigenspaces for all the T_p (though not for all the W_q , see below) but have dimension greater than 1. In order to recognize and discard oldforms as early as possible, we can create a cumulative database of the number of newforms and the first few Hecke eigenvalues (including all W_q -eigenvalues) of each newform at each level. If we proceed systematically through the levels N in order, we will thus always know about the newforms at levels M dividing N but less than N .

An alternative approach might be possible here, in which we use further operators at level N , such as the U_q of [1], to eliminate all but newforms. We have not devised such a scheme which works in full generality; the advantage would be that each level could then be treated in isolation, independently of lower levels, but this was not necessary in our systematic investigations which resulted in the tables in this volume.

Before starting to split $H^+(N)$ we have the following data: the number of rational newforms g in $S_2(M)$ for proper divisors M of N ; and for each such g , the W_q -eigenvalue ε_q for all primes q dividing M and the T_p -eigenvalue a_p for several primes p not dividing N . Each form g generates an “oldclass” in $S_2(N)$: a subspace of forms which have the same eigenvalue a_p for all primes p not dividing N . A basis for this oldclass consists of the forms $g(Dz)$ for all positive divisors D of N/M ; hence its dimension is $d(N/M)$, the number of positive divisors of N/M . The forms in the oldclass do not necessarily, however, have the same W_q -eigenvalue for primes q dividing N . We now proceed to find these eigenvalues explicitly.

To simplify the following exposition, observe that the W_q operators may be defined for *all* primes q , not just those dividing the level N , using the matrices $W_q = \begin{pmatrix} q^\alpha x & y \\ Nz & q^\alpha w \end{pmatrix}$ of determinant q^α where $q^\alpha \parallel N$; for if in fact $q \nmid N$, then $\alpha = 0$, so that $W_q \in \Gamma_0(N)$ and $f|W_q = f$ for all $f \in S_2(N)$. Thus in such a case, W_q reduces to the identity.

We first consider the case where N/M is a prime power.

LEMMA 2.7.1. *Let g be a newform in $S_2(M)$, let l be a prime with $g|W_l = \varepsilon g$, and let $N = q^\beta M$ where q is also prime. Thus g determines an oldclass of dimension $\beta + 1$, spanned by the forms $g_i(z) = q^i g(q^i z) \in S_2(N)$, for $0 \leq i \leq \beta$.*

- (1) *If $l \neq q$, then $g_i|W_l = \varepsilon g_i$ for all i ;*
- (2) *If $l = q$, then $g_i|W_q = \varepsilon g_{\beta-i}$.*

In case (1), all members of the oldclass have the same W_q -eigenvalue ε as g , so ε has multiplicity $\beta + 1$ (as an eigenvalue of W_l acting on this oldclass). In case (2), the ε -eigenspace for W_q has dimension $[(2 + \beta)/2]$ (that is, $\frac{1}{2}(\beta + 1)$ if β is odd, or $\frac{1}{2}(\beta + 2)$ if β is even).

PROOF. Suppose $l^\alpha \parallel M$. In case (1) we have $l^\alpha \parallel N$ also. Let $W_l^{(N)} = \begin{pmatrix} l^\alpha x & y \\ Nz & l^\alpha w \end{pmatrix}$, with $\det W_l^{(N)} = l^\alpha$. Then for $0 \leq i \leq \beta$ we have

$$\begin{pmatrix} q^i & 0 \\ 0 & 1 \end{pmatrix} W_l^{(N)} = W_l^{(M)} \begin{pmatrix} q^i & 0 \\ 0 & 1 \end{pmatrix}$$

where $W_l^{(M)} = \begin{pmatrix} l^\alpha x & q^i y \\ Mq^{\beta-i} z & l^\alpha w \end{pmatrix}$ also has determinant l^α . Hence

$$\begin{aligned} g_i \left| W_l^{(N)} \right. &= g \left| \begin{pmatrix} q^i & 0 \\ 0 & 1 \end{pmatrix} W_l^{(N)} \right. \\ &= g \left| W_l^{(M)} \begin{pmatrix} q^i & 0 \\ 0 & 1 \end{pmatrix} \right. \\ &= \varepsilon g \left| \begin{pmatrix} q^i & 0 \\ 0 & 1 \end{pmatrix} \right. = \varepsilon g_i. \end{aligned}$$

In case (2), when $q = l$, we have $q^{\alpha+\beta} \parallel N$. Let $W_q^{(N)} = \begin{pmatrix} q^{\alpha+\beta} x & y \\ Nz & q^{\alpha+\beta} w \end{pmatrix}$ with $\det W_q^{(N)} = q^{\alpha+\beta}$. Then for $0 \leq i \leq \beta$ we have

$$\begin{pmatrix} q^i & 0 \\ 0 & 1 \end{pmatrix} W_q^{(N)} = W_q^{(M)} \begin{pmatrix} q^{\beta-i} & 0 \\ 0 & 1 \end{pmatrix}$$

(modulo scalar matrices, which act trivially), where $W_q^{(M)} = \begin{pmatrix} q^{\alpha+i} x & y \\ Mz & q^{\alpha+\beta-i} w \end{pmatrix}$ has determinant q^α . Hence $g_i \left| W_q^{(N)} \right. = \varepsilon g_{\beta-i}$ as required. As a basis for the ε -eigenspace for $W_q^{(N)}$ we may take the forms $g_i + g_{\beta-i}$ for $0 \leq i \leq \beta/2$, and for the $(-\varepsilon)$ -eigenspace, $g_i - g_{\beta-i}$ for $0 \leq i < \beta/2$. Hence the multiplicities are as stated. \square

Using this result we can easily extend to the general case by induction on the number of prime divisors of N/M , giving the following result.

PROPOSITION 2.7.2. *Let g be a newform in $S_2(M)$ where $M \mid N$. Write $N/M = \prod_{i=1}^k q_i^{\beta_i}$, so that the oldclass in $S_2(N)$ coming from g has dimension $d(N/M) = \prod(1 + \beta_i)$.*

(1) *For every prime q not dividing N/M , the W_q -eigenvalue of every form in the oldclass is the same as that of g .*

(2) *Suppose $g \left| W_{q_i} = \varepsilon_i g$ for $1 \leq i \leq k$. Let*

$$(2.7.1) \quad n_i^\pm = \begin{cases} \frac{1}{2}(\beta_i + 1) & \text{if } \beta_i \text{ is odd,} \\ \frac{1}{2}(\beta_i + 2) & \text{if } \beta_i \text{ is even and } \varepsilon_i = +1, \\ \frac{1}{2}\beta_i & \text{if } \beta_i \text{ is even and } \varepsilon_i = -1, \end{cases}$$

and put $n_i^- = 1 + \beta_i - n_i^+$, so that $\prod(n_i^+ + n_i^-) = \prod(\beta_i + 1) = d(N/M)$. If $(\delta_1, \delta_2, \dots, \delta_k)$ is any k -vector with each $\delta_i = \pm 1$, then the subspace of oldforms in the oldclass on which W_{q_i} has eigenvalue δ_i for $1 \leq i \leq k$ has dimension $\prod_{i=1}^k n_i^{\delta_i}$. \square

Hence we are able to compute from our database a complete set of “sub-oldclasses”—that is, subspaces of oldclasses which have the same eigenvalues for *all* the operators T_p and W_q —with their dimensions.

Having thus computed a list of sub-oldclasses with their dimensions, W_q -eigenvalues and first few T_p -eigenvalues, we now proceed to find “new” one-dimensional rational eigenspaces of $H^+(N)$ as follows. We consider each prime in turn, starting with the q which divide N , then moving on to the p which do not divide N , computing W_q or T_p as appropriate. For each, we consider all possible integer eigenvalues ($\varepsilon_q = \pm 1$ for W_q , and a_p with $|a_p| < 2\sqrt{p}$ for T_p) and restrict all subsequent operations to each nonzero eigenspace in turn. At any given stage we have a subspace of $H^+(N)$ on which all the operators so far considered act as

scalars. Comparing with the oldform data we can tell whether this subspace consists entirely of oldforms: if so, we discard it. If not, and the subspace is one-dimensional, we have found a rational one-dimensional eigenspace corresponding to a newform. We then record a basis vector and a list of the (prime–eigenvalue) pairs needed to isolate this subspace. Otherwise we proceed recursively to the next prime and the next operator.

At the end of this stage of the computation in $H^+(N)$, we have found the number of rational one-dimensional “new” eigenspaces in $H^+(N)$, or equivalently, the number of rational newforms in $S_2(N)$. For each we have a dual (integer) eigenvector, which we will use to compute a large number of Hecke eigenvalues in Section 2.9.

Implementation. In preparation for splitting off the one-dimensional eigenspaces of $H^+(N)$ we compute the matrices of all the W -operators acting on $H^+(N)$, and store their transposes. We also collect from the “oldform database” information about the newforms at all levels M dividing and less than N . For each oldclass we must compute the eigenvalue multiplicities for each W_q using the formula (2.7.1) above.

The splitting itself is done recursively. At the general stage, at depth n , we have the following data:

- a particular subspace S of $H^+(N)$ (initially the whole of $H^+(N)$);
- a list of n primes (starting with the q dividing N , and initially empty);
- a list of eigenvalues, one for each of the primes in the list.

Here S is precisely the subspace of $H^+(N)$ on which the first n operators have the given eigenvalues.

Given this data, the recursive procedure does the following:

- (1) check whether S consists entirely of oldforms, by comparing the list of eigenvalues which determine S with those of each “suboldclass”; if so, terminate this branch;
- (2) otherwise, if $\dim S = 1$ then store the (single) basis vector for S in a cumulative list and terminate;
- (3) otherwise, take the next operator T in sequence (computing and storing its matrix if it has not been used before) and compute the matrix T_S of its restriction to S ; for all possible eigenvalues a of T , compute the kernel of $T_S - aI$; if non-trivial, pass the accumulated data, together with this kernel as a new working subspace, to the procedure at the next depth.

This procedure has been found to work extremely efficiently in practice. The only practical difficulty is the possibility of overflow during Gaussian elimination; it was found that the early use of W -operators was an efficient way of avoiding this for as long as possible. However, for larger values of N we were forced to abandon single-precision integer arithmetic for the linear algebra at this stage, and instead use a modular method, working in $\mathbb{Z}/P\mathbb{Z}$ for some large prime P , instead of in \mathbb{Z} . Alternatively, one could use multiprecision arithmetic, but this is likely to be slower.

In all subsequent calculations in $H^+(N)$, we will be interested only in the one-dimensional eigenspaces corresponding to rational newforms. To enhance the speed we now change the main M-symbol lookup tables: each vector in the table is replaced by the vector of its projections onto each of the subspaces, computed simply by taking the dot product with each dual eigenvector.

For each one-dimensional rational eigenspace found, we also compute the eigenvalue ε_N of the Fricke involution W_N , which is the product of all the W_q involutions. The significance of this is that $w = -\varepsilon_N$ is the sign of the functional equation of the L -series $L(f, s)$ attached to the newform f (see [64] and the next section).

2.8 $L(f, s)$ and the evaluation of $L(f, 1)/\Omega(f)$

Attached to each newform f in $S_2(N)$ there is an L -function $L(f, s)$, defined as follows via Mellin transform:

$$(2.8.1) \quad L(f, s) = (2\pi)^s \Gamma(s)^{-1} \int_0^{i\infty} (-iz)^s f(z) \frac{dz}{z}.$$

This gives an entire function of the complex variable s . Substitute the Fourier expansion $f(z) = \sum_{n=1}^{\infty} a(n, f) \exp(2\pi inz)$ and integrate term by term; provided that $\operatorname{Re}(s) > 3/2$ (for convergence), we obtain a representation of $L(f, s)$ as a Dirichlet series:

$$(2.8.2) \quad L(f, s) = \sum_{n=1}^{\infty} \frac{a(n, f)}{n^s}.$$

This L -function is one of the key links between the newform f and the modular elliptic curve E_f defined in Section 2.6 by its periods. First of all, the multiplicative relations satisfied by the coefficients $a(n, f)$, given above in Section 2.6, are equivalent to the statement that the Dirichlet series in (2.8.2) has an Euler product expansion:

$$(2.8.3) \quad \sum_{n=1}^{\infty} \frac{a(n, f)}{n^s} = \prod_{p \nmid N} (1 - a(p, f)p^{-s} + p^{1-2s})^{-1} \prod_{p|N} (1 - a(p, f)p^{-s})^{-1}.$$

This is exactly the form of the L -function of an elliptic curve of conductor N defined over \mathbb{Q} , and in fact the fundamental result (see [7], though partial results were known considerably earlier) is that

$$(2.8.4) \quad L(f, s) = L(E_f, s).$$

Thus (2.8.1) provides an analytic continuation to the entire plane of the L -function attached to the curve E_f , such as is conjectured to exist for all elliptic curves E defined over \mathbb{Q} .

Instead of the function $L(f, s)$ defined above by (2.8.1), it is sometimes convenient to use the variant with extra ‘infinite’ Euler factors:

$$(2.8.5) \quad \Lambda(f, s) = N^{s/2} (2\pi)^{-s} \Gamma(s) L(f, s) = \int_0^{\infty} f(iy/\sqrt{N}) y^{s-1} dy.$$

Thus for $\operatorname{Re}(s) > 3/2$ we have

$$\Lambda(f, s) = N^{s/2} (2\pi)^{-s} \Gamma(s) \sum_{n=1}^{\infty} \frac{a(n, f)}{n^s}.$$

The functions $L(f, s)$ and $\Lambda(f, s)$ also satisfy functional equations relating their values at s and $2 - s$. For since f is an eigenform for the Hecke algebra \mathbb{T} , it is in particular an eigenform for the Fricke involution W_N . Suppose that $f|W_N = \varepsilon_N f$ with $\varepsilon_N = \pm 1$: that is, $f(-1/(Nz)) = \varepsilon_N N z^2 f(z)$. With $z = iy/\sqrt{N}$ this gives $f(i/y\sqrt{N}) = -\varepsilon_N y^2 f(iy/\sqrt{N})$. Hence the substitution of $1/y$ for y in (2.8.5) yields the functional equation

$$(2.8.6) \quad \Lambda(f, 2 - s) = -\varepsilon_N \Lambda(f, s)$$

(note the change of sign). In view of (2.8.4), this gives a functional equation for $L(E_f, s)$ too, of the form conjectured for all elliptic curves over \mathbb{Q} .

From (2.8.6), we deduce that $L(f, 1) = \Lambda(f, 1) = 0$ when $\varepsilon_N = +1$; more generally, $L(f, s)$ has a zero of odd order when $\varepsilon_N = +1$, and a zero of even order (or no zero) when $\varepsilon_N = -1$. The significance of this is that the Birch–Swinnerton-Dyer conjectures predict that the order of the zero of $L(E, s)$ is equal to the rank of $E(\mathbb{Q})$, for an elliptic curve E defined over \mathbb{Q} . Thus we will be able to compare this order with the rank of the modular curves E_f , once we have found their equations explicitly.

The Birch–Swinnerton-Dyer conjectures also predict the value of $L(E, 1)/\Omega(E)$, which in the case of our modular curve $E = E_f$ is $L(f, 1)/\Omega(f)$, where $\Omega(f)$ is a certain period of f . We now discuss the relationship between $L(f, 1)$ and the periods of f (by which we will always mean the periods of the differential $2\pi i f(z) dz$).

Substituting $s = 1$ into the Mellin transform formula (2.8.1), we obtain

$$(2.8.7) \quad L(f, 1) = -2\pi i \int_0^{i\infty} f(z) dz = - \langle \{0, \infty\}, f \rangle.$$

The modular symbol $\{0, \infty\}$ is in the rational homology, so that $L(f, 1)$ is a rational multiple of some period of f . To find the rational factor, we use the trick of “closing the path” (see [38, page 286] or [37]).

For each prime p not dividing N we have, by (2.4.1),

$$T_p(\{0, \infty\}) = \{0, \infty\} + \sum_{k=0}^{p-1} \{k/p, \infty\} = (1+p)\{0, \infty\} + \sum_{k=0}^{p-1} \{k/p, 0\},$$

and hence

$$(2.8.8) \quad (1+p - T_p) \cdot \{0, \infty\} = \sum_{k=0}^{p-1} \{0, k/p\}.$$

Let a_p be the T_p -eigenvalue of f , so that $T_p f = a_p f$. Integrating the differential $2\pi i f(z) dz$ along both sides of (2.8.8) gives

$$(2.8.9) \quad (1+p - a_p) \cdot \langle \{0, \infty\}, f \rangle = \sum_{k=0}^{p-1} \langle \{0, k/p\}, f \rangle.$$

Since p does not divide N , each modular symbol $\{0, k/p\}$ on the right of (2.8.9) is *integral*: that is, in $H_1(X_0(N), \mathbb{Z})$. Thus the right-hand side of (2.8.9) is a period of f . It is even a real period, since

$$\overline{\langle \{0, k/p\}, f \rangle} = \langle \{0, -k/p\}, f \rangle = \langle \{0, (p-k)/p\}, f \rangle.$$

Let $\Omega_0(f)$ denote the least positive real period of f , and set

$$\Omega(f) = \begin{cases} 2\Omega_0(f) & \text{if the period lattice of } f \text{ is rectangular,} \\ \Omega_0(f) & \text{otherwise.} \end{cases}$$

Thus $\Omega(f)/\Omega_0(f)$ is the number of components of the real locus of the elliptic curve E_f . Also note that in each case, $\Omega(f)$ is twice the least real part of a period of f . This is useful since, as we are working in $H^+(N)$, we can only (at this stage) determine the projection of the period lattice Λ_f onto the real axis.

In both cases, (2.8.9) becomes

$$(2.8.10) \quad \frac{L(f, 1)}{\Omega(f)} = \frac{n(p, f)}{2(1 + p - a_p)},$$

where $n(p, f)$ is an integer. Note that $1 + p - a_p$ is non-zero, since, by well-known estimates, $|a_p| < 2\sqrt{p}$.

Formula (2.8.10) is significant in several ways. On the one hand, let E_f be the modular elliptic curve attached to f as above. Then $L(E_f, 1) = L(f, 1)$, and $\Omega_0(f) = \Omega_0(E_f)$, the least positive real period of E_f . Thus, once we know a_p and $n(p, f)$ for a single prime p , we can evaluate the rational number $L(E_f, 1)/\Omega(E_f)$, whose value is predicted by the Birch–Swinnerton-Dyer conjecture for E_f . In particular, we should have $L(f, 1) = 0$ if and only if $E_f(\mathbb{Q})$ is infinite. In the tables we give the value of $L(f, 1)/\Omega(f)$ for each rational newform f computed, and observe that the value is consistent with the Birch–Swinnerton-Dyer conjecture in each case.

Secondly, having computed the right-hand side of (2.8.10) for a single prime p , we may (if $L(f, 1) \neq 0$) use the fact that $n(p, f)/(1 + p - a_p)$ is independent of p to compute the eigenvalue a_p quickly for other p , by computing $n(p, f)$. This is discussed in the next section.

2.9 Computing Fourier coefficients

For each one-dimensional rational eigenspace of $H^+(N)$ we will need to know many Fourier coefficients $a(n, f)$ of the corresponding newform $f(z) = \sum a(n, f) \exp(2\pi inz)$. These are obtained from the Hecke eigenvalues by the recurrence formulae given in Section 2.6. We already have the eigenvalue ε_q of each W_q operator, and at least one eigenvalue a_{p_0} for the smallest prime p_0 not dividing N , which we recorded as we found the one-dimensional eigenspaces earlier.

It remains to compute a large number of the Hecke eigenvalues a_p for primes p not dividing N . If $L(f, 1) \neq 0$ then the most efficient method is to use (2.8.10). First we compute $n(p_0, f)$ from the right-hand side of (2.8.8). (This integer is nonzero if and only if $L(f, 1) \neq 0$, by (2.8.10)). For other primes p we then have

$$\frac{n(p, f)}{2(1 + p - a_p)} = \frac{n(p_0, f)}{2(1 + p_0 - a_{p_0})},$$

and hence

$$a_p = 1 + p - \frac{n(p, f)(1 + p_0 - a_{p_0})}{n(p_0, f)}.$$

The integers $n(p, f)$ may be computed by expressing the right-hand side of (2.8.8) as a linear combination of the M -symbols which generate $H^+(N)$, and then projecting onto the one-dimensional subspace corresponding to f : here we take the dot product with the dual eigenvector computed previously, normalized so that its components are relatively prime integers. The integer this produces is then actually too big by a scaling factor $d_1 d_2$, where d_1 and d_2 are the denominators defined in Section 2.2; this factor can be ignored at this stage, where it cancels out in the computation of a_p , but must be included when we need the actual ratio $L(f, 1)/\Omega(f)$ from (2.8.10).

If $L(f, 1) = 0$ then a variation of this method may be used. For $\alpha \in \mathbb{Q}$ we have

$$(2.9.1) \quad \begin{aligned} (1 + p - T_p)\{\alpha, \infty\} &= \{\alpha, p\alpha\} + \sum_{k=0}^{p-1} \left\{ \alpha, \frac{\alpha + k}{p} \right\} \\ &= \{0, p\alpha\} + \sum_{k=0}^{p-1} \left\{ 0, \frac{\alpha + k}{p} \right\} - (p + 1)\{0, \alpha\}. \end{aligned}$$

If p does not divide N and $\alpha = n/d$ with $\gcd(d, N) = 1$ then $[0] = [p\alpha] = [(\alpha + k)/p]$ for all k , so that the right-hand side of (2.9.1) lies in the integral homology $H_1(X_0(N), \mathbb{Z})$. Hence we can express it as an integral linear combination of the generating M-symbols. Projecting onto the rational one-dimensional subspace of $H^+(N)$ corresponding to f , we find that

$$(2.9.2) \quad \frac{\operatorname{Re} \langle \{\alpha, \infty\}, f \rangle}{\Omega(f)} = \frac{n(\alpha, p, f)}{2(1 + p - a_p)}$$

for some integer $n(\alpha, p, f)$, where the left-hand side is independent of p . Thus we can compute each a_p from $n(\alpha, p, f)$, given a_{p_0} and $n(\alpha, p_0, f)$, provided that the latter is nonzero.

It is slightly simpler to use a modular symbol of the form $\{0, \alpha\}$ here instead of $\{\alpha, \infty\}$, since (for suitable α) this will be integral. However the formula analogous to (2.9.1) has more terms of the form $\{0, \beta\}$ on the right, so this is slower in practice.

REMARK. Equation (2.9.1) and the remarks following it show that the modular symbol $\{\alpha, \infty\}$ lies in the rational homology $H_1(X_0(N), \mathbb{Q})$ provided that the denominator of α is coprime to N . More generally, for an arbitrary rational number α , the right-hand side of (2.9.1) will be integral provided that $p \equiv 1 \pmod{N}$; this proves that $\{\alpha, \infty\} \in H_1(X_0(N), \mathbb{Q})$ in all cases, which is the Manin-Drinfeld Theorem (Theorem 2.1.3) for $\Gamma_0(N)$.

Implementation. In practice we only use the first method if $L(f, 1) \neq 0$ for *all* the rational newforms f in $S_2(N)$. Otherwise we find a rational α such that $n(\alpha, p_0, f) \neq 0$ for all f , where p_0 is the smallest prime not dividing N .

We have already discussed computation of the integers $n(p, f)$. The $n(\alpha, p, f)$ are computed similarly by expressing the right-hand side of (2.9.1) in terms of the generating M-symbols and projecting onto each eigenspace. Note that the term $\{0, \alpha\}$ of (2.9.1) need only be computed once.

The Hecke eigenvalues which we have computed are stored in a data file for use both in subsequent steps of the calculations at level N , and also as part of the cumulative database which will be accessed when levels which are multiples of N are reached.

The exact number of a_p needed depends on N , and on the form f , and will not be known until the numerical calculation of periods is carried out in the next phase. Our strategy here was first to compute a_p for all p up to some predetermined bound (we used all $p < 1000$ for $N \leq 200$, $p < 2000$ for $200 < N \leq 400$, and $p < 3000$ for $401 < N \leq 1000$). We may also store extra information, so that if more eigenvalues are needed later, these can be computed without having to repeat the time-consuming steps described in Sections 2.1–2.7. Specifically, we may store the following: the M-symbols which generate $H^+(N)$; a table giving each M-symbol as a linear combination of these generators; a basis for $\ker(\delta)$; and a (dual) basis vector for each rational one-dimensional eigenspace.

Recapitulation. At this point we have completed the first phase of the computation at level N , in which we have been working in the space $H^+(N)$. To summarize, we know

- (1) the number of rational newforms f in $S_2(N)$; and, for each f ,
- (2) the sign w of the functional equation for $L(f, s)$;
- (3) the ratio $L(f, 1)/\Omega(f)$;
- (4) all W_q -eigenvalues ε_q of f ;
- (5) a large number of T_p -eigenvalues a_p of f .

In particular, we know the number of modular elliptic curves E_f of conductor N (up to isogeny); for each curve, we know the sign of its functional equation and whether or not its L -series vanishes at $s = 1$.

All computations carried out so far are exact and algebraic. In addition, we can also in this first phase compute approximations to the value $L(f, 1)$ (when it is non-zero) and to the

period $\Omega(f)$, though we do not know at this stage whether $\Omega(f)/\Omega_0(f) = 1$ or 2 . In other words, we can compute the projection of the period lattice Λ_f onto the real axis. Of course, this is insufficient information from which to construct the curve E_f .

In the second phase, which we describe in Sections 2.10–2.14, we compute the period lattice Λ_f of each rational newform f , and hence obtain an (approximate) equation for the curve \mathbb{C}/Λ_f .

These “analytic” quantities (periods) will necessarily be computed approximately, by summing certain infinite series whose coefficients involve the Fourier coefficients of f (see below). In order to achieve sufficient accuracy, we may have to compute many thousands of these Fourier coefficients, and it is therefore necessary to have efficient ways of doing this, such as the method described in this section.

2.10 Computing periods I

In order to compute the full period lattice Λ_f for each rational newform f found earlier, we have to work in the full space $H(N)$. By working in $H^+(N)$ we could only compute the real period $\Omega_0(f)$. Although we could also compute the least imaginary period $\Omega_{\text{im}}(f)$ by working similarly in $H^-(N)$ (which would be slightly faster), the lattice spanned by $\Omega_0(f)$ and $\Omega_{\text{im}}(f)$ may have index 2 in Λ_f . Hence from now on we work in $H(N)$.

We begin by computing $H(N)$ using M-symbols as in Section 2.2 (omitting relations (2.5.1)). Let $\gamma_1, \gamma_2, \dots, \gamma_{2g}$ be a \mathbb{Z} -basis for $H_1(X_0(N), \mathbb{Z})$ (and hence also a \mathbb{Q} -basis for $H(N)$). Using this basis we will identify $H(N)$ with the space of rational column vectors, and dual vectors will be represented by row vectors. Next we read from the data file (created during the first phase) the number of rational newforms and, for each, the eigenvalues a_p and ε_q . For each form f we now compute two integer dual (row) eigenvectors with eigenvalues a_p and ε_q for all p and q : one, v^+ , with eigenvalue $+1$ for the $*$ operator, and one, v^- , with eigenvalue -1 . This is much faster than repeating the splitting step described in Section 2.7, since we already know the eigenvalues which determine each one-dimensional eigenspace. As before, the eigenvectors v^\pm we compute must be dual eigenvectors, since we will use them for projecting onto the eigenspaces in question.

Let $\gamma^\pm \in H^\pm(N)$ (respectively) be eigenvectors with the same eigenvalues as v^\pm , such that $v^+\gamma^+ = v^-\gamma^- = 1$. We view γ^\pm as column vectors in \mathbb{Q}^{2g} by expressing them as linear combinations of the basis $\gamma_1, \gamma_2, \dots, \gamma_{2g}$ for $H(N)$. Thus the product $v^+\gamma^+$ is the product of a row vector by a column vector: essentially a dot product. Set $x = \langle \gamma^+, f \rangle$ and $y = -i \langle \gamma^-, f \rangle$ (so that $x, y \in \mathbb{R}$). We do not actually compute these vectors γ^\pm in practice; they are only needed for this exposition, as they determine the real numbers x and y . Moreover, although the eigenvectors v^\pm which we do use are only determined up to a scalar multiple, we shall see that this choice does not (as it should not) affect the specific period lattice we obtain.

Let $\gamma = \sum_{j=1}^{2g} c_j \gamma_j$ be an arbitrary integral cycle in $H(N)$. We identify γ with the column vector with component c_j . Then we have

$$(2.10.1) \quad \langle \gamma, f \rangle = \langle (v^+\gamma)\gamma^+ + (v^-\gamma)\gamma^-, f \rangle = (v^+\gamma)x + (v^-\gamma)yi.$$

The period lattice Λ_f is the set of all such integral periods $\langle \gamma, f \rangle$. To determine a \mathbb{Z} -basis for Λ_f we proceed as follows. Write $v^+ = (a_1, a_2, \dots, a_{2g})$ and $v^- = (b_1, b_2, \dots, b_{2g})$ with $a_j, b_j \in \mathbb{Z}$. Then as a special case of (2.10.1) we have

$$\langle \gamma_j, f \rangle = a_j x + b_j y i,$$

since $v^+\gamma_j = a_j$ and $v^-\gamma_j = b_j$. Hence Λ_f is spanned over \mathbb{Z} by the $2g$ periods $\langle \gamma_j, f \rangle = a_j x + b_j y i$. Let Λ be the \mathbb{Z} -span in \mathbb{Z}^2 of the $2g$ pairs (a_j, b_j) , and let $(\lambda_1, \mu_1), (\lambda_2, \mu_2)$ be a

\mathbb{Z} -basis for Λ . Then we find that

$$\Lambda_f = \{\langle \gamma, f \rangle \mid \gamma \in H(N)\} = \mathbb{Z}\omega_1 + \mathbb{Z}\omega_2,$$

where

$$(2.10.2) \quad \omega_j = \lambda_j x + \mu_j yi \quad (j = 1, 2).$$

Thus ω_1 and ω_2 form a \mathbb{Z} -basis for Λ_f .

We may compute (λ_1, μ_1) and (λ_2, μ_2) from v^+ and v^- using the Euclidean algorithm in \mathbb{Z} . In fact it is easy to see that there are only two possibilities, since v^\pm are determined within the subspace they generate by being the $+1$ and -1 eigenvectors for an involution. Normalize v^\pm so that each is primitive in \mathbb{Z}^{2g} ; that is, $\gcd(a_1, \dots, a_{2g}) = \gcd(b_1, \dots, b_{2g}) = 1$. In the first case (which we will call ‘‘Type 1’’), $v^+ \equiv v^- \pmod{2}$, and we may take $(\lambda_1, \mu_1) = (2, 0)$ and $(\lambda_2, \mu_2) = (1, 1)$, so that $\omega_1 = 2x$ and $\omega_2 = x + yi$. In this case $\Omega(f) = \Omega_0(f)$ and the elliptic curve has negative discriminant.

In the second case (‘‘Type 2’’), when v^+ and v^- are independent modulo 2, we will be able to take $(\lambda_1, \mu_1) = (1, 0)$ and $(\lambda_2, \mu_2) = (0, 1)$, so that $\omega_1 = x$ and $\omega_2 = yi$. In this case the period lattice is rectangular, $\Omega(f) = 2\Omega_0(f)$, and the elliptic curve has positive discriminant.

It remains to compute the real numbers x and y . We describe two methods: the first computes periods directly, while the second computes them indirectly by computing $L(f \otimes \chi, 1)$ for suitable quadratic characters χ . The latter method is in certain cases more accurate (in that fewer a_p are needed for the same accuracy) but cannot be used when N is a perfect square, as we shall see below.

Observe that the cycles γ^\pm do not enter into the calculations directly, but are merely used to define x and y . Also, if either v^+ or v^- is replaced by a scalar multiple of itself, then γ^+ and γ^- (and hence x and y) are scaled down by the same amount, but λ_j and μ_j are scaled up. In particular, it is no loss of generality to assume that v^\pm are primitive integer vectors. Thus (2.10.2) defines ω_1 and ω_2 unambiguously, as generators of the full period lattice of f .

Direct method. The simplest method is essentially the same as that used by Tingley in [67]. Using a recent improvement (see [18]), this method can now be made to converge as well as the indirect method described later.

From (2.10.1) it suffices to compute $\langle \gamma, f \rangle$ for a single cycle γ such that $v^+\gamma$ and $v^-\gamma$ are both nonzero; then by taking real and imaginary parts we can solve (2.10.1) for x and y and compute the periods ω_1 and ω_2 from (2.10.2). (In some cases it may be better in practice to use two different cycles, one for the real period and one for the imaginary period, but for simplicity we will assume that this is not the case.)

We denote by $I_f(\alpha, \beta)$ the integral $I_f(\alpha, \beta) = \int_\alpha^\beta 2\pi i f(z) dz$, and set $I_f(\alpha) = I_f(\alpha, \infty)$. Let $M \in \Gamma_0(N)$; since f is holomorphic, the period integral $I_f(\alpha, M(\alpha))$ is independent of the basepoint α , and can be expressed as $I_f(\alpha) - I_f(M(\alpha))$. We will denote this period of f by $P_f(M)$. Note that any cycle $\gamma \in H(N)$ can be expressed as $\{\alpha, M(\alpha)\}$ for a suitable matrix $M \in \Gamma_0(N)$, and then $\langle \gamma, f \rangle = P_f(M)$. The map $M \mapsto P_f(M)$ is (by Corollary 2.1.2) a group homomorphism from $\Gamma_0(N)$ to the additive group of complex numbers, whose image is the period lattice Λ_f .

Our basic tool for computing periods is the following easy result.

PROPOSITION 2.10.1. *Let $z_0 = x_0 + iy_0 \in \mathcal{H}$, so that $y_0 > 0$. Let f be a cusp form of weight 2 with Fourier coefficients $a(n, f)$. Then*

$$(2.10.3) \quad I_f(z_0) = \int_{z_0}^{\infty} 2\pi i f(z) dz = - \sum_{n=1}^{\infty} \frac{a(n, f)}{n} e^{2\pi i n x_0} e^{-2\pi n y_0}.$$

PROOF. Using the Fourier expansion $f(z) = \sum_{n=1}^{\infty} a(n, f) \exp(2\pi inz)$, we can integrate term-by-term over a vertical path from z_0 to ∞ to obtain the result. The term-by-term integration is justified since the series converges absolutely, since $|a(n, f)| \ll n$ and $y_0 > 0$. \square

We can sum the series (2.10.3) to obtain an approximation to $I_f(z_0)$, provided that we have sufficiently many Fourier coefficients $a(n, f)$. The important point to notice is that this series is a power series in $\exp(-2\pi y_0)$ (with bounded coefficients since $|a(n, f)| < n$ for all n), so will converge best when y_0 is large (or at least, not too small).

Suppose we are given a matrix $M = \begin{pmatrix} a & b \\ cN & d \end{pmatrix} \in \Gamma_0(N)$, where $a, b, c, d \in \mathbb{Z}$, and we wish to compute the associated period $P_f(M) = I_f(\alpha) - I_f(M(\alpha))$ of f . How should we choose α ? If α has large imaginary part, then $M(\alpha)$ will tend to have a small imaginary part; we would like to maximize both of these simultaneously. The simplest solution, used by Tingley in his thesis [67] for the original computations of modular elliptic curves², is to choose

$$\alpha = \frac{-d+i}{cN}, \quad \text{so that} \quad M(\alpha) = \frac{a+i}{cN}.$$

Thus both α and $M(\alpha)$ have imaginary part $(cN)^{-1}$. (Note that, by replacing M by $-M$ if necessary, we may assume that $c > 0$; we are not interested in M with $c = 0$ since these are parabolic, and hence have zero period.) Hence we obtain the following.

PROPOSITION 2.10.2. *Let $f \in S_2(N)$. Then, for all $M = \begin{pmatrix} a & b \\ cN & d \end{pmatrix} \in \Gamma_0(N)$, the period $P_f(M)$ is given by*

$$(2.10.4) \quad P_f(M) = I_f(\alpha) - I_f(M(\alpha)),$$

where $\alpha \in \mathcal{H}$ is arbitrary. Taking $\alpha = \frac{-d+i}{cN}$, we have:

$$(2.10.5) \quad \begin{aligned} P_f(M) &= I_f\left(\frac{-d+i}{cN}\right) - I_f\left(\frac{a+i}{cN}\right) \\ &= \sum_{n=1}^{\infty} \frac{a(n, f)}{n} e^{-2\pi n/cN} \left(e^{2\pi ina/cN} - e^{-2\pi ind/cN} \right). \end{aligned}$$

To use this result, we take a rational number b/d with denominator d coprime to N , solve $ad - bcN = 1$ for a and c , and set $M = \begin{pmatrix} a & b \\ cN & d \end{pmatrix}$. The integral cycle $\gamma = \{0, b/d\}$ should have the properties that $v^+\gamma$ and $v^-\gamma$ are both nonzero; also, since $y_0 = 1/(Nc)$ with $c > 0$ we should also choose b/d so that c is as small as possible, to speed convergence in the series (2.10.5). This series converges adequately quickly for small N , but when N increases we require too many terms in order to obtain the periods to sufficient precision. (Not only does it take longer to sum the series when we use more terms, but more significantly, computing the coefficients $a(n, f)$ by modular symbols becomes more expensive as n increases.)

The series (2.10.5) is a power series in $\exp(-2\pi/cN)$ for some small positive integer c ; at best we might hope to use $c = 1$ and have a power series in $\exp(-2\pi/N)$. We can improve this, however, to give a better formula which involves power series in $\exp(-2\pi/d\sqrt{N})$ for a small positive integer d . This greatly improves the convergence of the series.

²and also used in the first edition of this book

In order to do this, we make use of the fact that the newform f is an eigenform for the Fricke involution W_N , which for brevity we will here denote simply W . Thus (as in Section 2.8 above) we have

$$f(z) = \varepsilon (f | W)(z)$$

where $\varepsilon = \pm 1$ is the Fricke eigenvalue. By changing variables in the integrals, we see that

$$(2.10.6) \quad I_f(W(\alpha), W(\beta)) = I_{f|W}(\alpha, \beta) = \varepsilon I_f(\alpha, \beta).$$

In particular, if $\beta = W(\alpha)$ we obtain $I_f(\alpha, W(\alpha)) = -\varepsilon I_f(\alpha, W(\alpha))$, so that when $\varepsilon = +1$ we have $I_f(\alpha, W(\alpha)) = 0$ for all α .

Assume we are in this case ($\varepsilon = +1$). Then in any period integral, we may replace an endpoint α with $W(\alpha)$ without affecting the value of the integral. In particular,

$$P_f(M) = I_f(\alpha, M(\alpha)) = I_f(W(\alpha), M(\alpha)).$$

Setting $\alpha = di/(\sqrt{N} - cNi)$ we find that

$$M(\alpha) = \frac{b}{d} + \frac{i}{d\sqrt{N}} \quad \text{and} \quad W(\alpha) = \frac{c}{d} + \frac{i}{d\sqrt{N}},$$

which both have the same imaginary part $1/d\sqrt{N}$. (We may assume that $d > 0$, again by replacing M by $-M$ if necessary.) Hence $P_f(M) = I_f(W(\alpha)) - I_f(M(\alpha)) = I_f(\frac{c}{d} + \frac{i}{d\sqrt{N}}) - I_f(\frac{b}{d} + \frac{i}{d\sqrt{N}})$, where both integrals converge relatively well.

When $\varepsilon = -1$, we can obtain a slightly more complicated result which is just as good in practice. Combining both cases gives the following.

PROPOSITION 2.10.3. *Let $f \in S_2(N)$, such that $f | W = \varepsilon f$ with $\varepsilon = \pm 1$. Then for all $M = \begin{pmatrix} a & b \\ cN & d \end{pmatrix} \in \Gamma_0(N)$ the period $P_f(M)$ is given by*

$$(2.10.7) \quad P_f(M) = (1 - \varepsilon)I_f(i/\sqrt{N}) + \varepsilon I_f(W(\alpha)) - I_f(M(\alpha)),$$

where $\alpha \in \mathcal{H}$ is arbitrary. Taking $\alpha = M^{-1}(\frac{b}{d} + \frac{i}{d\sqrt{N}})$, so that $W(\alpha) = \frac{c}{d} + \frac{i}{d\sqrt{N}}$, we have

$$(2.10.8) \quad \begin{aligned} P_f(M) &= (1 - \varepsilon)I_f(i/\sqrt{N}) + \varepsilon I_f\left(\frac{c}{d} + \frac{i}{d\sqrt{N}}\right) - I_f\left(\frac{b}{d} + \frac{i}{d\sqrt{N}}\right) \\ &= \sum_{n=1}^{\infty} \frac{a(n, f)}{n} \left((\varepsilon - 1)e^{-2\pi n/\sqrt{N}} + e^{-2\pi n/d\sqrt{N}} \left(e^{2\pi inb/d} - \varepsilon e^{2\pi inc/d} \right) \right). \end{aligned}$$

PROOF. Using $W(i/\sqrt{N}) = i/\sqrt{N}$, we simply compute:

$$\begin{aligned} I_f(\alpha, M(\alpha)) &= I_f(\alpha, i/\sqrt{N}) + I_f(i/\sqrt{N}, W(\alpha)) + I_f(W(\alpha), M(\alpha)) \\ &= \varepsilon I_f(W(\alpha), i/\sqrt{N}) + I_f(i/\sqrt{N}, W(\alpha)) + I_f(W(\alpha), M(\alpha)) \\ &= (1 - \varepsilon)(I_f(i/\sqrt{N}) - I_f(W(\alpha))) + I_f(W(\alpha)) - I_f(M(\alpha)) \\ &= (1 - \varepsilon)I_f(i/\sqrt{N}) + \varepsilon I_f(W(\alpha)) - I_f(M(\alpha)) \end{aligned}$$

which establishes (2.10.7). Then (2.10.8) follows from (2.10.3), using the value of α defined before. \square

Note that the term $(1 - \varepsilon)I_f(i/\sqrt{N})$, which appears in (2.10.7), is equal to $-L(f, 1)$, by (2.11.1) below. Hence this term is zero unless the analytic rank of f is zero.

When we use this method for computing the periods, before proceeding to the next stage we store the following data:

$$\mathbf{type}, M, v^+\gamma, v^-\gamma.$$

Here $\mathbf{type} = 1$ or 2 denotes the lattice type, M is a matrix in $\Gamma_0(N)$ such that $\gamma = \{0, M(0)\}$, and the integers $v^+\gamma$ and $v^-\gamma$ are nonzero. Then we will be able to compute the periods from stored data quickly without having to recompute $H(N)$ or the eigenvectors v^\pm . We compute the period $P_f(M)$ using (2.10.8), set $x = \operatorname{Re}(P_f(M))/v^+\gamma$ and $y = \operatorname{Im}(P_f(M))/v^-\gamma$ from (2.10.1), and take the period lattice Λ_f to be the lattice with \mathbb{Z} -basis $2x, x + yi$ (if type 1) or x, yi (if type 2). If we later find that we need greater accuracy here, then after computing more a_p , we can obtain more accurate values for the periods ω_1 and ω_2 very quickly, without having to repeat the expensive calculation in $H(N)$.

2.11 Computing periods II: Indirect method

The idea here is to compute $\Omega(f)$ indirectly by computing $L(f, 1)$ and dividing by the ratio $L(f, 1)/\Omega(f)$, which we know from (2.8.10). If $L(f, 1) = 0$, and in any case to find the imaginary period, we can use the technique of twisting by a quadratic character χ , since the value $L(f \otimes \chi, 1)$ is a rational multiple of a real or imaginary period of f (depending on whether $\chi(-1) = +1$ or -1), and is non-zero for suitable χ .

We are also interested in the value of $L(f, 1)$ for its own sake, in relation to the Birch–Swinnerton-Dyer conjecture for the modular curve E_f . We will return to this, and the method of computing $L^{(r)}(f, 1)$ for $r > 0$, in Section 2.13.

If $L(f, 1) \neq 0$, then we may compute $L(f, 1)$ accurately from (2.8.7) as follows. Let $\varepsilon_N = \pm 1$ be the eigenvalue of the Fricke involution W_N on f . Then in the notation of the previous section, using (2.10.6) and $W_N(i/\sqrt{N}) = i/\sqrt{N}$:

$$\begin{aligned} L(f, 1) &= - \int_0^{i\infty} 2\pi i f(z) dz = I_f(\infty, 0) \\ (2.11.1) \qquad &= I_f(\infty, i/\sqrt{N}) + I_f(i/\sqrt{N}, 0) \\ &= I_f(\infty, i/\sqrt{N}) + \varepsilon_N I_f(i/\sqrt{N}, \infty) \\ &= (\varepsilon_N - 1) I_f(i/\sqrt{N}). \end{aligned}$$

Thus if $L(f, 1) \neq 0$, then necessarily $\varepsilon_N = -1$, and in this case $L(f, 1) = -2I_f(i/\sqrt{N})$. Using Proposition 2.10.1 then gives the following result.

PROPOSITION 2.11.1. *If $f(z) = \sum_{n=1}^{\infty} a(n, f) \exp(2\pi i n z) \in S_2(N)$ and $f|W_N = -f$ then*

$$(2.11.2) \qquad L(f, 1) = 2 \sum_{n=1}^{\infty} \frac{a(n, f)}{n} \exp(-2\pi n/\sqrt{N}).$$

REMARK. If in (2.11.1) we split the range of integration at Ai/\sqrt{N} for some positive real number A (instead of taking $A = 1$) then we obtain the more general formula

$$L(f, 1) = \sum_{n=1}^{\infty} \frac{a(n, f)}{n} \left(\exp(-2\pi An/\sqrt{N}) - \varepsilon_N \exp(-2\pi n/A\sqrt{N}) \right),$$

where the right-hand side is independent of A . This can be useful in situations where we do not know the value of ε_N , since we can evaluate this expression for two values of A , say $A = 1$ and $A = 1.1$, and check that the values obtained are approximately the same. For only one of the two possible values of ε_N will this happen. This idea is due to H. Cohen (see [9, Section 7.5]).

More generally, let l be an odd prime not dividing N , and χ the quadratic character modulo l . Define

$$(f \otimes \chi)(z) = \sum_{n=1}^{\infty} \chi(n)a(n, f) \exp(2\pi inz)$$

and

$$L(f \otimes \chi, s) = (2\pi)^s \Gamma(s)^{-1} \int_0^{i\infty} (-iz)^s (f \otimes \chi)(z) \frac{dz}{z};$$

then for $\operatorname{Re}(s) > 3/2$ we can integrate term-by-term to obtain

$$L(f \otimes \chi, s) = \sum_{n=1}^{\infty} \chi(n)a(n, f)n^{-s}.$$

Suppose, as above, that $f|W_N = \varepsilon_N f$. Then $f \otimes \chi$ is in $S_2(Nl^2)$, and

$$(f \otimes \chi)|W_{Nl^2} = \chi(-N)\varepsilon_N f \otimes \chi$$

(special case of equation (14) in [64]). Hence we can immediately generalize Proposition 2.11.1 to obtain the following.

PROPOSITION 2.11.2. *Let f be as above. Let l be an odd prime not dividing N . If $\chi(-N) = \varepsilon_N$ then $L(f \otimes \chi, 1) = 0$, while if $\chi(-N) = -\varepsilon_N$, then*

$$(2.11.3) \quad L(f \otimes \chi, 1) = 2 \sum_{n=1}^{\infty} \frac{\chi(n)a(n, f)}{n} \exp(-2\pi n/l\sqrt{N}).$$

The values $L(f \otimes \chi, 1)$ are related to the periods of f by a formula similar to (2.8.10). Let $g(\chi)$ be the Gauss sum attached to χ : if $l \equiv 1 \pmod{4}$ then $\chi(-1) = +1$ and $g(\chi) = \sqrt{l}$, while if $l \equiv 3 \pmod{4}$ then $\chi(-1) = -1$ and $g(\chi) = i\sqrt{l}$. If we set $l^* = \chi(-1)l$ then in all cases we have $g(\chi) = \sqrt{l^*}$. By [64, equation(12)] we have

$$f \otimes \chi = \frac{g(\chi)}{l} \sum_{k=0}^{l-1} \chi(-k) f \left| \begin{pmatrix} l & k \\ 0 & l \end{pmatrix} \right.$$

Hence

$$\begin{aligned} L(f \otimes \chi, 1) &= -\langle \{0, \infty\}, f \otimes \chi \rangle \\ &= -\frac{g(\chi)}{l} \sum \chi(-k) \left\langle \{0, \infty\}, f \left| \begin{pmatrix} l & k \\ 0 & l \end{pmatrix} \right. \right\rangle \\ &= -\frac{g(\chi)}{l} \sum \chi(-k) \langle \{k/l, \infty\}, f \rangle \\ &= \chi(-1) \frac{g(\chi)}{l} \langle \gamma_l, f \rangle \\ &= \frac{1}{\sqrt{l^*}} \langle \gamma_l, f \rangle, \end{aligned}$$

where

$$\gamma_l = \sum_{k=0}^{l-1} \chi(k) \{0, k/l\}.$$

Here we have used the identity $\sum \chi(k) = 0$. Since l does not divide N , the cycle γ_l is in the integral homology. Thus for each prime l not dividing $2N$ we can define an integral period

$$P(l, f) = \langle \gamma_l, f \rangle,$$

and we have shown that

$$P(l, f) = \sqrt{l^*} L(f \otimes \chi, 1).$$

Clearly $(\gamma_l)^* = \chi(-1)\gamma_l$, since $\{0, k/l\}^* = \{0, -k/l\}$. So, if $\chi(-1) = +1$, then $\gamma_l \in H^+(N)$, hence $P(l, f)$ is an integer multiple of the real period $\Omega_0(f)$, and thus of the form $m^+(l, f)x$ for some integer $m^+(l, f)$. So, provided that $m^+(l, f) \neq 0$, we have

$$(2.11.4) \quad x = \sqrt{l} \frac{L(f \otimes \chi, 1)}{m^+(l, f)} = \frac{P(l, f)}{m^+(l, f)}.$$

In practice, if we express γ_l as a linear combination of the basis cycles γ_j and thus view it as a column vector, then $m^+(l, f) = v^+ \gamma_l$.

Similarly, if $\chi(-1) = -1$ then $\gamma_l \in H^-(N)$, and $P(l, f) = m^-(l, f)yi$, where $m^-(l, f) = v^- \gamma_l$ is an integer, so that if $m^-(l, f) \neq 0$ then

$$(2.11.5) \quad y = \sqrt{l} \frac{L(f \otimes \chi, 1)}{m^-(l, f)} = \frac{P(l, f)}{im^-(l, f)}.$$

Assuming that N is not a perfect square, we find the smallest primes $l^+ \equiv 1 \pmod{4}$ and $l^- \equiv 3 \pmod{4}$ (not dividing N) such that $m^+ = m^+(l^+, f)$ and $m^- = m^-(l^-, f)$ are nonzero. A necessary (but not sufficient) condition for this to be true is that for the associated quadratic characters, $\chi_1(-N) = \chi_2(-N) = -\varepsilon_N$; for if $\chi(-N) = \varepsilon_N$ then the sign of the functional equation for $L(f \otimes \chi, s)$ is -1 , and hence $L(f \otimes \chi, 1) = 0$. Suitable primes always exist, provided that N is not a perfect square, by a theorem of Murty and Murty (see [44]). We then compute $L(f \otimes \chi_j, 1)$ for $j = 1, 2$ from (2.11.3), obtain x and y from (2.11.4) and (2.11.5), and finally substitute in (2.10.2) as before to obtain the periods ω_1 and ω_2 .

If N is a square, however, then $\chi(-N) = \chi(-1)$ for all primes l not dividing $2N$; hence we will only be able to find the real period this way if $\varepsilon_N = -1$, and only the imaginary period if $\varepsilon_N = +1$. Rather than seek a way round this difficulty we always use the ‘‘direct’’ method to compute the periods when N is square.

To assist convergence in (2.11.3) we clearly want to choose l as small as possible. It is a simple matter to estimate the error obtained in truncating the series (2.11.3) for $L(f \otimes \chi, 1)$ at a certain point $n = n_{\max}$. In practice we may use this to estimate the number of eigenvalues a_p needed to obtain the desired accuracy. However, to save time, we did not in all cases compute this many a_p , if the computed values of c_4 and c_6 (see Section 2.14) were close to integers, and when rounded led us to the coefficients of an elliptic curve of conductor N .

Note that, apart from the numerical evaluation of the periods $P(l^\pm, f)$ (using the series (2.11.3) for $L(f \otimes \chi, 1)$), all these computations are purely algebraic: we express the cycles γ_l in terms of our homology basis using continued fractions, and take the dot products of the resulting column vectors with our dual eigenvectors v^\pm to obtain the integers m^\pm .

The result of this algebraic computation consists of the following data for each rational newform f : primes l^\pm congruent respectively to ± 1 modulo 4; nonzero integers m^\pm ; and the

type (1 or 2) of the lattice. As in the direct method, before proceeding we store the following data for each newform f :

$$\text{type}, l^+, m^+, l^-, m^-.$$

To compute the lattice from this data set of five integers, we compute the periods $P(l^\pm, f)$ using formula (2.11.3), divide by m^\pm respectively to obtain x and y , and take Λ_f to be the lattice with \mathbb{Z} -basis $2x, x + yi$ (if type 1) or x, yi (if type 2). In practice we store just these five integers, and recompute the periods when we need them. In particular, if at the first attempt we are unable to compute the integer invariants c_4, c_6 of the curve E_f to sufficient precision to recognize them, then we will return to $H^+(N)$ in order to compute more Hecke eigenvalues, and then recompute the periods to greater precision without having to recompute $H(N)$.

Tricks and shortcuts.

In fact, the data l^+ and m^+ can be computed earlier in the first $H^+(N)$ phase, since they only depend on the real projection of the period lattice. Hence we can already compute the real period x from the data we have from the first phase. Moreover, it is easy to find a suitable prime l^- once we know the Hecke eigenvalues of f , by numerically computing $P(l, f)$ for several primes $l \equiv -1 \pmod{4}$ until we find a value which is clearly non-zero.

It follows that the only purpose of the extremely expensive second phase of the computation, working in $H(N)$, is to determine the integer factor m^- and the type of the lattice. An alternative approach, which we have used systematically for larger levels ($N > 3200$), is simply to guess the value of m^- by trying each positive integer m in turn. For each $m \geq 1$ we set $y = P(l^-, f)/m$ and test the two possible lattices (one of each type). If either lattice has approximate integer invariants c_4 and c_6 , and the rounded integral values are valid invariants of an elliptic curve over \mathbb{Q} , and the resulting curve has conductor N , then we store for later use the successful value m^- of m and the type, and consider the curve E'_f we have found as a possible candidate for the actual modular elliptic curve E_f .

The curves E_f and E'_f are certainly isogenous; they even have the same real period. In many cases, the curve E'_f has no rational isogenies; in such a case we can conclude that $E_f = E'_f$ with no ambiguity. In any case, we can compute the isogeny class of curves isogenous to E'_f via rational isogenies, and the only loss is that we do not always know exactly which curve in the class is the “strong Weil curve” E_f . (A further disadvantage is that we cannot compute the degree of the modular parametrization of E_f , as this requires knowledge of $H(N)$: see Section 2.15 below.)

The great advantage of this method is that in only a few seconds computation time, as soon as we have a rational newform, we can (almost always) write down an associated curve E'_f ; before this was implemented, it could take many hours of computation time to determine $H(N)$, find the eigenvectors v^\pm , and hence determine the factor m^- and the lattice type, before we could compute E_f .

Finally we discuss some variants of the trick just described.

1. We may use the same trick to find l^+ and m^+ if we have not computed them earlier. Then we are obtaining the period lattice and equation of the curve using only the Fourier coefficients of f (i.e. the coefficients of the L -series of the curve); the sign of the functional equation; and the conductor N . No modular symbol information at all is needed in this case. In fact, one may even guess the sign of the functional equation if all one has is the L -series; see the remark after Proposition 2.11.1.

2. Let l_1 and l_2 be two primes $\equiv -1 \pmod{4}$ for which $-N$ has the correct quadratic character, so that $P(l_1, f)$ and $P(l_2, f)$ are both not trivially zero. We may compute the periods $P(l_j, f)$; assume that these are nonzero (or use different primes l_j). We know that there exist nonzero integers m_j such that $P(l_j, f) = m_j yi$ for $j = 1, 2$. Therefore $P(l_2, f)/P(l_1, f) =$

m_2/m_1 , and we may compute a floating point approximation to this rational number. In practice (provided we have many Fourier coefficients, and the primes l_j are small) we will be able to recognize this rational number (say by using continued fractions). Its denominator is a factor of the unknown integer m_1 . If we do this for several different values of l_2 (with the same l_1) then the least common multiple of the denominators may give us a nontrivial factor of m_1 , and then in our search for the exact value we may restrict to multiples of this factor. This is useful in practice.

3. Another possibility, which we have not implemented, is to compute $H^-(N)$ in order to determine m^- exactly, as we do m^+ from $H^+(N)$. This would be no harder than the original computation of $H^+(N)$, and in fact it would be easier to find the eigenvector corresponding to each newform f , since we already know its eigenvalues. The result would be that we would have computed exactly all the data we need in a shorter time than would be required for computing $H(N)$, except for the type of the lattice. Then the only ambiguity is that we would not know the type, and would have to try both types to see which results in a curve with integral coefficients. If both types succeeded (as does happen), we would only know the curve E_f up to a 2-isogeny.

2.12 Computing periods III: Evaluation of the sums

The results of the previous two sections express the periods of a rational newform $f(z) = \sum a(n, f) \exp(2\pi i n z)$, and the value $L(f, 1)$, in terms of various infinite series, each of the form $\sum a(n, f) c(n)$. In each case the factor $c(n)$ is a simple function of n , but the coefficient $a(n, f)$ must be computed more indirectly from the $a(p, f)$ for prime p as in Section 2.9.

In practice we will know $a(p, f)$ for the first few primes, say $p \leq \mathbf{pmax}$. An elegant and efficient recursive procedure for summing a series of the form $\sum a(n) c(n)$ over

$$\{n : 1 \leq n \leq \mathbf{nmax}, \text{ and } p|n \Rightarrow p \leq \mathbf{pmax}\},$$

with $a(n)$ defined in a similar recursive manner, was described in [5, pages 27–28]. This method has the advantage of minimizing the number of multiplications involved and the number of $a(n)$ which must be stored. Also, if some $a(n) = 0$ then there is a whole class of integers m for which $a(m) = 0$ that the procedure avoids automatically. Although in our program this part of the computation was not critical for either time or storage space, we found this algorithm to be very useful. It may also be applied in other similar situations for other kinds of modular forms: we have ourselves used it in [14], with cusp forms of weight 2 for $\Gamma_1(N)$, and also in our work over imaginary quadratic fields.

To evaluate such a sum, assume that the array $\mathbf{p}[i]$ hold the first \mathbf{pmax} primes p_i , and that the array $\mathbf{ap}[i]$ holds the coefficients $\mathbf{ap}[i] = a(p_i)$ for $p_i \leq \mathbf{pmax}$. We can evaluate the sum $a(n)c(n)$ over all $n \leq \mathbf{nmax}$ all of whose prime divisors are less than or equal to \mathbf{pmax} with the following pseudo-code.

Algorithm for recursively computing a multiplicative sum

1. BEGIN
2. Sum = c(1);
3. FOR i WHILE $\mathbf{p}[i] \leq \mathbf{pmax}$ DO
4. BEGIN
5. add($\mathbf{p}[i], i, \mathbf{ap}[i], 1$)
6. END
7. END

(Subroutine to add the terms dependent on p)

```

subroutine add(n,i,a,last_a)
1. BEGIN
2. IF a=0 THEN j0 = i ELSE Sum = Sum + a*c(n); j0 = 1 FI;
3. FOR j FROM j0 TO i WHILE p[j]*n ≤ nmax DO
4. BEGIN
5.     next_a = a*ap[j];
6.     IF j=i AND (N ≠ 0 (mod p[j])) THEN
7.         next_a = next_a - p[j]*last_a
8.     FI;
9.     add(p[j]*n,j,next_a,a)
10. END
11. END

```

Here the recursive function $\text{add}(n, i, a, \text{last_a})$ is always called under the following conditions: (i) $p_i = p[i]$ is the smallest prime dividing $n = n$; (ii) $a = a(n)$; (iii) $\text{last_a} = a(n/p_i)$. The procedure for n calls itself with pn in place of n , for all primes $p \leq p_i$, having first computed $\text{next_a} = a(pn)$ using the recurrence formulae from Section 2.6; if $a(n) = 0$ then only $p = p_i$ need be used, since then $a(pn) = a(p)a(n) = 0$ for all $p < p_i$.

2.13 Computing $L^{(r)}(f, 1)$

In investigating the Birch–Swinnerton-Dyer conjecture for the modular curves E_f we will need to compute the numerical value of the r th derivative $L^{(r)}(E_f, 1) = L^{(r)}(f, 1)$, where r is the order of $L(f, s)$ at $s = 1$. This integer r is sometimes called the ‘analytic rank’ of the curve E_f , since it is also, according to the Birch–Swinnerton-Dyer conjecture, the rank of $E_f(\mathbb{Q})$. Following earlier work with examples of rank 0 and 1, this computation was carried out by Buhler, Gross and Zagier in [6], for the curve of conductor 5077 and rank 3. Their method works in general, and we describe it here.

Recall the definition of $\Lambda(f, s)$ from Section 2.8:

$$(2.8.5) \quad \Lambda(f, s) = N^{s/2} (2\pi)^{-s} \Gamma(s) L(f, s) = \int_0^\infty f(iy/\sqrt{N}) y^{s-1} dy.$$

Let the W_N -eigenvalue of f be ε . Using $f(-1/Nz) = \varepsilon N z^2 f(z)$ we obtain

$$\Lambda(f, s) = \int_1^\infty f(iy/\sqrt{N}) (y^{s-1} - \varepsilon y^{1-s}) dy$$

(from which the functional equation (2.8.6) follows immediately). Differentiating k times with respect to s gives

$$\Lambda^{(k)}(f, s) = \int_1^\infty f(iy/\sqrt{N}) (\log y)^k (y^{s-1} - \varepsilon (-1)^k y^{1-s}) dy,$$

so at $s = 1$ we have

$$\Lambda^{(k)}(f, 1) = (1 - (-1)^k \varepsilon) \int_1^\infty f(iy/\sqrt{N}) (\log y)^k dy.$$

Trivially this gives $\Lambda^{(k)}(f, 1) = 0$ if $\varepsilon = (-1)^k$. In particular, since $\Lambda^{(r)}(f, 1) \neq 0$, by definition of r , we must have $(-1)^r = -\varepsilon$ so that r is even if and only if $\varepsilon = -1$. Hence setting $k = r$, we have

$$\begin{aligned} \Lambda^{(r)}(f, 1) &= 2 \int_1^\infty f(iy/\sqrt{N})(\log y)^r dy \\ (2.13.1) \quad &= 2 \sum_{n=1}^\infty a(n, f) \int_1^\infty \exp(-2\pi ny/\sqrt{N})(\log y)^r dy. \end{aligned}$$

If $r = 0$, of course, we recover the formula

$$\Lambda(f, 1) = \frac{\sqrt{N}}{\pi} \sum_{n=1}^\infty \frac{a(n, f)}{n} \exp(-2\pi n/\sqrt{N})$$

which agrees with (2.11.2) since $\Lambda(f, 1) = (\sqrt{N}/2\pi)L(f, 1)$. Now assume that $r \geq 1$. Integrating (2.13.1) by parts gives

$$\Lambda^{(r)}(f, 1) = \frac{r\sqrt{N}}{\pi} \sum_{n=1}^\infty \frac{a(n, f)}{n} \int_1^\infty \exp(-2\pi ny/\sqrt{N})(\log y)^{r-1} \frac{dy}{y}.$$

Since $\Lambda(f, s)$ vanishes to order r at $s = 1$ we have $L^{(r)}(f, 1) = (2\pi/\sqrt{N})\Lambda^{(r)}(f, 1)$, and hence the following result.

PROPOSITION 2.13.1. *Let f be a newform in $S_2(N)$ with W_N -eigenvalue ε , and suppose that the order of $L(f, s)$ at $s = 1$ is at least r , where $\varepsilon = (-1)^{r-1}$. Then*

$$(2.13.2) \quad L^{(r)}(f, 1) = 2r! \sum_{n=1}^\infty \frac{a(n, f)}{n} G_r\left(\frac{2\pi n}{\sqrt{N}}\right)$$

where

$$G_r(x) = \frac{1}{(r-1)!} \int_1^\infty e^{-xy} (\log y)^{r-1} \frac{dy}{y}.$$

In order to evaluate the series in (2.13.2) we may use the summation procedure of the preceding section, provided that we are able to compute the function $G_r(x)$. When $r = 1$, $G_1(x)$ is the exponential integral $\int_1^\infty e^{-xy} dy/y$, which may be evaluated for small x (say $x < 3$) by the power series

$$(2.13.3) \quad G_1(x) = \left(\log \frac{1}{x} - \gamma\right) - \sum_{n=1}^\infty \frac{(-x)^n}{n \cdot n!}$$

where γ is Euler's constant $0.577\dots$. For larger x (say $x > 2$) it is better to use the continued fraction expansion

$$G_1(x) = \frac{e^{-x}}{x + \frac{1}{1 + \frac{1}{x + \frac{2}{1 + \frac{2}{x + \frac{3}{1 + \dots}}}}}}.$$

To generalize the series (2.13.3) for $G_1(x)$, we observe that the functions $G_r(x)$ satisfy the functional equations $G'_r(x) = (-1/x)G_{r-1}(x)$, with $G_0(x) = e^{-x}$. It follows that

$$G_r(x) = P_r\left(\log \frac{1}{x}\right) + \sum_{n=1}^{\infty} \frac{(-1)^{n-r}}{n^r \cdot n!} x^n$$

where $P_r(t)$ is a polynomial of degree r satisfying $P'_r(t) = P_{r-1}(t)$ and $P_0(t) = 0$. From our earlier expression for $G_1(x)$ we see that $P_1(t) = t - \gamma$. In general $P_r(t) = Q_r(t - \gamma)$ where

$$\begin{aligned} Q_1(t) &= t; \\ Q_2(t) &= \frac{1}{2}t^2 + \frac{\pi^2}{12}; \\ Q_3(t) &= \frac{1}{6}t^3 + \frac{\pi^2}{12}t - \frac{\zeta(3)}{3}; \\ Q_4(t) &= \frac{1}{24}t^4 + \frac{\pi^2}{24}t^2 - \frac{\zeta(3)}{3}t + \frac{\pi^4}{160}; \\ Q_5(t) &= \frac{1}{120}t^5 + \frac{\pi^2}{72}t^3 - \frac{\zeta(3)}{6}t^2 + \frac{\pi^4}{160}t - \frac{\zeta(5)}{5} - \frac{\zeta(3)\pi^2}{36}. \end{aligned}$$

For $N < 5077$ we always found that $r \leq 2$, and determining the value of r in such cases is easy. Certainly $r = 0$ if and only if $L(f, 1) \neq 0$, which can be determined algebraically by (2.8.10). When $L(f, 1) = 0$ and $\varepsilon = +1$ we know that r is odd; by computing $L'(f, 1)$ to sufficient precision using (2.13.2) we could verify that $L'(f, 1) \neq 0$, so that $r = 1$. Similarly, when $L(f, 1) = 0$ and $\varepsilon = -1$, we know that r is even and at least 2, and we could check that $r = 2$ by computing $L''(f, 1)$ to sufficient precision to be certain that $L''(f, 1) \neq 0$.

In higher rank cases we have the problem of deciding whether $L^{(k)}(f, 1) = 0$, since no approximate calculation can by itself determine this. The first case where this occurs is for $N = 5077$, the rank 3 case considered in [6]. Here one finds that $L'(f, 1) = 0$ to 13 decimal places using (2.13.2) with 250 terms; then it is possible to conclude that $L'(f, 1) = 0$ exactly, by applying the theorem of Gross and Zagier concerning modular elliptic curves of rank 1 (see [25] or [26]) which relates the value of $L'(f, 1)$ to the height of a certain Heegner point on E_f . In this case no point on E_f has sufficiently small positive height, and one can therefore deduce that $L'(f, 1) = 0$, so that $r \geq 3$. Finally the value of $L^{(3)}(f, 1)$ can be computed numerically and hence shown to be non-zero (approximately 1.73 in this case). See [6] for more details. Using more recent work of Kolyvagin (see [29]) this argument can be simplified, since it is now known that when $L(f, s)$ has a simple zero at $s = 1$, the curve E_f has rank exactly 1. But in this case E_f has rank 3 (computed via two-descent, though finding three independent points of infinite order is easy and shows that the rank is at least 3), so again the analytic rank must be at least 3, and is therefore exactly 3 as before.

The results of Kolyvagin in [29] imply that when $L(f, s)$ has a zero of order $r = 0$ or 1 at $s = 1$ then³ the rank of E_f is exactly r . For the tables we also verified that the rank of $E_f(\mathbb{Q})$ was r directly in almost all cases (the exceptions being curves where the coefficients were so large that the two-descent algorithm, described in the next chapter, would have taken too long to run). These results apply to all but 18 of the rational newforms f we found at levels up to 1000. The remaining cases all had $r = 2$ (determined as above) and we verified that the rank of $E_f(\mathbb{Q})$ was 2 in each case. For a summary of the ranks found in the extended computations to $N = 5077$, see Chapter IV.

³In fact, Kolyvagin's result in the rank 0 case was conditional on a certain technical hypothesis, which was later proved independently by Murty and Murty and by Bump, Friedberg and Hoffstein. See [44]. The analogous hypothesis in the rank 1 case was already known as a consequence of a theorem of Waldspurger. The rank 0 result was previously proved in the case of complex multiplication by Coates and Wiles.

2.14 Obtaining equations for the curves

So far we have described how to compute, to a certain precision, the periods ω_1 and ω_2 which generate the period lattice Λ_f of the modular curve $E_f = \mathbb{C}/\Lambda_f$ attached to each rational newform f in $S_2(N)$. Now we turn to the question of finding an equation for E_f .

Set $\tau = \omega_1/\omega_2$. Interchanging ω_1 and ω_2 if necessary, we may assume that $\text{Im}(\tau) > 0$. By applying the well-known algorithm for moving a point in the upper half-plane \mathcal{H} into the usual fundamental region for $\text{SL}(2, \mathbb{Z})$ we may assume that $|\text{Re}(\tau)| \leq 1/2$ and $|\tau| \geq 1$, so that $\text{Im}(\tau) \geq \sqrt{3}/2$. One merely replaces (ω_1, ω_2) by $(\omega_1 - n\omega_2, \omega_2)$ for suitable $n \in \mathbb{Z}$ and (ω_1, ω_2) by $(-\omega_2, \omega_1)$ until both conditions are satisfied. In practice one must be careful about rounding errors, as it is quite possible to have both $|\tau| < 1$ and $|-1/\tau| < 1$ after rounding, which is liable to prevent the algorithm from terminating.

Set $q = \exp(2\pi i\tau)$. Then the lattice invariants $c_4 (= 12g_2)$ and $c_6 (= 216g_3)$ are given by

$$(2.14.1) \quad c_4 = \left(\frac{2\pi}{\omega_2}\right)^4 \left(1 + 240 \sum_{n=1}^{\infty} \frac{n^3 q^n}{1 - q^n}\right) \quad \text{and} \quad c_6 = \left(\frac{2\pi}{\omega_2}\right)^6 \left(1 - 504 \sum_{n=1}^{\infty} \frac{n^5 q^n}{1 - q^n}\right)$$

(see, for example, [31, p.47]). Since $|q| = \exp(-2\pi\text{Im}(\tau)) \leq \exp(-\pi\sqrt{3}) < 0.005$, these series converge extremely rapidly. Thus, assuming that ω_1 and ω_2 are known to sufficient precision, we can compute c_4 and c_6 as precisely as required.

Since E_f is defined over \mathbb{Q} , the numbers c_4 and c_6 are rational, but there is no simple reason why they should be integral. Fortunately, a result of Edixhoven (see [21]) states that in fact they are integral. Hence, provided that we have computed the periods and then c_4 and c_6 to sufficient precision, we will be able to recognize the corresponding exact integer values.

This only presents practical difficulties when c_4 and c_6 are large, since standard double precision arithmetic only yields around 16 decimal places. In several cases this means that we can recognize c_4 , but the last digit or digits of c_6 are undetermined. One obvious way round these difficulties is to use multiprecision arithmetic, though the resulting programs are slower, which can be an important consideration when large numbers of curves are being processed. In these situations, we are helped by the fact that we know that c_4 and c_6 are the invariants of an elliptic curve of conductor N . This implies the following congruence conditions (see [30], [10] or Section 3.2 below):

- (1) $c_4^3 - c_6^2 = 1728\Delta$, where Δ is a non-zero integer divisible by the primes dividing N ;
- (2) for primes $p \geq 5$ which divide N , we have $p \mid c_4 \iff p \mid c_6 \iff p^2 \mid N$;
- (3) $c_6 \not\equiv 9 \pmod{27}$;
- (4) either $c_6 \equiv -1 \pmod{4}$, or $c_4 \equiv 0 \pmod{16}$ and $c_6 \equiv 0, 8 \pmod{32}$.

Note that in condition (1) we should not assume that Δ is only divisible by the ‘‘bad primes’’ which divide N , since we do not know that c_4 and c_6 are the invariants of a minimal model. However, Edixhoven’s result does bound the non-minimality, and in practice all the equations of curves we have constructed are minimal, verifying the conjecture (Manin’s ‘‘ $c = 1$ ’’ conjecture) that this should always be the case. Conditions (2)–(4) do assume minimality at the relevant primes.

Since c_4 tends to be smaller than c_6 , the common situation is that we know c_4 , but may need to use the above congruence conditions to help us find c_6 in case it has more than 16 digits.

Given integral invariants c_4, c_6 satisfying (1), (3) and (4) above, the coefficients of a standard Weierstrass equation for the curve may be obtained as follows (see Section 3.1), where all divisions are exact:

$$b_2 = -c_6 \pmod{12} \in \{-5, \dots, 6\};$$

$$b_4 = (b_2^2 - c_4)/24;$$

$$b_6 = (-b_2^3 + 36b_2b_4 - c_6)/216;$$

$$a_1 = b_2 \pmod{2} \in \{0, 1\};$$

$$a_3 = b_6 \pmod{2} \in \{0, 1\};$$

$$a_2 = (b_2 - a_1)/4;$$

$$a_4 = (b_4 - a_1a_3)/2;$$

$$a_6 = (b_6 - a_3)/4.$$

Having the coefficients $[a_1, a_2, a_3, a_4, a_6]$ of a curve E , we may apply Tate's algorithm (see Section 3.2 below) to check that E has conductor N . We also check whether this model for E is minimal. These conditions do hold for all the cases we have computed to date ($N \leq 5077$). We also verify in each case that the traces of Frobenius of E_f and E for all primes under 1000 agree in each case, and in nearly all cases (see the previous section) that the rank of E , computed via two-descent, agrees with the 'analytic rank' of E_f . Finally, we can compute all curves isogenous to E over \mathbb{Q} : see Section 3.8 for one way to do this. This final list of curves will, according to the Shimura–Taniyama–Weil conjectures, contain all elliptic curves defined over \mathbb{Q} with conductor N (up to isomorphism). At the time of writing⁴ this has been proved (by Wiles and Taylor, following Ribet, Frey and others) provided that N is divisible by neither 4 nor 25.

Our computations do not give any verification of the Shimura–Taniyama–Weil conjecture, since if there did exist elliptic curves over \mathbb{Q} which were not modular, then we would simply not find them. We could only verify the conjecture if we had an independent method for listing all curves of conductor N , up to isogeny. For example, this has been done

- when N is a power of 2 (Ogg) or of the form 2^a3^b (Coghlan, see [2, Table 4]);
- when $N = 11$ (by Agrawal, Coates, Hunt and Van der Poorten, using the theory of Baker; and independently by Serre, using a variant of Faltings's method based on quartic fields [54]);
- for certain prime values of N (see [4]).

Our results are compatible with those of Brumer and Kramer in [4] for curves of prime conductor under 1000.

The algorithms we used to study these curves E further will be the subject of the next chapter.

2.15 Computing the degree of a modular parametrization

The modular elliptic curves we have shown how to construct in this chapter can be parametrized by modular functions for the subgroup $\Gamma_0(N)$ of the modular group $\Gamma = \mathrm{PSL}(2, \mathbb{Z})$. Equivalently, there is a non-constant map φ from the modular curve $X_0(N)$ to E . In the paper [17], we presented a method of computing the degree of such a map φ for arbitrary N . Our method is derived from a method of Zagier in [69]; by using those ideas, together with the modular symbol and M-symbol techniques which have been used above, we are able to derive an explicit formula for $\deg(\varphi)$ which is in general much simpler to implement than Zagier's, for arbitrary subgroups of finite index in Γ . To implement this formula one needs to have explicit coset representatives for the subgroup, but it is not necessary to determine an explicit fundamental domain for its action on the upper half-plane \mathcal{H} . In particular, it is

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simple to implement for $\Gamma_0(N)$ for arbitrary N , in contrast with Zagier's formula which is only completely explicit for N prime.

In this section we present the algorithm described in [17]. For more details and proofs, see [17]. Worked examples are given in the appendix to this chapter, and results for $N \leq 1000$ may be found in Chapter IV.

2.15.1. Modular Parametrizations.

Let G be a congruence subgroup of the modular group $\Gamma = \mathrm{PSL}(2, \mathbb{Z})$. The quotient $X = X_G = G \backslash \mathcal{H}^*$ is a Riemann surface, and an algebraic curve defined over a number field, and is called a modular curve.

An elliptic curve E defined over \mathbb{Q} is called a modular elliptic curve if there is a non-constant map $\varphi: X_G \rightarrow E$ for some modular curve X_G . The pull-back of the (unique up to scalar multiplication) holomorphic differential on E is then of the form $2\pi i f(z) dz$, where $f \in S_2(G)$. According to the Shimura–Taniyama–Weil conjecture, this should be the case for every elliptic curve defined over \mathbb{Q} , with $G = \Gamma_0(N)$, where N is the conductor of E . Moreover, the cusp form f should be a newform in the usual sense.

We will suppose that we are given a cusp form $f \in S_2(G)$. Since the differential $f(z) dz$ is holomorphic, the function

$$z_0 \mapsto I_f(z_0) = 2\pi i \int_{z_0}^{\infty} f(z) dz$$

is well-defined for $z_0 \in \mathcal{H}^*$ (independent of the path from z_0 to ∞). Also, for $M \in G$, the function

$$M \mapsto P_f(M) = I_f(z_0) - I_f(M(z_0)) = 2\pi i \int_{z_0}^{M(z_0)} f(z) dz$$

is independent of z_0 , and defines a function $P_f: G \rightarrow \mathbb{C}$ which is a group homomorphism. The image Λ_f of this map will, under suitable hypotheses on f which we will assume to hold, be a lattice of rank 2 in \mathbb{C} , so that $E_f = \mathbb{C}/\Lambda_f$ is an elliptic curve. Hence I_f induces a map

$$\begin{aligned} \varphi: X = G \backslash \mathcal{H}^* &\rightarrow E_f = \mathbb{C}/\Lambda_f \\ z \bmod G &\mapsto I_f(z) \bmod \Lambda_f. \end{aligned}$$

The period map $P_f: G \rightarrow \Lambda_f$ is surjective (by definition) and its kernel contains all elliptic and parabolic elements of G . We may write $\Lambda_f = \mathbb{Z}\omega_1 + \mathbb{Z}\omega_2$ with $\mathrm{Im}(\omega_2/\omega_1) > 0$. Then

$$P_f(M) = n_1(M)\omega_1 + n_2(M)\omega_2$$

where $n_1, n_2: G \rightarrow \mathbb{Z}$ are homomorphisms. These functions are explicitly computable in terms of modular symbols as seen in earlier sections. Alternatively, given sufficiently many Fourier coefficients of the cusp form $f(z)$ we may evaluate the period integrals $I_f(z)$ (using the formula (2.10.8), for example) to sufficient precision that (assuming that the fundamental periods ω_1 and ω_2 are also known to some precision) one can determine the integer values of $n_1(M)$ and $n_2(M)$ for any given $M \in G$. The latter approach is used in [69]. The advantage of the modular symbol approach is that exact values are obtained directly, and that it is not necessary to compute (or even know) any Fourier coefficients of f . On the other hand, it becomes computationally infeasible to carry out the modular symbol computations when the index of G in Γ is too large, whereas the approximate approach can still be used, provided that one has an explicit equation for the curve E to hand, from which one can compute the periods and the Fourier coefficients in terms of traces of Frobenius (assuming that E is modular and defined over \mathbb{Q}). This method was used, for example, to compute $\mathrm{deg}(\varphi)$ for the curve of rank 3

with conductor 5077, in [69]; we verified the value obtained (namely 1984) using our modular symbol implementation.

The special case we are particularly interested in is where $G = \Gamma_0(N)$ and $f(z)$ is a normalized newform for $\Gamma_0(N)$. Then the periods of $2\pi if(z)$ do form a suitable lattice Λ_f , and the modular elliptic curve $E_f = \mathbb{C}/\Lambda_f$ is defined over \mathbb{Q} and has conductor N .

In order to compute the degree of the map $\varphi: X \rightarrow E_f$, the idea used in [69] is to compute the Petersson norm $\|f\|$ in two ways. The first way involves $\deg(\varphi)$ explicitly, while the second expresses it as a sum of terms involving periods, which can be evaluated as above.

PROPOSITION 2.15.1. *Let $f(z)$ be a cusp form of weight 2 for G as above, and $\varphi: X \rightarrow E_f$ the associated modular parametrization. Then*

$$4\pi^2 \|f\|^2 = \deg(\varphi) \text{Vol}(E_f).$$

REMARK. In terms of the fundamental periods ω_1, ω_2 of E_f , the volume is given by $\text{Vol}(E_f) = |\text{Im}(\overline{\omega_1}\omega_2)|$. More generally, if $\omega, \omega' \in \Lambda_f$, with $\omega = n_1(\omega)\omega_1 + n_2(\omega)\omega_2$ and $\omega' = n_1(\omega')\omega_1 + n_2(\omega')\omega_2$, then (up to sign) we have

$$\text{Im}(\overline{\omega}\omega') = \text{Vol}(E_f) \cdot \begin{vmatrix} n_1(\omega) & n_1(\omega') \\ n_2(\omega) & n_2(\omega') \end{vmatrix}.$$

2.15.2. Coset representatives and Fundamental Domains.

Let $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ and $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ be the usual generators for Γ , so that S has order 2 and TS has order 3. Let \mathcal{F} be the usual fundamental domain for Γ defined above in (2.1.1), and \mathcal{T} the “ideal triangle” with vertices at 0, 1 and ∞ . Recall from Section 2.1 that $\langle M \rangle$ denotes the transform of \mathcal{T} by M for $M \in \Gamma$, which is the ideal triangle with vertices at the cusps $M(0)$, $M(1)$ and $M(\infty)$. These triangles form a triangulation of the upper half-plane \mathcal{H} , whose vertices are precisely the cusps $\mathbb{Q} \cup \{\infty\}$. Recall that

$$\langle M \rangle = \langle MTS \rangle = \langle M(TS)^2 \rangle$$

but that otherwise the triangles are distinct. The triangle $\langle M \rangle$ has three (oriented) edges; these are the modular symbols (M) , (MTS) and $(M(TS)^2)$.

Assume, for simplicity, that G has no non-trivial elements of finite order, *i.e.*, no conjugates of either S or TS . (This assumption is merely for ease of exposition; in fact, it is easy to see that elliptic elements of G contribute nothing to the formula in Theorem 2.15.4 below in any case.) Choose, once and for all, a set \mathcal{S} of right coset representatives for G in Γ , such that $M \in \mathcal{S} \Rightarrow MTS \in \mathcal{S}$; this is possible since, by hypothesis, G contains no conjugates of TS .

Let \mathcal{S}' be a subset of \mathcal{S} which contains exactly one of each triple $M, MTS, M(TS)^2$, so that $\mathcal{S} = \mathcal{S}' \cup \mathcal{S}'TS \cup \mathcal{S}'(TS)^2$. Then a fundamental domain for the action of G on \mathcal{H} is given by

$$\mathcal{F}_G = \bigcup_{M \in \mathcal{S}'} \langle M \rangle.$$

In general, this set need not be connected, but this does not matter for our purposes: it can be treated as a disjoint union of triangles, whose total boundary is the sum of the oriented edges (M) for $M \in \mathcal{S}$.

The key idea in the algebraic reformulation of Zagier’s method is to make use of the coset action of Γ on the set \mathcal{S} . We now introduce notation for the actions of the generators S and T .

Action of S . For each $M \in \mathcal{S}$ we set $MS = s(M)\sigma(M)$, where $s: \mathcal{S} \rightarrow G$ is a function and $\sigma: \mathcal{S} \rightarrow \mathcal{S}$ is a permutation. Since S^2 is the identity, the same is true of σ , and $s(\sigma(M)) = s(M)^{-1}$. For brevity we will write $M^* = \sigma(M)$, so that $M^{**} = M$ for all $M \in \mathcal{S}$. (This conflicts with an earlier use of the notation M^* in Section 2.1, but this should not cause confusion.)

Note that the triangles $\langle M \rangle$ and $\langle MS \rangle$ are adjacent in the triangulation of \mathcal{H} , since they share the common side $(M) = \{M(0), M(\infty)\} = -(MS)$. However, since in general we do not have $MS \in \mathcal{S}$, in the fundamental domain \mathcal{F}_G for G it is the triangles $\langle M \rangle$ and $\langle M^* \rangle$ which are glued together by the element $s(M) \in G$ which takes (M^*) to $-(M)$ (the orientation is reversed).

Action of T . Similarly, for $M \in \mathcal{S}$ we set $MT = t(M)\tau(M)$ with $t(M) \in G$ and $\tau(M) \in \mathcal{S}$. The permutation τ of \mathcal{S} plays a vital part in what follows. The following lemma will not be used later, but is included for its own interest as it explains the geometric significance of this algebraic permutation.

LEMMA 2.15.2.

(a) *Two elements M and M' of \mathcal{S} are in the same τ -orbit if and only if the cusps $M(\infty)$ and $M'(\infty)$ are G -equivalent; hence the number of τ -orbits on \mathcal{S} is the number of G -equivalence classes of cusps.*

(b) *The length of the τ -orbit containing $M \in \mathcal{S}$ is the width of the cusp $M(\infty)$ of G .*

PROOF. (a) M and M' are in the same τ -orbit if and only if $M_0 = M'T^jM^{-1} \in G$ for some j , which is if and only if $M_0M(\infty) = M'(\infty)$, since the stabilizer of ∞ in Γ is the subgroup generated by T .

(b) The length of the orbit of M is the least $k > 0$ such that $MT^kM^{-1} = (MTM^{-1})^k \in G$, which is the width of the cusp $M(\infty)$, since the stabilizer of $M(\infty)$ in Γ is generated by MTM^{-1} . \square

Thus there is a one-one correspondence between the orbits of τ on \mathcal{S} and the classes of G -inequivalent cusps, with the length of each orbit being the width of the corresponding cusp.

In each τ -orbit in \mathcal{S} , we choose an arbitrary base-point M_1 , and set $M_{j+1} = \tau(M_j)$ for $1 \leq j \leq k$, where k is the length of the orbit and $M_{k+1} = M_1$. Thus $M_jT = t(M_j)M_{j+1}$, so that

$$M_1T^j = t(M_1)t(M_2)\dots t(M_j)M_{j+1}.$$

In particular, $M_1T^k = M_0M_1$, where $M_0 = t(M_1)t(M_2)\dots t(M_k) \in G$. Since M_0 is parabolic and P_f is a homomorphism, we obtain the following.

LEMMA 2.15.3.

$$\sum_{j=1}^k P_f(t(M_j)) = 0.$$

Write $M \prec M'$ if M and M' are in the same τ -orbit in \mathcal{S} , and M precedes M' in the fixed ordering determined by choosing a base-point for each orbit. In the notation above, $M \prec M'$ if and only if $M = M_i$ and $M' = M_j$ where $1 \leq i < j \leq k$.

We can now state the main results of this section.

THEOREM 2.15.4. *Let f be a cusp form of weight 2 for G with associated period function $P_f: G \rightarrow \mathbb{C}$. Then (the square of) the Petersson norm of f is given by*

$$\|f\|^2 = \frac{1}{8\pi^2} \sum_{M \prec M'} \operatorname{Im}(P_f(t(M))\overline{P_f(t(M'))}).$$

Here the sum is over all ordered pairs $M \prec M'$ in \mathcal{S} which are in the same orbit of the permutation τ of \mathcal{S} induced by right multiplication by T .

Combining this result with Proposition 2.15.1, we immediately obtain our explicit formula for the degree of the modular parametrization.

THEOREM 2.15.5. *With the above notation,*

$$\deg(\varphi) = \frac{1}{2\text{Vol}(E_f)} \sum_{M \prec M'} \text{Im}(\overline{P_f(t(M))} P_f(t(M'))) = \frac{1}{2} \sum_{M \prec M'} \begin{vmatrix} n_1(t(M)) & n_1(t(M')) \\ n_2(t(M)) & n_2(t(M')) \end{vmatrix}.$$

Hence to compute $\deg(\varphi)$, we only have to compute the right coset action of T on an explicit set \mathcal{S} of coset representatives for G in Γ , and evaluate the integer-valued functions n_1 and n_2 on each of the matrices $t(M)$ for $M \in \mathcal{S}$. In the case of $\Gamma_0(N)$, these steps can easily be carried out using M-symbols, and we will give some further details below.

REMARKS. 1. The formula given in Theorem 2.15.5 expresses $\deg(\varphi)$ explicitly as a sum which can be grouped as a sum of terms, one term for each cusp, by collecting together the terms for each τ -orbit. It is not at all clear what significance, if any, can be given to the individual contributions of each cusp to the total.

2. The form of our formula is identical to the one in [69]. However, we stress that in [69], the analogue of our coset action τ is defined not algebraically, as here, but geometrically, as a permutation of the edges of a fundamental polygonal domain for G (and dependent on the particular fundamental domain used). Then it becomes necessary to have an explicit picture of such a fundamental domain, including explicit matrices which identify the edges of the domain in pairs. This is only carried out explicitly in [69] in the case $G = \Gamma_0(N)$ where N is a prime. In our formulation, the details are all algebraic rather than geometric, which makes the evaluation of the formula more practical to implement. Also, we have the possibility of evaluating the functions n_1 and n_2 exactly using modular symbols, instead of using numerical evaluation of the periods, which reduces the computation of $\deg(\varphi)$ entirely to linear algebra and integer arithmetic.

2.15.3. Implementation for $\Gamma_0(N)$.

We now discuss the case $G = \Gamma_0(N)$ in greater detail, using M-symbols to represent the coset representatives. The right coset action of Γ on $P^1(N)$ is given by (2.2.4), so we have $\sigma(c : d) = (c : d)S = (d : -c)$ and $\tau(c : d) = (c : d)T = (c : c + d)$.

LEMMA 2.15.6. *The length of the τ -orbit of $(c : d)$ is $N/\gcd(N, c^2)$.*

PROOF. $\tau^k(c : d) = (c : d) \iff (c : kc + d) = (c : d) \iff cd \equiv c(kc + d) \pmod{N} \iff kc^2 \equiv 0 \pmod{N} \iff k \equiv 0 \pmod{N/\gcd(N, c^2)}$. \square

In earlier sections, it was immaterial exactly which coset representatives were used, or in practice which pair $(c, d) \in \mathbb{Z}^2$ was used to represent the M-symbol $(c : d)$. For the application of Theorem 2.15.5, however, we must ensure that our set is closed under right multiplication by TS , where $(c : d)TS = (c + d : -c)$, unless $(c : d)$ is fixed by TS , which is if and only if $c^2 + cd + d^2 \equiv 0 \pmod{N}$. Thus each M-symbol $(c : d)$ will be represented by a specific pair $(c, d) \in \mathbb{Z}^2$ with $\gcd(c, d) = 1$, in such a way that our set \mathcal{S} of representatives contains the pairs $(c + d, -c)$ and $(-d, c + d)$ whenever it contains (c, d) , unless $(c : d)$ is fixed by TS . (Even when working with pairs $(c, d) \in \mathbb{Z}^2$ we will identify (c, d) and $(-c, -d)$.)

Fixing these triples of pairs (c, d) corresponds to fixing the triangles $\langle M \rangle$ which form a (possibly disconnected) fundamental domain for $\Gamma_0(N)$. If $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the pair (c, d) corresponds to the directed edge $\{M(0), M(\infty)\} = \{b/d, a/c\}$. For this reason, we will refer

to the pairs (c, d) as edges, and the triples of pairs as triangles. Right multiplication by TS corresponds geometrically to moving round to the next edge of the triangle, while right multiplication by S corresponds to moving across to the next triangle $\langle M^* \rangle$ adjacent to the current one. The τ -action is given by composing these, taking $(c : d)$ (or edge $\{b/d, a/c\}$) to the symbol $(c : d)T = (c : c + d)$ with corresponding edge $\{(a + b)/(c + d), a/c\}$, up to translation by an element of $\Gamma_0(N)$. Note how in this operation the endpoint at the cusp $M(\infty) = a/c$ is fixed, in accordance with Lemma 2.15.2 above.

We may therefore proceed as follows. For each orbit, start with a standard pair (c, d) , chosen in an M-symbol class $(c : d)$ not yet handled. Apply T to obtain the pair $(c, c + d)$. If this pair is the standard representative for the class $(c : c + d)$, we need take no action and may continue with the orbit. But if $(c, c + d) \equiv (r, s)$, say, with $(r, s) \in \mathcal{S}$, then we must record the “gluing matrix”

$$M = \begin{pmatrix} a & a + b \\ c & c + d \end{pmatrix} \begin{pmatrix} p & q \\ r & s \end{pmatrix}^{-1} \in \Gamma_0(N),$$

where $ad - bc = ps - qr = 1$, whose period $P_f(M)$ will contribute to the partial sum for this orbit. When this happens, we say that the orbit has a “jump” at this point. Different choices for a, b, p and q only change M by parabolic elements, and so do not affect the period $P_f(M)$. We continue until we return to the starting pair, and then move to another orbit, until all M-symbols have been used. As checks on the computation we may use Lemmas 2.15.2 and 2.15.6: the length of the orbit starting at (c, d) can be precomputed as $N/\gcd(N, c^2)$, and the number of orbits is the number of $\Gamma_0(N)$ -inequivalent cusps.

A worked example for the case $N = 11$ is included in the appendix to this chapter.

ELLIPTIC CURVE ALGORITHMS

3.1 Terminology and notation

For reference in the following sections, we collect here the notation, terminology and formulae concerning elliptic curves which we will use throughout this chapter.

An elliptic curve E defined over \mathbb{Q} has an equation or *model* of the form

$$(3.1.1) \quad E: \quad y^2 + a_1xy + a_3y = x^3 + a_2x^2 + a_4x + a_6$$

where the coefficients $a_i \in \mathbb{Q}$. We call such an equation a *Weierstrass equation* for E , and denote this model by $[a_1, a_2, a_3, a_4, a_6]$. We say that (3.1.1) is *integral* or *defined over \mathbb{Z}* if all the a_i are in \mathbb{Z} . From these coefficients we derive the auxiliary quantities

$$\begin{aligned} b_2 &= a_1^2 + 4a_2, \\ b_4 &= a_1a_3 + 2a_4, \\ b_6 &= a_3^2 + 4a_6, \\ b_8 &= a_1^2a_6 - a_1a_3a_4 + 4a_2a_6 + a_2a_3^2 - a_4^2, \end{aligned}$$

the *invariants*

$$\begin{aligned} c_4 &= b_2^2 - 24b_4, \\ c_6 &= -b_2^3 + 36b_2b_4 - 216b_6, \end{aligned}$$

the *discriminant*

$$\Delta = -b_2^2b_8 - 8b_4^3 - 27b_6^2 + 9b_2b_4b_6,$$

and the *j -invariant*

$$j = c_4^3/\Delta,$$

which are related by the identities

$$4b_8 = b_2b_6 - b_4^2 \quad \text{and} \quad 1728\Delta = c_4^3 - c_6^2.$$

The discriminant Δ must be non-zero for the curve defined by equation (3.1.1) to be non-singular and hence an elliptic curve. The j -invariant is (as its name suggests) invariant under isomorphism; elliptic curves with the same j are called *twists*: they are isomorphic over an algebraic extension, but not necessarily over \mathbb{Q} . The invariants c_4 and c_6 are sufficient to determine E up to isomorphism (over \mathbb{Q}) since E is isomorphic to

$$Y^2 = X^3 - 27c_4X - 54c_6.$$

The most general isomorphism from E to a second curve E' given by an equation of the form (3.1.1), which we usually think of as a change of coordinates on E itself, is $T(r, s, t, u)$, given by

$$(3.1.2) \quad \begin{aligned} x &= u^2x' + r \\ y &= u^3y' + su^2x' + t \end{aligned}$$

where $r, s, t \in \mathbb{Q}$ and $u \in \mathbb{Q}^*$. The effect of $T(r, s, t, u)$ on the coefficients a_i is given by

$$(3.1.3) \quad \begin{aligned} ua'_1 &= a_1 + 2s \\ u^2 a'_2 &= a_2 - sa_1 + 3r - s^2 \\ u^3 a'_3 &= a_3 + ra_1 + 2t \\ u^4 a'_4 &= a_4 - sa_3 + 2ra_2 - (t + rs)a_1 + 3r^2 - 2st \\ u^6 a'_6 &= a_6 + ra_4 + r^2 a_2 + r^3 - ta_3 - t^2 - rta_1 \end{aligned}$$

so that

$$u^4 c'_4 = c_4, \quad u^6 c'_6 = c_6, \quad u^{12} \Delta' = \Delta \quad \text{and} \quad j' = j.$$

The transformations $T(0, 0, 0, u)$ we will refer to as scaling transformations; these have the effect of dividing each coefficient a_i by u^i , and similarly for each of the other quantities, according to its weight. Here a_i , b_i and c_i have weight i , while Δ has weight 12 and j has weight 0. By applying $T(0, 0, 0, u)$ for suitable u we can always transform to an integral model; all the invariants are then integral, except (possibly) for j . Among such integral models, those for which the positive integer $|\Delta|$ is minimal are called *global minimal models* for E . We will give in the next section a simple algorithm for finding such a model, given the invariants c_4 and c_6 of any model. Clearly, isomorphisms between minimal models must have $u = \pm 1$ and $r, s, t \in \mathbb{Z}$. We may normalize so that $a_1, a_3 \in \{0, 1\}$ and $a_2 \in \{-1, 0, 1\}$, by suitable choice of s, r and t (in that order), as may be seen from (3.1.3). Such an equation will be called *reduced*, and it is not hard to show that it is unique: the only transformation other than the identity $T(0, 0, 0, 1)$ from a reduced model to any another reduced model is the transformation $T(0, -a_1, -a_3, -1)$, which takes any model to itself; this is just the negation map $(x, y) \mapsto (x, y - a_1 x - a_3)$ from the curve to itself. Thus every elliptic curve E defined over \mathbb{Q} has a *unique* reduced minimal model. This fact makes it very easy to recognize curves: in Table 1 we give the coefficients of such a model for each of the curves there.

Given integers c_4 and c_6 , two questions arise: is there a curve over \mathbb{Q} with these invariants, and is it minimal? Clearly we must have $c_4^3 - c_6^2 = 1728\Delta$ with $\Delta \neq 0$. A solution to the first problem is given by Kraus in Proposition 2 of [30], which states the following.

PROPOSITION 3.1.1. *Let c_4, c_6 be integers such that $\Delta = (c_4^3 - c_6^2)/1728$ is a non-zero integer. In order for there to exist an elliptic curve E with a model (3.1.1) defined over \mathbb{Z} having invariants c_4 and c_6 , it is necessary and sufficient that*

- (1) $c_6 \not\equiv \pm 9 \pmod{27}$;
- (2) *either* $c_6 \equiv -1 \pmod{4}$, *or* $c_4 \equiv 0 \pmod{16}$ *and* $c_6 \equiv 0, 8 \pmod{32}$.

The conditions of Proposition 3.1.1 will be referred to as *Kraus's conditions*. If we are given integers c_4 and c_6 satisfying these conditions, we can recover the coefficients a_i of the reduced model of the curve with c_4 and c_6 as invariants, using the formulae already given in Chapter 2, Section 14, which we repeat here for convenience:

$$\begin{aligned} b_2 &= -c_6 \pmod{12} \in \{-5, \dots, 6\}; \\ b_4 &= (b_2^2 - c_4)/24; \\ b_6 &= (-b_2^3 + 36b_2 b_4 - c_6)/216; \\ a_1 &= b_2 \pmod{2} \in \{0, 1\}; \\ a_3 &= b_6 \pmod{2} \in \{0, 1\}; \\ a_2 &= (b_2 - a_1)/4; \\ a_4 &= (b_4 - a_1 a_3)/2; \\ a_6 &= (b_6 - a_3)/4. \end{aligned}$$

To see this, we may assume that we are seeking coefficients of a reduced model; then $b_2 \in \{-4, -3, 0, 1, 4, 5\}$, and we have $-c_6 \equiv b_2^3 \equiv b_2 \pmod{12}$. The rest is easy; provided that c_4 and c_6 satisfy Kraus's conditions, all the divisions will be exact.

In the following section we answer the second question by giving an algorithm for computing the reduced coefficients of a *minimal* model for any curve E , given either integral invariants satisfying Kraus's conditions, or any integral model for E . We simply determine the maximal integer u such that $c'_4 = c_4/u^4$ and $c'_6 = c_6/u^6$ satisfy Kraus's conditions, and then compute the reduced coefficients a'_i from these. As with many questions concerning elliptic curves, most of the work goes into determining the powers of 2 and 3 which divide u .

We will assume without further discussion that on any given curve E , points may be added and multiples taken, using standard formulae. The Mordell–Weil group of all rational points on E will be denoted $E(\mathbb{Q})$ as usual. If n is a positive integer, we denote by $E(\mathbb{Q})[n]$ the subgroup of rational points of order dividing n , which is the kernel of the multiplication map from E to itself.

3.2 The Kraus–Laska–Connell algorithm and Tate's algorithm

In this section we give two algorithms. The first was originally given by Laska in [34], and finds a minimal model for a curve E , starting from an integral equation. Essentially the algorithm was to test all positive integers u such that $u^{-4}c_4$ and $u^{-6}c_6$ are integral, to see if they are the invariants of a curve defined over \mathbb{Z} . Using Kraus's conditions (see Proposition 3.1.1 above), this procedure can be simplified, since it is possible to compute in advance the exponent d_p of each prime p in the minimal discriminant, and hence compute u at the start. The usual formulae then give the coefficients a_i of the reduced model. Our formulation of the resulting algorithm over \mathbb{Z} is similar to that given in [10], where more general rings are considered: in particular an explicit algorithm is given there for finding local minimal models over arbitrary number fields, and hence global minimal models where they exist. Over \mathbb{Z} , the algorithm is extremely simple.

In the pseudocode below,

`ord(p,n)` gives the power of the prime p which divides the non-zero integer n ;

`floor(x)` gives the integral part of the real number x ;

`a mod p` gives the residue of a modulo p lying in the range $-\frac{1}{2}p < a \leq \frac{1}{2}p$; in particular, when $p = 2$ or 3 this gives a residue in $\{0, 1\}$ or $\{-1, 0, 1\}$ respectively. Also `inv(a,p)` gives the inverse of a modulo p , assuming that $\gcd(a,p)=1$.

The Laska–Kraus–Connell Algorithm

INPUT: c_4, c_6 (integer invariants of an elliptic curve E).

OUTPUT: a_1, a_2, a_3, a_4, a_6 (coefficients of a reduced minimal model for E).

```

1. BEGIN
2.  $\Delta = (c_4^3 - c_6^2) / 1728$ ;
(Compute scaling factor u)
3.  $u = 1$ ;  $g = \gcd(c_6^2, \Delta)$ ;
4. p_list = prime_divisors(g);
5. FOR p IN p_list DO
6. BEGIN
7.    $d = \text{floor}(\text{ord}(p, g) / 12)$ ;
8.   IF p=2 THEN
9.      $a = c_4 / 2^{(4*d)} \pmod{16}$ ;  $b = c_6 / 2^{(6*d)} \pmod{32}$ ;
10.    IF  $(b \pmod{4} \neq -1)$  AND NOT  $(a=0$  AND  $(b=0$  OR  $b=8))$ 

```

```

11.         THEN d = d-1
12.         FI
13.     ELIF p=3 THEN IF ord(3,c6)=6*d+2 THEN d = d-1 FI
14.     FI;
15.     u = u*pd
16. END;
(Compute minimal equation)
17. c4 = c4/u4; c6 = c6/u6;
18. b2 = -c6 mod 12; b4 = (b22-c4)/24; b6 = (-b23+36*b2*b4-c6)/216;
19. a1 = b2 mod 2;
20. a3 = b6 mod 2;
21. a2 = (b2-a1)/4;
22. a4 = (b4-a1*a3)/2;
23. a6 = (b6-a3)/4
24. END

```

Next we turn to Tate’s algorithm itself. The standard reference for this is Tate’s ‘letter to Cassels’ [65], which appeared in the Antwerp IV volume [2]. There is also a full account in the second volume of Silverman’s book [61, Section IV.9]. It may be applied to an integral model of a curve E and a prime p , to give the following data:

- The exponent f_p of p in the conductor N of E (see below);
- the Kodaira symbol of E at p , which classifies the type of reduction of E at p (see [47] or [61, Section IV.9]); these are: I_0 for good reduction; I_n ($n > 0$) for bad multiplicative reduction; and types I_n^* , II, III, IV, II^* , III^* and IV^* for bad additive reduction.
- the local index $c_p = [E(\mathbb{Q}_p) : E^0(\mathbb{Q}_p)]$, where $E^0(\mathbb{Q}_p)$ is the subgroup of the group $E(\mathbb{Q}_p)$ of p -adic points of E , consisting of those points whose reduction modulo p is non-singular. (That this index is finite is implied by the correctness of the algorithm, as observed by Tate in [65].)

In addition, the algorithm detects whether the given model is non-minimal at p , and if so, returns a model which is minimal at p . Thus by applying it in succession with all the primes dividing the discriminant of the original model, one can compute a minimal model at the same time as computing the conductor and the other local reduction data. In practice this makes the Laska–Kraus–Connell algorithm redundant, though much simpler to implement and use if all one needs is the standard model for a curve E .

The conductor N of an elliptic curve E defined over \mathbb{Q} is defined to be

$$N = \prod_p p^{f_p}$$

where $f_p = \text{ord}_p(\Delta) + 1 - n_p$ and n_p is the number of irreducible components on the special fibre of the minimal Néron model of E at p . This Néron model is a more sophisticated object than we wish to discuss here (see [47] or [61] for details): one has to consider E as a scheme over $\text{Spec}(\mathbb{Z}_p)$, and then resolve the singularity at p , to obtain a scheme whose generic fibre is E/\mathbb{Q}_p and whose special fibre is a union of curves over $\mathbb{Z}/p\mathbb{Z}$. In terms of a minimal model for E over \mathbb{Z} , all may be computed very simply except when $p = 2$ or $p = 3$ as follows:

- $f_p = 0$ if $p \nmid \Delta$;
- $f_p = 1$ if $p \mid \Delta$ and $p \nmid c_4$ (then $n_p = \text{ord}_p(\Delta)$);
- $f_p \geq 2$ if $p \mid \Delta$ and $p \mid c_4$; moreover, $f_p = 2$ in this case when $p \neq 2, 3$.

To obtain the value of f_p in the remaining cases, and to obtain the Kodaira symbol and the local index c_p , we use Tate’s algorithm itself.

In [65], the algorithm is given for curves defined over an arbitrary discrete valuation ring. To apply it to a curve defined over the ring of integers R of a number field K at a prime ideal \mathfrak{p} , one would in general have to work in the localization of R at \mathfrak{p} ; here we can work entirely over \mathbb{Z} , since \mathbb{Z} is a principal ideal domain. We have added to the presentation in [65] the explicit coordinate transformations $T(r, s, t, u)$ which are required during the course of the algorithm to achieve divisibility of the coefficients a_i by various power of p . In practice one would ignore the transformations which had taken place while processing each p , unless a scaling by p had taken place on discovering that the model was non-minimal. The most complicated part of the algorithm is the branch for reduction type I_m^* , where one successively refines the model p -adically until certain auxiliary quadratics have distinct roots modulo p . This requires careful book-keeping. The presentation given here closely follows our own implementation of the algorithm, which in turn owes much to an earlier Fortran program written by Pinch. The following sub-procedures are used:

`compute_invariants` computes the b_i , c_i and Δ from the coefficients a_i . Note that c_4 , c_6 and Δ do not change unless a scaling is required, since all other transformations have $u = 1$.

`transcoord(r,s,t,u)` applies the coordinate transformation formulae of the previous section to obtain new values for the a_i and other quantities. All calls to this procedure have $u = 1$ except when rescaling a non-minimal equation. In each case we first compute suitable values of r , s and t ; usually this requires a separate branch if $p = 2$ or $p = 3$.

`quadroots(a,b,c,p)` returns TRUE if the quadratic congruence $ax^2 + bx + c \equiv 0 \pmod{p}$ has a solution, and FALSE otherwise. This is used in determining the value of the index c_p .

`nrootscubic(b,c,d,p)` returns the number of roots of the cubic congruence $x^3 + bx^2 + cx + d \equiv 0 \pmod{p}$.

Tate's Algorithm

INPUT: a1, a2, a3, a4, a6 (integer coefficients of E); p (prime).

OUTPUT: Kp (Kodaira symbol)
 fp (Exponent of p in conductor)
 cp (Local index)

1. BEGIN
2. `compute_invariants(b2,b4,b6,b8,c4,c6,Δ)`;
3. `n = ord(p,Δ)`;

(Test for type I_0)

4. IF `n=0` THEN `Kp = "I0"`; `fp = 0`; `cp = 1`; EXIT FI;

(Change coordinates so that $p \mid a_3, a_4, a_6$)

5. IF `p=2` THEN
6. IF `p|b2`
7. THEN `r = a4 mod p`; `t = r*(1+a2+a4)+a6 mod p`
8. ELSE `r = a3 mod p`; `t = r+a4 mod p`
9. FI
10. ELIF `p=3` THEN
11. IF `p|b2` THEN `r = -b6 mod p` ELSE `r = -b2*b4 mod p` FI;
12. `t = a1*r+a3 mod p`
13. ELSE
14. IF `p|c4` THEN `r = -inv(12,p)*b2` ELSE `r = -inv(12*c4,p)*(c6+b2*c4)` FI;
15. `t = -inv(2,p)*(a1*r+a3)`;
16. `r = r mod p`; `t = t mod p`
17. FI;

```

18. transcoord(r,0,t,1);
(Test for types In, II, III, IV)
19. IF p|c4 THEN
20.     IF quadroots(1,a1,-a2,p) THEN cp = n ELIF 2|n THEN cp = 2 ELSE cp = 1 FI;
21.     Kp = "In"; fp = 1; EXIT
22. FI;
23. IF p2|a6 THEN Kp = "II"; fp = n; cp = 1; EXIT;
24. IF p3|b8 THEN Kp = "III"; fp = n-1; cp = 2; EXIT;
25. IF p3|b6 THEN
26.     IF quadroots(1,a3/p,-a6/p2,p) THEN cp = 3 ELSE cp = 1 FI;
27.     Kp = "IV"; fp = n-2; EXIT
28. FI;
(Change coordinates so that p | a1, a2; p2 | a3, a4; p3 | a6)
29. IF p=2
30. THEN s = a2 mod 2; t = 2*(a6/4 mod 2)
31. ELSE s = -a1*inv(2,p); t = -a3*inv(2,p)
32. FI;
33. transcoord(0,s,t,1);
(Set up auxiliary cubic T3 + bT2 + cT + d)
34. b = a2/p; c = a4/p2; d = a6/p3;
35. w = 27*d2-b2*c2+4*b3*d-18*b*c*d+4*c3;
36. x = 3*c-b2;
(Test for distinct roots: type I0*)
37. IF p|w THEN Kp = "I*0"; fp = n-4; cp = 1+nrootscubic(b,c,d,p); EXIT
(Test for double root: type Im*)
38. ELIF p|x THEN
(Change coordinates so that the double root is T ≡ 0)
39.     IF p=2 THEN r = c ELIF p=3 THEN r = b*c ELSE r = (b*c-9*d)*inv(2*x,p) FI;
40.     r = p*(r mod p);
41.     transcoord(r,0,0,1);
(Make a3, a4, a6 repeatedly more divisible by p)
42.     m = 1; mx = p2; my = p2; cp = 0;
43.     WHILE cp=0 DO
44.     BEGIN
45.         xa2 = a2/p; xa3 = a3/my; xa4 = a4/(p*mx); xa6 = a6/(mx*my);
46.         IF p|(xa32+4*xa6) THEN
47.             IF quadroots(1,xa3,-xa6,p) THEN cp = 4 ELSE cp = 2 FI
48.         ELSE
49.             IF p=2 THEN t = my*xa6 ELSE t = my*((-xa3*inv(2,p)) mod p) FI;
50.             transcoord(0,0,t,1);
51.             my = my*p; m = m+1;
52.             xa2 = a2/p; xa3 = a3/my; xa4 = a4/(p*mx); xa6 = a6/(mx*my);
53.             IF p|(xa42-4*xa2*xa6) THEN
54.                 IF quadroots(xa2,xa4,xa6,p) THEN cp = 4 ELSE cp = 2 FI
55.             ELSE
56.                 IF p=2 THEN r = mx*(xa6*xa2 mod 2)

```

```

57.             ELSE r = mx*(-xa4*inv(2*xa2,p) mod p)
58.             FI;
59.             transcoord(r,0,0,1);
60.             mx = mx*p; m = m+1
61.             FI
62.             FI
63.             END;
64.             fp = n-m-4; Kp = "I*m"; EXIT
65. ELSE
(Triple root case: types II*, III*, IV* or non-minimal)
(Change coordinates so that the triple root is  $T \equiv 0$ )
66.             IF p=3 THEN rp = -d ELSE rp = -b*inv(3,p) FI;
67.             r = p*(rp mod p);
68.             transcoord(r,0,0,1);
69.             x3 = a3/p2; x6 = a6/p4;
(Test for type IV*)
70.             IF p|(x32+4*x6) THEN
71.                 IF quadroots(1,x3,-x6,p) THEN cp = 3 ELSE cp = 1 FI;
72.                 Kp = "IV*"; fp = n-6; EXIT
73.             ELSE
(Change coordinates so that  $p^3 | a_3, p^5 | a_6$ )
74.                 IF p=2 THEN t = x6 ELSE t = x3*inv(2,p) FI;
75.                 t = -p2*(t mod p);
76.                 transcoord(0,0,t,1);
(Test for types III*, II*)
77.                 IF p4∤a4 THEN Kp = "III*"; fp = n-7; cp = 2; EXIT
78.                 ELIF p6∤a6 THEN Kp = "II*"; fp = n-8; cp = 1; EXIT
79.                 ELSE
(Equation non-minimal: divide each  $a_i$  by  $p^i$  and start again)
80.                 transcoord(0,0,0,p); restart
81.                 FI
82.             FI
83.             END

```

In Table 1 we will give the local reduction data for each curve at each ‘bad’ prime (dividing the discriminant of the minimal model). We also give the factorization of the minimal discriminant and of the denominator of j , as in the earlier tables. To save space we omit the c_4 and c_6 invariants, which are easily computable from the coefficients a_i .

3.3 Computing the Mordell–Weil group I: finding torsion points

In this and the next three sections we will discuss the question of determining the Mordell–Weil group $E(\mathbb{Q})$ of rational points on an elliptic curve E defined over \mathbb{Q} . This group is finitely generated, by Mordell’s Theorem, and hence has the structure

$$E(\mathbb{Q}) = T \times F$$

where T is the finite torsion subgroup $E(\mathbb{Q})_{\text{tors}}$ of $E(\mathbb{Q})$ consisting of the points of finite order, and F is free abelian of some rank $r \geq 0$:

$$F \cong \mathbb{Z}^r.$$

The problem of computing $E(\mathbb{Q})$ thus subdivides into several parts:

- computing the torsion T ;
- computing the rank r ;
- finding r independent points of infinite order;
- computing a \mathbb{Z} -basis for the free part F .

A related task is to compute the regulator $R(E(\mathbb{Q}))$ (defined below); for this and for the latter two steps we will also need to compute the canonical height $\hat{h}(P)$ of points $P \in E(\mathbb{Q})$, and hence the height pairing $\hat{h}(P, Q)$.

In this section we will treat the easiest of these problems, that of finding the torsion points. In fact, these can be found as a byproduct of the more general search for points on the curve, since their naive height can be bounded (see the remark before Lemma 3.5.2). However, it is also useful to have a self-contained method for determining the torsion.

Using the fact that $E(\mathbb{R})$ is isomorphic either to the circle group S^1 (when $\Delta < 0$) or to $S^1 \times C_2$ (when $\Delta > 0$), where C_k denotes a cyclic group of order k , together with the fact that all finite subgroups of S^1 are cyclic, we see that T is isomorphic either to C_k or to $C_{2k} \times C_2$ for some $k \geq 1$, the latter only being possible when Δ is positive. The number of possible values of k is finite: by a theorem of Mazur [39],[40], a complete list of possible structures of T is

$$\begin{array}{ll} C_k & \text{for } 1 \leq k \leq 10 \text{ or } k = 12; \\ C_{2k} \times C_2 & \text{for } 1 \leq k \leq 4. \end{array}$$

To determine the torsion subgroup of an elliptic curve defined over \mathbb{Q} , we may use a form of the Lutz–Nagell Theorem. (The situation is more complicated over number fields other than \mathbb{Q} , on account of the ramified primes.) The first step is to find a model for the curve in which all torsion points are integral. For this it suffices to complete the square (if necessary) to eliminate the xy and y terms, at the expense of a scaling by $u = 2$. Then for $P = (x, y)$ a torsion point, we can use the fact that both P and $2P$ are integral to bound y . For the first step, the following result may be found in [33, Section III.1] and [28, Theorem 5.1]. The original form of this result, due independently to Lutz [36] and Nagell [46], was for curves of the form $y^2 = x^3 + ax + b$, with no x^2 term. While such an equation may be obtained by completing the cube, this would involve a further scaling of coordinates, and so would lead to larger numbers. If $a_1 = a_3 = 0$ we can apply the following result directly; otherwise, put $a = b_2$, $b = 8b_4$ and $c = 16b_6$.

PROPOSITION 3.3.1. *Let E be an elliptic curve defined over \mathbb{Q} , given by an equation*

$$(3.3.1) \quad y^2 = f(x) = x^3 + ax^2 + bx + c$$

where $a, b, c \in \mathbb{Z}$. If $P = (x, y) \in E(\mathbb{Q})$ has finite order, then $x, y \in \mathbb{Z}$.

Next we bound the y coordinate of a torsion point $P = (x, y)$ (see [33, Theorem 1.4]).

PROPOSITION 3.3.2. *Let E be as in (3.3.1). If $P = (x_1, y_1)$ has finite order in $E(\mathbb{Q})$ then either $y_1 = 0$ or $y_1^2 \mid \Delta_0$, where*

$$\Delta_0 = 27c^2 + 4a^3c + 4b^3 - a^2b^2 - 18abc.$$

PROOF. If $2P = 0$ then $y_1 = 0$, since $-P = (x_1, -y_1)$. Otherwise $2P = (x_2, y_2)$ with $x_2, y_2 \in \mathbb{Z}$ by Proposition 3.3.1. Using the addition formula on E we find that $2x_1 + x_2 = m^2 - a$ where $m = f'(x_1)/2y_1$ is the slope of the tangent to E at P . Hence $m \in \mathbb{Z}$, so that $y_1 \mid f'(x_1)$. Using $y_1^2 = f(x_1)$, this implies that $y_1^2 \mid \Delta_0$, since

$$\Delta_0 = (-27f(x) + 54c + 4a^3 - 18ab)f(x) + (f'(x) + 3b - a^2)f'(x)^2. \quad \square$$

This gives us a finite number of values of y to check; for each, we attempt to solve the cubic for $x \in \mathbb{Z}$, to obtain all torsion points on E . Note that we are actually determining all points P such that both P and $2P$ are integral (in the possibly scaled model for E), which includes all torsion points, but may also include points of infinite order. To determine whether a given integral point has finite or infinite order, we simply compute multiples mP successively until either $mP = 0$, in which case P has order m , or mP is not integral, in which case P has infinite order. This does not take long, as the maximum possible order for a torsion point is 12 by Mazur's theorem. If we find points of infinite order at this stage we keep a note of them for later use (see Section 3.5).

The quantity Δ_0 is related to the discriminant Δ of the curve (3.3.1) by $\Delta = -16\Delta_0$. If this is large, there may be many values of y_0 to check when we apply the preceding Proposition to determine the torsion on a given curve. It is possible to save time by using a further result, which states that for an odd prime p of good reduction (that is, $p \nmid 2\Delta$), the reduction map from $E(\mathbb{Q})_{\text{tors}}$ to $E(\mathbb{Z}/p\mathbb{Z})$ is injective. For more details, and worked examples, see either [58, Section VIII.7] or [28, Section V.1].

If we want to know the structure of T and not just its order, note that from Mazur's theorem the only ambiguous cases are when T has order $4k = 4, 8$ or 12 and $\Delta > 0$; we can always tell apart the groups C_{4k} and $C_2 \times C_{2k}$ as the former has only one element of order 2 while the latter has three, and this number is the number of rational (integer) roots of $f(x)$.

To solve the cubic equations $f(x) = y^2$ for x , given y , we use the classical formula of Cardano (see any algebra textbook) to find the complex roots (which we also need in computing the periods in section 3.7 below), and if any of these are real and close to integers we check them using exact integer arithmetic. Testing all divisors of the constant term can be too time-consuming, as it involves factorization of the numbers $y^2 - c$ which may be very large.

Here is the algorithm in pseudocode; for simplicity we only give it for curves with no xy or y terms; in the general case, one works internally with points on a scaled model (including the calculation of the order), converting back to the original model on output. Since we know in advance that no point will have order greater than 12, when computing the order of a point we simply use repeated addition until we reach a non-integral point or the identity 0 . The subroutine `order(P)` returns 0 for a point of infinite order. Also: `square_part(Δ)` returns the largest integer whose square divides Δ ; `integer_roots` returns a list of the integer roots of a cubic with integral coefficients; and `integral(x)` tests whether its (rational) argument is integral.

Algorithm for finding all torsion points

INPUT: `a,b,c` (integer coefficients of a nonsingular cubic).
 OUTPUT: A list of all torsion points on $y^2 = x^3 + ax^2 + bx + c$, with orders.

1. BEGIN
2. $\Delta = 27*c^2 + 4*a^3*c + 4*b^3 - a^2*b^2 - 18*a*b*c$;
3. `y_list = positive_divisors(square_part(Δ)) \cup {0}`;
4. FOR `y` IN `y_list` DO
5. BEGIN

```

6.      x_list=integer_roots(x3+a*x2+b*x+c-y2);
7.      FOR x IN x_list DO
8.      BEGIN
9.          P=point(x,y);
10.         n=order(P);
11.         IF n>0 THEN OUTPUT P,n FI
12.     END
13. END
14. END

```

(Subroutine to compute order of a point)

```

SUBROUTINE order(P)
1.  BEGIN
2.  n=1; Q=P;
3.  WHILE integral(x(Q)) AND Q≠0 DO
4.  BEGIN
5.      n = n+1; Q = Q+P
6.  END;
7.  IF Q≠0 THEN n=0 FI;
8.  RETURN n
9.  END

```

3.4 Heights and the height pairing

In this section we will show how to compute the canonical height $\hat{h}(P)$ of a point $P \in E(\mathbb{Q})$, and hence the height pairing

$$\hat{h}(P, Q) = \frac{1}{2}(\hat{h}(P + Q) - \hat{h}(P) - \hat{h}(Q)).$$

We will use this in the following section to find dependence relations among finite sets of points of infinite order, when we are computing a \mathbb{Z} -basis $\{P_1, \dots, P_r\}$ for the free abelian group $E(\mathbb{Q})/T$. Also, the regulator $R(E)$ is given by the determinant of the height pairing matrix:

$$R(E) = \left| \det(\hat{h}(P_i, P_j)) \right|.$$

The canonical height \hat{h} is a real-valued quadratic form on $E(\mathbb{Q})$. It differs by a bounded amount (with a bound dependent on E but not on the point P) from the naive or Weil height $h(P)$. For a point $P = (x, y) = (a/c^2, b/c^3) \in E(\mathbb{Q})$ with $a, b, c \in \mathbb{Z}$ and $\gcd(a, c) = 1 = \gcd(b, c)$, the latter is defined to be

$$h(P) = \log \max\{|a|, c^2\}.$$

Now the canonical height may be defined as $\hat{h}(P) = \lim_{n \rightarrow \infty} 4^{-n} h(2^n P)$, but this is not practical for computational purposes. For the theory of heights on elliptic curves, see [58, Chapter VIII]. Later (in the next section) we will need an explicit bound on the difference between $\hat{h}(P)$ and $h(P)$.

The height algorithms in this section are taken from Silverman's paper [59]. The *global height* $\hat{h}(P)$ is defined as a sum of *local heights*:

$$(3.4.1) \quad \hat{h}(P) = \sum_{p \leq \infty} \hat{h}_p(P).$$

Here the sum is over all finite primes p and the ‘infinite prime’ ∞ coming from the real embedding of \mathbb{Q} . (Over a general number field, there would in general be several of these infinite primes, including complex ones, and the local heights need to be multiplied by certain multiplicities: see [59]).

A remark about normalization¹: the canonical height must be suitably normalized. In the literature there are two normalizations used, one of which is double the other and is the one appropriate for the Birch–Swinnerton-Dyer conjecture (resulting in a regulator 2^r times as large). In Silverman’s paper he uses the other (smaller) normalization. Thus all the formulae here are double those in the paper [59].

The following proposition, which is Theorem 5.2 of [59] (for curves over general number fields) specialized to the case of a curve defined over \mathbb{Q} , also applies to a curve defined over \mathbb{Q}_p and to a point $P = (x, y) \in E(\mathbb{Q}_p)$. In the proposition, we refer to the functions ψ_2 and ψ_3 defined on E by

$$\psi_2(P) = 2y + a_1x + a_3, \quad \text{and} \quad \psi_3(P) = 3x^4 + b_2x^3 + 3b_4x^2 + 3b_6x + b_8;$$

thus, ψ_2 vanishes at the 2-torsion points of E and ψ_3 at the 3-torsion.

PROPOSITION 3.4.1. *Let E be an elliptic curve defined over \mathbb{Q} given by a standard Weierstrass equation (3.1.1) which is minimal at p , and let $P = (x, y) \in E(\mathbb{Q})$.*

(a) *If*

$$\text{ord}_p(3x^2 + 2a_2x + a_4 - a_1y) \leq 0 \quad \text{or} \quad \text{ord}_p(2y + a_1x + a_3) \leq 0$$

then

$$\hat{h}_p(P) = \max\{0, -\text{ord}_p(x)\} \log p.$$

(b) *Otherwise, if $\text{ord}_p(c_4) = 0$ then set $N = \text{ord}_p(\Delta)$ and $M = \min\{\text{ord}_p(\psi_2(P), \frac{1}{2}N\}$; then*

$$\hat{h}_p(P) = \frac{M(M - N)}{N} \log p.$$

(c) *Otherwise, if $\text{ord}_p(\psi_3(P)) \geq 3\text{ord}_p(\psi_2(P))$ then*

$$\hat{h}_p(P) = -\frac{2}{3}\text{ord}_p(\psi_2(P)) \log p.$$

(d) *Otherwise*

$$\hat{h}_p(P) = -\frac{1}{4}\text{ord}_p(\psi_3(P)) \log p.$$

The first case in Proposition 3.4.1 covers primes p where the point P has good reduction (including all primes where E has good reduction, as well as those where the reduced curve is singular but P does not reduce to the singular point). In the other three cases, P has singular reduction, and the reduction of E at p is multiplicative, additive of types IV or IV*, and additive of types III, III* and I_m* respectively.

Hence for each point P , the local height $\hat{h}_p(P) = 0$ if p divides neither the discriminant Δ nor c , where c^2 is the denominator of the x -coordinate of the point P . In all cases, $\hat{h}_p(P)$ is a rational multiple of $\log(p)$. The total contribution from the primes dividing c in the global height $\hat{h}(P)$ is therefore (from case (a) of the Proposition) simply $2 \log(c)$, and we have

¹I am grateful to Gross for explaining this to me, after I found that apparently the two sides of the Birch–Swinnerton-Dyer conjecture disagreed by a factor of 2^r !

the following formula, better for practical computation than (3.4.1) since we do not have to factorize c :

$$(3.4.2) \quad \hat{h}(P) = \hat{h}_\infty(P) + 2 \log(c) + \sum_{p|\Delta, p \nmid c} \hat{h}_p(P).$$

This formula appears in [62], where it is shown how to compute $\hat{h}(P)$ using little (or no) factorization of Δ , which can be useful in certain situations. We refer the reader to [62] for details.

An algorithm for computing the local height at a finite prime p is given by the following:

Silverman's algorithm for computing local heights: finite primes

INPUT: a1, a2, a3, a4, a6 (integer coefficients of a minimal model for E).
 x,y (rational coordinates of a point P on E).
 p (a prime).

OUTPUT: the local height of P at p .

1. BEGIN
 2. compute_invariants(b2,b4,b6,b8,c4, Δ);
 3. N = ord(p, Δ);
 4. A = ord(p,3*x²+2*a2*x+a4-a1*y);
 5. B = ord(p,2*y+a1*x+a3);
 6. C = ord(p,3*x⁴+b2*x³+3*b4*x²+3*b6*x+b8);
 7. M = min(B,N/2);
 8. IF A \leq 0 OR B \leq 0 THEN L = max(0,-ord(p,x))
 9. ELSE IF ord(p,c4)=0 THEN L = M*(M-N)/N
 10. ELSE IF C \geq 3*B THEN L = -2*B/3
 11. ELSE L = -C/4
 12. FI;
 13. RETURN L*log(p)
 14. END
-

We must also compute the local component of the height at the infinite prime, $\hat{h}_\infty(P)$. The method here originated with Tate, but was amended by Silverman in [59] to improve convergence, and to apply also to complex valuations. Tate in [66] expressed $\hat{h}_\infty(P)$ as a series

$$\hat{h}_\infty(P) = \log|x| + \frac{1}{4} \sum_{n=0}^{\infty} 4^{-n} c_n$$

where the coefficients c_n are bounded provided that no point on $E(\mathbb{R})$ has x -coordinate zero. Of course, over \mathbb{R} one can shift coordinates to ensure that this condition holds, but the resulting series can have poor convergence properties, and this trick will not work over \mathbb{C} . Silverman's solution is to use alternately the parameters x and $x' = x + 1$, switching between them (and between the two associated series c_n and c'_n) whenever $|x|$ or $|x'|$ becomes small (less than $1/2$). The series of coefficients c_n is obtained by repeated doubling of the point P , working with $t = 1/x$ or $t' = 1/x'$ as local parameter. The result is a new series of the above type in which the error in truncating before the N th term is $O(4^{-N})$, with an explicit constant. In fact (see [59, Theorem 4.2]) the error is less than $\frac{1}{2}10^{-d}$, giving a result correct to d decimal places, if

$$N \geq \frac{5}{3}d + \frac{1}{2} + \frac{3}{4} \log(7 + \frac{4}{3} \log H + \frac{1}{3} \log \max\{1, |\Delta|^{-1}\})$$

where

$$H = \max\{4, |b_2|, 2|b_4|, 2|b_6|, |b_8|\}.$$

The last term vanishes for curves defined over \mathbb{Z} , since then we have $|\Delta| > 1$.

In the algorithm which we now give, the quantities b_2' , b_4' , b_6' and b_8' are those associated with the shifted model of E with $x' = x + 1$; the switching flag `beta` indicates which model we are currently working on; `mu` holds the current partial sum; `f` holds the negative power of 4.

Silverman's algorithm for computing local heights: real component

INPUT: `a1, a2, a3, a4, a6` (integer coefficients of a minimal model for E).
`x` (x-coordinate of a point P on E).
`d` (number of decimal places required).
OUTPUT: the real local height of P .

```

1. BEGIN
2. compute_invariants(b2,b4,b6,b8);
3. H = max(4,|b2|,2*|b4|,2*|b6|,|b8|);
4. b2' = b2-12; b4' = b4-b2+6; b6' = b6-2*b4+b2-4; b8' = b8-3*b6+3*b4-b2+3;
5. N = ceiling((5/3)*d + (1/2) + (3/4)*log(7+(4/3)*log(H)));
6. IF |x|<0.5 THEN t = 1/(x+1); beta = 0 ELSE t = 1/x; beta = 1 FI;
7. mu = -log|t|; f = 1;
8. FOR n = 0 TO N DO
9. BEGIN
10. f = f/4;
11. IF beta=1 THEN
12. w = b6*t4+2*b4*t3+b2*t2+4*t;
13. z = 1-b4*t2-2*b6*t3-b8*t4;
14. zw = z+w
15. ELSE
16. w = b6'*t4+2*b4'*t3+b2'*t2+4*t;
17. z = 1-b4'*t2-2*b6'*t3-b8'*t4;
18. zw = z-w
19. FI;
20. IF |w| ≤ 2*|z|
21. THEN mu = mu+f*log|z|; t = w/z
22. ELSE mu = mu+f*log|zw|; t = w/zw; beta = 1-beta
23. FI
24. END;
25. RETURN mu
26. END

```

Finally, to compute the global height $\hat{h}(P)$, we simply add to the infinite local height $\hat{h}_\infty(P)$ the finite local heights $\hat{h}_p(P)$ for all primes p dividing either Δ or the denominator of $x(P)$. Using (3.4.2) this leads to the following algorithm.

Algorithm for computing global canonical heights

INPUT: a_1, a_2, a_3, a_4, a_6 (integer coefficients of a minimal model for E).
 $P=(x,y)$ (a rational point P on E).

OUTPUT: the global canonical height $\hat{h}(P)$ of P .

```

1. BEGIN
2.  $\Delta = \text{discr}(a_1, a_2, a_3, a_4, a_6)$ ;
3.  $d = \text{denom}(x)$ ;
4.  $h = \text{real\_height}(P) + \log(d)$ ;
5.  $p\_list = \text{prime\_divisors}(\Delta)$ ;
6. FOR  $p$  IN  $p\_list$  DO
7. BEGIN
8.     IF  $p \nmid d$  THEN  $h = h + \text{local\_height}(p, P)$  FI
9. END;
10. RETURN  $h$ 
11. END

```

3.5 The Mordell–Weil group II: generators

In this section we will show how we look for rational points of infinite order on an elliptic curve E . In compiling the tables, we usually knew the rank r in advance so that we knew how many independent points to expect to find (and only looked for such points when we knew that $r > 0$); however, this procedure is also useful as an open-ended search when we do not know the rank, as obviously it can provide us with a lower bound for r .

The procedure divides into two parts. First, we have a searching routine which looks for points up to some bound on the naive height (equivalently, some bound on the numerator and denominator of the x -coordinate). As this routine finds points, it gives them to the second routine, which has at each stage a \mathbb{Z} -basis for a subgroup A of $E(\mathbb{Q})/T$: initially $A = 0$. This second routine uses the height pairing to determine one of three possibilities: the new point P may be independent of those already found and can then be added to our cumulative list of independent points; the rank of A is thus increased by 1. Secondly, P may be an integral combination of the current basis (modulo torsion) and can then be ignored. Finally, if a multiple kP of P is an integral combination of the current basis for some $k > 1$, we can find a basis for a new subgroup A which contains the old A with index k . Even when we know the rank r in advance, we do not stop as soon as we have a subgroup A of rank r , since A might still have finite index in $E(\mathbb{Q})/T$. To close this final gap we use explicit bounds for the difference between the naive and canonical heights, such as Silverman’s result (Proposition 3.5.1) below.

The algorithm we use for the second procedure is a very general one, which can be used in many other similar situations; for example, as part of an algorithm for finding the unit group of a number field, where the first routine somehow finds units. Our algorithm is essentially the same as the ‘Algorithm for enlarging sublattices’ in the book by Pohst and Zassenhaus [50, Chapter 3.3].

A rational point P on E (given by a standard Weierstrass equation) may be written uniquely as $P = (x, y) = (a/c^2, b/c^3)$ with integers a, b , and c satisfying $\gcd(a, c) = \gcd(b, c) = 1$ and $c \geq 1$. The naive or Weil height of P is $h(P) = \log \max\{|a|, c^2\}$. Initially, we find the point of order 2 in $E(\mathbb{R})$ with minimal x -coordinate x_0 ; this gives a lower bound for the x -coordinates of all real points on E . We then search for points P with naive height up to some bound B by looping through positive integers $c \leq \exp(B/2)$ and through a coprime to c in the range²

²If $E(\mathbb{R})$ has three points of order two, with x -coordinates $x_0 < x_1 < x_2$, then we also omit those a for which $c^2 x_1 < a < c^2 x_2$.

$\max\{c^2x_0, -\exp(B)\} \leq a \leq \exp(B)$. Given a and c , we attempt to solve the appropriate quadratic equation for $b \in \mathbb{Z}$. To speed up this procedure, we use a quadratic sieve: for each denominator c we precompute for about 10 auxiliary sieving primes p the residue classes modulo p to which a must belong if the equation for b is to be soluble modulo p . Each candidate value of a can then first be checked to see if it is admissible modulo each sieving prime before the more time-consuming step of attempting to solve for b . This improvement to the search results in a major time saving in most cases, though for most of the curves in our tables on which we expected to find points of infinite order, such a point was found very quickly anyway. (In some cases we had already found such a point during the search for torsion points.) In practice it may be better to use composite moduli for the sieving.

Each point P found by this search is passed to the second procedure, which tests whether it has infinite order, discarding it if not. At the general stage we will have k independent points P_i for $1 \leq i \leq k$ (initially $k = 0$) which generate a subgroup A of rank k , and will have stored the $k \times k$ height pairing matrix $M = (h(P_i, P_j))$ and its determinant R . Now we set $P_{k+1} = P$ and compute $\hat{h}(P_i, P_{k+1})$ for $i \leq k + 1$ to obtain a new height pairing matrix of order $k + 1$. If the determinant of this new matrix is non-zero, the new point is independent of the previous ones and we add it to the current list of generators, increment k , replace R by the new determinant, and go on with the point search. If the new determinant is zero, however, we use the values $h(P_i, P)$ to express P_{k+1} as a linear combination of the P_i for $i \leq k$, with approximate real coefficients: in fact we have

$$a_1P_1 + a_2P_2 + \dots + a_kP_k + a_{k+1}P_{k+1} = 0 \quad (\text{modulo torsion})$$

where for $1 \leq i \leq k + 1$ the coefficient a_i is the $(i, k + 1)$ cofactor of the enlarged matrix, which we will have stored during the computation of the new determinant. In particular, a_{k+1} is (up to sign) the previous value of R , and hence is non-zero. Next we find rational approximations to these floating-point coefficients a_i (using continued fractions, or MLLL if available), and clear denominators to obtain a new equation of the same form with coprime integer coefficients a_i , which we can check holds exactly. In this relation we still have $a_{k+1} \neq 0$ (the first k points are independent). The simplest case now is when $a_{k+1} = \pm 1$, for then P_{k+1} is redundant and can be discarded. Similarly, if $a_i = \pm 1$ for some $i \leq k$, then we may discard P_i , replacing it by P_{k+1} , and gaining index $|a_{k+1}|$. In general let a_i be the minimal non-zero coefficient (in absolute value); if $|a_i| > 1$, we find a coefficient a_j not divisible by a_i (which must exist since the coefficients are coprime) and write $a_j = a_iq + b$ where $0 < b < |a_i|$. Now since

$$a_iP_i + a_jP_j = a_iP_i + (a_iq + b)P_j = a_i(P_i + qP_j) + bP_j$$

we may replace the generator P_i by $P_i + qP_j$, replace the coefficient a_j by b (which is smaller than $|a_i|$), and replace i by j . After a finite number of steps we obtain a minimal coefficient $a_i = 1$ and can discard the current generator P_i , leaving a new set of k independent generators which generate a group larger than before by a finite index equal to the original value of $|a_{k+1}|$.

In this way, we will be able to find a \mathbb{Z} -basis for the subgroup A of the Mordell–Weil group (modulo torsion) which is generated by the points of naive height less than the bound B . Often we know the rank r of our curve in advance, so that we can increase B until A has rank r . Then A has finite index in $E(\mathbb{Q})$, and we must enlarge it to give the whole of $E(\mathbb{Q})$. There are various methods one can use here, all of which rely on having explicit bounds for the difference between the naive and canonical heights on the curve E . The simplest general bound here is a result of Silverman (see [60]). One can certainly often obtain better bounds for individual curves, and there are also more complicated results which apply in general and which usually give much better bounds, such as the main result of [57].

For simplicity we will only give Silverman’s version of the bound. In the following proposition,³ the height of a rational number a/b with $\gcd(a, b) = 1$ is $h(a/b) = \log \max\{|a|, |b|\}$, and $\log^+(x) = \log \max\{1, |x|\}$ for $x \in \mathbb{R}$.

PROPOSITION 3.5.1. *Let E be an elliptic curve defined by a standard Weierstrass equation over \mathbb{Z} , with discriminant Δ and j -invariant j . Set $2^* = 2$ if $b_2 \neq 0$, or $2^* = 1$ if $b_2 = 0$. Define*

$$\mu(E) = \frac{1}{6} (\log |\Delta| + \log^+(j)) + \log^+(b_2/12) + \log(2^*).$$

Then for all $P \in E(\mathbb{Q})$,

$$-\frac{1}{12}h(j) - \mu(E) - 1.922 \leq \hat{h}(P) - h(P) \leq \mu(E) + 2.14.$$

This result is easiest to apply in the rank 1 case, as follows. Suppose we have a rational point P of infinite order on E , of height $\hat{h}(P)$. If P is not a generator it is a multiple $P = kQ$ (modulo torsion) of some generator Q , where $k \geq 2$, so that $\hat{h}(Q) \leq \frac{1}{k}\hat{h}(P)$. By the preceding proposition we can bound the naive height of Q and adjust the bound B in our search accordingly. If a further search up to this bound finds no more points, then P was a generator after all; otherwise we are sure to find a generator.

Similar techniques are possible in higher rank situations, using estimates from the geometry of numbers. See the papers [70] and [57] for more details.

We may also remark that since P has finite order if and only if $\hat{h}(P) = 0$, the proposition implies that all torsion points have naive height $h(P) \leq \frac{1}{12}h(j) + \mu(E) + 1.922$, giving us another way of finding all the rational torsion points.

For the general case, the following simple result⁴ may be used.

LEMMA 3.5.2. *Let $B > 0$ be such that*

$$S = \{P \in E(\mathbb{Q}) \mid \hat{h}(P) \leq B\}$$

contains a complete set of coset representatives for $2E(\mathbb{Q})$ in $E(\mathbb{Q})$. Then S generates $E(\mathbb{Q})$.

PROOF. Let A be the subgroup of $E(\mathbb{Q})$ modulo torsion generated by the points in S . Suppose that A is a proper subgroup; then we may choose $Q \in E(\mathbb{Q}) - A$ with $\hat{h}(Q)$ minimal, since \hat{h} takes a discrete set of values. By hypothesis, there exist $P \in A$ and R such that $Q = P + 2R$; certainly $R \notin A$, so that $\hat{h}(R) \geq \hat{h}(Q)$ by minimality. Now using the fact that \hat{h} is quadratic and non-negative we obtain a contradiction:

$$\begin{aligned} \hat{h}(P) &= \frac{1}{2}(\hat{h}(Q + P) + \hat{h}(Q - P)) - \hat{h}(Q) \\ &\geq \frac{1}{2}\hat{h}(2R) - \hat{h}(Q) \\ &= 2\hat{h}(R) - \hat{h}(Q) \geq \hat{h}(Q) > B. \quad \square \end{aligned}$$

We have two ways of using this in practice. First of all, it is possible to obtain from the two-descent procedure which we use to determine the rank (see the next section), a set of coset representatives for $E(\mathbb{Q})$ modulo $2E(\mathbb{Q})$. Computing the heights of these points we can find

³When referring to [60], recall that our \hat{h} is double Silverman’s; also, the constant 1.922 appearing here is a (normalized) correction, due to Bremner, of the constant in Silverman’s paper.

⁴Attributed in [60] to Zagier, it is also exercise 5 on page 84 of Cassels’ book [8].

a B for which the Lemma holds, to which we add the maximum difference between naive and canonical heights from the preceding proposition to get a bound on the naive heights of a set of generators.

Alternatively, assuming that we know the rank r , we first run our search until we find r independent points P_i . Now it is easy to check whether a point P is twice another: if any subset of the P_i sums to $2Q$ for some Q we replace one of the P_i in the sum by Q and gain index 2. After a finite number of steps (since we are in a finitely-generated group) we obtain independent points which are independent modulo 2, and proceed as before.

Again, we have only presented here the most straightforward strategies for enlarging a set of r independent points in $E(\mathbb{Q})$ to a full \mathbb{Z} -basis; this is a topic of active research, with new ideas being developed rapidly: see the paper [57] for some recent advances.

Putting the pieces together, we can determine a set of generators for $E(\mathbb{Q})$ modulo torsion, and then compute the regulator, provided that we know its rank. If we do not know the rank, we at least can obtain lower bounds for the rank. Together with the torsion points found in section 3.3, we will have determined the Mordell–Weil group $E(\mathbb{Q})$ explicitly. Computing the rank is the subject of the next section.

3.6 The Mordell–Weil group III: the rank

For an elliptic curve E defined over the rationals, the rank of the Mordell–Weil group $E(\mathbb{Q})$ is by far the hardest of the elementary quantities associated with E to compute, both theoretically and in terms of implementation. Strictly speaking, the two-descent algorithms we will describe are not algorithms at all, as they are not guaranteed to terminate in all cases. One part of the procedure involves establishing whether or not certain curves of genus one have rational points, when they are known to have points everywhere locally (that is, over \mathbb{R} and over the p -adic field \mathbb{Q}_p for all primes p): there is no known algorithm to decide this in general. Moreover, even without this difficulty, for curves with large coefficients and no rational points of order two, the general two-descent algorithm takes too long to run in practice. For simplicity, we will refer to the procedures as rank algorithms, although their output in certain cases will be bounds on the rank rather than its actual value.

We originally decided to implement a general two-descent procedure in order to check that the modular curves we had computed did have their rank equal to the analytic rank, which we knew, as described in the previous chapter. This was a somewhat thankless task, as it involved a large programming effort, and a large amount of computer time to run the resulting program, in order to verify that approximately 2500 numbers did in fact have the values 0, 1 or 2 which we were already sure were correct. Since the project started, the major theoretical advances by Kolyvagin, Rubin and others meant that all the cases of rank 0 or 1 were known anyway, which left just 18 cases of conjectured rank 2 to verify. In the end we were able to verify these cases, and to check all but a few dozen of the rank 0 or 1 curves; we also obtained extra information by the two-descent procedure, such as the 2-rank of the Tate–Shafarevich group III, and a set of coset representatives for $E(\mathbb{Q})/2E(\mathbb{Q})$.

Since the original implementation, the algorithm has been much improved in many ways (notably the syzygy sieve in the search for quartics, the systematic use of group structure in the 2-isogeny case, and the use of quadratic sieving in searching for rational points on homogeneous spaces: see below for details). Our program `mwrnk`,⁵ based on the algorithm, now works well on a much larger set of curves, including some of fairly high rank such as a curve of Fermigier [23] with rank 13 and 2-torsion (see the example below), and several curves with no 2-torsion and ranks 6, 7 and 8. However, curves with extremely large coefficients, such as Nagao’s curve of rank (at least) 21 (see [45]), are beyond the reach of this algorithm owing to the enormous

⁵Available from the author’s ftp site: see the Introduction for details.

search regions required. One can also use the program `mwrnk` to find points on curves which are too large to find by the search methods of the previous section.

We will not describe here the theory of two-descent, which is the basis of the algorithm, in great detail. Roughly speaking, one has an injective homomorphism from $E(\mathbb{Q})/2E(\mathbb{Q})$ into a finite elementary abelian 2-group, the 2-Selmer group, and attempts to determine the image; if this has order 2^t then the rank of $E(\mathbb{Q})$ is t , $t - 1$ or $t - 2$ according to whether the number of points of order 2 in $E(\mathbb{Q})$ is 0, 1 or 3 (respectively). This procedure applies to arbitrary curves, and is called *general two-descent*. When E has a rational point P of order two, there is a rational 2-isogeny $\phi : E \rightarrow E' = E/\langle P \rangle$ and a dual isogeny $\phi' : E' \rightarrow E$. We may then proceed differently, using a procedure we call *two-descent via 2-isogeny*: we embed each of $E'/\phi(E)$ and $E/\phi'(E')$ into finite subgroups of $\mathbb{Q}^*/(\mathbb{Q}^*)^2$, which are easy to write down. This is in contrast to the general two-descent, where one has to work hard to find the Selmer group itself. A full description of two-descent can be found in the standard references such as the books by Silverman [58], Husemöller [27], Knapp [28], or Cassels [8], but the descriptions given there are only easy to apply when E has all its 2-torsion rational. For the general case where there are no rational points of order 2, the main reference is one of the original papers [3] by Birch and Swinnerton–Dyer on their Conjecture, and we followed that paper closely in writing the first version of our program. More detail on the invariant theory, which has resulted in substantial improvements to the general two-descent algorithm, can be found in the paper [20]; a very full description of the algorithm, together with its extension to real quadratic number fields (see also [19]), can be found in Serf’s thesis [52].

Both algorithms involve the classification of certain curves, associated with the given elliptic curve E , called *principal homogeneous spaces*. These are twists of E : curves of genus 1 isomorphic to E over an extension field, but not (necessarily) over \mathbb{Q} itself; they need not have rational points, so need not themselves be elliptic curves. When they do have rational points, these map to rational points on E ; the maps $H \rightarrow E$ are called *2-coverings* and have degree 4 (in the general two-descent) or 2 (in the 2-isogeny descent). The homogeneous spaces which arise in both algorithms have equations of the form

$$(3.6.1) \quad H : \quad y^2 = g(x) = ax^4 + bx^3 + cx^2 + dx + e$$

where $g(x)$ is a quartic polynomial with rational coefficients. For brevity we will usually refer to these principal homogeneous spaces simply as *quartics*. The invariants I and J of $g(x)$ (see below for their definition) are related to the invariants c_4 and c_6 of either E or the 2-isogenous curve E' . In the case of descent via 2-isogeny, $g(x)$ will in fact be a quadratic in x^2 . We will be interested in whether the quartic H has points over \mathbb{Q} or one of its completions, the p -adics \mathbb{Q}_p or the reals \mathbb{R} . Such a point will either be an affine point (x, y) satisfying the equation (3.6.1), or one of the two points at infinity on the projective completion of H , which are rational if and only if a is a square.

In all cases, a quartic with a (global) rational point (x, y) will lead to a rational point on the original curve E , and the set of all the rational points thus obtained will cover the cosets of $2E(\mathbb{Q})$ in $E(\mathbb{Q})$; thus we will be able to determine the rank of $E(\mathbb{Q})$, and at the same time obtain a set of points which generates a subgroup of the Mordell-Weil group $E(\mathbb{Q})$ of odd, finite index. Quartics with no global rational point which are everywhere locally soluble arise from non-trivial elements in the Tate–Shafarevich group of E (or of E'); if these exist, we will only obtain upper and lower bounds for the rank. This is because we currently have no general procedure for proving that a quartic with no rational points does have none. In practice, moreover, it is often impossible to distinguish between such a quartic and one with rational points which are all very large, and hence outside the search region; this happens when a curve has some very large generators, and in such cases also we may only be able to give bounds for

the rank. Further work on these questions is clearly needed, and is currently the focus of much active research.

Since the covering maps $H \rightarrow E$ have degree 2 or 4, the rational points on H tend to be smaller (in the sense of naive height) than the rational points they map to on E ; this makes them easier to find by search. Here is an example of this: the curve $y^2 = x^3 - 673$ has rank 2, with generators $P_1 = (29, 154)$ and $P_2 = (33989323537/61761^2, -1384230292401340/61761^3)$. The second generator, which would take a very long time to find by searching on the curve itself, is obtained from the rational point $(x, y) = (191/97, 123522/97^2)$ on the quartic with coefficients $(a, b, c, d, e) = (-2, 4, -24, 164, -58)$. This is much easier to find: our program takes less than a second to find the rank and both generators of this curve (but in this time it does not prove that they are generators, only that they generate a subgroup of finite odd index in the Mordell-Weil group).

Before we describe the two main two-descent algorithms, we will present algorithms for determining local solubility and for attempting to determine global solubility of a quartic equation such as (3.6.1), as these are used in both the algorithms.

Checking local solubility.

Here we present an algorithm for determining the local solubility of a curve of the form (3.6.1), where $g(x)$ is a square-free quartic polynomial with integer coefficients. It is easy to generalize this algorithm in two ways: firstly, one might be interested in polynomials of higher degree (when studying curves of higher genus, for example); secondly, working over a general number field K , one would replace the p -adic field \mathbb{Q}_p here with the appropriate completion of K . These extensions are quite straightforward.

Solubility at the infinite prime (that is, over the reals) is easily determined. If $g(x)$ has a real root then it certainly takes positive values, so that H has real points; if $g(x)$ has no real roots, then the values of $g(x)$ have constant sign, and we merely have to check that $a > 0$.

Regarding the finite primes, we first observe that there are only a finite number which need checking in each case, for if p is an odd prime not dividing the discriminant of g , then H certainly has points modulo p which are nonsingular and hence lift to p -adic points. For the other primes, we present an algorithm first given in [3].

It suffices to determine solubility in \mathbb{Z}_p for either $g(x)$ or $g^*(x) = ex^4 + dx^3 + cx^2 + bx + a$, and in the latter case we may assume $x \in p\mathbb{Z}_p$. Given x_k modulo p^k , one tries to lift to a p -adic point (x, y) with $x \equiv x_k \pmod{p^k}$. In [3], conditions are given for this to be possible; more precisely, one of three possibilities may occur (given k and x_k): either a lifting is definitely possible, and we may terminate the algorithm with a positive result; or it is definitely not possible, and we reject this value of x_k ; or it is impossible to decide without considering x_k modulo a higher power of p . The test for this lifting is given below in the two subroutines called `lemma6` and `lemma7`, named after the corresponding results in [3]. This leads to a recursive algorithm which is guaranteed to terminate since in any given case there is an exponent k such that it is possible⁶ to determine p -adic solubility by considering solubility modulo p^k . All this is an exercise in Hensel's Lemma; the prime $p = 2$ needs to be considered separately. For the details, we refer to the pseudocode below, or to [3]. Further information on local solubility may be found in [56] and [57].

Here is the pseudocode for these algorithms. Note that for any given elliptic curve, all the homogeneous spaces considered will have the same discriminant as the curve (up to a power of 2), so that in practice we would not need to factorize the discriminant of each quartic.

⁶In fact, if $k > \text{ord}_p(\text{disc}(g))$, and also $k \geq 2$ when $p = 2$, then the third possibility cannot occur in algorithms `lemma6` and `lemma7`.

Algorithm for determining local solubility of a quartic

INPUT: a, b, c, d, e (integer coefficients of a quartic $g(x)$)
 OUTPUT: TRUE/FALSE (solubility of $y^2=g(x)$ in \mathbb{R} and in \mathbb{Q}_p for all p)

```

1. BEGIN
2. IF NOT R_soluble(a,b,c,d,e) THEN RETURN FALSE FI;
3. IF NOT Qp_soluble(a,b,c,d,e,2) THEN RETURN FALSE FI;
4.  $\Delta$  = discriminant(a,b,c,d,e);
5. p_list = odd_prime_factors( $\Delta$ );
6. FOR p IN p_list DO
7. BEGIN
8.     IF NOT Qp_soluble(a,b,c,d,e,p) THEN RETURN FALSE FI
9. END;
10. RETURN TRUE
11. END

```

(Subroutine for determining real solubility)

SUBROUTINE R_soluble(a,b,c,d,e)

INPUT: a, b, c, d, e (integer coefficients of a quartic $g(x)$)
 OUTPUT: TRUE/FALSE (solubility of $y^2=g(x)$ in \mathbb{R})

```

1. BEGIN
2. IF a>0 THEN RETURN TRUE FI;
3. x_list = real_roots(a*x4+b*x3+c*x2+d*x+e=0);
4. IF length(x_list)>0 THEN RETURN TRUE FI;
5. RETURN FALSE
6. END

```

(Subroutine for determining p-adic solubility)

SUBROUTINE Qp_soluble(a,b,c,d,e,p)

INPUT: a, b, c, d, e (integer coefficients of a quartic $g(x)$)
 p (a prime)
 OUTPUT: TRUE/FALSE (solubility of $y^2=g(x)$ in \mathbb{Q}_p)

```

1. BEGIN
2. IF Zp_soluble(a,b,c,d,e,0,p,0) THEN RETURN TRUE FI;
3. IF Zp_soluble(e,d,c,b,a,0,p,1) THEN RETURN TRUE FI;
4. RETURN FALSE
5. END

```

(Recursive \mathbb{Z}_p -solubility subroutine)

SUBROUTINE Zp_soluble(a,b,c,d,e,x_k,p,k)

INPUT: a, b, c, d, e (integer coefficients of a quartic $g(x)$)
 p (a prime)
 x_k (an integer)
 k (a non-negative integer)
 OUTPUT: TRUE/FALSE (solubility of $y^2=g(x)$ in \mathbb{Z}_p , with $x \equiv x_k \pmod{p^k}$)

```

1. BEGIN
2. IF p=2
3. THEN code = lemma7(a,b,c,d,e,x_k,k)
4. ELSE code = lemma6(a,b,c,d,e,x_k,p,k)

```

```

5. FI;
6. IF code=+1 THEN RETURN TRUE FI;
7. IF code=-1 THEN RETURN FALSE FI;
8. FOR t = 0 TO p-1 DO
9. BEGIN
10.     IF Zp_soluble(a,b,c,d,e,x_k+t*p^k,p,k+1) THEN RETURN TRUE FI
11. END;
12. RETURN FALSE
13. END

```

(\mathbb{Z}_p lifting subroutine: odd p)

```

SUBROUTINE lemma6(a,b,c,d,e,x,p,n)
1. BEGIN
2. gx = a*x^4+b*x^3+c*x^2+d*x+e;
3. IF p_adic_square(gx,p) THEN RETURN +1 FI;
4. gdx = 4*a*x^3+3*b*x^2+2*c*x+d;
5. l = ord(p,gx); m = ord(p,gdx);
6. IF (l>=m+n) AND (n>m) THEN RETURN +1 FI;
7. IF (l>=2*n) AND (m>=n) THEN RETURN 0 FI;
8. RETURN -1
9. END

```

(\mathbb{Z}_2 lifting subroutine)

```

SUBROUTINE lemma7(a,b,c,d,e,x,n)
1. BEGIN
2. gx = a*x^4+b*x^3+c*x^2+d*x+e;
3. IF p_adic_square(gx,2) THEN RETURN +1 FI;
4. gdx = 4*a*x^3+3*b*x^2+2*c*x+d;
5. l = ord(p,gx); m = ord(p,gdx);
6. gxodd = gx; WHILE even(gxodd) DO gxodd = gxodd/2;
7. gxodd = gxodd (mod 4);
8. IF (l>=m+n) AND (n>m) THEN RETURN +1 FI;
9. IF (n>m) AND (l=m+n-1) AND even(l) THEN RETURN +1 FI;
10. IF (n>m) AND (l=m+n-2) AND (gxodd=1) AND even(l) THEN RETURN +1 FI;
11. IF (m>=n) AND (l>=2*n) THEN RETURN 0 FI;
12. IF (m>=n) AND (l=2*n-2) AND (gxodd=1) THEN RETURN 0 FI;
13. RETURN -1
14. END

```

A few further remarks on these algorithms: firstly, only trivial changes need to be made to the algorithms `Qp_soluble` and `Zp_soluble` to make them apply to more general equations of the form $y^2 = g(x)$ where $g(x)$ is a non-constant squarefree integer polynomial. This is relevant for work on curves of higher genus, and was observed by S. Siksek. Secondly, extensions to more general p -adic fields are also useful in studying curves over number fields, and again the extensions of Lemma 6 and Lemma 7 in [3] are not difficult. See the theses [56] and [52] for details of such extensions.

Lastly, D. Simon observed that in our application of the algorithms `lemma6` and `lemma7`, we only care whether there is a solution, not necessarily that there is a solution congruent to the given $x \pmod{p^k}$; hence line 6 of subroutine `lemma6` and line 8 of subroutine `lemma7` can both be replaced by:

```
IF l>2*m THEN RETURN +1 FI.
```

Checking global solubility.

To determine whether an equation (3.6.1) has a rational point is much harder than the corresponding local question. All we can do at present is search (efficiently) for a point up to a certain height, after checking that there is no local obstruction. The only satisfactory way known at present to decide on the existence of rational points on these homogeneous spaces is to carry out so-called higher descents; as mentioned above, this is the subject of current work (see [63], for example), and we will not consider it further here.

Our strategy is to look first for a small rational point, using a very simple procedure with low overheads; if this fails, we check for local solubility; if this passes, we start a much more thorough search for a global point, using a quadratic sieving procedure rather similar to the one described in the previous section for finding points on the elliptic curve itself. (In fact, such a sieve-assisted search may be used to find rational points on any curve given by an equation of the form $y^2 = g(x)$ where $g(x)$ is a polynomial in x .) The philosophy here is that there is no point in looking hard for rational points unless one is sure of local solubility, but also that there is no point in checking local solubility when there is an obvious global point.

To carry out the sieve-assisted search, for each possible denominator of x one precomputes, for each of several sieving moduli m , the residues to which the numerator of x must belong if the right-hand side of the equation is to be a square modulo m . In addition, it is easy to see that for every odd prime p dividing the denominator of the x -coordinate of a rational point, we must have $(\frac{a}{p}) = +1$; so provided that the leading coefficient a is not a square (in which case the points at infinity are rational anyway), we precompute a list of primes p for which $(\frac{a}{p}) = -1$, and discard possible denominators divisible by any of these primes. For $p = 2$ a similar condition holds.⁷ One also obviously restricts the search to ranges of x for which $g(x)$ is positive; depending on the number of real roots of g and the sign of a , this splits the search into up to three intervals. Finally, in the case of two-descent via 2-isogeny, where the quartics are polynomials in x^2 and thus even, we may restrict to positive x .

For reasons of space, we will only give here the code for a simple point search with no sieving.

Algorithm for searching for a rational point on a quartic: simple version

```

INPUT:      a, b, c, d, e (integer coefficients of a quartic g(x))
            k1, k2      (lower and upper bounds)
OUTPUT:     TRUE/FALSE (solubility of  $y^2=g(x)$  in  $\mathbb{Q}$  with  $x=u/w$ 
                    and  $k1 \leq |u|+w \leq k2$ )

1.  BEGIN
2.  FOR n = k1 TO k2 DO
3.  BEGIN
4.      IF n=1 THEN
5.          IF square(a) RETURN TRUE FI;
6.          IF square(e) RETURN TRUE FI
7.      ELSE
8.          FOR u = 1 TO n-1 DO
9.              BEGIN
10.                 IF gcd(u,n)=1
11.                 THEN
12.                     w = n-u;
13.                     IF square(a*u4+b*u3w+c*u2w2+d*uw3+e*w4) RETURN TRUE FI;

```

⁷I am grateful to J. Gebel for this idea, which saves considerable time in practice.

```

14.          IF square(a*u4-b*u3+c*u2-d*u+e) RETURN TRUE FI
15.          FI
16.          END
17.          FI
18. END;
19. RETURN FALSE
20. END

```

We will now describe the two main two-descent algorithms: two-descent via 2-isogeny for use when E has a rational point of order 2, and general two-descent in the general case. We only use general two-descent when there is no point of order 2, so that the first method does not apply. The situation is not appreciably simpler when E has all three of its points of order two rational than when there is just one rational point of order two, and so we will not bother to consider this case separately.

Method 1: descent using 2-isogeny.

Suppose that E has a rational point P of order 2. By a change of coordinates we may assume that E has equation

$$E : y^2 = x(x^2 + cx + d)$$

where $P = (0, 0)$, and $c, d \in \mathbb{Z}$. Explicitly, in terms of a Weierstrass equation, let x_0 be a root of the cubic $x^3 + b_2x^2 + 8b_4x + 16b_6$, and set $c = 3x_0 + b_2$, $d = (c + b_2)x_0 + 8b_4$. If $a_1 = a_3 = 0$, then we can avoid a scaling factor of 2 by letting x_0 be a root of $x^3 + a_2x^2 + a_4x + a_6$, and setting $c = 3x_0 + a_2$, $d = (c + a_2)x_0 + a_4$. The 2-isogenous curve $E' = E / \langle P \rangle$ has equation

$$E' : y^2 = x(x^2 + c'x + d')$$

where

$$c' = -2c \quad \text{and} \quad d' = c^2 - 4d.$$

The nonsingularity condition on E is equivalent to $dd' \neq 0$. The 2-isogeny $\phi: E \rightarrow E'$ has kernel $\{0, P\}$ and in general maps (x, y) to $\left(\frac{y^2}{x^2}, \frac{y(x^2-d)}{x^2}\right)$. The dual isogeny $\phi': E' \rightarrow E$ maps (x, y) to $\left(\frac{y^2}{4x^2}, \frac{y(x^2-d')}{8x^2}\right)$.

For each factorization $d = d_1d_2$, with d_1 square-free, we consider the homogeneous space

$$H(d_1, c, d_2) : v^2 = d_1u^4 + cu^2 + d_2.$$

Let $n_1 = n_1(c, d)$ be the number of factorizations of d for which the quartic $H(d_1, c, d_2)$ has a rational point, and $n_2 = n_2(c, d)$ the number for which the quartic has a point everywhere locally. Define $n'_1 = n_1(c', d')$ and $n'_2 = n_2(c', d')$ similarly. Then it is not hard to show by rather explicit calculation (see below and the references given) that $E(\mathbb{Q})/\phi'(E'(\mathbb{Q}))$ is isomorphic to the subgroup of $\mathbb{Q}^*/(\mathbb{Q}^*)^2$ generated by the factors d_1 for which $H(d_1, c, d_2)$ has a rational point. Thus

$$|E(\mathbb{Q})/\phi'(E'(\mathbb{Q}))| = n_1,$$

which must therefore be a power of 2, say $n_1 = 2^{e_1}$; similarly,

$$|E'(\mathbb{Q})/\phi(E(\mathbb{Q}))| = n'_1 = 2^{e'_1}.$$

It then follows (see below) that

$$(3.6.2) \quad \text{rank}(E(\mathbb{Q})) = \text{rank}(E'(\mathbb{Q})) = e_1 + e'_1 - 2.$$

With luck one will find rational points on all the quartics which have them everywhere locally; then $n_1 = n_2$, and there is no ambiguity in the result. However there will be cases in which the number \tilde{n}_1 of quartics on which we can find a rational point is strictly less than n_2 . In such cases, we will only have upper and lower bounds for n_1 , and similarly for n'_1 , leading to upper and lower bounds for the rank. This can happen for two reasons: either there is a rational point on some quartic, but our search bound was too small to find it; or the quartic has points everywhere locally but no global rational point.

The quartics H which have points everywhere locally but not globally come from elements of order 2 in the Tate–Shafarevich groups $\text{III}(E/\mathbb{Q})$ and $\text{III}(E'/\mathbb{Q})$. There is an exact sequence

$$0 \rightarrow E(\mathbb{Q})/\phi'(E'(\mathbb{Q})) \rightarrow S^{(\phi')}(E'/\mathbb{Q}) \rightarrow \text{III}(E'/\mathbb{Q})[\phi'] \rightarrow 0$$

coming from Galois cohomology; here $S^{(\phi')}(E'/\mathbb{Q})$ is the Selmer group of order n_2 whose elements are represented by the homogeneous spaces $H(d_1, c, d_2)$ which are everywhere locally soluble, and $\text{III}(E'/\mathbb{Q})$ is the Tate–Shafarevich group of E' . The injective map $E(\mathbb{Q})/\phi'(E'(\mathbb{Q})) \rightarrow S^{(\phi')}(E'/\mathbb{Q})$ is induced by taking a point (x, y) in $E(\mathbb{Q})$ with $x \neq 0$ to the space $H(d_1, c, d_2)$ where $d_1 = x$ modulo squares: if $x = d_1 u^2$ and $v = uy/x$ then (u, v) is a rational point on $H(d_1, c, d_2)$. The point $P = (0, 0)$ maps to d modulo squares. Conversely, if (u, v) is a rational point on $H(d_1, c, d_2)$ then $(x, y) = (d_1 u^2, d_1 uv)$ is a rational point on E . (In proving these statements, one has to check that two rational points on E have the same x -coordinate modulo squares if and only if their difference is in $\phi'(E'(\mathbb{Q}))$; for example, the image of P is d , which is a square if and only if $P \in \phi'(E'(\mathbb{Q}))$.) It follows that n_1 is the order of $E(\mathbb{Q})/\phi'(E'(\mathbb{Q}))$, as stated above, and hence that

$$|\text{III}(E'/\mathbb{Q})[\phi']| = n_2/n_1.$$

Similarly, from the exact sequence

$$0 \rightarrow E'(\mathbb{Q})/\phi(E(\mathbb{Q})) \rightarrow S^{(\phi)}(E/\mathbb{Q}) \rightarrow \text{III}(E/\mathbb{Q})[\phi] \rightarrow 0$$

with similarly defined maps, we obtain

$$|\text{III}(E/\mathbb{Q})[\phi]| = n'_2/n'_1.$$

Thus the result is only genuinely ambiguous when either $\text{III}(E/\mathbb{Q})[\phi]$ or $\text{III}(E'/\mathbb{Q})[\phi']$ is non-trivial, so that not all elements of the Selmer groups are obtained from rational points on the elliptic curves. This is rare for the curves in the tables, but obviously must be taken into account in general. A typical situation is to have $n_2 n'_2 = 16$ and $n_1 n'_1 \geq 4$, when one suspects that $r = 0$ with $|\text{III}(E/\mathbb{Q})[2]| = 4$ or $|\text{III}(E'/\mathbb{Q})[2]| = 4$, but where it is possible instead that $r = 2$ and $|\text{III}(E/\mathbb{Q})[2]| = |\text{III}(E'/\mathbb{Q})[2]| = 1$. Curve 960D1 in the tables is an example of this, although in this case since the curve is modular and we know that $L(E, 1) \neq 0$, it must have rank 0 by the result of Kolyvagin mentioned earlier. We can also deduce this by working with the 2-isogenous curves 960D3 and 960D2, where there is no ambiguity: here $n_1 = n_2 = n'_1 = n'_2 = 2$, showing that the rank is certainly 0. (Note that isogenous curves have the same rank, but not necessarily the same order of III , which can work to our advantage in cases like this.) Returning to the pair 960D1–960D2 where we compute $n_1 = n_2 = 1$, $n'_2 = 16$ and $n'_1 \geq 4$, now we know that the rank is in fact zero we can conclude that $n'_1 = 4$, and that $|\text{III}(E/\mathbb{Q})[\phi]| = 4$. The nontriviality of $\text{III}(E/\mathbb{Q})$ in this case is confirmed by the Birch–Swinnerton-Dyer conjecture, which for this curve predicts that III has order 4 (see Table 4).

Local solubility of $H(d_1, c, d_2)$ is automatic for all primes p which do not divide $2dd'$; for those p which do divide $2dd'$ we may apply the general criteria of Birch and Swinnerton-Dyer.

Local solubility in \mathbb{R} is easy to determine here: if $d' < 0$ then we require $d_1 > 0$, while if $d' > 0$ then either $d_1 > 0$ or $c + \sqrt{d'} > 0$ is necessary. Thus if either $d' < 0$, or $d' > 0$ and $c + \sqrt{d'} < 0$, then we only consider positive divisors d_1 of d , and need not apply the general test for solubility in \mathbb{R} .

Each rational point (u, v) on $H(d_1, c, d_2)$ maps, as observed above, to the point $(d_1 u^2, d_1 uv)$ on E ; modulo $\phi'(E'(\mathbb{Q}))$, this is independent of the rational point (u, v) , and only depends on d_1 modulo squares. Similarly, a rational point (u, v) on $H(d'_1, c', d'_2)$ maps to a point on E' , and hence via the dual isogeny ϕ' to the point

$$\left(\frac{v^2}{4u^2}, \frac{v(d'_1 u^4 - d'_2)}{8u^3} \right)$$

in $E(\mathbb{Q})$. The set of $n_1 n'_1$ points in $E(\mathbb{Q})$ thus determined (by adding the points constructed in this way) cover the cosets of $E(\mathbb{Q})/2E(\mathbb{Q})$, either once each, when $|E(\mathbb{Q})[2]| = 4$, which is when d' is a square, or twice, when d' is not a square and $|E(\mathbb{Q})[2]| = 2$. Thus, when $|E(\mathbb{Q})[2]| = 2$ we have

$$\frac{n_1 n'_1}{2} = |E(\mathbb{Q})/2E(\mathbb{Q})| = 2^{r+1},$$

while if $|E(\mathbb{Q})[2]| = 4$ we have

$$n_1 n'_1 = |E(\mathbb{Q})/2E(\mathbb{Q})| = 2^{r+2};$$

hence $2^r = n_1 n'_1 / 4$ in both cases, proving (3.6.2).

When counting n_1 and n_2 (and similarly, n'_1 and n'_2), it is very useful to use the fact that each is a power of 2, being the order of an elementary abelian 2-group. This is particularly important when d (or d') has many distinct prime factors. Let A_0 be the group of all divisors of d modulo squares, of order n_0 (say). Then A_0 is generated by -1 and the primes dividing d , so that $n_0 = 2^{e_0}$ where e_0 is the number of distinct prime factors of d , plus 1. Within A_0 we must determine the subgroups A_1 and A_2 of orders n_1 and n_2 , consisting of those divisors d_1 of d for which the corresponding homogeneous space is everywhere locally or globally soluble, respectively.

We can effectively reduce the size of the set A_0 of divisors to be searched by a factor up to 8 as follows: as observed above, if either $d' < 0$, or $d' > 0$ and $c + \sqrt{d'} < 0$, then we need only consider positive divisors d_1 of d , cutting in two the number of elements of A_0 which may lie in A_1 . Secondly, we may take advantage of the fact that we know the rational point $(0, 0)$ on E ; thus we know that d is in A_2 (though possibly just the identity if d is a square); similarly, if d' is a square then $x^2 + cx + d$ factorizes, say as $(x - x_2)(x - x_3)$, and we know that x_2 and x_3 also lie in A_2 .

More generally, whenever we find in the course of our systematic search through the elements of A_0 that the element d_1 lies in A_2 , we can effectively factor out d_1 and reduce the number of remaining values to check by a factor of 2. Of course, this requires careful book-keeping in the implementation; for simplicity, we omit these refinements from the pseudocode below, where we simply loop over all square-free divisors of d and d' .

As an example of the saving that can be made, consider the curve of rank 13 constructed by Fermigier in [23]; this is of the form $y^2 = x(x^2 + cx + d)$ with

$$c = 36861504658225 \quad \text{and}$$

$$d = 1807580157674409809510400 = 2^{15} \cdot 3^4 \cdot 5^2 \cdot 7^2 \cdot 17 \cdot 23 \cdot 29 \cdot 41 \cdot 103 \cdot 113 \cdot 127 \cdot 809,$$

so that d has 12 distinct prime factors and $2^{13} = 8192$ square-free divisors. Since d is non-square we can cut the set in half, say by excluding all d_1 divisible by the largest prime factor 809, leaving 4096 values to test. In our implementation, the results of the test are as follows:

- 7 non-trivial values of d_1 give rational points after searching, as well as $d_1 = 1$ which gives the trivial point;
- 120 further values are in the subgroup A_2 generated by these 7 values and need not be tested;
- 122 further values were tested and found to be not everywhere locally soluble, hence not in A_1 ;
- 3846 further values were discarded as being a product of an element of A_2 and an element not in A_1 , and hence not in A_1 .

Thus in this case we find that $n_1 = n_2 = 256$, after only having to search for points on seven homogeneous spaces. Working with the isogenous curve, we obtain $n'_1 = n'_2 = 128$ after only searching six homogeneous spaces for points. Thus $e_1 = 8$, $e'_1 = 7$ and the rank is 13. Note that in the course of computing this value, we have searched precisely 13 homogeneous spaces, and the points we thereby construct give 13 generators of $E(\mathbb{Q})/2E(\mathbb{Q})$ modulo torsion. Adding $P = (0, 0)$ to this list gives 14 points which generate $E(\mathbb{Q})/2E(\mathbb{Q})$ (which has order 2^{14}), and which therefore generate a subgroup of finite odd index in the full Mordell-Weil group $E(\mathbb{Q})$.

The situation is not always this simple, however, even for curves where $\text{III}[2]$ is trivial, since there may be homogeneous spaces with rational points which are hard to find. For example, consider Fermigier's curve of rank 14 from [23], with $c = 2429469980725060$ and $d = 275130703388172136833647756388$ (which has 14 prime factors). When we run our program using a (logarithmic) bound of 10 in the search for rational points on the quartics, we find $n_1 \geq 64$, $n'_1 \geq 128$, while $n_2 = n'_2 = 256$. Here the correct values are $n_1 = n'_1 = 256$, giving $r = 14$, but we only find $11 \leq r \leq 14$; and in the process, we have had to search many more homogeneous spaces for rational points.

Here is the pseudo-code which implements the algorithm just described. The main routine aborts if either the input curve is singular (this is useful if one wants to apply the algorithm systematically to a range of inputs) or if there is no point of order two. The latter is detected in lines 6–7, where an integer root to a monic cubic with integer coefficients is found (if it exists). Most of the work is done in the subroutine `count(c,d,p_list)` which determines $n_2(c, d)$ and, as far as possible, $n_1(c, d)$. Here `p_list` is the set of 'bad' primes dividing $2dd'$ where local solubility needs to be checked, which we only compute once. There are two calls to the subroutine `rational_point(a,b,c,d,e,k1,k2)`, which seeks a rational u/w with $k_1 \leq |u| + w \leq k_2$ such that $g(u/w)$ is a rational square, where $g(x) = ax^4 + bx^3 + cx^2 + dx + e$. (Here $w > 0$ and $\gcd(u, w) = 1$.) In the first call we carry out a quick check for 'small' points; then we look further, having first checked for everywhere local solubility. The particular parameters `lim1`, `lim2` for the search will probably be decided at run time. The subroutines `Qp_soluble` and `rational_point` are implementations of the algorithms given earlier (though in practice we would use a more efficient algorithm for the second call to `rational_point`, as explained above).

Algorithm for computing rank: rational 2-torsion case

INPUT: a1, a2, a3, a4, a6 (coefficients of E)
 OUTPUT: r_min, r_max (bounds for rank of E)
 S, S' (upper bounds for $\#\text{III}(E)[\phi]$ and $\#\text{III}(E')[\phi']$)

1. BEGIN
2. IF a1=a3=0
3. THEN s2 = a2; s4 = a4; s6 = a6
4. ELSE s2 = a1*a1+4*a2; s4 = 8*(a1*a3+2*a4); s6 = 16*(a3*a3+4*a6)
5. FI;
6. x_list = integer_roots(x³+s2*x²+s4*x+s6=0);

```

7. IF length(x_list)=0 THEN abort ELSE x0 = x_list[1] FI;
8. c = 3*x0+s2; d = (c+s2)*x0 + s4;
9. c' = -2*c; d' = c2-4*d;
10. IF d*d'=0 THEN abort FI;
11. p_list = prime_divisors(2*d*d');
12. (n1,n2) = count(c,d,p_list);
13. (n1',n2') = count(c',d',p_list);
14. e1 = log2(n1); e2 = log2(n2);
15. e1' = log2(n1'); e2' = log2(n2');
16. r_min = e1+e1'-2; r_max = e2+e2'-2;
17. S = n2'/n1'; S' = n2/n1;
18. RETURN r_min, r_max, S, S'
19. END

```

(Main counting subroutine)

SUBROUTINE count(c,d,p_list)

```

1. BEGIN
2. n1 = n2 = 1; d' = c2-4*d;
3. d1_list = squarefree_divisors(d);
4. FOR d1 IN d1_list DO
5. BEGIN
6. IF rational_point(d1,0,c,0,d/d1,1,lim1)
7. THEN n1 = n1+1; n2 = n2+1
8. ELSE
9. IF everywhere_locally_soluble(c,d,d',d1,p_list)
10. THEN
11. n2 = n2+1;
12. IF rational_point(d1,0,c,0,d/d1,lim1+1,lim2)
13. THEN n1 = n1+1
14. FI
15. FI
16. FI
17. END;
18. RETURN (n1, n2)
19. END

```

(Subroutine to check for everywhere local solubility)

```

1. SUBROUTINE everywhere_locally_soluble(c,d,d',d1,p_list)
2. BEGIN
3. IF d'<0 AND d1<0 THEN RETURN FALSE FI;
4. IF d'>0 AND d1<0 AND (c+sqrt(d'))<0 THEN RETURN FALSE FI;
5. FOR p IN p_list DO
6. BEGIN
7. IF NOT Qp_soluble(d1,0,c,0,d/d1,p) THEN RETURN FALSE FI
8. END;
9. RETURN TRUE
10. END

```

Method 2: general two-descent.

We now turn to the general two-descent, which applies whether or not E has a rational point of order 2. Again, the basic idea is to associate to E a collection of 2-covering quartic

curves (or homogeneous spaces) H . These have equations of the form

$$(3.6.1) \quad H : \quad y^2 = g(x) = ax^4 + bx^3 + cx^2 + dx + e$$

with $a, b, c, d, e \in \mathbb{Q}$, such that the *invariants*

$$I = 12ae - 3bd + c^2 \quad \text{and} \quad J = 72ace + 9bcd - 27ad^2 - 27eb^2 - 2c^3$$

are related to the c_4 and c_6 invariants of E via

$$I = \lambda^4 c_4 \quad \text{and} \quad J = 2\lambda^6 c_6$$

for some $\lambda \in \mathbb{Q}^*$. Two such quartics $g_1(x)$, $g_2(x)$ are *equivalent* if

$$g_2(x) = \mu^2(\gamma x + \delta)^4 g_1\left(\frac{\alpha x + \beta}{\gamma x + \delta}\right)$$

for some $\alpha, \beta, \gamma, \delta$ and $\mu \in \mathbb{Q}$, with μ and $\alpha\delta - \beta\gamma$ non-zero. The invariants of $g_1(x)$ and $g_2(x)$ are then related by the scaling factor $\lambda = \mu(\alpha\delta - \beta\gamma)$:

$$\begin{aligned} I(g_2) &= \mu^4(\alpha\delta - \beta\gamma)^4 I(g_1), \\ J(g_2) &= \mu^6(\alpha\delta - \beta\gamma)^6 J(g_1). \end{aligned}$$

We set $\Delta = 4I^3 - J^2 = 27\text{disc}(g)$, and call Δ the discriminant.

In particular, by scaling up the coefficients, we may assume that the invariants I and J are integral. The number of equivalence classes of quartics with given invariants (up to a scaling factor λ) which are everywhere locally soluble is finite. One of our tasks will be to determine, for a given integral quartic, an equivalent integral one with minimal invariants. This process is closely analogous to the one considered earlier in this chapter, using Kraus's conditions or Tate's algorithm to determine minimal models for elliptic curves. Indeed, we will see below that if c_4 and c_6 are invariants of a minimal model for the elliptic curve E , then $I = c_4$ and $J = 2c_6$ are also minimal, except possibly at the prime 2. (We may lose minimality at 2 because the equations (3.6.1) we use for homogeneous spaces are not completely general, not having terms in y , xy or x^2y ; to remove these by completing the square involves a scaling by a factor of 2.)

We now explain the relationship between equivalence classes of soluble quartics with invariants I and J and rational points on the elliptic curve. More details of this relationship, including proofs, may be found in [20]. For convenience, we again start by making a coordinate transformation: if c_4 and c_6 are the integral invariants of our curve E , we set $I = c_4$ and $J = 2c_6$, and replace E by the isomorphic curve

$$(3.6.3) \quad E_{I,J} : \quad Y^2 = F(X) = X^3 - 27IX - 27J.$$

This is the model on which the rational points we construct will naturally lie; it is then a simple matter to transfer them back to the original model for E . For simplicity, we will still continue to refer to the curve simply as E when this will not cause confusion.

Associated to each quartic g there are two so-called *covariants*, which we denote g_4 and g_6 :

$$(3.6.4) \quad \begin{aligned} g_4(X, Y) &= (3b^2 - 8ac)X^4 + 4(bc - 6ad)X^3Y + 2(2c^2 - 24ae - 3bd)X^2Y^2 \\ &\quad + 4(cd - 6be)XY^3 + (3d^2 - 8ce)Y^4, \\ g_6(X, Y) &= (b^3 + 8a^2d - 4abc)X^6 + 2(16a^2e + 2abd - 4ac^2 + b^2c)X^5Y \\ &\quad + 5(8abe + b^2d - 4acd)X^4Y^2 + 20(b^2e - ad^2)X^3Y^3 \\ &\quad - 5(8ade + bd^2 - 4bce)X^2Y^4 - 2(16ae^2 + 2bde - 4c^2e + cd^2)XY^5 \\ &\quad - (d^3 + 8be^2 - 4cde)Y^6. \end{aligned}$$

Wherever convenient, we will also denote by g the homogenized polynomial $g(X, Y) = aX^4 + bX^3Y + cX^2Y^2 + dXY^3 + eY^4$. These three homogeneous polynomials satisfy an algebraic identity, or syzygy:

$$(3.6.5) \quad 27g_6^2 = g_4^3 - 48Ig^2g_4 - 64Jg^3.$$

Later we will also need a simpler form of this syzygy; set

$$(3.6.6) \quad p = g_4(1, 0) = 3b^2 - 8ac \quad \text{and} \quad r = g_6(1, 0) = b^3 + 8a^2d - 4abc;$$

these quantities are called *seminvariants* of g . Substituting $(X, Y) = (1, 0)$ in the covariant syzygy (3.6.5) gives an identity (the seminvariant syzygy) between these seminvariants:

$$(3.6.7) \quad 27r^2 = p^3 - 48Ia^2p - 64Ja^3.$$

We will make use of this equation in our search for quartics with given invariants, where it will allow us to set up a quadratic sieve.

It follows from the covariant syzygy (3.6.5), by simple substitution, that the map

$$(3.6.8) \quad \xi : (x, y) \mapsto \left(\frac{3g_4(x, 1)}{(2y)^2}, \frac{27g_6(x, 1)}{(2y)^3} \right)$$

maps rational points (x, y) on H (satisfying $y^2 = g(x, 1)$) to rational points on $E_{I,J}$, thus defining a rational map ξ , of degree 4, from $H(\mathbb{Q})$ to $E_{I,J}(\mathbb{Q})$. We are using affine coordinates here; the points at infinity on H map to $\left(\frac{3p}{4a}, \frac{\pm 27r}{(4a)^{3/2}} \right)$, which are rational if and only if a is a square.

We now have the following facts (see [20] for details):

- If $R \in H(\mathbb{Q})$ with $P = \xi(R) \in E_{I,J}(\mathbb{Q})$, then the coset of P modulo $2E_{I,J}(\mathbb{Q})$ is independent of R , and of the particular quartic g up to equivalence; in fact, equivalences between quartics induce rational maps between the associated homogeneous spaces, and the covariant property of g_4 and g_6 ensures that corresponding rational points on the homogeneous spaces have the same image in $E_{I,J}(\mathbb{Q})$.

- Each rational point $P = (x, y) \in E_{I,J}(\mathbb{Q})$ arises as the image of a rational point on some quartic g with invariants I and J : explicitly, one can take the rational point at infinity on the quartic with coefficients $(a, b, c, d, e) = (1, 0, -x/6, y/27, I/12 - x^2/432)$; the equivalence class of g depends only on the coset of P modulo $2E_{I,J}(\mathbb{Q})$.

- The equivalence classes of everywhere locally soluble quartics with invariants I and J form a finite elementary abelian 2-group, isomorphic to the 2-Selmer group $S^{(2)}(E/\mathbb{Q})$.

- The equivalence classes of soluble quartics with invariants I and J form a finite elementary abelian 2-group isomorphic to $E(\mathbb{Q})/2E(\mathbb{Q})$; the identity is the *trivial* class, consisting of quartics with a rational root.

- More generally, when E has no 2-torsion, for any extension field K of \mathbb{Q} there is a bijection between the roots of $g(x)$ in K and the solutions $Q \in E_{I,J}(K)$ to the equation $2Q = P$ (where $P = \xi(R)$ for $R \in H(\mathbb{Q})$ as above). In particular, non-trivial quartics are irreducible in this case. We will use this fact with $K = \mathbb{R}$ later.

We therefore classify the set of equivalence classes of quartics with invariants I and J as follows:

- (0) the trivial class consists of those quartics $g(x)$ which have a rational root. These are elliptic curves isomorphic to E over \mathbb{Q} .
- (1) those which have a rational point: these are also elliptic curves, isomorphic to E over \mathbb{Q} .
- (2) those which have points everywhere locally.
- (3) those which fail to have points everywhere locally.

Let the number of inequivalent quartics in the first three sets be $n_0 = 1$, n_1 and n_2 . (Those in the last set will not be used.) Because of the group structure, each of these numbers is a power of 2. We write $n_i = 2^{e_i}$ for $i = 1, 2$.

As in the case of descent via 2-isogeny, Galois cohomology gives an exact sequence

$$0 \rightarrow E(\mathbb{Q})/2E(\mathbb{Q}) \rightarrow S^{(2)}(E/\mathbb{Q}) \rightarrow \text{III}(E/\mathbb{Q})[2] \rightarrow 0.$$

Thus the quotient of $S^{(2)}(E/\mathbb{Q})$ by the image of $E(\mathbb{Q})$ is isomorphic to $\text{III}(E/\mathbb{Q})[2]$, the 2-torsion subgroup of the Tate–Shafarevich group $\text{III}(E/\mathbb{Q})$. So it is the points of order 2 in $\text{III}(E/\mathbb{Q})$, if any, which account for the possible existence of homogeneous spaces which have points everywhere locally but not globally, and we have

$$|\text{III}(E/\mathbb{Q})[2]| = n_2/n_1.$$

As before, the potential practical difficulty lies in determining whether each homogeneous space H has a rational point, as there is no known algorithm to do this in general. Again, for the vast majority of the curves in the tables, we found a rational point easily on each space which was everywhere locally soluble, which not only determined the rank of E , but also implied that the Tate–Shafarevich group had no 2-torsion. The only example with $n_1 < n_2$ in the tables (for a curve with no 2-torsion) is curve 571A1, where $n_1 = 1$ and $n_2 = 4$; here the rank is 0, and $|\text{III}(E/\mathbb{Q})[2]| = 4$; the Birch–Swinnerton-Dyer conjecture predicts $|\text{III}(E/\mathbb{Q})| = 4$.

The steps of the algorithm are as follows: first we determine the pair or pairs of integral invariants (I, J) such that every quartic associated with our curve E is equivalent to one with integer coefficients and these invariants. There will be either one or two such pairs. For each pair (I, J) , we find a finite set of quartics with invariants (I, J) such that every non-trivial, everywhere locally soluble quartic with these invariants is equivalent to one in the list. This is the most time-consuming step, as the search region can be very large when I and J are large. Now we must test the quartics in our list pairwise for equivalence, discarding those equivalent to earlier ones; look for rational points; and test everywhere local solubility. Again, there may be quartics where we do not find rational points despite their having points everywhere locally, so that although we can always (given enough time) determine n_2 , we may in some cases only find bounds on n_1 . Since $n_1 = |E(\mathbb{Q})/2E(\mathbb{Q})|$, we can then compute the rank r , or bounds on the rank. Usually, E will have no rational 2-torsion, or we would probably be using descent via 2-isogeny, and then simply $2^r = n_1$.

We now consider each of these steps in more detail.

Step 1: Determining the invariants (I, J) .

Given an integral quartic g with invariants I and J , we must consider the question of whether there exists an equivalent integral quartic with smaller invariants. The smaller invariants will have the form $\lambda^{-4}I$, $\lambda^{-6}J$ with $\lambda \in \mathbb{Q}^*$. In [3, Lemmas 3–5], conditions are stated under which g is equivalent to an integral quartic with invariants $p^{-4}I$, $p^{-6}J$ for a prime p ; we call such a quartic *p-reducible*, otherwise *p-minimal*. Clearly a necessary condition for reducibility is that $p^4 \mid I$ and $p^6 \mid J$. We say that the pair (I, J) is *p-reducible* if every integral quartic with these invariants which is *p-adically soluble* is equivalent to an integral quartic with invariants $p^{-4}I$ and $p^{-6}J$.

The question of *p-reducibility* is almost completely settled by the following proposition. The result is simplest for primes greater than 3, but even for these it is important to note that the assumption of *p-adic solubility* is necessary for reduction to be possible when the divisibility conditions are satisfied.

PROPOSITION 3.6.1. *Let I and J be integers such that $\Delta = 4I^3 - J^2 \neq 0$.*

- (1) *If p is a prime and $p \geq 5$, then (I, J) is *p-reducible* if and only if $p^4 \mid I$ and $p^6 \mid J$.*

- (2) (I, J) is 3-reducible if and only if either $3^5 \mid I$ and $3^9 \mid J$, or $3^4 \parallel I$, $3^6 \parallel J$ and $3^{15} \mid \Delta$.
(3) (I, J) is 2-reducible if $2^6 \mid I$, $2^9 \mid J$ and $2^{10} \mid 8I + J$.

This proposition is stated in [3] as Lemmas 3–5, but only the proof of Lemma 3 (covering the case $p \geq 5$) is given there. Complete proofs in all cases (which are elementary though somewhat lengthy) can be found in [52].

Note that for $p = 2$ we only have sufficient conditions for reducibility. Because of this, we will sometimes have to consider two pairs of invariants, a smaller pair (I_0, J_0) and a larger pair $(16I_0, 64J_0)$. However, when searching for integral quartics with the larger invariants, we may assume that the quartic cannot be 2-reduced, and this provides us with useful congruence conditions on the coefficients of such a quartic. We state these here.

PROPOSITION 3.6.2. *Let g be an integral 2-adically soluble quartic whose invariants satisfy $2^4 \mid I$ and $2^6 \mid J$, such that*

- (1) g is not equivalent to an integral quartic with invariants $2^{-4}I$ and $2^{-6}J$;
(2) g is not equivalent to an integral quartic with the same invariants I and J and smaller leading coefficient a .

Then the coefficients of g satisfy

- (a) $2 \nmid a$, $2^2 \mid b$, $2 \mid c$, $2^4 \nmid e$ and $2^4 \nmid a + b + c + d + e$; or
(b) $2 \parallel a$, $2^2 \mid b$, $2^2 \mid c$, $2^3 \nmid e$ and $2^3 \nmid a + b + c + d + e$.

Moreover, if $2^6 \mid I$ and $2^7 \mid J$, then we must have

- (a') $2 \nmid a$, $2^2 \mid b$, $2^2 \parallel c$, $2^3 \mid d$, and $2^2 \parallel e$; or
(b') $2 \nmid a$, $2^2 \mid b$, $2^2 \parallel c - 2a + 3b$, $2^3 \mid d - b$ and $2^2 \parallel a + c + e$.

The first set of conditions stated here were given in [3]; the second set are from [52], which contains complete proofs in both cases.

Using this proposition, we may ensure that few of the quartics we find when searching the larger pair of invariants are equivalent to one with smaller invariants. More significantly in terms of running time, we have extra congruence conditions to apply when searching for the larger invariants, which speeds up this search.

It would appear that rational points in $E(\mathbb{Q})$ whose quartics have the larger pair of invariants lie in certain components of the 2-adic locus $E(\mathbb{Q}_2)$. Further study of this would be very useful, since if the search for quartics with the larger pair of invariants could be eliminated or curtailed, it could result in a major saving of time in the algorithm.

In practice, suppose that our original curve E is given by a minimal equation, with invariants c_4 and c_6 . We set $I = c_4$ and $J = 2c_6$. Clearly the pair (I, J) is p -minimal for $p \geq 5$: for if $p^4 \mid I$ and $p^6 \mid J$ then $p^{-4}c_4$ and $p^{-6}c_6$ would be integral invariants of an elliptic curve, contradicting minimality of E , and similarly the pair (p^4I, p^6J) is certainly p -reducible by Proposition 3.6.1(1). Less obvious is that (I, J) is also 3-minimal; using Kraus's conditions, it is easy to check first that $(3^4I, 3^6J)$ is certainly 3-reducible (one needs here that $\text{ord}_3(c_6) \neq 2$), and then that (I, J) itself is not 3-reducible, using Proposition 3.6.1(2).

For $p = 2$, the best we can do is the following. First set $I = c_4$ and $J = 2c_6$. Replace (I, J) by $(2^{-4}I, 2^{-6}J)$ if $2^4 \mid I$ and $2^6 \mid J$; the resulting pair (I, J) (which will not be further divisible by 2) will be the basic pair of invariants. Then we also use the pair $(16I, 64J)$ unless $4 \mid I$, $8 \mid J$ and $16 \mid (2I + J)$.

The result of this step is then to produce either one or two pairs of invariants (I, J) . In the latter case, the following steps must be carried out with both pairs separately.

Step 2: Finding the quartics with given I and J .

We now have a fixed pair of invariants (I, J) with $\Delta = 4I^3 - J^2 \neq 0$, and we wish to find all integral quartics with these invariants, up to equivalence. We classify the quartics $g(x)$ into

types, according as $g(x)$ has no real roots (type 1), four real roots (type 2) or two real roots (type 3). When $\Delta < 0$ only type 3 is possible, while if $\Delta > 0$, only types 1 and 2 are possible. For each relevant type, we now determine a finite list of quartics of that type with the given invariants such that every soluble quartic with these invariants is equivalent to at least one on the list. We can ignore quartics which are negative definite (type 1 with $a < 0$), since they will not be soluble over \mathbb{R} . For each type, we will determine a finite region of (a, b, c) -space such that every quartic with invariants I and J is equivalent to at least one in this region.

As observed above, the number of real roots of $g(x)$ is equal to the number of points $Q \in E(\mathbb{R})$ satisfying $2Q = P$, where $P \in E(\mathbb{R})$ is the image under the map ξ of any real point on the homogeneous space H with equation $y^2 = g(x)$. When $\Delta < 0$, the real locus is in one component, and $E(\mathbb{R})$ is isomorphic to the circle group, which is 2-divisible with two 2-torsion points, so in this case the equation $2Q = P$ has exactly two solutions for all $P \in E(\mathbb{R})$. This agrees with the observation just made, that quartics with negative discriminant Δ will all have exactly two real roots.

Consider further the case $\Delta > 0$. Now $E(\mathbb{R})$ has two components, the connected component of the identity $E^0(\mathbb{R})$ and a second component which we call the ‘egg’. There are four 2-torsion points, and $2E(\mathbb{R}) = E^0(\mathbb{R})$. There are therefore two possibilities for a point $P \in E(\mathbb{R})$ and its associated real quartic: if $P \in E^0(\mathbb{R})$, then there are four solutions Q to $2Q = P$, and P will be associated to a quartic of type 2 with four real roots. On the other hand, if $P \notin E^0(\mathbb{R})$, then there are no solutions and the quartic associated to P will be of type 1, with no real roots.

The image of $E(\mathbb{Q})$ in $E(\mathbb{R})/2E(\mathbb{R})$ has order 2 or 1, depending on whether or not there are any rational points on the egg. Thus there are two sub-cases to the case $\Delta > 0$: if $E(\mathbb{Q}) \subset E^0(\mathbb{R})$, then there are no rational points on the egg, the index is 1, and there will be *no* soluble quartics of type 1; on the other hand, if $E(\mathbb{Q}) \not\subset E^0(\mathbb{R})$, then there are rational points on the egg, the index is 2, and there are equal numbers of (equivalence classes of) soluble quartics of types 1 and 2. Those of type 2 will lead to rational points on $E(\mathbb{Q}) \cap E^0(\mathbb{R})$, while those of type 1 will lead to rational points on the egg.

To take advantage of this in practice, when $\Delta > 0$ we will first look for quartics of type 2; let the number of these be n_1^+ , where n_1/n_1^+ is either 1 or 2. At this stage we will already know the rank to within one, since if we set $r^+ = \log_2(n_1^+)$ then (assuming no rational 2-torsion) we have either $r = r^+$ or $r = r^+ + 1$. Then we start to look for quartics of type 1; as soon as we find one which is soluble, then we may abort the search for type 1 quartics at that point, and assert that $r = r^+ + 1$. On the other hand, if we complete the search for quartics of type 1 without finding any soluble ones, then we will know that $r = r^+$, and we will have proved that there are no rational points on the egg. An example of the second possibility happens with the curve $E = [0, 0, 1, -529, -3042]$ (which is the -23 -twist of the curve $[0, 0, 1, -1, 0]$ with conductor 37 and rank 1), which has rank 1 with generator $(46, 264)$ on the identity component, and no rational points on the egg.⁸

If we happened to know in advance that there were rational points on the egg (perhaps by a short preliminary search for such points with small height), then we would already know that $r = r^+ + 1$, and we would not need to search for type 1 quartics at all.

In order to find all integral quartics of a given type (up to equivalence) we proceed as follows. First, following [3], we determine bounds on the coefficients a , b and c . We also set up a sieve based on the seminvariant syzygy (3.6.7) to speed up our search through this region of (a, b, c) -space. For triples (a, b, c) in the region which pass the sieve, we solve for d and e and ensure that they are integral. Finally, we check that the quartic we have constructed satisfies any further congruence conditions we require (for example, when we are using the larger pair of invariants).

⁸Thanks to Nelson Stephens for this example.

The method for bounding the coefficients which is developed in [3] involves using the auxiliary (resolvent) cubic equation

$$(3.6.9) \quad \phi^3 - 3I\phi + J = 0$$

which will have one real root (type 3) or three real roots (types 1 and 2), since its discriminant is 27Δ . Indeed, ϕ is a root of (3.6.9) if and only if $(-3\phi, 0)$ is a point of order 2 on the curve $E_{I,J}$.

In each case, the bound for b arises simply from the fact that the quartics $g(x)$ and $g(x+k)$ are equivalent, and the coefficients of the latter are $(a, b+4ak, \dots)$, so that we may assume that b is reduced modulo $4a$. Also, note that the bounds on c are effectively bounds on the seminvariant $8ac - 3b^2 = -p$, which is how they arise in [3].

Bounds for (a, b, c) : Type 1. Here we may assume $a > 0$ for real solubility. Order the three real roots of (3.6.9) as $\phi_1 > \phi_2 > \phi_3$, and set $K = (4I - \phi_1^2)/3$. Then the bounds on a, b, c are

$$\begin{aligned} 0 < a &\leq \frac{K + K^{\frac{1}{2}}\phi_1}{3K^{\frac{1}{2}} + \phi_1 + 2\phi_2}; \\ -2a < b &\leq 2a; \\ \frac{4a\phi_2 + 3b^2}{8a} &\leq c \leq \frac{4a\phi_1 + 3b^2}{8a}. \end{aligned}$$

Bounds for (a, b, c) : Type 2. This subdivides into subtypes according as $a > 0$ or $a < 0$. For $a > 0$ we take $\phi_1 > \phi_2 > \phi_3$ and search the region

$$\begin{aligned} 0 < a &\leq \frac{I - \phi_2^2}{3(\phi_2 - \phi_3)}; \\ -2a < b &\leq 2a; \\ \frac{4a\phi_2 - \frac{4}{3}(I - \phi_2^2) + 3b^2}{8a} &\leq c \leq \frac{4a\phi_3 + 3b^2}{8a}. \end{aligned}$$

Then for $a < 0$ we take $\phi_1 < \phi_2 < \phi_3$ and search over

$$\begin{aligned} 0 < -a &\leq \frac{I - \phi_2^2}{3(\phi_3 - \phi_2)}; \\ -2|a| < b &\leq 2|a|; \\ \frac{4a\phi_2 - \frac{4}{3}(I - \phi_2^2) + 3b^2}{8a} &\geq c \geq \frac{4a\phi_3 + 3b^2}{8a}. \end{aligned}$$

Bounds for (a, b, c) : Type 3. Here we let ϕ be the unique real root of (3.6.9), and search

$$\begin{aligned} \frac{1}{3}\phi - \sqrt{\frac{4}{27}(\phi^2 - I)} &\leq a \leq \frac{1}{3}\phi + \sqrt{\frac{4}{27}(\phi^2 - I)}; \\ -2|a| < b &\leq 2|a|; \\ \frac{9a^2 - 2a\phi + \frac{1}{3}(4I - \phi^2) + 3b^2}{8|a|} &\leq c \cdot \text{sign}(a) \leq \frac{4a\phi + 3b^2}{8|a|}. \end{aligned}$$

The syzygy sieve. Recall the seminvariant syzygy

$$(3.6.7) \quad 27r^2 = p^3 - 48Ia^2p - 64Ja^3 = s(a, p),$$

say, where $p = 3b^2 - 8ac$ and $r = b^3 + 8a^2d - 4abc$. For fixed I, J the expression $s(a, p)$ is a polynomial in a, b and c , which we require to be 27 times an integer square. We can set up a quadratic sieve as follows: for each of several sieving moduli m we create and initialize an $m \times m$ array indicating whether $s(a, p)$ is 27 times a square modulo m , for each pair (a, p) modulo m . We take one of the moduli to be 9 and use it to force the right-hand side of (3.6.7) to be divisible by 27; it will certainly be positive, as this is ensured by the bounds on c .

For each (a, b, c) in the region searched, we check that it passes the sieving test; it is then quite likely that $s(a, p)$ will be 27 times a square, since it is so modulo a large modulus and is positive. We then test whether this is the case, discarding (a, b, c) if not, and if so we then find r . We can take $r > 0$, since the quartics with coefficients (a, b, c, d, e) and $(a, -b, c, -d, e)$ are equivalent, with opposite signs of their respective r -seminvariants. In fact, we treat the triples $(a, \pm b, c)$ together in practice.

Implementation note: It is worth pointing out that a large proportion of the running time of our algorithm is spent testing whether large integers are squares (given that they are positive and congruent to squares modulo several carefully chosen moduli), and find their integer square root if so. This is needed here, and in our searches for rational points, both on the elliptic curve directly, and on the homogeneous spaces. Hence it is crucial that we have access to efficient procedures for this in the multiprecision integer package we use.

Solving for d and e . Given integers a, b, c, r satisfying (3.6.7) with $p = 3b^2 - 8ac$, we can solve for d and e , setting

$$d = (r - b^3 + 4abc)/(8a^2) \quad \text{and} \quad e = (I + 3bd - c^2)/(12a).$$

This will certainly give rational values for d and e ; we must check that they are integral, discarding the triple (a, b, c) if not. If they are, we have integral coefficients (a, b, c, d, e) of a quartic $g(x)$ with invariants I and J in the search region, which we add to our list for further processing.

Solving for the roots of $g(x)$. For later use, when we check for triviality, and again when we search for rational points on the homogeneous spaces, we will need to know the real roots of the quartic $g(x)$ we have constructed. Although the formulae for finding the roots of quartic are well-known, we give them here: since we already know the roots of the resolvent cubic, there is very little work remaining.

For $i = 1, 2, 3$ we set $z_i = (4a\phi_i + p)/3$ where the ϕ_i are the three roots of (3.6.9). The product of these quantities is r^2 (from (3.6.7) again), and we form their square roots with product r by setting $w_1 = \sqrt{z_1}$, $w_2 = \sqrt{z_2}$, and $w_3 = r/(w_1w_2)$. Then the roots of $g(x)$ are

$$\begin{aligned} x_1 &= (w_1 + w_2 - w_3 - b)/(4a), \\ x_2 &= (w_1 - w_2 + w_3 - b)/(4a), \\ x_3 &= (-w_1 + w_2 + w_3 - b)/(4a), \\ x_4 &= (-w_1 - w_2 - w_3 - b)/(4a). \end{aligned}$$

We will not give here a pseudo-code algorithm for the search for quartics, as it is straightforward in principle, although in practice it needs careful book-keeping. As this is the most

time-consuming part of the whole procedure, particularly when the second, larger, pair of invariants must be used, it is important to make the implementation code as efficient as possible.

At the end of this step we will have a list of quartics with the desired invariants. We now discard any which are equivalent to earlier ones, or are not locally soluble at some prime p , and try to find rational roots on the remainder. In practice we may choose to apply these tests in a different order, such as not bothering to check equivalences between quartics which are not locally soluble.

Step 3: Testing triviality.

For each quartic $g(x)$ in the list, we already know its roots x to reasonable precision. If x is rational, then ax is integral, which we can test. If we suspect that ax is equal to an integer n to within some working tolerance, we can check whether n/a is a root of $g(x)$ using exact arithmetic.

Step 3: Testing equivalence of quartics.

With each quartic we find with the right invariants, we store its coefficients, type, roots and seminvariants p and r . We also compute and store the number of roots of the quartic (including roots at infinity) modulo each of several primes not dividing its discriminant, as these numbers are clearly invariant under equivalence.⁹

When testing equivalence of two quartics, we first check that their invariants and type are the same, as well as their numbers of roots modulo these primes. If this is the case, we use a general test for equivalence (valid over any field) from [20], which we state here.¹⁰

PROPOSITION 3.6.3. *Let g_1 and g_2 be quartics over the field K , both having the same invariants I and J , and with leading coefficients a_i and seminvariants p_i and r_i for $i = 1, 2$. Then g_1 is equivalent to g_2 over K if and only if the quartic $u^4 - 2pu^2 - 8ru + s$ has a root in K , where*

$$\begin{aligned} p &= (32a_1a_2I + p_1p_2)/3, \\ r &= r_1r_2, \quad \text{and} \\ s &= (64I(a_1^2p_2^2 + a_2^2p_1^2 + a_1a_2p_1p_2) - 256a_1a_2J(a_1p_2 + a_2p_1) - p_1^2p_2^2)/27. \end{aligned}$$

The quantities p , r and s in this proposition will be integers when g_1 and g_2 are integral. Converting the proposition into an algorithm is straightforward.

Step 5: Testing local and global solubility.

This is carried out using the procedures and strategy described earlier.

Step 6: Final computation of the rank.

The number of quartics found (up to equivalence) which are everywhere locally soluble is n_2 , the order of the 2-Selmer group. This must be a power of 2, say $n_2 = 2^{e_2}$, which serves as a check on our procedures. The number n_1 with a rational point is also a power of 2, say $n_1 = 2^{e_1}$, equal to the order of $E(\mathbb{Q})/2E(\mathbb{Q})$. If we have found rational points on all n_2 locally soluble quartics, then certainly $n_1 = n_2$, so that $\text{III}(E/\mathbb{Q})[2]$ is trivial, and the rank of $E(\mathbb{Q})$ is $e_1 - e_0$ where $|E(\mathbb{Q})[2]| = 2^{e_0}$ with $e_0 = 0, 1$ or 2 . The rank is equal to the Selmer rank $e_2 - e_0$ in this case. (Usually $e_0 = 0$ when we are using this method.)

As before, we may not have found global points on all the locally soluble quartics; if the number on which we have points is \tilde{n}_1 with $\tilde{n}_1 < n_2$ then we only know that $\tilde{n}_1 \leq n_1 \leq n_2$. If \tilde{n}_1 is not a power of 2, we will know that $n_1 > \tilde{n}_1$, so that at least some of our locally soluble

⁹This was suggested to us by S. Siksek.

¹⁰The algorithm presented here only applies to quartics. In the First Edition we presented a different algorithm, described in [3], which is messier to implement, but which generalizes more readily to more general situations, such as testing the equivalence of binary forms of higher degree.

quartics must have rational points which we have not found. In this case, we replace \tilde{n}_1 by the next highest power of 2, say $\tilde{n}_1 = 2^{\tilde{e}_1}$. Then we have bounds on the rank, namely

$$\tilde{e}_1 - e_0 \leq e_1 - e_0 = \text{rank}(E(\mathbb{Q})) \leq e_2 - e_0,$$

and on the order of $\text{III}(E/\mathbb{Q})[2]$:

$$|\text{III}(E/\mathbb{Q})[2]| \leq n_2/\tilde{n}_1.$$

One final point: from the Selmer conjecture, we expect the Selmer rank $e_2 - e_0$ to differ from the actual rank $e_1 - e_0$ by an even number, so that $e_2 \equiv e_1 \pmod{2}$. This would also follow from the conjecture that $\text{III}(E/\mathbb{Q})$ is finite, since then its order is known to be a perfect square, so that n_2/n_1 must be a square. So if we find that $e_2 \not\equiv \tilde{e}_1 \pmod{2}$, then we suspect that the rank is at least one more than our lower bound, and can output a comment to this effect, though of course we will not have proved that the rank is greater than our lower bound. In some cases, such as for a modular curve where we know the sign of the functional equation, we may have other conjectural evidence for the parity of the rank.

Step 7: Recovering points on E .

Each quartic $g(x)$ for which the homogeneous space $y^2 = g(x)$ has a rational point R leads to a rational point $P = \xi(R)$ on the model $E_{I,J}$ of our curve E , via the formula (3.6.8) given above. If we apply this formula to all the inequivalent quartics with rational points which we found in computing the rank of E , we will have a complete set of coset representatives for $2E(\mathbb{Q})$ in $E(\mathbb{Q})$, provided that $\tilde{n}_1 = n_1$. In cases where we have rounded up \tilde{n}_1 to the nearest power of 2, we will still have generators for $E(\mathbb{Q})/2E(\mathbb{Q})$, and can fill in the missing coset representatives if we wish.

This completes our description of algorithms for determining the Mordell-Weil group $E(\mathbb{Q})$.

3.7 The period lattice

In this section we show how to compute the complex periods for an elliptic curve defined over the complex numbers. We used this in our investigation of modular curves to check that the exact integral equations we found (after rounding the approximate computed values of c_4 and c_6) did have the correct periods; and also in our method for computing isogenous curves, which we describe in the following section.

Let E be an elliptic curve defined over the complex numbers \mathbb{C} , given by a Weierstrass equation. We wish to compute periods λ_1 and λ_2 which are a \mathbb{Z} -basis for the period lattice Λ of E . We do this using Gauss's arithmetic-geometric mean (AGM) algorithm. Write the equation for E in the form

$$\left(y + \frac{a_1x + a_3}{2}\right)^2 = x^3 + \frac{b_2}{4}x^2 + \frac{b_4}{2}x + \frac{b_6}{4} = (x - e_1)(x - e_2)(x - e_3),$$

where the roots e_i are found as complex floating-point approximations (using Cardano's formula, say). Then the periods are given by

$$(3.7.1) \quad \begin{aligned} \lambda_1 &= \frac{\pi}{\text{AGM}(\sqrt{e_3 - e_1}, \sqrt{e_3 - e_2})}, \\ \lambda_2 &= \frac{\pi i}{\text{AGM}(\sqrt{e_3 - e_1}, \sqrt{e_2 - e_1})}. \end{aligned}$$

Notice that in general this involves the AGM of pairs of complex numbers. This is a multi-valued function: at each stage of the AGM algorithm we replace the pair (z, w) by $(\sqrt{zw}, \frac{1}{2}(z+w))$, and must make a choice of complex square root. It follows from work of Cox (see [11]) that while a different set of choices does lead to a different value for the AGM, the periods we obtain this way will nevertheless always be a \mathbb{Z} -basis for the full period lattice Λ . We have found this to be the case in practice, where we always choose a square root with positive real part, or with positive imaginary part when the real part is zero. The computation of λ_1 and λ_2 by this method is very fast, as the AGM algorithm converges extremely quickly, even in its complex form. As a check on the values obtained, in each case we recomputed the invariants c_4 and c_6 of each curve from these computed periods λ_1 and λ_2 , using the standard formulae given in Chapter II; in every case we obtained the correct values (known exactly from the coefficients of the minimal Weierstrass equation) to within computational accuracy.

If the curve is defined over \mathbb{R} , we can avoid the use of the complex AGM, and also arrange that λ_1 is a positive real period, as follows. First suppose that all three roots e_i are real; order the roots so that $e_3 > e_2 > e_1$, and take the positive square root in the above formulae. Then we may use the usual AGM of positive reals in (3.7.1), and thus obtain a positive real value for λ_1 and a pure imaginary value for λ_2 . This is the case where the discriminant $\Delta > 0$ and the period lattice is rectangular. When $\Delta < 0$ there is one real root, say e_3 , and $e_2 = \bar{e}_1$. If $\sqrt{e_3 - e_1} = z = s + it$ with $s > 0$ then $\sqrt{e_3 - e_2} = \bar{z} = s - it$, so that $\lambda_1 = \pi/\text{AGM}(z, \bar{z}) = \pi/\text{AGM}(|z|, s)$ which is also real and positive.

3.8 Finding isogenous curves

Given an elliptic curve E defined over \mathbb{Q} , we now wish to find all curves E' isogenous to E over \mathbb{Q} . The set of all such curves is finite (up to isomorphism), and any two curves in the isogeny class are linked by a chain of isogenies of prime degree l . Thus it suffices to be able to compute l -isogenies for prime l , if we can determine those l for which rational l -isogenies exist. The latter question can be rather delicate in general, and we have to have a completely automatic algorithmic procedure if we are to apply it to several thousand curves, such as we had to when preparing the tables.

When the conductor N of E is square-free, so that E has good or multiplicative reduction at all primes, E is called semi-stable. In this case, a result of Serre (see [53]) says that either E or the isogenous curve E' has a rational point of order l , and so by Mazur's result already mentioned, l can only be 2, 3, 5 or 7. Moreover, if a curve E possesses a rational point of order l , then the congruence $1 + p - a_p \equiv 0 \pmod{l}$ holds for all primes p not dividing Nl , so the presence of such a point is easy to determine, even if it is not E itself but the isogenous curve E' which possesses the rational l -torsion, since the trace of Frobenius a_p is isogeny-invariant.

If E is not semi-stable we argue as follows. The existence of a rational l -isogeny is purely a function of the j -invariant j of E : in fact, pairs (E, E') of l -isogenous curves parametrize the modular curve $X_0(l)$ whose non-cuspidal points are given by the pairs $(j(E), j(E'))$. For $l = 2, 3, 5, 7$ or 13 the genus of $X_0(l)$ is zero, and infinitely many rational j occur. The only other values of l for which rational l -isogenies occur are $l = 11, 17, 19, 37, 43, 67$, and 163 , and these occur for only a small finite number of j -invariants (see below). The fact that no other l occur is a theorem of Mazur (see [39] and [40]), related to the theorem limiting the rational torsion which we quoted earlier in Section 3.3 of this chapter. These extra values occur only for curves with CM (complex multiplication, see the next section), apart from $l = 17$ (where $X_0(l)$ has genus 1) and the exotic case $l = 37$ studied by Mazur and Swinnerton-Dyer in [41] (where $X_0(l)$ has genus 2).

For isogenies of non-prime degree m , the degrees which occur are: $m \leq 10$, and $m = 12, 16, 18$, and 25 (where $X_0(l)$ has genus 0, infinitely many cases); and finally $m = 14, 15, 21$, and

27. The latter occur first for conductors $N = 49$ (with CM), $N = 50$, $N = 162$ and $N = 27$ (with CM) respectively. See [2, pages 78–80] for more details.

Thus our procedure is:

- If N is square-free, try $l = 2, 3, 5, 7$ only;
- else try $l = 2, 3, 5, 7$ and 13 in all cases; and
- if $j(E) = -2^{15}$, -11^2 , or $-11 \cdot 131^3$, try also $l = 11$;
- if $j(E) = -17^2 \cdot 101^3/2$ or $-17 \cdot 373^3/2^{17}$, try also $l = 17$;
- if $j(E) = -96^3$, try also $l = 19$;
- if $j(E) = -7 \cdot 11^3$ or $-7 \cdot 137^3 \cdot 2083^3$, try also $l = 37$;
- if $j(E) = -960^3$, try also $l = 43$;
- if $j(E) = -5280^3$, try also $l = 67$;
- if $j(E) = -640320^3$, try also $l = 163$.

Now we turn to the question of finding all curves (if any) which are l -isogenous to our given curve E for a specific prime l . The kernel of the isogeny is a subgroup A of $E(\overline{\mathbb{Q}})$ which is defined over \mathbb{Q} , but the points of A may not be individually rational points. If we have the coordinates of the points of a subgroup of E of order l defined over K , we may use Vélú's formulae in [68] to find the corresponding l -isogenous curve. Finding such coordinates by algebraic means is troublesome, except when the subgroup is point-wise defined over K , and instead we resort to a floating-point method.

The case $l = 2$ is simpler to describe separately. Obviously in this case the subgroup of order 2 defined over \mathbb{Q} must consist of a single rational point P of order 2 together with the identity. We have already found such points, if any, in computing the torsion. There will be 0, 1 or 3 of them according to the number of rational roots of the cubic $4x^3 + b_2x^2 + 2b_4x + b_6$. If x_1 is such a root, then $P = (x_1, y_1)$ has order 2, where $y_1 = -(a_1x_1 + a_3)/2$. As a special case of Vélú's formulae we find that the isogenous curve E' has coefficients $[a'_1, a'_2, a'_3, a'_4, a'_6] = [a_1, a_2, a_3, a_4 - 5t, a_6 - b_2t - 7w]$ where

$$t = (6x_1^2 + b_2x_1 + b_4)/2 \quad \text{and} \quad w = x_1t.$$

Note that the point (x_1, y_1) need not be integral even when E has integral coefficients a_i , but that $4x_1$ and $8y_1$ are certainly integral, by the formulae given; thus the model just given for the isogenous curve may need scaling by a factor of 2 to make it integral.

The simpler formula for a curve in the form $y^2 = x^3 + cx^2 + dx$ and the point $P = (0, 0)$ was given in the previous section: the formulae just given take the curve $[0, c, 0, d, 0]$ to $[0, c, 0, -4d, -4cd]$, which transforms to $[0, -2c, 0, c^2 - 4d, 0]$ after replacing x by $x - c$. The relation between the two formulae is given by $c = 12x_1 + b_2$ and $d = 16t$.

For reference we give here similar algebraic formulae for l -isogenies for $l = 3$ and $l = 5$, from Laska's book [35]. In each case we assume that the curve E is given by an equation of the form $y^2 = x^3 + ax + b$, and the isogenous curve E' by $y^2 = x^3 + Ax + B$. Each subgroup of E of order l is determined by a rational factor of degree $(l - 1)/2$ of the l -division polynomial of degree $(l^2 - 1)/2$, whose roots are the x -coordinates of the points in the subgroup. The simplest case is $l = 3$, where there is just one x -coordinate, which must be rational.

$l = 3$. Let ξ be a root of the 3-division polynomial $3x^4 + 6ax^2 + 12bx - a^2$. Then the 3-isogenous curve E' is given by

$$\begin{aligned} A &= -3(3a + 10\xi^2) \\ B &= -(70\xi^3 + 42a\xi + 27b). \end{aligned}$$

$l = 5$. Let $x^2 + h_1x + h_2$ be a rational factor of the 5-division polynomial $5x^{12} + 62ax^{10} + 380bx^9 - 105a^2x^8 + 240abx^7 - (300a^3 + 240b^2)x^6 - 696a^2bx^5 - (125a^4 + 1920ab^2)x^4 - (1600b^3 + 80a^3b)x^3 - (50a^5 + 240a^2b^2)x^2 - (100a^4b + 640ab^3)x + (a^6 - 32a^3b^2 - 256b^4)$. Then the 5-isogenous curve E' is given by

$$\begin{aligned} A &= -19a - 30(h_1^2 - 2h_2) \\ B &= -55b - 14(15h_1h_2 - 5h_1^3 - 3ah_1). \end{aligned}$$

A similar formula is given in [35] for $l = 7$, where A and B are given in terms of a , b and the coefficients of a factor $x^3 + h_1x^2 + h_2x + h_3$ of the 7-division polynomial. Rather than take up space by giving the latter here, we refer the reader to [35, page 72].

Now we turn to Vélú's formulae in the case of an odd prime l . Let $P = (x_1, y_1)$ be a point of order l in $E(\overline{\mathbb{Q}})$, and set $kP = (x_k, y_k)$ for $1 \leq k \leq (l-1)/2$. Define

$$t_k = 6x_k^2 + b_2x_k + b_4 \quad \text{and} \quad u_k = 4x_k^3 + b_2x_k^2 + 2b_4x_k + b_6,$$

and then set

$$t = \sum_{k=1}^{(l-1)/2} t_k \quad \text{and} \quad w = \sum_{k=1}^{(l-1)/2} (u_k + x_k t_k).$$

Then the isogenous curve E' has coefficients $[a_1, a_2, a_3, a_4 - 5t, a_6 - b_2t - 7w]$ as before. Again, these may not be integral, even when the original coefficients were; but since the x_k are the roots of a polynomial of degree $(l-1)/2$ with integral coefficients and leading coefficient l^2 (the so-called l -division equation), we must have l^2x_k integral. Thus a scaling factor of l will certainly produce an integral equation.

We make these remarks on integrality as our method is to find the coordinates x_k and y_k as real floating-point approximations, and thus to determine the coefficients of any curves l -isogenous to E over \mathbb{R} ; there will always be exactly two such curves over \mathbb{R} , but of course they will not necessarily be defined over \mathbb{Q} . As we will only know the coefficients a'_i of the isogenous curves approximately, we wish to ensure that if they are rational then they will in fact be integral, so that we will be able to recognize them as such.

First we find the period lattice Λ of E , as described in the previous section. The \mathbb{Z} -basis $[\lambda_1, \lambda_2]$ of Λ is normalized as follows: there are two cases to consider, according as $\Delta > 0$ (first or 'harmonic' case) or $\Delta < 0$ (second or 'anharmonic' case). In both cases λ_1 is real (the least positive real period); in the first case, λ_2 is pure imaginary, while in the second case, $2\lambda_1 - \lambda_2$ is pure imaginary. We can also ensure that $\tau = \lambda_2/\lambda_1$ is in the upper half-plane; however we can not simultaneously arrange that τ is in the usual fundamental region for $\text{SL}(2, \mathbb{Z})$, and this needs to be remembered when evaluating the Weierstrass functions below.

Of the $l+1$ subgroups of \mathbb{C}/Λ of order l , the two defined over \mathbb{R} are the one generated by $z = \lambda_1/l$ (in both cases), and in the first case, the one generated by $z = \lambda_2/l$, or in the second case, the one generated by $z = (\lambda_1 - 2\lambda_2)/l$. Thus z/λ_1 is either $1/l$, τ/l , or $(1-2\tau)/l$. Let $\wp(z; \tau)$ denote the Weierstrass \wp -function relative to the lattice $[1, \tau]$. Then we have

$$x_k = \wp(kz\lambda_1^{-1}; \tau)\lambda_1^{-2} - \frac{1}{12}b_4 \quad \text{and} \quad y_k = \frac{1}{2}(\wp'(kz\lambda_1^{-1}; \tau)\lambda_1^{-3} - a_1x_k - a_3).$$

Here we have had to take account of the lattice scaling $[\lambda_1, \lambda_2] = \lambda_1[1, \tau]$, and also of the fact that $(\wp(z), \wp'(z))$ is a point on the model of E of the form $y^2 = 4x^3 - g_2x - g_3 = 4x^3 - (c_4/12)x - (c_6/216)$ rather than a standard model where the coefficient of x^3 is 1.

We evaluate these points of order l numerically for $k = 1, 2, \dots, (l-1)/2$, for each of the two values of z (depending on whether we are in case 1 or case 2). Substituting into Vélú's

formulae, we obtain in each case the real coefficients a'_i of a curve which is l -isogenous to E over \mathbb{R} . If these coefficients are close to integers we round them and check that the resulting curve over \mathbb{Q} has the same conductor N as the original curve E . If not, we also test the curve with coefficients $l^i a'_i$.

The resulting program finds l -isogenous curves very quickly for any given prime l . We run it for all primes l in the set determined previously, applying it recursively to each new curve found until we have a set of curves closed under l -isogeny for these values of l . Since the set of primes l for which a rational l -isogeny exists is itself an isogeny invariant, once we have finished processing the first curve in the class, we will already know which primes l to use for all the remaining curves.

Some care needs to be taken with a method of computation such as this, where we use floating-point arithmetic to find integers. The series we use to compute the periods and the Weierstrass function and its derivative all converge very quickly, so that we can compute the a'_i to whatever precision is available, though of course in practice some rounding error is bound to arise. When we test whether a floating-point number is ‘approximately an integer’ in the program, we must make a judgement on how close is close enough. With too relaxed a test, we will find too many curves are ‘approximately integral’; usually these will fail the next hurdle, where we test the conductor, but this takes time to check (using Tate’s algorithm). On the other hand, too strict a test might mean that we miss some rational isogenies altogether, which is far more serious. In compiling the tables, there was only one case which caused trouble after the program had been finely tuned. The resulting error resulted in a curve (916B1) being erroneously listed as 3-isogenous to itself in the first (preprint) edition of the tables; this is possible only when a curve has complex multiplication, which is not the case here, though it does not often occur even in the complex multiplication case (see the remarks in the next section). Unfortunately the error was not noticed in the automatic generation of the typeset tables, and I am grateful to Elkies for spotting it.¹¹ The curve $E = [0, 0, 0, -1013692, 392832257]$ has three real points of order 2, two of which are equal to seven significant figures; the period ratio is approximately $7i$. One of the curves 2-isogenous to E over \mathbb{R} has coefficients $[0, 0, 0, -1013691.999999999992, 392832257.000000006]$, which are extremely close to those of E itself. Thus this new curve, which is not defined over \mathbb{Q} , passed both our original tests (the coefficients are extremely close to integers, and the rounded coefficients are those of a curve of the right conductor, namely E itself). After this example was discovered, we inserted an extra line in the program, to print a warning whenever a supposedly isogenous curve was the original curve itself, and reran the program on all 2463 isogeny classes (which only takes a few minutes of machine time). The result was that expected, namely that 916B1 is the only curve for which this phenomenon occurs within the range of the tables¹². There is no example of a curve actually l -isogenous to itself with conductor less than 1000.

Our original implementation of this algorithm in Algol68 used a precision of approximately 30 significant figures for its real and complex arithmetic, which was sufficient to find all the isogenous curves up to conductor 1000. However, our implementation in C++ misses several isogenous curves when using standard double precision, with approximately 15 digits (though this runs very quickly); we need to use a multiprecision floating-point package (such as the one included in LiDIA) to obtain a satisfactory working program, though the resulting code runs very much slower. In our extended computations to conductor 5077, we have computed the isogenies independently using both a C++/LiDIA program and a PARI program, and the results agree.

When we were initially persuaded to extend the tables to include isogenous curves as well as the modular curves themselves, we were afraid that the total number of resulting curves

¹¹This error also somehow survived into the first edition of this book, despite these comments in the text.

¹²Another example of the same type occurs for curve 1342C3, where the period ratio is approximately $9.5i$.

would be rather larger than it turned out to be. On average, we found that the number of curves per isogeny class was $5113/2463$, or just under 2.08. We do not know of any asymptotic analysis, or even a heuristic argument, which would predict an average number of two curves per class. However, it is dangerous to generalize from the limited amount of data which we have available. In the extended computations to conductor 5077, the ratio slowly diminishes; for all curves up to this conductor, the ratio is $31570/17583 = 1.795$.

3.9 Twists and complex multiplication

Traces of Frobenius.

If E is given by a standard minimal Weierstrass equation over \mathbb{Z} , then for all primes p of good reduction the trace of Frobenius a_p is given by

$$a_p = 1 + p - |E(\mathbb{F}_p)|.$$

If E has bad reduction at p , this same formula gives the correct value for the p th Fourier coefficient of the L -series of E .

Since in our applications we never needed to compute a_p for large primes p , we used a very simple method to count the number of points on E modulo p . First, for all primes p in the desired range (say $3 \leq p \leq 1000$; $p = 2$ would be dealt with separately), we precompute the number $n(t, p)$ of solutions to the congruence $s^2 \equiv t \pmod{p}$. Then we simply compute

$$a_p = p - \sum_{x=0}^{p-1} n(4x^3 + b_2x^2 + 2b_4x + b_6, p).$$

This was sufficient for us to compute a_p for all $p < 1000$ for all the curves in the table, which we did to compare with the corresponding Hecke eigenvalues. For large p , there are far more efficient methods, such as the baby-step giant-step method or Schoof's algorithm (see [51]). Details of these may be found in [9]. More recently, even better algorithms have been developed, by Atkin, Elkies, Morain, Müller and others. For example, Morain and Lercier in 1995 successfully computed the number of points on the curve $[0, 0, 0, 4589, 91228]$ over \mathbb{F}_p for $p = 10^{499} + 153$, a prime with 500 decimal digits. This took 4200 hours of computer time.

Twists.

A twist of a curve E over \mathbb{Q} is an elliptic curve defined over \mathbb{Q} and isomorphic to E over $\overline{\mathbb{Q}}$ but not necessarily over \mathbb{Q} itself. Thus the set of all twists of E is the set of all curves with the same j -invariant as E . These can be simply described, as follows.

First suppose that $c_4 \neq 0$ and $c_6 \neq 0$; equivalently, $j \neq 1728$ and $j \neq 0$ (respectively). Then the twists of E are all quadratic, in that they become isomorphic to E over a quadratic extension of \mathbb{Q} . For each integer d (square-free, not 0 or 1), there is a twisted curve $E * d$ with invariants d^2c_4 and d^3c_6 , which is isomorphic to E over $\mathbb{Q}(\sqrt{d})$. If E has a model of the form $y^2 = f(x)$ with $f(x)$ cubic, then $E * d$ has equation $dy^2 = f(x)$. A minimal model for $E * d$ may be found easily by the Laska–Kraus–Connell algorithm. The conductor of $E * d$ is only divisible by primes dividing ND , where D is the discriminant of $\mathbb{Q}(\sqrt{d})$. The simplest case is when $\gcd(D, N) = 1$; then $E * d$ has conductor ND^2 . More generally, if $D^2 \nmid N$ then $E * d$ has conductor $\text{lcm}(N, D^2)$, but if $D^2 \mid N$ then the conductor may be smaller; for example, $(E * d) * d$ is isomorphic to E , so has conductor N again.

Twisting commutes with isogenies, in the sense that if two curves E, F are l -isogenous then so are their twists $E * d, F * d$. If E has no complex multiplication (see below), then the structure of the isogeny class of E is a function of $j(E)$ alone.

The trace of Frobenius of $E * d$ at a prime p not dividing N is $\chi(p)a_p$, where χ is the quadratic character associated to $\mathbb{Q}(\sqrt{d})$ and a_p is the trace of Frobenius of E . Thus if E is modular, attached to the newform f , then $E * d$ is also modular and attached to the twisted form $f \otimes \chi$, in the notation of Chapter 2.

When $j = 0$ (or equivalently $c_4 = 0$), E has an equation of the form $y^2 = x^3 + k$ with $k \in \mathbb{Z}$ non-zero and free of sixth powers. Such curves have complex multiplication by $\mathbb{Z}[(1 + \sqrt{-3})/2]$. Two such curves with parameters k, k' are isomorphic over $\mathbb{Q}(\sqrt[6]{k/k'})$.

Similarly, when $j = 1728$ (or equivalently $c_6 = 0$), E has an equation of the form $y^2 = x^3 + kx$ with $k \in \mathbb{Z}$ non-zero and free of fourth powers. Such curves have complex multiplication by $\mathbb{Z}[\sqrt{-1}]$. Two such curves with parameters k, k' are isomorphic over $\mathbb{Q}(\sqrt[4]{k/k'})$.

Complex multiplication.

Each of the 13 imaginary quadratic orders \mathfrak{D} of class number 1 has a rational value of $j(\mathfrak{D}) = j(\omega_1/\omega_2)$, where $\mathfrak{D} = \mathbb{Z}\omega_1 + \mathbb{Z}\omega_2$. Elliptic curves E with $j(E) = j(\mathfrak{D})$ have complex multiplication: their ring of endomorphisms defined over \mathbb{C} is isomorphic to \mathfrak{D} . In all other cases the endomorphism ring of an elliptic curve defined over \mathbb{Q} is isomorphic to \mathbb{Z} , since an elliptic curve with complex multiplication by an order of class number $h > 1$ has a j -invariant which is not rational, but algebraic of degree h over \mathbb{Q} .

We give here a table of triples (D, j, N) where $j = j(\mathfrak{D})$ for an order of discriminant D , and N is the smallest conductor of an elliptic curve defined over \mathbb{Q} with this j -invariant. All but the last three values ($D = -43, -67, -163$) have $N < 1000$ and so occur in the tables.

D	-4	-16	-8	-3	-12	-27	-7	-28	-11	-19	-43	-67	-163
j	12^3	66^3	20^3	0	$2 \cdot 30^3$	$-3 \cdot 160^3$	-15^3	255^3	-32^3	-96^3	-960^3	-5280^3	-640320^3
N	32	32	256	27	36	27	49	49	121	361	43^2	67^2	163^2

If E has complex multiplication by the order \mathfrak{D} of discriminant D , then the twist $E * D$ is isogenous to E , though not usually isomorphic to E (over \mathbb{Q}). Indeed, the only cases where E is isomorphic to $E * D$ are $D = -4$ and $D = -16$ with $j(E) = 1728$: the curves $y^2 = x^3 + 16kx$ and $y^2 + 256kx$ are twists of, and isomorphic to, $y^2 = x^3 + kx$. Since curves are isogenous if and only if they have the same L -series by Faltings's Theorem (see [22]), this implies that E has complex multiplication if and only if $a_p = \chi(p)a_p$ for all primes p , where χ is the quadratic character as above. Thus $a_p = 0$ for half the primes p , namely those for which $\chi(p) = -1$. This gives an alternative way of recognizing a curve with complex multiplication, from its traces of Frobenius. This is particularly convenient in the case of modular curves, where we compute the a_p first, and will always know when a newform f , and hence the associated curve E_f , has complex multiplication. For, in such a case, we must have $D^2 \mid N$ and $f = f \otimes \chi$, which we may easily check from the tables.

EXAMPLES

We give here some worked examples of the methods described in the preceding chapter, to illustrate and clarify the different situations which arise. The first example is $N = 11$, which is the first non-trivial level; here we give most detail. Then we consider $N = 33$, where we encounter oldforms and more complicated M-symbols, and $N = 37$, where there are two newforms, one of which has $L(f, 1) = 0$, necessitating a different method of computing Hecke eigenvalues. Finally we look at a square level, $N = 49$, to illustrate the direct method of computing periods.

Example 1: $N = 11$

For simplicity we will only work in $H(11)$, rather than the smaller quotient space $H^+(11)$. The M-symbols for $N = 11$ are $(c : 1)$ for c modulo 11 and $(1 : 0)$, which we abbreviate as (c) and (∞) respectively, with $|c| \leq 5$. (Similarly with other prime levels). The 2-term and 3-term relations (2.2.6) and (2.2.7) are as follows.

$$\begin{aligned} (0) + (\infty) &= 0 \\ (1) + (-1) &= 0 & (0) + (\infty) + (-1) &= 0 \\ (2) + (5) &= 0 & (1) + (-2) + (5) &= 0 \\ (-2) + (-5) &= 0 & (2) + (4) + (-4) &= 0 \\ (3) + (-4) &= 0 & (3) + (-5) + (-3) &= 0 \\ (-3) + (4) &= 0 \end{aligned}$$

Solving these equations we can express all 12 symbols in terms of $A = (2)$, $B = (3)$ and $C = (0)$:

$$\begin{aligned} (0) &= C & (3) &= B \\ (\infty) &= -C & (-3) &= A - B \\ (1) &= (-1) = 0 & (4) &= B - A \\ (2) &= (-2) = A & (-4) &= -B \\ (5) &= (-5) = -A \end{aligned}$$

There are two classes of cusps, $[0]$ and $[\infty]$, with $[a/b] = [0]$ if $11 \nmid b$ and $[a/b] = [\infty]$ if $11 \mid b$. Hence $\delta((c)) = \delta(\{0, 1/c\}) = [1/c] - [0] = 0$ for $c \not\equiv 0$. It follows that

$$H(11) = \ker(\delta) = \langle A, B \rangle,$$

with $2g = \dim H(11) = 2$, so that the genus is 1. There is therefore one newform f . This makes the rest of the calculation simpler, as we do not have to find and split off eigenspaces.

The conjugation $*$ involution maps $(c) \mapsto (-c)$, so $A^* = A$ and $B^* = A - B$. This has matrix $\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$ with respect to the basis A, B . The $+1$ - and -1 -eigenspaces are generated

by A and $A - 2B$ respectively, and we have left eigenvectors $v^+ = (2, 1)$ and $v^- = (0, 1)$. Thus the period lattice is Type 1 (non-rectangular), and $\Omega(f) = \Omega_0(f) = \langle A, f \rangle$.

If we had worked in $H^+(11)$, viewed as the quotient $H(11)/H^-(11)$, by including relations $(c) = (-c)$, the effect would be to identify (c) and $(-c)$. This gives a 1-dimensional space generated by \overline{B} with $\overline{A} = 2\overline{B}$, where the bars denote the projections to the quotient. Notice that although \overline{B} is a generator here, the integral of f over B is not a real period; its real part is half the real period. However we do still have $\Omega(f) = \langle B + B^*, f \rangle = 2\operatorname{Re} \langle B, f \rangle$, so we could compute $\Omega(f)$ in this context without actually knowing whether it was 1 or 2 times the smallest real period.

To compute Hecke eigenvalues we may work in the subspace $\langle A \rangle$; since this subspace is conjugation invariant (being the +1-eigenspace) we will have $T_p(A) = a_p A$ for all $p \neq 11$. We first compute T_2 explicitly. The first method, converting the M-symbol $A = (2 : 1)$ to the modular symbol $\{0, 1/2\}$, gives:

$$\begin{aligned} T_2(A) &= T_2\left(\left\{0, \frac{1}{2}\right\}\right) = \{0, 1\} + \left\{0, \frac{1}{4}\right\} + \left\{\frac{1}{2}, \frac{3}{4}\right\} \\ &= \{0, 1\} + \left\{0, \frac{1}{4}\right\} + \left\{\frac{1}{2}, 1\right\} + \left\{1, \frac{3}{4}\right\} \\ &= (1 : 1) + (4 : 1) + (1 : 2) + (-4 : 1) \\ &= (1) + (4) + (-5) + (-4) \\ &= 0 + (B - A) + (-A) + (-B) = -2A, \end{aligned}$$

so that $a_2 = -2$. Alternatively, using the Heilbronn matrices from Section 2.4, we compute:

$$\begin{aligned} T_2(A) &= T_2((2 : 1)) = (2 : 1)R_2 \\ &= (2 : 1) \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} + (2 : 1) \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} + (2 : 1) \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix} + (2 : 1) \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} \\ &= (2 : 2) + (4 : 1) + (4 : 3) + (3 : 2) \\ &= (1) + (4) + (5) + (-4) \\ &= 0 + (B - A) + (-A) + (-B) \\ &= -2A. \end{aligned}$$

Now $(1 + 2 - a_2)L(f, 1) = \langle \{0, 1/2\}, f \rangle = \langle A, f \rangle = \Omega(f)$, giving

$$\frac{L(f, 1)}{\Omega(f)} = \frac{1}{5}.$$

For all primes $p \neq 11$ we will evaluate $\mu_p = \sum_{a=0}^{p-1} \{0, a/p\} = n_p A$ for a certain integer n_p , since then also $1/5 = n_p/(1 + p - a_p)$, giving

$$a_p = 1 + p - 5n_p.$$

At this stage we already know that the corresponding elliptic curve has rank 0, and that $1 + p - a_p \equiv 0 \pmod{5}$ for all $p \neq 11$, so that it will possess a rational 5-isogeny.

To save time, we can use the fact that $\{0, a/p\}^* = \{0, -a/p\}$; thus for odd p we need only evaluate half the sum, say

$$\mu'_p = \sum_{a=1}^{(p-1)/2} \left\{0, \frac{a}{p}\right\},$$

and then set $\mu_p = \mu'_p + (\mu'_p)^*$.

For $p = 3$, we have $\mu'_3 = \{0, 1/3\} = (3 : 1) = (3) = B$, so $\mu_3 = B + B^* = A$, giving $n_3 = 1$ and $a_3 = 1 + 3 - 5n_3 = -1$.

For $p = 5$ we compute:

$$\begin{aligned} \left\{0, \frac{1}{5}\right\} &= (5 : 1) = (5) = -A; \\ \left\{0, \frac{2}{5}\right\} &= \left\{0, \frac{1}{2}\right\} + \left\{\frac{1}{2}, \frac{2}{5}\right\} \\ &= (2 : 1) + (-5 : 2) = (2) + (3) = A + B; \\ \mu'_5 &= (-A) + (A + B) = B; \\ \mu_5 &= B + B^* = A, \quad \text{so that } n_5 = 1; \\ a_5 &= 1 + 5 - 5n_5 = 1. \end{aligned}$$

Similarly, with $p = 7$ we have $n_7 = 2$, so that $a_7 = 1 + 7 - 5n_7 = -2$, and with $p = 13$ we have $n_{13} = 2$ so that $a_{13} = 4$.

These computations can also be carried out using Heilbronn matrices, by applying the Hecke operators directly to $C = (0 : 1) = \{0, \infty\}$, as follows. Once we know that $a_2 = -2$, we have

$$-2C = T_2(C) = T_2((0 : 1)) = (0 : 1)R_2 = (0 : 2) + (0 : 1) + (0 : 1) + (1 : 2) = 3C - A,$$

giving $C = \frac{1}{5}A$ in agreement with the ratio $L(f, 1)/\Omega(f)$ found earlier. Similarly, using the Heilbronn matrices R_3 listed in Section 2.4, we find

$$\begin{aligned} a_3C &= T_3(C) = (0 : 1)R_3 \\ &= (0 : 3) + (0 : 1) + (1 : 3) + (0 : 1) + (0 : 1) + (1 : -3) \\ &= C + C + (B - A) + C + C + (-B) \\ &= 4C - A = -C, \end{aligned}$$

giving $a_3 = -1$ again.

For the prime $q = 11$ we compute the involution W_{11} induced by the action of the matrix $\begin{pmatrix} 0 & -1 \\ 11 & 0 \end{pmatrix}$:

$$\begin{aligned} W_{11}(A) &= \begin{pmatrix} 0 & -1 \\ 11 & 0 \end{pmatrix} \left\{0, \frac{1}{2}\right\} = \left\{\infty, \frac{-2}{11}\right\} \\ &= \{\infty, 0\} + \left\{0, \frac{-1}{5}\right\} + \left\{\frac{-1}{5}, \frac{-2}{11}\right\} \\ &= (1 : 0) + (-5 : 1) + (11 : 5) \\ &= (\infty) + (-5) + (0) \\ &= -A, \end{aligned}$$

so that the eigenvalue ε_{11} of W_{11} is -1 . In fact, this was implicit earlier, since $L(f, 1) \neq 0$ implies that the sign of the functional equation is $+1$, which is minus the eigenvalue of the Fricke involution W_{11} .

The Fourier coefficients $a(n) = a(n, f)$ for $1 \leq n \leq 16$ are now given by the following table.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$a(n)$	1	-2	-1	2	1	2	-2	0	-2	-2	1	-2	4	4	-1	-4

Here we have used multiplicativity, and:

$$\begin{aligned}
 a(11) &= -\varepsilon_{11} = +1; \\
 a(4) &= a(2)^2 - 2a(1) = 2; \\
 a(8) &= a(2)a(4) - 2a(2) = 0; \\
 a(16) &= a(2)a(8) - 2a(4) = -4; \\
 a(9) &= a(3)^2 - 3a(1) = -2.
 \end{aligned}$$

We know that the period lattice Λ_f has a \mathbb{Z} -basis of the form $[\omega_1, \omega_2] = [2x, x + iy]$, where $\omega_1 = P_f(A)$ and $\omega_2 = P_f(B)$. We can compute the real period $\omega_1 = \Omega(f) = 5L(f, 1)$ by computing $L(f, 1)$:

$$L(f, 1) = 2 \sum_{n=1}^{\infty} \frac{a(n)}{n} t^n$$

where $t = \exp(-2\pi/\sqrt{11}) = 0.15\dots$. Using the first 16 terms which we have, already gives this to 13 decimal places:

$$L(f, 1) = 0.2538418608559\dots;$$

thus

$$\omega_1 = \Omega(f) = 1.269209304279\dots$$

For the imaginary period y we twist with a prime $l \equiv 3 \pmod{4}$. Here $l = 3$ will do, since

$$\gamma_3 = \left\{0, \frac{1}{3}\right\} - \left\{0, \frac{-1}{3}\right\} = (3) - (-3) = -A + 2B \neq 0.$$

To project onto the minus eigenspace we take the dot product of this cycle (expressed as a row vector $(-1, 2)$) with $v^- = (0, 1)$ to get $m^-(3) = 2$. Hence

$$y = \frac{1}{2i}P(3, f) = \frac{\sqrt{3}}{2}L(f \otimes 3, 1).$$

Summing the series for $L(f \otimes 3, 1)$ to 16 terms gives only 4 decimals:

$$L(f \otimes 3, 1) = 1.6845\dots$$

This is less accurate than $L(f, 1)$ since this series is a power series in $\exp(-2\pi/3\sqrt{11}) = 0.53\dots$, compared with $0.15\dots$. Hence $y = 1.4588\dots$, so that

$$\omega_2 = 0.634604652139\dots + 1.4588\dots i.$$

So far we have only used the Hecke eigenvalues a_p for $p \leq 13$, and only 16 terms of each series. If we use these approximate values for the period lattice generators ω_1 and ω_2 we already find the approximate values $c_4 = 495.99$ and $c_6 = 20008.09$ which round to the integer values $c_4 = 496$ and $c_6 = 20008$. Taking the first 25 a_p and the first 100 terms of the series gives

$$c_4 = 495.99999999999954\dots \quad \text{and} \quad c_6 = 20008.00000000085.$$

The exact values $c_4 = 496$ and $c_6 = 20008$ are the invariants of an elliptic curve of conductor 11, which is in fact the modular curve E_f :

$$y^2 + y = x^3 - x^2 - 10x - 20.$$

This is the first curve in the tables, with code 11A1 (or Antwerp code 11B). The value $L(f, 1)/\Omega(f) = 1/5$ agrees with the value predicted by the Birch–Swinnerton-Dyer conjecture for $L(E_f, 1)/\Omega(E_f)$, provided that E_f has trivial Tate–Shafarevich group.

We now illustrate the shortcut method presented in Section 2.11, where we guess the imaginary period and lattice type without computing $H(11)$. Having computed $P(3, f) = 2.9176\dots i$ which is certainly non-zero, we consider the lattices $\langle x, yi \rangle$ and $\langle 2x, x + yi \rangle$, where $2x = 1.2692\dots$ (from above) and $yi = P(3, f)/m^-$, for $m^- = 1, 2, 3, \dots$. With $m^- = 1$ we do not find integral invariants, but for $m^- = 2$ and lattice type 1 we find the curve $E_f = [0, -1, 1, -10, -20]$ given above¹.

Using the first variant of the method, where we do not even know x , we can take $l^+ = 5$ since $P(5, f) = 6.346\dots \neq 0$. The correct value of m^+ here is 10; if we do not know this, but try $m^+ = 1, 2, 3, \dots$ in a double loop with m^- , the first valid lattice we come across is with $(m^+, m^-) = (2, 2)$ and type 1, which leads to the curve $E' = [0, -1, 1, 0, 0]$, also of conductor 11; this is 5-isogenous to the “correct” curve E_f , which comes from $(m^+, m^-) = (10, 2)$ and type 1.

We may also consider the ratios $P(l, f)/P(3, f)$ for other primes $l \equiv 3 \pmod{4}$; we restrict to those l satisfying $\left(\frac{-11}{l}\right) = \left(\frac{l}{11}\right) = +1$, since otherwise $P(l, f)$ is trivially 0 (since the sign of the functional equation for the corresponding $L(f \otimes \chi, s)$ is then -1). We find the following table of values (rounded: they are only computed approximately):

l	3	23	31	47	59	67	71	103	163	179	191	199	223	251
$\frac{P(l, f)}{P(3, f)}$	1	1	1	0	1	9	1	0	4	25	1	4	1	1

The zero values for $l = 47$ and $l = 103$ indicate that the corresponding twists of the newform f have positive even analytic rank (one can check that the corresponding twists of the curve E_f do indeed have rank 2). As all these values are integral here (*a priori* they are only known to be rational) we do not find any nontrivial divisor of m^- (which we know in fact equals 2). The fact that all the integers are perfect squares is an amusing observation, but has a simple explanation in terms of the numbers appearing in the Birch–Swinnerton-Dyer conjecture for the twists of E_f .

There is one other curve E'' isogenous to E_f in addition to E' (found above). If the period lattice of $E_f = [0, -1, 1, -10, -20]$ is $\langle 2x, yi \rangle$ with $x = 0.6346\dots$ and $y = 1.4588\dots$, then $E' = [0, -1, 1, 0, 0]$ has period lattice $\langle 10x, 5x + yi \rangle$, and $E'' = [0, -1, 1, -7820, 263580]$ has lattice $\langle x/5, 2x/5 + yi \rangle$. These curves are linked by 5-isogenies $E_f \leftrightarrow E'$ and $E_f \leftrightarrow E''$.

Finally, we compute the degree of the modular parametrization $\varphi: X_0(11) \rightarrow E_f$. Of course, this is obviously 1, since the modular curve $X_0(11)$ has genus 1, so that φ is the identity map in this case; but this example will serve to illustrate the general method.

The twelve M-symbols form 4 triangles which we choose as follows:

$$\begin{aligned} &(1, 0), (-1, 1), (0, 1); && (1, 1), (-2, 1), (-1, 2); \\ &(1, 2), (-3, 1), (-2, 3); && (1, 3), (-4, 1), (-3, 4). \end{aligned}$$

¹Here, $[a_1, a_2, a_3, a_4, a_6]$ denote the Weierstrass coefficients of the curve; see Chapter 3.

There are two τ -orbits, corresponding to the two cusps at ∞ (of width 1) and at 0 (of width 11). The first contributes nothing. The second is as follows:

$$\begin{aligned} (1, 0) &\mapsto (1, 1) \mapsto (1, 2) \mapsto (1, 3) \mapsto (1, 4) \equiv (-2, 3) \mapsto (-2, 1) \mapsto (-2, -1) \\ &\equiv (-3, 4) \mapsto (-3, 1) \mapsto (-3, -2) \equiv (-4, 1) \mapsto (-4, -3) \equiv (-1, 2) \mapsto (-1, 1) \mapsto (1, 0). \end{aligned}$$

There are four jump matrices coming from the above sequence. From $(1, 4) \equiv (-2, 3)$ we obtain

$$\delta_1 = \begin{pmatrix} 0 & -1 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} -2 & -1 \\ 11 & 5 \end{pmatrix};$$

the others are $\delta_2 = \begin{pmatrix} 4 & 1 \\ 11 & 3 \end{pmatrix}$, $\delta_3 = \begin{pmatrix} -5 & -1 \\ 11 & 2 \end{pmatrix}$ and $\delta_4 = \begin{pmatrix} -3 & 1 \\ 11 & -4 \end{pmatrix}$. Using modular symbols, we can compute the coefficients of $P_f(\delta_i)$ with respect to the period basis ω_1, ω_2 , to obtain $P_f(\delta_1) = -\omega_1$, $P_f(\delta_2) = \omega_2$, $P_f(\delta_3) = \omega_1$, and $P_f(\delta_4) = -\omega_2$. Hence

$$\begin{aligned} \deg(\varphi) &= \frac{1}{2} \left(\left| \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right| + \left| \begin{array}{cc} 0 & 0 \\ -1 & 1 \end{array} \right| + \left| \begin{array}{cc} 0 & -1 \\ -1 & 0 \end{array} \right| + \left| \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right| + \left| \begin{array}{cc} 1 & -1 \\ 0 & 0 \end{array} \right| + \left| \begin{array}{cc} 0 & -1 \\ 1 & 0 \end{array} \right| \right) \\ &= \frac{1}{2} (1 + 0 - 1 + 1 + 0 + 1) = 1, \end{aligned}$$

as expected.

Example 2: $N = 33$

Since $33 = 3 \cdot 11$, the number of M-symbols is $48 = 4 \cdot 12$, consisting of 33 symbols $(c) = (c : 1)$, 13 symbols $(1 : d)$ with $\gcd(d, 33) > 1$, and the symbols $(3 : 11)$ and $(11 : 3)$. (In fact, whenever N is a product pq of 2 distinct primes, the M-symbols have this form, with exactly two symbols, $(p : q)$ and $(q : p)$ not of the form $(c : 1)$ or $(1 : d)$).

There are four cusp classes represented by 0, $1/3$, $1/11$ and ∞ , with the class of a cusp a/b being determined by $\gcd(b, 33)$. (Similarly, whenever N is square-free, the cusp classes are in one-one correspondence with the divisors of N).

Using the two-term and three-term relations, and including the relations $(c : d) = (-c : d)$, we can express all the M-symbols in terms of six of them, and $\ker(\delta^+) = \langle (7), (2), (15) - (9) \rangle$. Hence $H^+(33)$ is three-dimensional. We know there will be a two-dimensional oldclass coming from the newform at level 11, so there will also be a single newform f at this level.

If we compute the images of the basis modular symbols $\{0, 1/7\}$, $\{0, 1/2\}$ and $\{1/9, 1/15\}$ under T_2 and W_{33} , we find that they have matrices

$$T_2 = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -2 \end{pmatrix} \quad \text{and} \quad W_{33} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}.$$

T_2 has a double eigenvalue of -2 , coming from the oldforms, which we ignore, and also the new eigenvalue $a_2 = 1$ with left eigenvector $v = (0, 1, 0)$. The corresponding eigenvalue for W_{33} is $\varepsilon_{33} = -1$. Hence the sign of the functional equation is $+$, and the analytic rank is even. Moreover since the eigencycle for a_2 is the second basis element, which is $\{0, 1/2\} = \mu_2$, we have $2(1 + 2 - a_2)L(f, 1) = \Omega(f)$, so that

$$\frac{L(f, 1)}{\Omega(f)} = \frac{1}{4}.$$

In particular, $L(f, 1) \neq 0$, so that the analytic rank is 0. Note that because we have factored out the pure imaginary component, we do not usually know at this stage whether the least real period $\Omega_0(f)$ is equal to $\Omega(f)$ or half this; all we can say is that $\Omega(f)/2 = \operatorname{Re} \langle \{0, 1/2\}, f \rangle$ is the least real part of a period (up to sign). But in this case, $\{0, 1/2\}$ is certainly an integral cycle, and since $\langle \{0, 1/2\}, f \rangle$ is real, we can in fact deduce already that the period lattice is of type 2 (rectangular) with $\Omega(f) = 2\Omega_0(f)$.

To compute more a_p we express each cycle μ_p as a linear combination of the basis and project to the eigenspace by taking the dot product with the left eigenvector v , which just amounts in this case to taking the second component. In this way we find $a_5 = -2$, $a_7 = 4$, $a_{13} = -2$, and so on. For the involutions W_3 and W_{11} we can either compute their 3×3 matrices or just apply them directly to the eigencycle $\{0, 1/2\}$, and we find that $\varepsilon_3 = +1$ and $\varepsilon_{11} = -1$. In fact we already knew that the product of these was $\varepsilon_{33} = -1$, so we need not have computed ε_{11} directly, though doing so serves as a check.

Now we go back and compute the full space $H(33)$, which is six-dimensional, with basis

$$\left\{0, \frac{1}{7}\right\}, \left\{0, \frac{1}{4}\right\}, \left\{0, \frac{-1}{4}\right\}, \left\{\frac{1}{12}, \frac{-1}{6}\right\}, \left\{\frac{1}{12}, \frac{-1}{3}\right\}, \left\{0, \frac{1}{10}\right\}.$$

By computing the 6×6 matrices of conjugation and T_2 , we may pick out the left eigenvectors

$$v^+ = (0, 1, -1, 1, 2, 0) \quad \text{and} \quad v^- = (-1, 0, 0, 2, 1, 1).$$

Since these vectors are independent modulo 2, it follows (as expected) that the period lattice is type 2, with a \mathbb{Z} -basis of the form $[\omega_1, \omega_2] = [x, yi]$.

Firstly, $x = \Omega_0(f) = \Omega(f)/2 = 2L(f, 1)$. Summing the series for $L(f, 1)$ we obtain $L(f, 1) = 0.74734\dots$, so that $\omega_1 = x = 1.49468\dots$ and $\Omega(f) = 2x = 2.98936\dots$. Then we use the twisting prime $l = 7$: the twisting cycle

$$\gamma_7 = \sum_{a=1}^6 \left(\frac{a}{7}\right) \left\{0, \frac{a}{7}\right\}$$

is evaluated in terms of our basis to be $(2, 2, 0, -2, 0, 0)$, whose dot product with v^- is -6 . Hence $y = \sqrt{7}L(f \otimes 7, 1)/6$. The value of $L(f \otimes 7, 1)$ is determined by summing the series to be $3.11212\dots$, so that $y = 1.37232\dots$ and $\omega_2 = 1.37232\dots i$. If we evaluate these from the first 100 terms of the series, using a_p for $p < 100$, we find the approximate values $c_4 = 552.99999\dots$ and $c_6 = -4084.99947\dots$. These round to $c_4 = 553$ and $c_6 = -4085$, which are the invariants of the curve 33A1: $y^2 + xy = x^3 + x^2 - 11x$. Notice that this curve has four rational points, which we could have predicted since the ratio $L(f, 1)/\Omega(f) = 1/4$ implies that $1 + p - a_p \equiv 0 \pmod{4}$ for all $p \neq 2, 3, 11$.

Example 3: $N = 37$

Since 37 is prime the M-symbols are simple here, as for $N = 11$. We find that $H^+(37)$ is two-dimensional, generated by $A = (8)$ and $B = (13)$. With this basis the matrices of T_2 and W_{37} are

$$T_2 = \begin{pmatrix} -2 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad W_{37} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Thus we have two one-dimensional eigenspaces, generated by A and B respectively, with eigenvalues $(a_2 = -2, \varepsilon_{37} = +1)$ for A and $(a_2 = 0, \varepsilon_{37} = -1)$ for B . The left eigenvectors are simply

$v_1 = (1, 0)$ and $v_2 = (0, 1)$. Let us denote the corresponding newforms by f and g respectively. Now $\{0, 1/2\} = 2B$, so

$$\frac{L(f, 1)}{\Omega(f)} = 0 \quad \text{and} \quad \frac{L(g, 1)}{\Omega(g)} = \frac{1}{3}.$$

The fact that $\varepsilon_{37}(f) = +1$ implies that f has odd analytic rank, while the previous line shows that g has analytic rank 0.

To compute Hecke eigenvalues, the method we used previously would only work for g , so instead we use the variation discussed in Section 2.9. The cycle $\{1/5, \infty\}$ projects non-trivially onto both eigenspaces. In fact $(1 + 2 - T_2)\{1/5, \infty\} = -5A - B$, so the components in the two eigenspaces are $(-5)/(1 + 2 - (-2)) = -1$ and $(-1)/(1 + 2 - 0) = -1/3$. Hence by computing $(1 + p - T_p)\{1/5, \infty\} = n_1(p)A + n_2(p)B$ for other primes $p \neq 37$, we may deduce that

$$a(p, f) = 1 + p + n_1(p) \quad \text{and} \quad a(p, g) = 1 + p + 3n_2(p).$$

In this way we find that the first few Hecke eigenvalues are as follows:

p	2	3	5	7	11	13	17	19	...
$a(p, f)$	-2	-3	-2	-1	-5	-2	0	0	...
$a(p, g)$	0	1	0	-1	3	-4	6	2	...

Two things can be noticed here: the preponderance of negative values amongst the first few $a(p, f)$ means that the curve E_f has many points modulo p for small p , which we might expect heuristically since we know that its analytic rank is odd, and hence positive. Secondly, since $1 + p - a(p, g) \equiv 0 \pmod{3}$ for all $p \neq 37$, we know that E_g will have a rational 3-isogeny.

Turning to the full space $H(37)$, we find that it has basis $\langle (8), (16), (20), (28) \rangle$. Conjugation and W_{37} have matrices

$$\begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & -1 & -1 & 0 \end{pmatrix}$$

respectively.

For the A eigenspace corresponding to f we find left eigenvectors $v_1^+ = (-1, 1, 0, 0)$ and $v_1^- = (-1, 0, -1, 1)$. These are independent modulo 2, so the period lattice is rectangular, say $[x, yi]$. To find x we must twist by a real quadratic character, using a prime $l \equiv 1 \pmod{4}$. Here $l = 5$ will do: the twisting cycle is $\{0, 1/5\} - \{0, 2/5\} - \{0, 3/5\} + \{0, 4/5\} = (2, -2, 0, 2)$, whose dot product with v_1^+ is -4 , so that $x = \sqrt{5}L(f \otimes 5, 1)/4$. For the imaginary period we use $l = 3$ with twisting cycle $(0, 0, -1, 1)$ and a dot product of 2 with v_1^- , so that $y = \sqrt{3}L(f \otimes 3, 1)/2$. Evaluating numerically, using 100 terms of the series and a_p for $p < 100$, we find the values

$$\begin{aligned} L(f \otimes 5, 1) &= 5.35486\dots, & \text{so that} & \quad x = 2.99346\dots; \\ L(f \otimes 3, 1) &= 2.83062\dots, & \text{so that} & \quad y = 2.45139\dots; \end{aligned}$$

and finally,

$$\begin{aligned} c_4 &= 47.9999999996\dots, \\ c_6 &= -216.000000004\dots \end{aligned}$$

The rounded values $c_4 = 48$ and $c_6 = -216$ are those of the curve 37A1, with equation $y^2 + y = x^3 - x$. This curve does have rank 1. We may also check that the analytic rank is 1

by computing $L'(f, 1)$ by summing the series given in Section 2.13: we find that $L'(f, 1) = 0.306\dots$, which is certainly non-zero.

The B eigenspace is handled similarly to the example at level 33. We find $v_2^+ = (0, 1, 1, 1)$ and $v_2^- = (1, 1, 0, 0)$. The period lattice is $[x, iy]$ with $x = 3L(g, 1)/2$ and $y = \sqrt{19}L(g \otimes 19, 1)/4$. The latter needs more terms to compute to sufficient accuracy, as 19 is larger than the twisting primes we have previously used. Using $p < 100$ as before we find $c_4 = 1119.878\dots$, which rounds to the correct (with hindsight) value 1120, but for c_6 we get 36304.495, and neither 36304 nor 36305 is correct. Going back to compute a_p for $100 < p < 200$ we reevaluate the series to 200 terms, and find

$$\begin{aligned} L(g, 1) &= 0.72568\dots, & \text{so that} & \quad x = 1.08852\dots; \\ L(g \otimes 19, 1) &= 1.62207\dots, & \text{so that} & \quad y = 1.76761\dots; \end{aligned}$$

and hence

$$c_4 = 1120.000008\dots, \quad \text{and} \quad c_6 = 36295.99943\dots$$

Now the rounded values $c_4 = 1120$ and $c_6 = 36296$ are the invariants of the curve 37B1 with equation $y^2 + y = x^3 + x^2 - 23x - 50$. As expected, this curve does admit a rational 3-isogeny.

Example 4: $N = 49$

$H(49)$ is two-dimensional, with a basis consisting of the M-symbols (11), (2). Hence there is a unique newform f at this level, which must be its own -7 -twist, or in other words have complex multiplication by -7 . The conjugation matrix with respect to this basis is $\begin{pmatrix} -1 & 0 \\ -1 & 1 \end{pmatrix}$, so we take $v^+ = (1, -2)$ and $v^- = (1, 0)$. Hence the period lattice has the form $[2x, x + yi]$ with $2x = \Omega_0(f) = \Omega(f)$. Also $a_2 = 1$, so we have $L(f, 1)/\Omega(f) = 1/2$. Hence we may compute the real period via $L(f, 1)$ as before, and find $L(f, 1) = 0.96666\dots$, so that $\Omega(f) = 1.9333\dots$. But the method we have used in the earlier examples to find the imaginary period will not work here, since for every prime $l \equiv 3 \pmod{4}$, $l \neq 7$, we have $L(f \otimes l, 1) = 0$, since $\chi(-49) = \chi(-1) = -1$ where χ is the associated quadratic character modulo l .

Instead, we compute periods directly, as in Section 2.10. The cycle (5) = $\{0, 1/5\}$ is equal to (11) + (2), from which it follows that $\langle (5), f \rangle = -x + yi$; the coefficients are the dot products of the vector $(1, 1)$ with v^\pm . Now $\{0, 1/5\} = \{0, M(0)\}$ with $M = \begin{pmatrix} 10 & 1 \\ 49 & 5 \end{pmatrix}$. Hence the simpler formula (2.10.5) gives

$$P_f(M) = \left\langle \left\{ 0, \frac{1}{5} \right\}, f \right\rangle = -x + yi = \sum_{n=1}^{\infty} \frac{a(n)}{n} e^{-2\pi n/49} (e^{2\pi i n x_2} - e^{2\pi i n x_1})$$

where $x_1 = -5/49$ and $x_2 = 10/49$. Summing the first 100 terms as before, we find the values

$$x = 0.96666\dots \quad \text{and} \quad y = 2.557536\dots$$

Of course, the value of x merely confirms the value we had previously obtained a different way. These values give, in turn,

$$c_4 = 104.99992\dots \quad \text{and} \quad c_6 = 1322.9994\dots$$

which round to the exact invariants $c_4 = 105$ and $c_6 = 1323$ of the curve 49A1, which has equation $y^2 + xy = x^3 - x^2 - 2x - 1$.

Using the improved formula (2.10.8) with better convergence, gives (also using 100 terms)

$$x = 0.96665585\dots \quad \text{and} \quad y = 2.55753099\dots,$$

which lead to the better values

$$c_4 = 104.999992\dots \quad \text{and} \quad c_6 = 1322.99998\dots.$$

In this computation, we have not exploited the presence of complex multiplication. Notice that, in fact, $y/x = \sqrt{7}$. Obviously if we had known this it would have given us an easier way of computing y from x , and hence from $L(f, 1)$. However not all newforms at square levels have complex multiplication. Some are twists of forms at lower levels (for example, 100A is the 5-twist of 20A, and 144B is the -3 -twist of 48A), which means that we could find the associated curves more easily by twisting the earlier curve. Others first appear at the square level in pairs which are twists of each other (for example, 121A and 121C are -11 -twists of each other, and 196B is the -7 -twist of 196A). One could probably find both periods of all such forms by looking at suitable twists to moduli not coprime to the level, but we have not done this systematically, as the more direct method was adequate in all the cases we came across in compiling the tables.

In practice we always computed the Hecke eigenvalues for $p < 1000$ at least, with a larger bound for higher levels. In some cases, particularly when the target values of c_4 or (more usually) c_6 were large, and especially when a large twisting prime was needed, we needed to sum the series to several thousand terms before obtaining the values of c_4 and c_6 to sufficient accuracy.

These four examples exhibit essentially all the variations which can occur. The only problem with the larger levels is one of scale, as the number of symbols and the dimensions of the spaces grow. A large proportion of the computation time, in practice, is taken up with Gaussian elimination. This is why we have tried wherever possible to reduce the size of the matrices which occur: first by carefully using the 2-term symbol relations to identify symbols in pairs as early as possible, and secondly by working in $H^+(N)$ during the stage where we are searching for Hecke eigenvalues. The symbol relation matrices are very sparse (with at most three entries per row); sparse matrix techniques, which we use in our implementation, help greatly here. For finding eigenvectors of the Hecke algebra, however, we use a completely general purpose exact Gaussian elimination procedure.

The second time-consuming stage is when we are computing a large number of Hecke eigenvalues, where we call a very large number of times the procedures to convert rational numbers (cusps) to M-symbols and look these up in tables to find their coordinates with respect to the symbol basis. It is vital that these procedures are written efficiently; during the preparation of the tables, many great improvements in the efficiency of the program were achieved over a period of several months.

TABLE 1

ELLIPTIC CURVES

The table is arranged in blocks by conductor. Each conductor is given in factorized form at the top of its block (repeated, if necessary, on continuation pages), together with the number of isogeny classes of curves with that conductor. Each block is subdivided into isogeny classes by a row of dashes.

The columns of the table give the following data for each curve E :

- (1) an identifying letter (A, B, C, . . .) for each isogeny class of curves with the same conductor, choosing consecutive letters for the curves in the order in which they were computed. Within each isogeny class we also number the curves in that class, with curve 1 being the “strong Weil curve”.¹ For ease of reference, when $N \leq 200$ we also give the identifying letter of each curve as given in Table 1 of [2].
- (2) The integer coefficients a_1, a_2, a_3, a_4 and a_6 of a minimal equation for E .
- (3) The rank r of $E(\mathbb{Q})$.
- (4) The order $|T|$ of the torsion subgroup T of $E(\mathbb{Q})$.
- (5) The sign of the discriminant Δ of E , and its factorization.
- (6) The prime factorization of the denominator of $j(E)$.
- (7) The local indices c_p for the primes of bad reduction.
- (8) The Kodaira symbols for E at each prime of bad reduction.
- (9) The curves isogenous to E via an isogeny of prime degree, with the degree l in bold face. For example, the entry “**2**: 3; **3**: 2, 6” for curve 448C4 indicates it is 2-isogenous to 448C3 and 3-isogenous to both 448C2 and 448C6. From these entries it is easy to draw isogeny diagrams for each isogeny class in the manner of the Antwerp tables [2]. We regret that we could not persuade Birch to draw little diagrams for us in this column, as he did for [2].

For convenience, we give the factorization of N at the head of each section of the table. This order of the ‘bad’ prime factors p_1, \dots, p_k of N is used within the table itself. We give the discriminant $\Delta = \pm p_1^{e_1} \dots p_k^{e_k}$ in factorized form as \pm, e_1, \dots, e_k in the columns headed $s, \text{ord}(\Delta)$. The column headed $\text{ord}_-(j)$ contains the exponents of these same primes in the denominator of the j -invariant, as in [2]. Finally the local factors c_p , and then the Kodaira symbols, are given for each of these primes in order.

¹For class 990H the “strong” curve is 990H3 and not 990H1.

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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11 $N = 11 = 11$ (1 isogeny class) **11**

A1(B)	0	-1	1	-10	-20	0	5	-	5	5	5	I_5	5 : 2, 3
A2(C)	0	-1	1	-7820	-263580	0	1	-	1	1	1	I_1	5 : 1
A3(A)	0	-1	1	0	0	0	5	-	1	1	1	I_1	5 : 1

14 $N = 14 = 2 \cdot 7$ (1 isogeny class) **14**

A1(C)	1	0	1	4	-6	0	6	-	6, 3	6, 3	2, 3	I_6, I_3	2 : 2; 3 : 3, 4
A2(D)	1	0	1	-36	-70	0	6	+	3, 6	3, 6	1, 6	I_3, I_6	2 : 1; 3 : 5, 6
A3(E)	1	0	1	-171	-874	0	2	-	18, 1	18, 1	2, 1	I_{18}, I_1	2 : 5; 3 : 1
A4(A)	1	0	1	-1	0	0	6	-	2, 1	2, 1	2, 1	I_2, I_1	2 : 6; 3 : 1
A5(F)	1	0	1	-2731	-55146	0	2	+	9, 2	9, 2	1, 2	I_9, I_2	2 : 3; 3 : 2
A6(B)	1	0	1	-11	12	0	6	+	1, 2	1, 2	1, 2	I_1, I_2	2 : 4; 3 : 2

15 $N = 15 = 3 \cdot 5$ (1 isogeny class) **15**

A1(C)	1	1	1	-10	-10	0	8	+	4, 4	4, 4	2, 4	I_4, I_4	2 : 2, 3, 4
A2(E)	1	1	1	-135	-660	0	4	+	8, 2	8, 2	2, 2	I_8, I_2	2 : 1, 5, 6
A3(B)	1	1	1	-5	2	0	8	+	2, 2	2, 2	2, 2	I_2, I_2	2 : 1, 7, 8
A4(F)	1	1	1	35	-28	0	8	-	2, 8	2, 8	2, 8	I_2, I_8	2 : 1
A5(H)	1	1	1	-2160	-39540	0	2	+	4, 1	4, 1	2, 1	I_4, I_1	2 : 2
A6(G)	1	1	1	-110	-880	0	2	-	16, 1	16, 1	2, 1	I_{16}, I_1	2 : 2
A7(D)	1	1	1	-80	242	0	4	+	1, 1	1, 1	1, 1	I_1, I_1	2 : 3
A8(A)	1	1	1	0	0	0	4	-	1, 1	1, 1	1, 1	I_1, I_1	2 : 3

17 $N = 17 = 17$ (1 isogeny class) **17**

A1(C)	1	-1	1	-1	-14	0	4	-	4	4	4	I_4	2 : 2
A2(B)	1	-1	1	-6	-4	0	4	+	2	2	2	I_2	2 : 1, 3, 4
A3(D)	1	-1	1	-91	-310	0	2	+	1	1	1	I_1	2 : 2
A4(A)	1	-1	1	-1	0	0	4	+	1	1	1	I_1	2 : 2

19 $N = 19 = 19$ (1 isogeny class) **19**

A1(B)	0	1	1	-9	-15	0	3	-	3	3	3	I_3	3 : 2, 3
A2(C)	0	1	1	-769	-8470	0	1	-	1	1	1	I_1	3 : 1
A3(A)	0	1	1	1	0	0	3	-	1	1	1	I_1	3 : 1

20 $N = 20 = 2^2 \cdot 5$ (1 isogeny class) **20**

A1(B)	0	1	0	4	4	0	6	-	8, 2	0, 2	3, 2	IV^*, I_2	2 : 2; 3 : 3
A2(A)	0	1	0	-1	0	0	6	+	4, 1	0, 1	3, 1	IV, I_1	2 : 1; 3 : 4
A3(D)	0	1	0	-36	-140	0	2	-	8, 6	0, 6	1, 2	IV^*, I_6	2 : 4; 3 : 1
A4(C)	0	1	0	-41	-116	0	2	+	4, 3	0, 3	1, 1	IV, I_3	2 : 3; 3 : 2

21 $N = 21 = 3 \cdot 7$ (1 isogeny class) **21**

A1(B)	1	0	0	-4	-1	0	8	+	4, 2	4, 2	4, 2	I_4, I_2	2 : 2, 3, 4
A2(D)	1	0	0	-49	-136	0	4	+	2, 4	2, 4	2, 2	I_2, I_4	2 : 1, 5, 6
A3(C)	1	0	0	-39	90	0	8	+	8, 1	8, 1	8, 1	I_8, I_1	2 : 1
A4(A)	1	0	0	1	0	0	4	-	2, 1	2, 1	2, 1	I_2, I_1	2 : 1
A5(F)	1	0	0	-784	-8515	0	2	+	1, 2	1, 2	1, 2	I_1, I_2	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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24 $N = 24 = 2^3 \cdot 3$ (1 isogeny class) **24**

A1(B)	0	-1	0	-4	4	0	8	+	8, 2	0, 2	4, 2	I_1^*, I_2	2 : 2, 3, 4
A2(C)	0	-1	0	-24	-36	0	4	+	10, 4	0, 4	2, 2	III^*, I_4	2 : 1, 5, 6
A3(D)	0	-1	0	-64	220	0	4	+	10, 1	0, 1	2, 1	III^*, I_1	2 : 1
A4(A)	0	-1	0	1	0	0	4	-	4, 1	0, 1	2, 1	III, I_1	2 : 1
A5(F)	0	-1	0	-384	-2772	0	2	+	11, 2	0, 2	1, 2	II^*, I_2	2 : 2
A6(E)	0	-1	0	16	-180	0	2	-	11, 8	0, 8	1, 2	II^*, I_8	2 : 2

26 $N = 26 = 2 \cdot 13$ (2 isogeny classes) **26**

A1(B)	1	0	1	-5	-8	0	3	-	3, 3	3, 3	1, 3	I_3, I_3	3 : 2, 3
A2(C)	1	0	1	-460	-3830	0	1	-	9, 1	9, 1	1, 1	I_9, I_1	3 : 1
A3(A)	1	0	1	0	0	0	3	-	1, 1	1, 1	1, 1	I_1, I_1	3 : 1
B1(D)	1	-1	1	-3	3	0	7	-	7, 1	7, 1	7, 1	I_7, I_1	7 : 2
B2(E)	1	-1	1	-213	-1257	0	1	-	1, 7	1, 7	1, 1	I_1, I_7	7 : 1

27 $N = 27 = 3^3$ (1 isogeny class) **27**

A1(B)	0	0	1	0	-7	0	3	-	9	0	3	IV^*	3 : 2, 3
A2(D)	0	0	1	-270	-1708	0	1	-	11	0	1	II^*	3 : 1
A3(A)	0	0	1	0	0	0	3	-	3	0	1	II	3 : 1, 4
A4(C)	0	0	1	-30	63	0	3	-	5	0	1	IV	3 : 3

30 $N = 30 = 2 \cdot 3 \cdot 5$ (1 isogeny class) **30**

A1(A)	1	0	1	1	2	0	6	-	4, 3, 1	4, 3, 1	2, 3, 1	I_4, I_3, I_1	2 : 2; 3 : 3
A2(B)	1	0	1	-19	26	0	12	+	2, 6, 2	2, 6, 2	2, 6, 2	I_2, I_6, I_2	2 : 1, 4, 5; 3 : 6
A3(C)	1	0	1	-14	-64	0	2	-	12, 1, 3	12, 1, 3	2, 1, 1	I_{12}, I_1, I_3	2 : 6; 3 : 1
A4(D)	1	0	1	-69	-194	0	6	+	1, 12, 1	1, 12, 1	1, 12, 1	I_1, I_{12}, I_1	2 : 2; 3 : 7
A5(E)	1	0	1	-289	1862	0	6	+	1, 3, 4	1, 3, 4	1, 3, 2	I_1, I_3, I_4	2 : 2; 3 : 8
A6(F)	1	0	1	-334	-2368	0	4	+	6, 2, 6	6, 2, 6	2, 2, 2	I_6, I_2, I_6	2 : 3, 7, 8; 3 : 2
A7(G)	1	0	1	-5334	-150368	0	2	+	3, 4, 3	3, 4, 3	1, 4, 1	I_3, I_4, I_3	2 : 6; 3 : 4
A8(H)	1	0	1	-454	-544	0	2	+	3, 1, 12	3, 1, 12	1, 1, 2	I_3, I_1, I_{12}	2 : 6; 3 : 5

32 $N = 32 = 2^5$ (1 isogeny class) **32**

A1(B)	0	0	0	4	0	0	4	-	12	0	4	I_3^*	2 : 2
A2(A)	0	0	0	-1	0	0	4	+	6	0	2	III	2 : 1, 3, 4
A3(C)	0	0	0	-11	-14	0	2	+	9	0	1	I_0^*	2 : 2
A4(D)	0	0	0	-11	14	0	4	+	9	0	2	I_0^*	2 : 2

33 $N = 33 = 3 \cdot 11$ (1 isogeny class) **33**

A1(B)	1	1	0	-11	0	0	4	+	6, 2	6, 2	2, 2	I_6, I_2	2 : 2, 3, 4
A2(A)	1	1	0	-6	-9	0	2	+	3, 1	3, 1	1, 1	I_3, I_1	2 : 1
A3(D)	1	1	0	-146	621	0	4	+	3, 4	3, 4	1, 4	I_3, I_4	2 : 1
A4(C)	1	1	0	44	55	0	2	-	12, 1	12, 1	2, 1	I_{12}, I_1	2 : 1

34 $N = 34 = 2 \cdot 17$ (1 isogeny class) **34**

A1(A)	1	0	0	-3	1	0	6	+	6, 1	6, 1	6, 1	I_6, I_1	2 : 2; 3 : 3
A2(B)	1	0	0	-43	105	0	6	+	3, 2	3, 2	3, 2	I_3, I_2	2 : 1; 3 : 4
A3(C)	1	0	0	-103	-411	0	2	+	2, 3	2, 3	2, 1	I_2, I_3	2 : 4; 3 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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35 $N = 35 = 5 \cdot 7$ (1 isogeny class) **35**

A1(B)	0	1	1	9	1	0	3	−	3, 3	3, 3	1, 3	I_3, I_3	3 : 2, 3
A2(C)	0	1	1	−131	−650	0	1	−	9, 1	9, 1	1, 1	I_9, I_1	3 : 1
A3(A)	0	1	1	−1	0	0	3	−	1, 1	1, 1	1, 1	I_1, I_1	3 : 1

36 $N = 36 = 2^2 \cdot 3^2$ (1 isogeny class) **36**

A1(A)	0	0	0	0	1	0	6	−	4, 3	0, 0	3, 2	IV, III	2 : 2; 3 : 3
A2(B)	0	0	0	−15	22	0	6	+	8, 3	0, 0	3, 2	IV*, III	2 : 1; 3 : 4
A3(C)	0	0	0	0	−27	0	2	−	4, 9	0, 0	1, 2	IV, III*	2 : 4; 3 : 1
A4(D)	0	0	0	−135	−594	0	2	+	8, 9	0, 0	1, 2	IV*, III*	2 : 3; 3 : 2

37 $N = 37 = 37$ (2 isogeny classes) **37**

A1(A)	0	0	1	−1	0	1	1	+	1	1	1	I_1	
B1(C)	0	1	1	−23	−50	0	3	+	3	3	3	I_3	3 : 2, 3
B2(D)	0	1	1	−1873	−31833	0	1	+	1	1	1	I_1	3 : 1
B3(B)	0	1	1	−3	1	0	3	+	1	1	1	I_1	3 : 1

38 $N = 38 = 2 \cdot 19$ (2 isogeny classes) **38**

A1(D)	1	0	1	9	90	0	3	−	9, 3	9, 3	1, 3	I_9, I_3	3 : 2, 3
A2(E)	1	0	1	−86	−2456	0	1	−	27, 1	27, 1	1, 1	I_{27}, I_1	3 : 1
A3(C)	1	0	1	−16	22	0	3	−	3, 1	3, 1	1, 1	I_3, I_1	3 : 1
B1(A)	1	1	1	0	1	0	5	−	5, 1	5, 1	5, 1	I_5, I_1	5 : 2
B2(B)	1	1	1	−70	−279	0	1	−	1, 5	1, 5	1, 1	I_1, I_5	5 : 1

39 $N = 39 = 3 \cdot 13$ (1 isogeny class) **39**

A1(B)	1	1	0	−4	−5	0	4	+	2, 2	2, 2	2, 2	I_2, I_2	2 : 2, 3, 4
A2(C)	1	1	0	−69	−252	0	2	+	4, 1	4, 1	2, 1	I_4, I_1	2 : 1
A3(D)	1	1	0	−19	22	0	4	+	1, 4	1, 4	1, 4	I_1, I_4	2 : 1
A4(A)	1	1	0	1	0	0	2	−	1, 1	1, 1	1, 1	I_1, I_1	2 : 1

40 $N = 40 = 2^3 \cdot 5$ (1 isogeny class) **40**

A1(B)	0	0	0	−7	−6	0	4	+	8, 2	0, 2	2, 2	I_1^*, I_2	2 : 2, 3, 4
A2(D)	0	0	0	−107	−426	0	2	+	10, 1	0, 1	2, 1	III*, I_1	2 : 1
A3(A)	0	0	0	−2	1	0	4	+	4, 1	0, 1	2, 1	III, I_1	2 : 1
A4(C)	0	0	0	13	−34	0	4	−	10, 4	0, 4	2, 4	III*, I_4	2 : 1

42 $N = 42 = 2 \cdot 3 \cdot 7$ (1 isogeny class) **42**

A1(A)	1	1	1	−4	5	0	8	−	8, 2, 1	8, 2, 1	8, 2, 1	I_8, I_2, I_1	2 : 2
A2(B)	1	1	1	−84	261	0	8	+	4, 4, 2	4, 4, 2	4, 2, 2	I_4, I_4, I_2	2 : 1, 3, 4
A3(C)	1	1	1	−104	101	0	4	+	2, 8, 4	2, 8, 4	2, 2, 2	I_2, I_8, I_4	2 : 2, 5, 6
A4(D)	1	1	1	−1344	18405	0	4	+	2, 2, 1	2, 2, 1	2, 2, 1	I_2, I_2, I_1	2 : 2
A5(F)	1	1	1	−914	−10915	0	2	+	1, 4, 8	1, 4, 8	1, 2, 2	I_1, I_4, I_8	2 : 3
A6(E)	1	1	1	386	1277	0	2	−	1, 16, 2	1, 16, 2	1, 2, 2	I_1, I_{16}, I_2	2 : 3

43 $N = 43 = 43$ (1 isogeny class) **43**

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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44 **44**
 $N = 44 = 2^2 \cdot 11$ (1 isogeny class)

A1(A)	0	1	0	3	-1	0	3	-	8, 1	0, 1	3, 1	IV^*, I_1	3 : 2
A2(B)	0	1	0	-77	-289	0	1	-	8, 3	0, 3	1, 1	IV^*, I_3	3 : 1

45 **45**
 $N = 45 = 3^2 \cdot 5$ (1 isogeny class)

A1(A)	1	-1	0	0	-5	0	2	-	7, 1	1, 1	2, 1	I_1^*, I_1	2 : 2
A2(B)	1	-1	0	-45	-104	0	4	+	8, 2	2, 2	4, 2	I_2^*, I_2	2 : 1, 3, 4
A3(D)	1	-1	0	-720	-7259	0	2	+	7, 1	1, 1	4, 1	I_1^*, I_1	2 : 2
A4(C)	1	-1	0	-90	175	0	4	+	10, 4	4, 4	4, 2	I_4^*, I_4	2 : 2, 5, 6
A5(E)	1	-1	0	-1215	16600	0	4	+	14, 2	8, 2	4, 2	I_8^*, I_2	2 : 4, 7, 8
A6(F)	1	-1	0	315	1066	0	2	-	8, 8	2, 8	2, 2	I_2^*, I_8	2 : 4
A7(H)	1	-1	0	-19440	1048135	0	2	+	10, 1	4, 1	2, 1	I_4^*, I_1	2 : 5
A8(G)	1	-1	0	-990	22765	0	2	-	22, 1	16, 1	4, 1	$\text{I}_{16}^*, \text{I}_1$	2 : 5

46 **46**
 $N = 46 = 2 \cdot 23$ (1 isogeny class)

A1(A)	1	-1	0	-10	-12	0	2	-	10, 1	10, 1	2, 1	$\text{I}_{10}, \text{I}_1$	2 : 2
A2(B)	1	-1	0	-170	-812	0	2	+	5, 2	5, 2	1, 2	I_5, I_2	2 : 1

48 **48**
 $N = 48 = 2^4 \cdot 3$ (1 isogeny class)

A1(B)	0	1	0	-4	-4	0	4	+	8, 2	0, 2	2, 2	I_0^*, I_2	2 : 2, 3, 4
A2(D)	0	1	0	-64	-220	0	2	+	10, 1	0, 1	2, 1	I_2^*, I_1	2 : 1
A3(C)	0	1	0	-24	36	0	8	+	10, 4	0, 4	4, 4	I_2^*, I_4	2 : 1, 5, 6
A4(A)	0	1	0	1	0	0	2	-	4, 1	0, 1	1, 1	II, I_1	2 : 1
A5(F)	0	1	0	-384	2772	0	4	+	11, 2	0, 2	2, 2	I_3^*, I_2	2 : 3
A6(E)	0	1	0	16	180	0	8	-	11, 8	0, 8	4, 8	I_3^*, I_8	2 : 3

49 **49**
 $N = 49 = 7^2$ (1 isogeny class)

A1(A)	1	-1	0	-2	-1	0	2	-	3	0	2	III	2 : 2; 7 : 3
A2(B)	1	-1	0	-37	-78	0	2	+	3	0	2	III	2 : 1; 7 : 4
A3(C)	1	-1	0	-107	552	0	2	-	9	0	2	III^*	2 : 4; 7 : 1
A4(D)	1	-1	0	-1822	30393	0	2	+	9	0	2	III^*	2 : 3; 7 : 2

50 **50**
 $N = 50 = 2 \cdot 5^2$ (2 isogeny classes)

A1(E)	1	0	1	-1	-2	0	3	-	1, 4	1, 0	1, 3	I_1, IV	3 : 2; 5 : 3
A2(F)	1	0	1	-126	-552	0	1	-	3, 4	3, 0	1, 1	I_3, IV	3 : 1; 5 : 4
A3(G)	1	0	1	-76	298	0	3	-	5, 8	5, 0	1, 3	I_5, IV^*	3 : 4; 5 : 1
A4(H)	1	0	1	549	-2202	0	1	-	15, 8	15, 0	1, 1	$\text{I}_{15}, \text{IV}^*$	3 : 3; 5 : 2
B1(A)	1	1	1	-3	1	0	5	-	5, 2	5, 0	5, 1	I_5, II	3 : 2; 5 : 3
B2(B)	1	1	1	22	-9	0	5	-	15, 2	15, 0	15, 1	I_{15}, II	3 : 1; 5 : 4
B3(C)	1	1	1	-13	-219	0	1	-	1, 10	1, 0	1, 1	I_1, II^*	3 : 4; 5 : 1
B4(D)	1	1	1	-3138	-68969	0	1	-	3, 10	3, 0	3, 1	I_3, II^*	3 : 3; 5 : 2

51 **51**
 $N = 51 = 3 \cdot 17$ (1 isogeny class)

A1(A)	0	1	1	1	-1	0	3	-	3, 1	3, 1	3, 1	I_3, I_1	3 : 2
A2(B)	0	1	1	-59	-196	0	1	-	1, 3	1, 3	1, 1	I_1, I_3	3 : 1

52 **52**
 $N = 52 = 2^2 \cdot 13$ (1 isogeny class)

A1(B)	0	0	0	1	-10	0	2	-	8, 2	0, 2	1, 2	IV^*, I_2	2 : 2
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	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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53 $N = 53 = 53$ (1 isogeny class) **53**

A1(A)	1	-1	1	0	0	1	1	-	1	1	1	I_1	
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54 $N = 54 = 2 \cdot 3^3$ (2 isogeny classes) **54**

A1(E)	1	-1	0	12	8	0	3	-	3, 9	3, 0	1, 3	I_3, IV^*	3 : 2, 3
A2(F)	1	-1	0	-123	-667	0	1	-	9, 11	9, 0	1, 1	I_9, II^*	3 : 1
A3(D)	1	-1	0	-3	3	0	3	-	1, 3	1, 0	1, 1	I_1, II	3 : 1
B1(A)	1	-1	1	1	-1	0	3	-	3, 3	3, 0	3, 1	I_3, II	3 : 2, 3
B2(C)	1	-1	1	-29	-53	0	1	-	1, 9	1, 0	1, 1	I_1, IV^*	3 : 1
B3(B)	1	-1	1	-14	29	0	9	-	9, 5	9, 0	9, 3	I_9, IV	3 : 1

55 $N = 55 = 5 \cdot 11$ (1 isogeny class) **55**

A1(B)	1	-1	0	-4	3	0	4	+	2, 2	2, 2	2, 2	I_2, I_2	2 : 2, 3, 4
A2(D)	1	-1	0	-29	-52	0	2	+	1, 4	1, 4	1, 2	I_1, I_4	2 : 1
A3(C)	1	-1	0	-59	190	0	4	+	4, 1	4, 1	4, 1	I_4, I_1	2 : 1
A4(A)	1	-1	0	1	0	0	2	-	1, 1	1, 1	1, 1	I_1, I_1	2 : 1

56 $N = 56 = 2^3 \cdot 7$ (2 isogeny classes) **56**

A1(C)	0	0	0	1	2	0	4	-	8, 1	0, 1	4, 1	I_1^*, I_1	2 : 2
A2(D)	0	0	0	-19	30	0	4	+	10, 2	0, 2	2, 2	III^*, I_2	2 : 1, 3, 4
A3(E)	0	0	0	-59	-138	0	2	+	11, 4	0, 4	1, 2	II^*, I_4	2 : 2
A4(F)	0	0	0	-299	1990	0	2	+	11, 1	0, 1	1, 1	II^*, I_1	2 : 2
B1(A)	0	-1	0	0	-4	0	2	-	10, 1	0, 1	2, 1	III^*, I_1	2 : 2
B2(B)	0	-1	0	-40	-84	0	2	+	11, 2	0, 2	1, 2	II^*, I_2	2 : 1

57 $N = 57 = 3 \cdot 19$ (3 isogeny classes) **57**

A1(E)	0	-1	1	-2	2	1	1	-	2, 1	2, 1	2, 1	I_2, I_1	
B1(B)	1	0	1	-7	5	0	4	+	2, 2	2, 2	2, 2	I_2, I_2	2 : 2, 3, 4
B2(A)	1	0	1	-2	-1	0	2	+	1, 1	1, 1	1, 1	I_1, I_1	2 : 1
B3(C)	1	0	1	-102	385	0	4	+	4, 1	4, 1	4, 1	I_4, I_1	2 : 1
B4(D)	1	0	1	8	29	0	2	-	1, 4	1, 4	1, 2	I_1, I_4	2 : 1
C1(F)	0	1	1	20	-32	0	5	-	10, 1	10, 1	10, 1	I_{10}, I_1	5 : 2
C2(G)	0	1	1	-4390	-113432	0	1	-	2, 5	2, 5	2, 1	I_2, I_5	5 : 1

58 $N = 58 = 2 \cdot 29$ (2 isogeny classes) **58**

A1(A)	1	-1	0	-1	1	1	1	-	2, 1	2, 1	2, 1	I_2, I_1	
B1(B)	1	1	1	5	9	0	5	-	10, 1	10, 1	10, 1	I_{10}, I_1	5 : 2
B2(C)	1	1	1	-455	-3951	0	1	-	2, 5	2, 5	2, 1	I_2, I_5	5 : 1

61 $N = 61 = 61$ (1 isogeny class) **61**

A1(A)	1	0	0	-2	1	1	1	-	1	1	1	I_1	
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62 $N = 62 = 2 \cdot 31$ (1 isogeny class) **62**

A1(A)	1	-1	1	-1	1	0	4	-	4, 1	4, 1	4, 1	I_4, I_1	2 : 2
A2(B)	1	-1	1	-21	41	0	4	+	2, 2	2, 2	2, 2	I_2, I_2	2 : 1, 3, 4
A3(C)	1	-1	1	-31	5	0	2	+	1, 4	1, 4	1, 2	I_1, I_4	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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63 $N = 63 = 3^2 \cdot 7$ (1 isogeny class) **63**

A1(A)	1	-1	0	9	0	0	2	-	8, 1	2, 1	2, 1	I_{2, I_1}^*	2 : 2
A2(B)	1	-1	0	-36	27	0	4	+	10, 2	4, 2	4, 2	I_{4, I_2}^*	2 : 1, 3, 4
A3(C)	1	-1	0	-351	-2430	0	2	+	14, 1	8, 1	4, 1	I_{8, I_1}^*	2 : 2
A4(D)	1	-1	0	-441	3672	0	4	+	8, 4	2, 4	4, 2	I_{2, I_4}^*	2 : 2, 5, 6
A5(F)	1	-1	0	-7056	229905	0	4	+	7, 2	1, 2	4, 2	I_{1, I_2}^*	2 : 4
A6(E)	1	-1	0	-306	5859	0	2	-	7, 8	1, 8	2, 2	I_{1, I_8}^*	2 : 4

64 $N = 64 = 2^6$ (1 isogeny class) **64**

A1(B)	0	0	0	-4	0	0	4	+	12	0	4	I_2^*	2 : 2, 3, 4
A2(C)	0	0	0	-44	-112	0	2	+	15	0	2	I_5^*	2 : 1
A3(D)	0	0	0	-44	112	0	4	+	15	0	4	I_5^*	2 : 1
A4(A)	0	0	0	1	0	0	2	-	6	0	1	II	2 : 1

65 $N = 65 = 5 \cdot 13$ (1 isogeny class) **65**

A1(A)	1	0	0	-1	0	1	2	+	1, 1	1, 1	1, 1	I_{1, I_1}	2 : 2
A2(B)	1	0	0	4	1	1	2	-	2, 2	2, 2	2, 2	I_{2, I_2}	2 : 1

66 $N = 66 = 2 \cdot 3 \cdot 11$ (3 isogeny classes) **66**

A1(A)	1	0	1	-6	4	0	6	+	2, 3, 1	2, 3, 1	2, 3, 1	I_2, I_3, I_1	2 : 2; 3 : 3
A2(B)	1	0	1	4	20	0	6	-	1, 6, 2	1, 6, 2	1, 6, 2	I_1, I_6, I_2	2 : 1; 3 : 4
A3(C)	1	0	1	-81	-284	0	2	+	6, 1, 3	6, 1, 3	2, 1, 1	I_6, I_1, I_3	2 : 4; 3 : 1
A4(D)	1	0	1	-41	-556	0	2	-	3, 2, 6	3, 2, 6	1, 2, 2	I_3, I_2, I_6	2 : 3; 3 : 2
B1(E)	1	1	1	-2	-1	0	4	+	4, 1, 1	4, 1, 1	4, 1, 1	I_4, I_1, I_1	2 : 2
B2(F)	1	1	1	-22	-49	0	4	+	2, 2, 2	2, 2, 2	2, 2, 2	I_2, I_2, I_2	2 : 1, 3, 4
B3(H)	1	1	1	-352	-2689	0	2	+	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	2 : 2
B4(G)	1	1	1	-12	-81	0	2	-	1, 4, 4	1, 4, 4	1, 2, 2	I_1, I_4, I_4	2 : 2
C1(I)	1	0	0	-45	81	0	10	+	10, 5, 1	10, 5, 1	10, 5, 1	I_{10}, I_5, I_1	2 : 2; 5 : 3
C2(J)	1	0	0	115	561	0	10	-	5, 10, 2	5, 10, 2	5, 10, 2	I_5, I_{10}, I_2	2 : 1; 5 : 4
C3(L)	1	0	0	-10065	-389499	0	2	+	2, 1, 5	2, 1, 5	2, 1, 5	I_2, I_1, I_5	2 : 4; 5 : 1
C4(K)	1	0	0	-10055	-390309	0	2	-	1, 2, 10	1, 2, 10	1, 2, 10	I_1, I_2, I_{10}	2 : 3; 5 : 2

67 $N = 67 = 67$ (1 isogeny class) **67**

A1(A)	0	1	1	-12	-21	0	1	-	1	1	1	I_1	
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69 $N = 69 = 3 \cdot 23$ (1 isogeny class) **69**

A1(A)	1	0	1	-1	-1	0	2	-	2, 1	2, 1	2, 1	I_{2, I_1}	2 : 2
A2(B)	1	0	1	-16	-25	0	2	+	1, 2	1, 2	1, 2	I_{1, I_2}	2 : 1

70 $N = 70 = 2 \cdot 5 \cdot 7$ (1 isogeny class) **70**

A1(A)	1	-1	1	2	-3	0	4	-	4, 2, 1	4, 2, 1	4, 2, 1	I_4, I_2, I_1	2 : 2
A2(B)	1	-1	1	-18	-19	0	4	+	2, 4, 2	2, 4, 2	2, 2, 2	I_2, I_4, I_2	2 : 1, 3, 4
A3(D)	1	-1	1	-268	-1619	0	2	+	1, 2, 4	1, 2, 4	1, 2, 2	I_1, I_2, I_4	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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72 $N = 72 = 2^3 \cdot 3^2$ (1 isogeny class) **72**

A1(A)	0	0	0	6	-7	0	4	-	4, 7	0, 1	2, 4	III, I ₁ *	2 : 2
A2(B)	0	0	0	-39	-70	0	4	+	8, 8	0, 2	2, 4	I ₁ *, I ₂ *	2 : 1, 3, 4
A3(D)	0	0	0	-579	-5362	0	2	+	10, 7	0, 1	2, 2	III*, I ₁ *	2 : 2
A4(C)	0	0	0	-219	1190	0	4	+	10, 10	0, 4	2, 4	III*, I ₄ *	2 : 2, 5, 6
A5(F)	0	0	0	-3459	78302	0	2	+	11, 8	0, 2	1, 2	II*, I ₂ *	2 : 4
A6(E)	0	0	0	141	4718	0	2	-	11, 14	0, 8	1, 4	II*, I ₈ *	2 : 4

73 $N = 73 = 73$ (1 isogeny class) **73**

A1(B)	1	-1	0	4	-3	0	2	-	2	2	2	I ₂	2 : 2
A2(A)	1	-1	0	-1	0	0	2	+	1	1	1	I ₁	2 : 1

75 $N = 75 = 3 \cdot 5^2$ (3 isogeny classes) **75**

A1(A)	0	-1	1	-8	-7	0	1	-	1, 4	1, 0	1, 1	I ₁ , IV	5 : 2
A2(B)	0	-1	1	42	443	0	1	-	5, 8	5, 0	1, 1	I ₅ , IV*	5 : 1
B1(E)	1	0	1	-1	23	0	2	-	1, 7	1, 1	1, 2	I ₁ , I ₁ *	2 : 2
B2(F)	1	0	1	-126	523	0	4	+	2, 8	2, 2	2, 4	I ₂ , I ₂ *	2 : 1, 3, 4
B3(G)	1	0	1	-251	-727	0	4	+	4, 10	4, 4	4, 4	I ₄ , I ₄ *	2 : 2, 5, 6
B4(H)	1	0	1	-2001	34273	0	2	+	1, 7	1, 1	1, 2	I ₁ , I ₁ *	2 : 2
B5(I)	1	0	1	-3376	-75727	0	4	+	8, 8	8, 2	8, 4	I ₈ , I ₂ *	2 : 3, 7, 8
B6(J)	1	0	1	874	-5227	0	2	-	2, 14	2, 8	2, 4	I ₂ , I ₈ *	2 : 3
B7(L)	1	0	1	-54001	-4834477	0	2	+	4, 7	4, 1	4, 4	I ₄ , I ₁ *	2 : 5
B8(K)	1	0	1	-2751	-104477	0	4	-	16, 7	16, 1	16, 4	I ₁₆ , I ₁ *	2 : 5
C1(C)	0	1	1	2	4	0	5	-	5, 2	5, 0	5, 1	I ₅ , II	5 : 2
C2(D)	0	1	1	-208	-1256	0	1	-	1, 10	1, 0	1, 1	I ₁ , II*	5 : 1

76 $N = 76 = 2^2 \cdot 19$ (1 isogeny class) **76**

A1(A)	0	-1	0	-21	-31	0	1	-	8, 1	0, 1	1, 1	IV*, I ₁	
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77 $N = 77 = 7 \cdot 11$ (3 isogeny classes) **77**

A1(F)	0	0	1	2	0	1	1	-	2, 1	2, 1	2, 1	I ₂ , I ₁	
B1(D)	0	1	1	-49	600	0	3	-	6, 3	6, 3	6, 1	I ₆ , I ₃	3 : 2, 3
B2(E)	0	1	1	441	-15815	0	1	-	2, 9	2, 9	2, 1	I ₂ , I ₉	3 : 1
B3(C)	0	1	1	-89	295	0	3	-	2, 1	2, 1	2, 1	I ₂ , I ₁	3 : 1
C1(A)	1	1	0	4	11	0	2	-	3, 2	3, 2	1, 2	I ₃ , I ₂	2 : 2
C2(B)	1	1	0	-51	110	0	2	+	6, 1	6, 1	2, 1	I ₆ , I ₁	2 : 1

78 $N = 78 = 2 \cdot 3 \cdot 13$ (1 isogeny class) **78**

A1(A)	1	1	0	-19	685	0	2	-	16, 5, 1	16, 5, 1	2, 1, 1	I ₁₆ , I ₅ , I ₁	2 : 2
A2(B)	1	1	0	-1299	17325	0	4	+	8, 10, 2	8, 10, 2	2, 2, 2	I ₈ , I ₁₀ , I ₂	2 : 1, 3, 4
A3(C)	1	1	0	-2339	-15747	0	2	+	4, 20, 1	4, 20, 1	2, 2, 1	I ₄ , I ₂₀ , I ₁	2 : 2
A4(D)	1	1	0	-20739	1140957	0	4	+	4, 5, 4	4, 5, 4	2, 1, 4	I ₄ , I ₅ , I ₄	2 : 2

79 $N = 79 = 79$ (1 isogeny class) **79**

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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80 $N = 80 = 2^4 \cdot 5$ (2 isogeny classes) **80**

A1(F)	0	0	0	-7	6	0	4	+	8, 2	0, 2	2, 2	I_0^*, I_2	2 : 2, 3, 4
A2(E)	0	0	0	-2	-1	0	2	+	4, 1	0, 1	1, 1	II, I_1	2 : 1
A3(H)	0	0	0	-107	426	0	4	+	10, 1	0, 1	4, 1	I_2^*, I_1	2 : 1
A4(G)	0	0	0	13	34	0	4	-	10, 4	0, 4	2, 4	I_2^*, I_4	2 : 1
B1(B)	0	-1	0	4	-4	0	2	-	8, 2	0, 2	1, 2	I_0^*, I_2	2 : 2; 3 : 3
B2(A)	0	-1	0	-1	0	0	2	+	4, 1	0, 1	1, 1	II, I_1	2 : 1; 3 : 4
B3(D)	0	-1	0	-36	140	0	2	-	8, 6	0, 6	1, 2	I_0^*, I_6	2 : 4; 3 : 1
B4(C)	0	-1	0	-41	116	0	2	+	4, 3	0, 3	1, 1	II, I_3	2 : 3; 3 : 2

82 $N = 82 = 2 \cdot 41$ (1 isogeny class) **82**

A1(A)	1	0	1	-2	0	1	2	+	2, 1	2, 1	2, 1	I_2, I_1	2 : 2
A2(B)	1	0	1	-12	-16	1	2	+	1, 2	1, 2	1, 2	I_1, I_2	2 : 1

83 $N = 83 = 83$ (1 isogeny class) **83**

A1(A)	1	1	1	1	0	1	1	-	1	1	1	I_1	
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84 $N = 84 = 2^2 \cdot 3 \cdot 7$ (2 isogeny classes) **84**

A1(C)	0	1	0	7	0	0	6	-	4, 3, 2	0, 3, 2	3, 3, 2	IV, I_3, I_2	2 : 2; 3 : 3
A2(D)	0	1	0	-28	-28	0	6	+	8, 6, 1	0, 6, 1	3, 6, 1	IV^*, I_6, I_1	2 : 1; 3 : 4
A3(E)	0	1	0	-113	-516	0	2	-	4, 1, 6	0, 1, 6	1, 1, 6	IV, I_1, I_6	2 : 4; 3 : 1
A4(F)	0	1	0	-1828	-30700	0	2	+	8, 2, 3	0, 2, 3	1, 2, 3	IV^*, I_2, I_3	2 : 3; 3 : 2
B1(A)	0	-1	0	-1	-2	0	2	-	4, 1, 2	0, 1, 2	1, 1, 2	IV, I_1, I_2	2 : 2
B2(B)	0	-1	0	-36	-72	0	2	+	8, 2, 1	0, 2, 1	1, 2, 1	IV^*, I_2, I_1	2 : 1

85 $N = 85 = 5 \cdot 17$ (1 isogeny class) **85**

A1(A)	1	1	0	-8	-13	0	2	+	2, 1	2, 1	2, 1	I_2, I_1	2 : 2
A2(B)	1	1	0	-3	-22	0	2	-	4, 2	4, 2	2, 2	I_4, I_2	2 : 1

88 $N = 88 = 2^3 \cdot 11$ (1 isogeny class) **88**

A1(A)	0	0	0	-4	4	1	1	-	8, 1	0, 1	4, 1	I_1^*, I_1	
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89 $N = 89 = 89$ (2 isogeny classes) **89**

A1(C)	1	1	1	-1	0	1	1	-	1	1	1	I_1	
B1(A)	1	1	0	4	5	0	2	-	2	2	2	I_2	2 : 2
B2(B)	1	1	0	-1	0	0	2	+	1	1	1	I_1	2 : 1

90 $N = 90 = 2 \cdot 3^2 \cdot 5$ (3 isogeny classes) **90**

A1(M)	1	-1	0	6	0	0	6	-	2, 3, 3	2, 0, 3	2, 2, 3	I_2, III, I_3	2 : 2; 3 : 3
A2(N)	1	-1	0	-24	18	0	6	+	1, 3, 6	1, 0, 6	1, 2, 6	I_1, III, I_6	2 : 1; 3 : 4
A3(O)	1	-1	0	-69	-235	0	2	-	6, 9, 1	6, 0, 1	2, 2, 1	I_6, III^*, I_1	2 : 4; 3 : 1
A4(P)	1	-1	0	-1149	-14707	0	2	+	3, 9, 2	3, 0, 2	1, 2, 2	I_3, III^*, I_2	2 : 3; 3 : 2
B1(A)	1	-1	1	-8	11	0	6	-	6, 3, 1	6, 0, 1	6, 2, 1	I_6, III, I_1	2 : 2; 3 : 3
B2(B)	1	-1	1	-128	587	0	6	+	3, 3, 2	3, 0, 2	3, 2, 2	I_3, III, I_2	2 : 1; 3 : 4
B3(C)	1	-1	1	52	-53	0	2	-	2, 9, 3	2, 0, 3	2, 2, 1	I_2, III^*, I_3	2 : 4; 3 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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90 **90**
 $N = 90 = 2 \cdot 3^2 \cdot 5$ (continued)

C1(E)	1	-1	1	13	-61	0	4	-	4, 9, 1	4, 3, 1	4, 4, 1	I_4, I_3^*, I_1	2 : 2 ; 3 : 3
C2(F)	1	-1	1	-167	-709	0	4	+	2, 12, 2	2, 6, 2	2, 4, 2	I_2, I_6^*, I_2	2 : 1, 4, 5; 3 : 6
C3(G)	1	-1	1	-122	1721	0	12	-	12, 7, 3	12, 1, 3	12, 4, 3	I_{12}, I_1^*, I_3	2 : 6 ; 3 : 1
C4(I)	1	-1	1	-2597	-50281	0	2	+	1, 9, 4	1, 3, 4	1, 2, 4	I_1, I_3^*, I_4	2 : 2 ; 3 : 7
C5(H)	1	-1	1	-617	5231	0	2	+	1, 18, 1	1, 12, 1	1, 4, 1	I_1, I_{12}^*, I_1	2 : 2 ; 3 : 8
C6(J)	1	-1	1	-3002	63929	0	12	+	6, 8, 6	6, 2, 6	6, 4, 6	I_6, I_2^*, I_6	2 : 3 , 7 , 8 ; 3 : 2
C7(L)	1	-1	1	-4082	14681	0	6	+	3, 7, 12	3, 1, 12	3, 2, 12	I_3, I_1^*, I_{12}	2 : 6 ; 3 : 4
C8(K)	1	-1	1	-48002	4059929	0	6	+	3, 10, 3	3, 4, 3	3, 4, 3	I_3, I_4^*, I_3	2 : 6 ; 3 : 5

91 **91**
 $N = 91 = 7 \cdot 13$ (2 isogeny classes)

A1(A)	0	0	1	1	0	1	1	-	1, 1	1, 1	1, 1	I_1, I_1	
B1(B)	0	1	1	-7	5	1	3	-	1, 1	1, 1	1, 1	I_1, I_1	3 : 2
B2(C)	0	1	1	13	42	1	3	-	3, 3	3, 3	3, 3	I_3, I_3	3 : 1, 3
B3(D)	0	1	1	-117	-1245	1	1	-	9, 1	9, 1	9, 1	I_9, I_1	3 : 2

92 **92**
 $N = 92 = 2^2 \cdot 23$ (2 isogeny classes)

A1(A)	0	1	0	2	1	0	3	-	4, 1	0, 1	3, 1	IV, I_1	3 : 2
A2(B)	0	1	0	-18	-43	0	1	-	4, 3	0, 3	1, 1	IV, I_3	3 : 1
B1(C)	0	0	0	-1	1	1	1	-	4, 1	0, 1	3, 1	IV, I_1	

94 **94**
 $N = 94 = 2 \cdot 47$ (1 isogeny class)

A1(A)	1	-1	1	0	-1	0	2	-	2, 1	2, 1	2, 1	I_2, I_1	2 : 2
A2(B)	1	-1	1	-10	-9	0	2	+	1, 2	1, 2	1, 2	I_1, I_2	2 : 1

96 **96**
 $N = 96 = 2^5 \cdot 3$ (2 isogeny classes)

A1(E)	0	1	0	-2	0	0	4	+	6, 2	0, 2	2, 2	III, I_2	2 : 2 , 3 , 4
A2(F)	0	1	0	-17	-33	0	2	+	12, 1	0, 1	2, 1	I_3^*, I_1	2 : 1
A3(H)	0	1	0	-32	60	0	2	+	9, 1	0, 1	1, 1	I_0^*, I_1	2 : 1
A4(G)	0	1	0	8	8	0	4	-	9, 4	0, 4	2, 4	I_0^*, I_4	2 : 1
B1(A)	0	-1	0	-2	0	0	4	+	6, 2	0, 2	2, 2	III, I_2	2 : 2 , 3 , 4
B2(D)	0	-1	0	-32	-60	0	2	+	9, 1	0, 1	2, 1	I_0^*, I_1	2 : 1
B3(B)	0	-1	0	-17	33	0	4	+	12, 1	0, 1	4, 1	I_3^*, I_1	2 : 1
B4(C)	0	-1	0	8	-8	0	2	-	9, 4	0, 4	1, 2	I_0^*, I_4	2 : 1

98 **98**
 $N = 98 = 2 \cdot 7^2$ (1 isogeny class)

A1(B)	1	1	0	-25	-111	0	2	-	2, 7	2, 1	2, 2	I_2, I_1^*	2 : 2 ; 3 : 3
A2(A)	1	1	0	-515	-4717	0	2	+	1, 8	1, 2	1, 4	I_1, I_2^*	2 : 1 ; 3 : 4
A3(D)	1	1	0	220	2192	0	2	-	6, 9	6, 3	2, 2	I_6, I_3^*	2 : 4 ; 3 : 1, 5
A4(C)	1	1	0	-1740	22184	0	2	+	3, 12	3, 6	1, 4	I_3, I_6^*	2 : 3 ; 3 : 2 , 6
A5(F)	1	1	0	-8355	291341	0	2	-	18, 7	18, 1	2, 2	I_{18}, I_1^*	2 : 6 ; 3 : 3
A6(E)	1	1	0	-133795	18781197	0	2	+	9, 8	9, 2	1, 4	I_9, I_2^*	2 : 5 ; 3 : 4

99 **99**
 $N = 99 = 3^2 \cdot 11$ (4 isogeny classes)

A1(A)	1	-1	1	-2	0	1	2	+	3, 1	0, 1	2, 1	III, I_1	2 : 2
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	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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99 $N = 99 = 3^2 \cdot 11$ (continued) **99**

B1(H)	1	-1	1	-59	186	0	4	+	9, 1	3, 1	4, 1	I_3^*, I_1	2 : 2
B2(I)	1	-1	1	-104	-102	0	4	+	12, 2	6, 2	4, 2	I_6^*, I_2	2 : 1, 3, 4
B3(K)	1	-1	1	-1319	-18084	0	2	+	9, 4	3, 4	2, 2	I_3^*, I_4	2 : 2
B4(J)	1	-1	1	391	-1092	0	2	-	18, 1	12, 1	4, 1	I_{12}^*, I_1	2 : 2
C1(F)	1	-1	0	-15	8	0	2	+	9, 1	0, 1	2, 1	III^*, I_1	2 : 2
C2(G)	1	-1	0	-150	-667	0	2	+	9, 2	0, 2	2, 2	III^*, I_2	2 : 1
D1(C)	0	0	1	-3	-5	0	1	-	6, 1	0, 1	1, 1	I_0^*, I_1	5 : 2
D2(D)	0	0	1	-93	625	0	1	-	6, 5	0, 5	1, 1	I_0^*, I_5	5 : 1, 3
D3(E)	0	0	1	-70383	7187035	0	1	-	6, 1	0, 1	1, 1	I_0^*, I_1	5 : 2

100 $N = 100 = 2^2 \cdot 5^2$ (1 isogeny class) **100**

A1(A)	0	-1	0	-33	62	0	2	+	4, 7	0, 1	1, 2	IV, I_1^*	2 : 2; 3 : 3
A2(B)	0	-1	0	92	312	0	2	-	8, 8	0, 2	1, 4	IV^*, I_2^*	2 : 1; 3 : 4
A3(C)	0	-1	0	-1033	-12438	0	2	+	4, 9	0, 3	3, 2	IV, I_3^*	2 : 4; 3 : 1
A4(D)	0	-1	0	-908	-15688	0	2	-	8, 12	0, 6	3, 4	IV^*, I_6^*	2 : 3; 3 : 2

101 $N = 101 = 101$ (1 isogeny class) **101**

A1(A)	0	1	1	-1	-1	1	1	+	1	1	1	I_1	
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102 $N = 102 = 2 \cdot 3 \cdot 17$ (3 isogeny classes) **102**

A1(E)	1	1	0	-2	0	1	2	+	2, 2, 1	2, 2, 1	2, 2, 1	I_2, I_2, I_1	2 : 2
A2(F)	1	1	0	8	10	1	2	-	1, 4, 2	1, 4, 2	1, 2, 2	I_1, I_4, I_2	2 : 1
B1(G)	1	0	0	-34	68	0	8	+	8, 4, 1	8, 4, 1	8, 4, 1	I_8, I_4, I_1	2 : 2
B2(H)	1	0	0	-114	-396	0	8	+	4, 8, 2	4, 8, 2	4, 8, 2	I_4, I_8, I_2	2 : 1, 3, 4
B3(J)	1	0	0	-1734	-27936	0	4	+	2, 4, 4	2, 4, 4	2, 4, 4	I_2, I_4, I_4	2 : 2, 5, 6
B4(I)	1	0	0	226	-2232	0	4	-	2, 16, 1	2, 16, 1	2, 16, 1	I_2, I_{16}, I_1	2 : 2
B5(L)	1	0	0	-27744	-1781010	0	2	+	1, 2, 2	1, 2, 2	1, 2, 2	I_1, I_2, I_2	2 : 3
B6(K)	1	0	0	-1644	-30942	0	2	-	1, 2, 8	1, 2, 8	1, 2, 8	I_1, I_2, I_8	2 : 3
C1(A)	1	0	1	-256	1550	0	6	+	6, 6, 1	6, 6, 1	2, 6, 1	I_6, I_6, I_1	2 : 2; 3 : 3
C2(B)	1	0	1	-216	2062	0	6	-	3, 12, 2	3, 12, 2	1, 12, 2	I_3, I_{12}, I_2	2 : 1; 3 : 4
C3(C)	1	0	1	-751	-6046	0	2	+	18, 2, 3	18, 2, 3	2, 2, 1	I_{18}, I_2, I_3	2 : 4; 3 : 1
C4(D)	1	0	1	1809	-37790	0	2	-	9, 4, 6	9, 4, 6	1, 4, 2	I_9, I_4, I_6	2 : 3; 3 : 2

104 $N = 104 = 2^3 \cdot 13$ (1 isogeny class) **104**

A1(A)	0	1	0	-16	-32	0	1	-	11, 1	0, 1	1, 1	II^*, I_1	
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105 $N = 105 = 3 \cdot 5 \cdot 7$ (1 isogeny class) **105**

A1(A)	1	0	1	-3	1	0	2	+	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	2 : 2
A2(B)	1	0	1	-8	-7	0	4	+	2, 2, 2	2, 2, 2	2, 2, 2	I_2, I_2, I_2	2 : 1, 3, 4
A3(D)	1	0	1	-113	-469	0	2	+	1, 4, 1	1, 4, 1	1, 4, 1	I_1, I_4, I_1	2 : 2
A4(C)	1	0	1	17	-37	0	4	-	4, 1, 4	4, 1, 4	4, 1, 4	I_4, I_1, I_4	2 : 2

106 $N = 106 = 2 \cdot 53$ (4 isogeny classes) **106**

A1(B)	1	0	0	1	1	0	3	-	3, 1	3, 1	3, 1	I_3, I_1	3 : 2
A2(C)	1	0	0	-9	-29	0	1	-	1, 3	1, 3	1, 1	I_1, I_3	3 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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106 **106**
 $N = 106 = 2 \cdot 53$ (continued)

C1(E)	1	0	0	-283	-2351	0	3	-	24, 1	24, 1	24, 1	I_{24}, I_1	3 : 2
C2(F)	1	0	0	-24603	-1487407	0	1	-	8, 3	8, 3	8, 1	I_8, I_3	3 : 1
D1(D)	1	1	0	-27	-67	0	1	-	5, 1	5, 1	1, 1	I_5, I_1	

108 **108**
 $N = 108 = 2^2 \cdot 3^3$ (1 isogeny class)

A1(A)	0	0	0	0	4	0	3	-	8, 3	0, 0	3, 1	IV^*, II	3 : 2
A2(B)	0	0	0	0	-108	0	1	-	8, 9	0, 0	1, 1	IV^*, IV^*	3 : 1

109 **109**
 $N = 109 = 109$ (1 isogeny class)

A1(A)	1	-1	0	-8	-7	0	1	-	1	1	1	I_1	
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110 **110**
 $N = 110 = 2 \cdot 5 \cdot 11$ (3 isogeny classes)

A1(C)	1	1	1	10	-45	0	5	-	5, 5, 1	5, 5, 1	5, 5, 1	I_5, I_5, I_1	5 : 2
A2(D)	1	1	1	-5940	-178685	0	1	-	1, 1, 5	1, 1, 5	1, 1, 5	I_1, I_1, I_5	5 : 1
B1(A)	1	0	0	-1	1	0	3	-	3, 1, 1	3, 1, 1	3, 1, 1	I_3, I_1, I_1	3 : 2
B2(B)	1	0	0	9	-25	0	1	-	1, 3, 3	1, 3, 3	1, 1, 1	I_1, I_3, I_3	3 : 1
C1(E)	1	0	1	-89	316	0	3	-	7, 1, 3	7, 1, 3	1, 1, 3	I_7, I_1, I_3	3 : 2
C2(F)	1	0	1	296	1702	0	1	-	21, 3, 1	21, 3, 1	1, 1, 1	I_{21}, I_3, I_1	3 : 1

112 **112**
 $N = 112 = 2^4 \cdot 7$ (3 isogeny classes)

A1(K)	0	1	0	0	4	1	2	-	10, 1	0, 1	4, 1	I_2^*, I_1	2 : 2
A2(L)	0	1	0	-40	84	1	2	+	11, 2	0, 2	4, 2	I_3^*, I_2	2 : 1
B1(A)	0	0	0	1	-2	0	2	-	8, 1	0, 1	2, 1	I_0^*, I_1	2 : 2
B2(B)	0	0	0	-19	-30	0	4	+	10, 2	0, 2	4, 2	I_2^*, I_2	2 : 1, 3, 4
B3(D)	0	0	0	-299	-1990	0	2	+	11, 1	0, 1	4, 1	I_3^*, I_1	2 : 2
B4(C)	0	0	0	-59	138	0	4	+	11, 4	0, 4	2, 4	I_3^*, I_4	2 : 2
C1(E)	0	-1	0	-8	-16	0	2	-	14, 1	2, 1	4, 1	I_6^*, I_1	2 : 2; 3 : 3
C2(F)	0	-1	0	-168	-784	0	2	+	13, 2	1, 2	2, 2	I_5^*, I_2	2 : 1; 3 : 4
C3(G)	0	-1	0	72	368	0	2	-	18, 3	6, 3	4, 1	I_{10}^*, I_3	2 : 4; 3 : 1, 5
C4(H)	0	-1	0	-568	4464	0	2	+	15, 6	3, 6	2, 2	I_7^*, I_6	2 : 3; 3 : 2, 6
C5(I)	0	-1	0	-2728	55920	0	2	-	30, 1	18, 1	4, 1	I_{22}^*, I_1	2 : 6; 3 : 3
C6(J)	0	-1	0	-43688	3529328	0	2	+	21, 2	9, 2	2, 2	I_{13}^*, I_2	2 : 5; 3 : 4

113 **113**
 $N = 113 = 113$ (1 isogeny class)

A1(B)	1	1	1	3	-4	0	2	-	2	2	2	I_2	2 : 2
A2(A)	1	1	1	-2	-2	0	2	+	1	1	1	I_1	2 : 1

114 **114**
 $N = 114 = 2 \cdot 3 \cdot 19$ (3 isogeny classes)

A1(A)	1	0	0	-8	0	0	6	+	6, 3, 1	6, 3, 1	6, 3, 1	I_6, I_3, I_1	2 : 2; 3 : 3
A2(B)	1	0	0	32	8	0	6	-	3, 6, 2	3, 6, 2	3, 6, 2	I_3, I_6, I_2	2 : 1; 3 : 4
A3(C)	1	0	0	-428	-3444	0	2	+	2, 1, 3	2, 1, 3	2, 1, 3	I_2, I_1, I_3	2 : 4; 3 : 1
A4(D)	1	0	0	-418	-3610	0	2	-	1, 2, 6	1, 2, 6	1, 2, 6	I_1, I_2, I_6	2 : 3; 3 : 2
B1(E)	1	1	0	-95	-399	0	2	+	2, 5, 1	2, 5, 1	2, 1, 1	I_2, I_5, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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114 $N = 114 = 2 \cdot 3 \cdot 19$ (continued) **114**

C1(G)	1	1	1	-352	-2431	0	4	+	20, 3, 1	20, 3, 1	20, 1, 1	I_{20}, I_3, I_1	2 : 2
C2(H)	1	1	1	-5472	-158079	0	4	+	10, 6, 2	10, 6, 2	10, 2, 2	I_{10}, I_6, I_2	2 : 1, 3, 4
C3(J)	1	1	1	-87552	-10007679	0	2	+	5, 3, 1	5, 3, 1	5, 1, 1	I_5, I_3, I_1	2 : 2
C4(I)	1	1	1	-5312	-167551	0	2	-	5, 12, 4	5, 12, 4	5, 2, 2	I_5, I_{12}, I_4	2 : 2

115 $N = 115 = 5 \cdot 23$ (1 isogeny class) **115**

A1(A)	0	0	1	7	-11	0	1	-	5, 1	5, 1	1, 1	I_5, I_1	
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116 $N = 116 = 2^2 \cdot 29$ (3 isogeny classes) **116**

A1(E)	0	0	0	-4831	-129242	0	1	-	8, 1	0, 1	3, 1	IV^*, I_1	
B1(A)	0	1	0	-4	4	0	3	-	8, 1	0, 1	3, 1	IV^*, I_1	3 : 2
B2(B)	0	1	0	36	-76	0	1	-	8, 3	0, 3	1, 1	IV^*, I_3	3 : 1
C1(D)	0	-1	0	-4	24	0	2	-	8, 2	0, 2	1, 2	IV^*, I_2	2 : 2
C2(C)	0	-1	0	-9	14	0	2	+	4, 1	0, 1	1, 1	IV, I_1	2 : 1

117 $N = 117 = 3^2 \cdot 13$ (1 isogeny class) **117**

A1(A)	1	-1	1	4	6	1	4	-	7, 1	1, 1	4, 1	I_1^*, I_1	2 : 2
A2(B)	1	-1	1	-41	96	1	4	+	8, 2	2, 2	4, 2	I_2^*, I_2	2 : 1, 3, 4
A3(D)	1	-1	1	-176	-768	1	2	+	7, 4	1, 4	2, 4	I_1^*, I_4	2 : 2
A4(C)	1	-1	1	-626	6180	1	2	+	10, 1	4, 1	4, 1	I_4^*, I_1	2 : 2

118 $N = 118 = 2 \cdot 59$ (4 isogeny classes) **118**

A1(A)	1	1	0	1	1	1	1	-	2, 1	2, 1	2, 1	I_2, I_1	
B1(B)	1	1	1	-25	39	0	5	-	10, 1	10, 1	10, 1	I_{10}, I_1	5 : 2
B2(C)	1	1	1	115	-2481	0	1	-	2, 5	2, 5	2, 1	I_2, I_5	5 : 1
C1(D)	1	1	1	-4	-5	0	1	-	1, 1	1, 1	1, 1	I_1, I_1	
D1(E)	1	1	0	56	-192	0	1	-	19, 1	19, 1	1, 1	I_{19}, I_1	

120 $N = 120 = 2^3 \cdot 3 \cdot 5$ (2 isogeny classes) **120**

A1(E)	0	1	0	-15	18	0	4	+	4, 2, 1	0, 2, 1	2, 2, 1	III, I_2, I_1	2 : 2
A2(F)	0	1	0	-20	0	0	8	+	8, 4, 2	0, 4, 2	4, 4, 2	I_1^*, I_4, I_2	2 : 1, 3, 4
A3(H)	0	1	0	-200	-1152	0	4	+	10, 2, 4	0, 2, 4	2, 2, 4	III^*, I_2, I_4	2 : 2, 5, 6
A4(G)	0	1	0	80	80	0	4	-	10, 8, 1	0, 8, 1	2, 8, 1	III^*, I_8, I_1	2 : 2
A5(J)	0	1	0	-3200	-70752	0	2	+	11, 1, 2	0, 1, 2	1, 1, 2	II^*, I_1, I_2	2 : 3
A6(I)	0	1	0	-80	-2400	0	2	-	11, 1, 8	0, 1, 8	1, 1, 8	II^*, I_1, I_8	2 : 3
B1(A)	0	1	0	4	0	0	2	-	8, 1, 1	0, 1, 1	2, 1, 1	I_1^*, I_1, I_1	2 : 2
B2(B)	0	1	0	-16	-16	0	4	+	10, 2, 2	0, 2, 2	2, 2, 2	III^*, I_2, I_2	2 : 1, 3, 4
B3(C)	0	1	0	-216	-1296	0	2	+	11, 4, 1	0, 4, 1	1, 4, 1	II^*, I_4, I_1	2 : 2
B4(D)	0	1	0	-136	560	0	2	+	11, 1, 4	0, 1, 4	1, 1, 2	II^*, I_1, I_4	2 : 2

121 $N = 121 = 11^2$ (4 isogeny classes) **121**

A1(H)	1	1	1	-30	-76	0	1	-	2	0	1	II	11 : 2
A2(I)	1	1	1	-305	7888	0	1	-	10	0	1	II^*	11 : 1
B1(D)	0	-1	1	-7	10	1	1	-	3	0	2	III	11 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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121 **121**
 $N = 121 = 11^2$ (continued)

C1(F)	1	1	0	-2	-7	0	1	-	4	0	1	IV	11 : 2
C2(G)	1	1	0	-3632	82757	0	1	-	8	0	1	IV*	11 : 1
D1(A)	0	-1	1	-40	-221	0	1	-	7	1	2	I ₁ *	5 : 2
D2(B)	0	-1	1	-1250	31239	0	1	-	11	5	2	I ₅ *	5 : 1, 3
D3(C)	0	-1	1	-946260	354609639	0	1	-	7	1	2	I ₁ *	5 : 2

122 **122**
 $N = 122 = 2 \cdot 61$ (1 isogeny class)

A1(A)	1	0	1	2	0	1	1	-	4, 1	4, 1	2, 1	I ₄ , I ₁	
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123 **123**
 $N = 123 = 3 \cdot 41$ (2 isogeny classes)

A1(A)	0	1	1	-10	10	1	5	-	5, 1	5, 1	5, 1	I ₅ , I ₁	5 : 2
A2(B)	0	1	1	20	-890	1	1	-	1, 5	1, 5	1, 5	I ₁ , I ₅	5 : 1
B1(C)	0	-1	1	1	-1	1	1	-	1, 1	1, 1	1, 1	I ₁ , I ₁	

124 **124**
 $N = 124 = 2^2 \cdot 31$ (2 isogeny classes)

A1(B)	0	1	0	-2	1	1	3	-	4, 1	0, 1	3, 1	IV, I ₁	3 : 2
A2(C)	0	1	0	18	-11	1	1	-	4, 3	0, 3	1, 3	IV, I ₃	3 : 1
B1(A)	0	0	0	-17	-27	0	1	-	4, 1	0, 1	1, 1	IV, I ₁	

126 **126**
 $N = 126 = 2 \cdot 3^2 \cdot 7$ (2 isogeny classes)

A1(A)	1	-1	1	-5	-7	0	2	-	2, 6, 1	2, 0, 1	2, 2, 1	I ₂ , I ₀ *, I ₁	2 : 2; 3 : 3
A2(B)	1	-1	1	-95	-331	0	2	+	1, 6, 2	1, 0, 2	1, 2, 2	I ₁ , I ₀ *, I ₂	2 : 1; 3 : 4
A3(C)	1	-1	1	40	155	0	6	-	6, 6, 3	6, 0, 3	6, 2, 3	I ₆ , I ₀ *, I ₃	2 : 4; 3 : 1, 5
A4(D)	1	-1	1	-320	1883	0	6	+	3, 6, 6	3, 0, 6	3, 2, 6	I ₃ , I ₀ *, I ₆	2 : 3; 3 : 2, 6
A5(E)	1	-1	1	-1535	23591	0	6	-	18, 6, 1	18, 0, 1	18, 2, 1	I ₁₈ , I ₀ *, I ₁	2 : 6; 3 : 3
A6(F)	1	-1	1	-24575	1488935	0	6	+	9, 6, 2	9, 0, 2	9, 2, 2	I ₉ , I ₀ *, I ₂	2 : 5; 3 : 4
B1(G)	1	-1	0	-36	-176	0	2	-	8, 8, 1	8, 2, 1	2, 2, 1	I ₈ , I ₂ *, I ₁	2 : 2
B2(H)	1	-1	0	-756	-7808	0	4	+	4, 10, 2	4, 4, 2	2, 4, 2	I ₄ , I ₄ *, I ₂	2 : 1, 3, 4
B3(J)	1	-1	0	-12096	-509036	0	2	+	2, 8, 1	2, 2, 1	2, 4, 1	I ₂ , I ₂ *, I ₁	2 : 2
B4(I)	1	-1	0	-936	-3668	0	4	+	2, 14, 4	2, 8, 4	2, 4, 2	I ₂ , I ₈ *, I ₄	2 : 2, 5, 6
B5(L)	1	-1	0	-8226	286474	0	2	+	1, 10, 8	1, 4, 8	1, 2, 2	I ₁ , I ₄ *, I ₈	2 : 4
B6(K)	1	-1	0	3474	-31010	0	2	-	1, 22, 2	1, 16, 2	1, 4, 2	I ₁ , I ₁₆ *, I ₂	2 : 4

128 **128**
 $N = 128 = 2^7$ (4 isogeny classes)

A1(C)	0	1	0	1	1	1	2	-	8	0	2	III	2 : 2
A2(D)	0	1	0	-9	7	1	2	+	13	0	4	I ₂ *	2 : 1
B1(F)	0	1	0	3	-5	0	2	-	14	0	2	III*	2 : 2
B2(E)	0	1	0	-2	-2	0	2	+	7	0	1	II	2 : 1
C1(A)	0	-1	0	1	-1	0	2	-	8	0	2	III	2 : 2
C2(B)	0	-1	0	-9	-7	0	2	+	13	0	2	I ₂ *	2 : 1
D1(G)	0	-1	0	3	5	0	2	-	14	0	2	III*	2 : 2
D2(H)	0	-1	0	-2	2	0	2	+	7	0	1	II	2 : 1

129 **129**
 $N = 129 = 3 \cdot 43$ (2 isogeny classes)

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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129 **129**
 $N = 129 = 3 \cdot 43$ (continued)

B1(B)	1	0	1	-30	-29	0	4	+	6, 2	6, 2	6, 2	I_{6, I_2}	$2 : 2, 3, 4$
B2(A)	1	0	1	-25	-49	0	2	+	3, 1	3, 1	3, 1	I_{3, I_1}	$2 : 1$
B3(C)	1	0	1	-245	1433	0	4	+	12, 1	12, 1	12, 1	I_{12, I_1}	$2 : 1$
B4(D)	1	0	1	105	-191	0	2	-	3, 4	3, 4	3, 2	I_{3, I_4}	$2 : 1$

130 **130**
 $N = 130 = 2 \cdot 5 \cdot 13$ (3 isogeny classes)

A1(E)	1	0	1	-33	68	1	6	+	4, 3, 1	4, 3, 1	2, 3, 1	I_4, I_3, I_1	$2 : 2; 3 : 3$
A2(F)	1	0	1	-13	156	1	6	-	2, 6, 2	2, 6, 2	2, 6, 2	I_2, I_6, I_2	$2 : 1; 3 : 4$
A3(G)	1	0	1	-208	-1122	1	2	+	12, 1, 3	12, 1, 3	2, 1, 3	I_{12, I_1, I_3}	$2 : 4; 3 : 1$
A4(H)	1	0	1	112	-4194	1	2	-	6, 2, 6	6, 2, 6	2, 2, 6	I_6, I_2, I_6	$2 : 3; 3 : 2$
B1(A)	1	-1	1	-7	-1	0	4	+	8, 1, 1	8, 1, 1	8, 1, 1	I_8, I_1, I_1	$2 : 2$
B2(B)	1	-1	1	-87	-289	0	4	+	4, 2, 2	4, 2, 2	4, 2, 2	I_4, I_2, I_2	$2 : 1, 3, 4$
B3(D)	1	-1	1	-1387	-19529	0	2	+	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	$2 : 2$
B4(C)	1	-1	1	-67	-441	0	4	-	2, 4, 4	2, 4, 4	2, 4, 4	I_2, I_4, I_4	$2 : 2$
C1(J)	1	1	1	-841	-9737	0	2	+	8, 5, 1	8, 5, 1	8, 1, 1	I_8, I_5, I_1	$2 : 2$
C2(I)	1	1	1	-761	-11561	0	2	-	4, 10, 2	4, 10, 2	4, 2, 2	I_4, I_{10}, I_2	$2 : 1$

131 **131**
 $N = 131 = 131$ (1 isogeny class)

A1(A)	0	-1	1	1	0	1	1	-	1	1	1	I_1	
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132 **132**
 $N = 132 = 2^2 \cdot 3 \cdot 11$ (2 isogeny classes)

A1(A)	0	1	0	3	0	0	2	-	4, 2, 1	0, 2, 1	1, 2, 1	IV, I_2, I_1	$2 : 2$
A2(B)	0	1	0	-12	-12	0	2	+	8, 1, 2	0, 1, 2	1, 1, 2	IV^*, I_1, I_2	$2 : 1$
B1(C)	0	-1	0	-77	330	0	2	-	4, 10, 1	0, 10, 1	1, 2, 1	IV, I_{10}, I_1	$2 : 2$
B2(D)	0	-1	0	-1292	18312	0	2	+	8, 5, 2	0, 5, 2	1, 1, 2	IV^*, I_5, I_2	$2 : 1$

135 **135**
 $N = 135 = 3^3 \cdot 5$ (2 isogeny classes)

A1(A)	0	0	1	-3	4	1	1	-	5, 2	0, 2	3, 2	IV, I_2	
B1(B)	0	0	1	-27	-115	0	1	-	11, 2	0, 2	1, 2	II^*, I_2	

136 **136**
 $N = 136 = 2^3 \cdot 17$ (2 isogeny classes)

A1(A)	0	1	0	-4	0	1	2	+	8, 1	0, 1	4, 1	I_1^*, I_1	$2 : 2$
A2(B)	0	1	0	16	16	1	2	-	10, 2	0, 2	2, 2	III^*, I_2	$2 : 1$
B1(C)	0	-1	0	-8	-4	0	2	+	10, 1	0, 1	2, 1	III^*, I_1	$2 : 2$
B2(D)	0	-1	0	-48	140	0	2	+	11, 2	0, 2	1, 2	II^*, I_2	$2 : 1$

138 **138**
 $N = 138 = 2 \cdot 3 \cdot 23$ (3 isogeny classes)

A1(E)	1	1	0	-1	1	1	2	-	2, 2, 1	2, 2, 1	2, 2, 1	I_2, I_2, I_1	$2 : 2$
A2(F)	1	1	0	-31	55	1	2	+	1, 1, 2	1, 1, 2	1, 1, 2	I_1, I_1, I_2	$2 : 1$
B1(G)	1	0	1	-36	82	0	6	-	4, 6, 1	4, 6, 1	2, 6, 1	I_4, I_6, I_1	$2 : 2; 3 : 3$
B2(H)	1	0	1	-576	5266	0	6	+	2, 3, 2	2, 3, 2	2, 3, 2	I_2, I_3, I_2	$2 : 1; 3 : 4$
B3(I)	1	0	1	189	190	0	2	-	12, 2, 3	12, 2, 3	2, 2, 1	I_{12}, I_2, I_3	$2 : 4; 3 : 1$
B4(J)	1	0	1	-771	1342	0	2	+	6, 1, 6	6, 1, 6	2, 1, 2	I_6, I_1, I_6	$2 : 3; 3 : 2$
C1(A)	1	1	1	3	3	0	4	-	4, 2, 1	4, 2, 1	4, 2, 1	I_4, I_2, I_1	$2 : 2$
C2(B)	1	1	1	-17	11	0	4	+	2, 4, 2	2, 4, 2	2, 2, 2	I_2, I_4, I_2	$2 : 1, 3, 4$
C3(D)	1	1	1	-107	-457	0	2	+	1, 2, 4	1, 2, 4	1, 2, 2	I_1, I_2, I_4	$2 : 2$

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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139 **139**
 $N = 139 = 139$ (1 isogeny class)

A1(A)	1	1	0	-3	-4	0	1	-	1	1	1	I_1	
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140 **140**
 $N = 140 = 2^2 \cdot 5 \cdot 7$ (2 isogeny classes)

A1(A)	0	1	0	-5	-25	0	3	-	8, 3, 1	0, 3, 1	3, 3, 1	IV^*, I_3, I_1	3 : 2
A2(B)	0	1	0	-805	-9065	0	1	-	8, 1, 3	0, 1, 3	1, 1, 3	IV^*, I_1, I_3	3 : 1
B1(C)	0	0	0	32	212	0	1	-	8, 1, 5	0, 1, 5	1, 1, 1	IV^*, I_1, I_5	

141 **141**
 $N = 141 = 3 \cdot 47$ (5 isogeny classes)

A1(E)	0	1	1	-12	2	1	1	+	7, 1	7, 1	7, 1	I_7, I_1	
B1(G)	1	1	1	-8	-16	0	2	-	6, 1	6, 1	2, 1	I_6, I_1	2 : 2
B2(F)	1	1	1	-143	-718	0	2	+	3, 2	3, 2	1, 2	I_3, I_2	2 : 1
C1(A)	1	0	0	-2	3	0	4	-	4, 1	4, 1	4, 1	I_4, I_1	2 : 2
C2(B)	1	0	0	-47	120	0	4	+	2, 2	2, 2	2, 2	I_2, I_2	2 : 1, 3, 4
C3(C)	1	0	0	-62	33	0	2	+	1, 4	1, 4	1, 2	I_1, I_4	2 : 2
C4(D)	1	0	0	-752	7875	0	2	+	1, 1	1, 1	1, 1	I_1, I_1	2 : 2
D1(I)	0	-1	1	-1	0	1	1	+	1, 1	1, 1	1, 1	I_1, I_1	
E1(H)	0	1	1	-26	-61	0	1	+	1, 1	1, 1	1, 1	I_1, I_1	

142 **142**
 $N = 142 = 2 \cdot 71$ (5 isogeny classes)

A1(F)	1	-1	1	-12	15	1	1	+	9, 1	9, 1	9, 1	I_9, I_1	
B1(E)	1	1	0	-1	-1	1	1	+	1, 1	1, 1	1, 1	I_1, I_1	
C1(A)	1	-1	0	-1	-3	0	2	-	6, 1	6, 1	2, 1	I_6, I_1	2 : 2
C2(B)	1	-1	0	-41	-91	0	2	+	3, 2	3, 2	1, 2	I_3, I_2	2 : 1
D1(C)	1	0	0	-8	8	0	3	+	3, 1	3, 1	3, 1	I_3, I_1	3 : 2
D2(D)	1	0	0	-58	-170	0	1	+	1, 3	1, 3	1, 1	I_1, I_3	3 : 1
E1(G)	1	-1	0	-2626	52244	0	1	+	27, 1	27, 1	1, 1	I_{27}, I_1	

143 **143**
 $N = 143 = 11 \cdot 13$ (1 isogeny class)

A1(A)	0	-1	1	-1	-2	1	1	-	1, 2	1, 2	1, 2	I_1, I_2	
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144 **144**
 $N = 144 = 2^4 \cdot 3^2$ (2 isogeny classes)

A1(A)	0	0	0	0	-1	0	2	-	4, 3	0, 0	1, 2	II, III	2 : 2; 3 : 3
A2(B)	0	0	0	-15	-22	0	2	+	8, 3	0, 0	1, 2	I_0^*, III	2 : 1; 3 : 4
A3(C)	0	0	0	0	27	0	2	-	4, 9	0, 0	1, 2	II, III*	2 : 4; 3 : 1
A4(D)	0	0	0	-135	594	0	2	+	8, 9	0, 0	1, 2	I_0^*, III^*	2 : 3; 3 : 2
B1(E)	0	0	0	6	7	0	2	-	4, 7	0, 1	1, 2	II, I_1^*	2 : 2
B2(F)	0	0	0	-39	70	0	4	+	8, 8	0, 2	2, 4	I_0^*, I_2^*	2 : 1, 3, 4
B3(G)	0	0	0	-219	-1190	0	4	+	10, 10	0, 4	4, 4	I_2^*, I_4^*	2 : 2, 5, 6
B4(H)	0	0	0	-579	5362	0	4	+	10, 7	0, 1	2, 4	I_2^*, I_1^*	2 : 2
B5(J)	0	0	0	-3459	-78302	0	2	+	11, 8	0, 2	4, 2	I_3^*, I_2^*	2 : 3
B6(I)	0	0	0	141	-4718	0	2	-	11, 14	0, 8	2, 4	I_3^*, I_8^*	2 : 3

145 **145**
 $N = 145 = 5 \cdot 29$ (1 isogeny class)

A1(A)	1	-1	1	-3	2	1	2	+	1, 1	1, 1	1, 1	I_1, I_1	2 : 2
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	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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147 **147**
 $N = 147 = 3 \cdot 7^2$ (3 isogeny classes)

A1(C)	1	1	1	48	48	0	4	−	2, 7	2, 1	2, 4	I_{2, I_1^*}	2 : 2
A2(D)	1	1	1	−197	146	0	4	+	4, 8	4, 2	2, 4	I_4, I_2^*	2 : 1, 3, 4
A3(E)	1	1	1	−1912	−32782	0	2	+	8, 7	8, 1	2, 2	I_8, I_1^*	2 : 2
A4(F)	1	1	1	−2402	44246	0	4	+	2, 10	2, 4	2, 4	I_2, I_4^*	2 : 2, 5, 6
A5(H)	1	1	1	−38417	2882228	0	2	+	1, 8	1, 2	1, 2	I_1, I_2^*	2 : 4
A6(G)	1	1	1	−1667	72764	0	2	−	1, 14	1, 8	1, 4	I_1, I_8^*	2 : 4

B1(I)	0	1	1	−114	473	0	1	−	1, 8	1, 0	1, 1	I_1, IV^*	13 : 2
B2(J)	0	1	1	−44704	−3655907	0	1	−	13, 8	13, 0	13, 1	I_{13}, IV^*	13 : 1

C1(A)	0	−1	1	−2	−1	0	1	−	1, 2	1, 0	1, 1	I_1, II	13 : 2
C2(B)	0	−1	1	−912	10919	0	1	−	13, 2	13, 0	1, 1	I_{13}, II	13 : 1

148 **148**
 $N = 148 = 2^2 \cdot 37$ (1 isogeny class)

A1(A)	0	−1	0	−5	1	1	1	+	8, 1	0, 1	3, 1	IV^*, I_1	
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150 **150**
 $N = 150 = 2 \cdot 3 \cdot 5^2$ (3 isogeny classes)

A1(A)	1	0	0	−3	−3	0	2	−	2, 1, 3	2, 1, 0	2, 1, 2	I_2, I_1, III	2 : 2; 5 : 3
A2(B)	1	0	0	−53	−153	0	2	+	1, 2, 3	1, 2, 0	1, 2, 2	I_1, I_2, III	2 : 1; 5 : 4
A3(C)	1	0	0	−28	272	0	10	−	10, 5, 3	10, 5, 0	10, 5, 2	I_{10}, I_5, III	2 : 4; 5 : 1
A4(D)	1	0	0	−828	9072	0	10	+	5, 10, 3	5, 10, 0	5, 10, 2	I_5, I_{10}, III	2 : 3; 5 : 2

B1(G)	1	1	0	−75	−375	0	2	−	2, 1, 9	2, 1, 0	2, 1, 2	I_2, I_1, III^*	2 : 2; 5 : 3
B2(H)	1	1	0	−1325	−19125	0	2	+	1, 2, 9	1, 2, 0	1, 2, 2	I_1, I_2, III^*	2 : 1; 5 : 4
B3(E)	1	1	0	−700	34000	0	2	−	10, 5, 9	10, 5, 0	2, 1, 2	I_{10}, I_5, III^*	2 : 4; 5 : 1
B4(F)	1	1	0	−20700	1134000	0	2	+	5, 10, 9	5, 10, 0	1, 2, 2	I_5, I_{10}, III^*	2 : 3; 5 : 2

C1(I)	1	1	1	37	281	0	4	−	4, 3, 7	4, 3, 1	4, 1, 4	I_4, I_3, I_1^*	2 : 2; 3 : 3
C2(J)	1	1	1	−463	3281	0	4	+	2, 6, 8	2, 6, 2	2, 2, 4	I_2, I_6, I_2^*	2 : 1, 4, 5; 3 : 6
C3(K)	1	1	1	−338	−7969	0	4	−	12, 1, 9	12, 1, 3	12, 1, 4	I_{12}, I_1, I_3^*	2 : 6; 3 : 1
C4(L)	1	1	1	−1713	−24219	0	2	+	1, 12, 7	1, 12, 1	1, 2, 4	I_1, I_{12}, I_1^*	2 : 2; 3 : 7
C5(M)	1	1	1	−7213	232781	0	2	+	1, 3, 10	1, 3, 4	1, 1, 4	I_1, I_3, I_4^*	2 : 2; 3 : 8
C6(N)	1	1	1	−8338	−295969	0	4	+	6, 2, 12	6, 2, 6	6, 2, 4	I_6, I_2, I_6^*	2 : 3, 7, 8; 3 : 2
C7(O)	1	1	1	−133338	−18795969	0	2	+	3, 4, 9	3, 4, 3	3, 2, 4	I_3, I_4, I_3^*	2 : 6; 3 : 4
C8(P)	1	1	1	−11338	−67969	0	2	+	3, 1, 18	3, 1, 12	3, 1, 4	I_3, I_1, I_{12}^*	2 : 6; 3 : 5

152 **152**
 $N = 152 = 2^3 \cdot 19$ (2 isogeny classes)

A1(A)	0	1	0	−1	3	1	1	−	8, 1	0, 1	4, 1	I_1^*, I_1	
B1(B)	0	1	0	−8	−16	0	1	−	11, 1	0, 1	1, 1	II^*, I_1	

153 **153**
 $N = 153 = 3^2 \cdot 17$ (4 isogeny classes)

A1(C)	0	0	1	−3	2	1	1	−	3, 1	0, 1	2, 1	III, I_1	
B1(A)	0	0	1	6	27	1	1	−	9, 1	3, 1	4, 1	I_3^*, I_1	3 : 2
B2(B)	0	0	1	−534	4752	1	3	−	7, 3	1, 3	4, 3	I_1^*, I_3	3 : 1

C1(E)	1	−1	0	−6	−1	0	2	+	6, 1	0, 1	2, 1	I_0^*, I_1	2 : 2
C2(F)	1	−1	0	−51	152	0	4	+	6, 2	0, 2	4, 2	I_0^*, I_2	2 : 1, 3, 4
C3(H)	1	−1	0	−816	9179	0	2	+	6, 1	0, 1	2, 1	I_0^*, I_1	2 : 2
C4(G)	1	−1	0	−6	377	0	2	−	6, 4	0, 4	2, 2	I_0^*, I_4	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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154 $N = 154 = 2 \cdot 7 \cdot 11$ (3 isogeny classes) **154**

A1(C)	1	-1	0	-29	69	1	2	-	6, 1, 2	6, 1, 2	2, 1, 2	I_{6, I_1, I_2}	2 : 2
A2(D)	1	-1	0	-469	4029	1	2	+	3, 2, 1	3, 2, 1	1, 2, 1	I_{3, I_2, I_1}	2 : 1
B1(E)	1	-1	1	-4	-89	0	4	-	12, 1, 2	12, 1, 2	12, 1, 2	I_{12, I_1, I_2}	2 : 2
B2(F)	1	-1	1	-324	-2137	0	4	+	6, 2, 4	6, 2, 4	6, 2, 2	I_{6, I_2, I_4}	2 : 1, 3, 4
B3(G)	1	-1	1	-5164	-141529	0	2	+	3, 4, 2	3, 4, 2	3, 2, 2	I_{3, I_4, I_2}	2 : 2
B4(H)	1	-1	1	-604	2343	0	2	+	3, 1, 8	3, 1, 8	3, 1, 2	I_{3, I_1, I_8}	2 : 2
C1(A)	1	1	0	-14	-28	0	2	-	4, 1, 2	4, 1, 2	2, 1, 2	I_{4, I_1, I_2}	2 : 2
C2(B)	1	1	0	-234	-1480	0	2	+	2, 2, 1	2, 2, 1	2, 2, 1	I_{2, I_2, I_1}	2 : 1

155 $N = 155 = 5 \cdot 31$ (3 isogeny classes) **155**

A1(D)	0	-1	1	10	6	1	5	-	5, 1	5, 1	5, 1	I_{5, I_1}	5 : 2
A2(E)	0	-1	1	-840	-9114	1	1	-	1, 5	1, 5	1, 5	I_{1, I_5}	5 : 1
B1(A)	1	1	1	-1	-2	0	2	-	2, 1	2, 1	2, 1	I_{2, I_1}	2 : 2
B2(B)	1	1	1	-26	-62	0	2	+	1, 2	1, 2	1, 2	I_{1, I_2}	2 : 1
C1(C)	0	-1	1	-1	1	1	1	-	1, 1	1, 1	1, 1	I_{1, I_1}	

156 $N = 156 = 2^2 \cdot 3 \cdot 13$ (2 isogeny classes) **156**

A1(E)	0	-1	0	-5	6	1	2	+	4, 2, 1	0, 2, 1	3, 2, 1	IV, I_2, I_1	2 : 2
A2(F)	0	-1	0	-20	-24	1	2	+	8, 1, 2	0, 1, 2	3, 1, 2	IV^*, I_1, I_2	2 : 1
B1(A)	0	1	0	-13	-4	0	6	+	4, 6, 1	0, 6, 1	3, 6, 1	IV, I_6, I_1	2 : 2; 3 : 3
B2(B)	0	1	0	-148	644	0	6	+	8, 3, 2	0, 3, 2	3, 3, 2	IV^*, I_3, I_2	2 : 1; 3 : 4
B3(C)	0	1	0	-733	-7888	0	2	+	4, 2, 3	0, 2, 3	1, 2, 3	IV, I_2, I_3	2 : 4; 3 : 1
B4(D)	0	1	0	-748	-7564	0	2	+	8, 1, 6	0, 1, 6	1, 1, 6	IV^*, I_1, I_6	2 : 3; 3 : 2

158 $N = 158 = 2 \cdot 79$ (5 isogeny classes) **158**

A1(E)	1	-1	1	-9	9	1	1	+	8, 1	8, 1	8, 1	I_8, I_1	
B1(D)	1	1	0	-3	1	1	1	+	2, 1	2, 1	2, 1	I_2, I_1	
C1(H)	1	1	1	-420	3109	0	5	+	20, 1	20, 1	20, 1	I_{20, I_1}	5 : 2
C2(I)	1	1	1	-23380	-1385691	0	1	+	4, 5	4, 5	4, 1	I_4, I_5	5 : 1
D1(B)	1	0	1	-82	-92	0	3	+	6, 3	6, 3	2, 3	I_6, I_3	3 : 2, 3
D2(C)	1	0	1	-5217	-145452	0	1	+	18, 1	18, 1	2, 1	I_{18, I_1}	3 : 1
D3(A)	1	0	1	-47	118	0	3	+	2, 1	2, 1	2, 1	I_2, I_1	3 : 1
E1(F)	1	1	1	1	1	0	2	-	2, 1	2, 1	2, 1	I_2, I_1	2 : 2
E2(G)	1	1	1	-9	5	0	2	+	1, 2	1, 2	1, 2	I_1, I_2	2 : 1

160 $N = 160 = 2^5 \cdot 5$ (2 isogeny classes) **160**

A1(D)	0	1	0	-6	4	1	2	+	6, 1	0, 1	2, 1	III, I_1	2 : 2
A2(C)	0	1	0	-1	15	1	2	-	12, 2	0, 2	4, 2	I_3^*, I_2	2 : 1
B1(A)	0	-1	0	-6	-4	0	2	+	6, 1	0, 1	2, 1	III, I_1	2 : 2
B2(B)	0	-1	0	-1	-15	0	2	-	12, 2	0, 2	2, 2	I_3^*, I_2	2 : 1

161 $N = 161 = 7 \cdot 23$ (1 isogeny class) **161**

A1(B)	1	-1	1	-9	8	0	4	+	2, 2	2, 2	2, 2	I_2, I_2	2 : 2, 3, 4
A2(A)	1	-1	1	-4	-2	0	2	+	1, 1	1, 1	1, 1	I_1, I_1	2 : 1
A3(C)	1	-1	1	-124	560	0	4	+	4, 1	4, 1	4, 1	I_4, I_1	2 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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162 $N = 162 = 2 \cdot 3^4$ (4 isogeny classes) **162**

A1(K)	1	-1	0	-6	8	1	3	-	2, 6	2, 0	2, 3	I_2, IV	3 : 2
A2(L)	1	-1	0	39	-19	1	1	-	6, 10	6, 0	2, 3	I_6, IV^*	3 : 1
B1(G)	1	-1	1	-5	5	0	3	-	3, 4	3, 0	3, 1	I_3, II	3 : 2; 7 : 3
B2(H)	1	-1	1	25	1	0	1	-	1, 12	1, 0	1, 1	I_1, II^*	3 : 1; 7 : 4
B3(I)	1	-1	1	-95	-697	0	3	-	21, 4	21, 0	21, 1	I_{21}, II	3 : 4; 7 : 1
B4(J)	1	-1	1	-9695	-364985	0	1	-	7, 12	7, 0	7, 1	I_7, II^*	3 : 3; 7 : 2
C1(A)	1	-1	0	3	-1	0	3	-	1, 6	1, 0	1, 3	I_1, IV	3 : 2; 7 : 3
C2(B)	1	-1	0	-42	-100	0	1	-	3, 10	3, 0	1, 1	I_3, IV^*	3 : 1; 7 : 4
C3(D)	1	-1	0	-1077	13877	0	3	-	7, 6	7, 0	1, 3	I_7, IV	3 : 4; 7 : 1
C4(C)	1	-1	0	-852	19664	0	1	-	21, 10	21, 0	1, 1	I_{21}, IV^*	3 : 3; 7 : 2
D1(E)	1	-1	1	4	-1	0	3	-	6, 4	6, 0	6, 1	I_6, II	3 : 2
D2(F)	1	-1	1	-56	-161	0	1	-	2, 12	2, 0	2, 1	I_2, II^*	3 : 1

163 $N = 163 = 163$ (1 isogeny class) **163**

A1(A)	0	0	1	-2	1	1	1	-	1	1	1	I_1	
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166 $N = 166 = 2 \cdot 83$ (1 isogeny class) **166**

A1(A)	1	1	0	-6	4	1	1	-	4, 1	4, 1	2, 1	I_4, I_1	
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168 $N = 168 = 2^3 \cdot 3 \cdot 7$ (2 isogeny classes) **168**

A1(B)	0	1	0	-7	-10	0	2	+	4, 1, 1	0, 1, 1	2, 1, 1	III, I_1, I_1	2 : 2
A2(A)	0	1	0	-12	0	0	4	+	8, 2, 2	0, 2, 2	2, 2, 2	I_1^*, I_2, I_2	2 : 1, 3, 4
A3(C)	0	1	0	-152	672	0	4	+	10, 4, 1	0, 4, 1	2, 4, 1	III^*, I_4, I_1	2 : 2
A4(D)	0	1	0	48	48	0	2	-	10, 1, 4	0, 1, 4	2, 1, 2	III^*, I_1, I_4	2 : 2
B1(E)	0	-1	0	-7	52	0	4	-	4, 3, 4	0, 3, 4	2, 1, 4	III, I_3, I_4	2 : 2
B2(F)	0	-1	0	-252	1620	0	4	+	8, 6, 2	0, 6, 2	2, 2, 2	I_1^*, I_6, I_2	2 : 1, 3, 4
B3(G)	0	-1	0	-392	-228	0	2	+	10, 12, 1	0, 12, 1	2, 2, 1	III^*, I_{12}, I_1	2 : 2
B4(H)	0	-1	0	-4032	99900	0	2	+	10, 3, 1	0, 3, 1	2, 1, 1	III^*, I_3, I_1	2 : 2

170 $N = 170 = 2 \cdot 5 \cdot 17$ (5 isogeny classes) **170**

A1(A)	1	0	1	-8	6	1	2	+	4, 2, 1	4, 2, 1	2, 2, 1	I_4, I_2, I_1	2 : 2
A2(B)	1	0	1	12	38	1	2	-	2, 4, 2	2, 4, 2	2, 4, 2	I_2, I_4, I_2	2 : 1
B1(H)	1	0	1	-2554	49452	0	6	+	8, 2, 3	8, 2, 3	2, 2, 3	I_8, I_2, I_3	2 : 2; 3 : 3
B2(I)	1	0	1	-2474	52716	0	6	-	4, 4, 6	4, 4, 6	2, 2, 6	I_4, I_4, I_6	2 : 1; 3 : 4
B3(J)	1	0	1	-4169	-20724	0	2	+	24, 6, 1	24, 6, 1	2, 2, 1	I_{24}, I_6, I_1	2 : 4; 3 : 1
B4(K)	1	0	1	16311	-159988	0	2	-	12, 12, 2	12, 12, 2	2, 2, 2	I_{12}, I_{12}, I_2	2 : 3; 3 : 2
C1(F)	1	0	0	399	-919	0	3	-	21, 3, 1	21, 3, 1	21, 1, 1	I_{21}, I_3, I_1	3 : 2
C2(G)	1	0	0	-6641	-215575	0	1	-	7, 9, 3	7, 9, 3	7, 1, 1	I_7, I_9, I_3	3 : 1
D1(D)	1	0	1	-3	6	0	3	-	3, 3, 1	3, 3, 1	1, 3, 1	I_3, I_3, I_1	3 : 2
D2(E)	1	0	1	22	-164	0	1	-	9, 1, 3	9, 1, 3	1, 1, 1	I_9, I_1, I_3	3 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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171 $N = 171 = 3^2 \cdot 19$ (4 isogeny classes)**171**

A1(D)	1	-1	1	-14	20	0	4	+	7, 1	1, 1	4, 1	I_1^*, I_1	2 : 2
A2(E)	1	-1	1	-59	-142	0	4	+	8, 2	2, 2	4, 2	I_2^*, I_2	2 : 1, 3, 4
A3(F)	1	-1	1	-914	-10402	0	2	+	10, 1	4, 1	4, 1	I_4^*, I_1	2 : 2
A4(G)	1	-1	1	76	-790	0	2	-	7, 4	1, 4	2, 2	I_1^*, I_4	2 : 2
B1(A)	0	0	1	6	0	1	1	-	6, 1	0, 1	2, 1	I_0^*, I_1	3 : 2
B2(B)	0	0	1	-84	315	1	3	-	6, 3	0, 3	2, 3	I_0^*, I_3	3 : 1, 3
B3(C)	0	0	1	-6924	221760	1	3	-	6, 1	0, 1	2, 1	I_0^*, I_1	3 : 2
C1(I)	0	0	1	177	1035	0	1	-	16, 1	10, 1	2, 1	I_{10}^*, I_1	5 : 2
C2(J)	0	0	1	-39513	3023145	0	1	-	8, 5	2, 5	2, 1	I_2^*, I_5	5 : 1
D1(H)	0	0	1	-21	-41	0	1	-	8, 1	2, 1	2, 1	I_2^*, I_1	

172 $N = 172 = 2^2 \cdot 43$ (1 isogeny class)**172**

A1(A)	0	1	0	-13	15	1	3	-	8, 1	0, 1	3, 1	IV^*, I_1	3 : 2
A2(B)	0	1	0	67	79	1	1	-	8, 3	0, 3	1, 3	IV^*, I_3	3 : 1

174 $N = 174 = 2 \cdot 3 \cdot 29$ (5 isogeny classes)**174**

A1(I)	1	0	1	-7705	1226492	0	3	-	11, 21, 1	11, 21, 1	1, 21, 1	I_{11}, I_{21}, I_1	3 : 2
A2(J)	1	0	1	68840	-31810330	0	1	-	33, 7, 3	33, 7, 3	1, 7, 1	I_{33}, I_7, I_3	3 : 1
B1(G)	1	0	0	-1	137	0	7	-	7, 7, 1	7, 7, 1	7, 7, 1	I_7, I_7, I_1	7 : 2
B2(H)	1	0	0	-6511	-203353	0	1	-	1, 1, 7	1, 1, 7	1, 1, 7	I_1, I_1, I_7	7 : 1
C1(F)	1	1	1	-5	-7	0	1	-	1, 3, 1	1, 3, 1	1, 1, 1	I_1, I_3, I_1	
D1(A)	1	0	1	0	-2	0	2	-	4, 1, 1	4, 1, 1	2, 1, 1	I_4, I_1, I_1	2 : 2
D2(B)	1	0	1	-20	-34	0	4	+	2, 2, 2	2, 2, 2	2, 2, 2	I_2, I_2, I_2	2 : 1, 3, 4
D3(C)	1	0	1	-310	-2122	0	2	+	1, 4, 1	1, 4, 1	1, 4, 1	I_1, I_4, I_1	2 : 2
D4(D)	1	0	1	-50	86	0	2	+	1, 1, 4	1, 1, 4	1, 1, 2	I_1, I_1, I_4	2 : 2
E1(E)	1	1	0	-56	-192	0	1	-	13, 1, 1	13, 1, 1	1, 1, 1	I_{13}, I_1, I_1	

175 $N = 175 = 5^2 \cdot 7$ (3 isogeny classes)**175**

A1(B)	0	-1	1	2	-2	1	1	-	3, 1	0, 1	2, 1	III, I_1	5 : 2
A2(A)	0	-1	1	-148	748	1	5	-	3, 5	0, 5	2, 5	III, I_5	5 : 1
B1(C)	0	-1	1	-33	93	1	1	-	7, 1	1, 1	4, 1	I_1^*, I_1	3 : 2
B2(D)	0	-1	1	217	-282	1	1	-	9, 3	3, 3	4, 1	I_3^*, I_3	3 : 1, 3
B3(E)	0	-1	1	-3283	-74657	1	1	-	15, 1	9, 1	4, 1	I_9^*, I_1	3 : 2
C1(F)	0	1	1	42	-131	0	1	-	9, 1	0, 1	2, 1	III^*, I_1	5 : 2
C2(G)	0	1	1	-3708	86119	0	1	-	9, 5	0, 5	2, 1	III^*, I_5	5 : 1

176 $N = 176 = 2^4 \cdot 11$ (3 isogeny classes)**176**

A1(C)	0	0	0	-4	-4	0	1	-	8, 1	0, 1	1, 1	I_0^*, I_1	
B1(D)	0	1	0	-5	-13	0	1	-	12, 1	0, 1	1, 1	II^*, I_1	5 : 2
B2(E)	0	1	0	-165	1427	0	1	-	12, 5	0, 5	1, 1	II^*, I_5	5 : 1, 3
B3(F)	0	1	0	-125125	16994227	0	1	-	12, 1	0, 1	1, 1	II^*, I_1	5 : 2
C1(A)	0	-1	0	3	1	1	1	-	8, 1	0, 1	2, 1	I^*, I_1	3 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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178 **178**
 $N = 178 = 2 \cdot 89$ (2 isogeny classes)

A1(A)	1	0	0	6	-28	0	3	-	12, 1	12, 1	12, 1	I_{12}, I_1	3 : 2
A2(B)	1	0	0	-554	-5068	0	1	-	4, 3	4, 3	4, 1	I_4, I_3	3 : 1
B1(C)	1	1	0	-44	80	0	2	+	14, 1	14, 1	2, 1	I_{14}, I_1	2 : 2
B2(D)	1	1	0	-684	6608	0	2	+	7, 2	7, 2	1, 2	I_7, I_2	2 : 1

179 **179**
 $N = 179 = 179$ (1 isogeny class)

A1(A)	0	0	1	-1	-1	0	1	-	1	1	1	I_1	
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180 **180**
 $N = 180 = 2^2 \cdot 3^2 \cdot 5$ (1 isogeny class)

A1(A)	0	0	0	-12	-11	0	2	+	4, 6, 1	0, 0, 1	1, 2, 1	IV, I_0^*, I_1	2 : 2; 3 : 3
A2(B)	0	0	0	33	-74	0	2	-	8, 6, 2	0, 0, 2	1, 2, 2	IV^*, I_0^*, I_2	2 : 1; 3 : 4
A3(C)	0	0	0	-372	2761	0	6	+	4, 6, 3	0, 0, 3	3, 2, 3	IV, I_0^*, I_3	2 : 4; 3 : 1
A4(D)	0	0	0	-327	3454	0	6	-	8, 6, 6	0, 0, 6	3, 2, 6	IV^*, I_0^*, I_6	2 : 3; 3 : 2

182 **182**
 $N = 182 = 2 \cdot 7 \cdot 13$ (5 isogeny classes)

A1(E)	1	-1	1	866	6445	0	4	-	20, 3, 2	20, 3, 2	20, 1, 2	I_{20}, I_3, I_2	2 : 2
A2(F)	1	-1	1	-4254	59693	0	4	+	10, 6, 4	10, 6, 4	10, 2, 2	I_{10}, I_6, I_4	2 : 1, 3, 4
A3(G)	1	-1	1	-31294	-2081875	0	2	+	5, 12, 2	5, 12, 2	5, 2, 2	I_5, I_{12}, I_2	2 : 2
A4(H)	1	-1	1	-59134	5547693	0	2	+	5, 3, 8	5, 3, 8	5, 1, 2	I_5, I_3, I_8	2 : 2
B1(A)	1	0	0	7	-7	0	3	-	9, 1, 1	9, 1, 1	9, 1, 1	I_9, I_1, I_1	3 : 2
B2(B)	1	0	0	-193	-1055	0	3	-	3, 3, 3	3, 3, 3	3, 3, 3	I_3, I_3, I_3	3 : 1, 3
B3(C)	1	0	0	-15663	-755809	0	1	-	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	3 : 2
C1(J)	1	0	1	-4609	120244	0	1	-	11, 7, 1	11, 7, 1	1, 1, 1	I_{11}, I_7, I_1	
D1(D)	1	-1	1	3	-5	0	1	-	1, 3, 1	1, 3, 1	1, 1, 1	I_1, I_3, I_1	
E1(I)	1	-1	0	-22	884	0	1	-	7, 1, 5	7, 1, 5	1, 1, 1	I_7, I_1, I_5	

184 **184**
 $N = 184 = 2^3 \cdot 23$ (4 isogeny classes)

A1(C)	0	-1	0	0	1	1	1	-	4, 1	0, 1	2, 1	III, I_1	
B1(B)	0	-1	0	-4	5	1	1	-	4, 1	0, 1	2, 1	III, I_1	
C1(D)	0	0	0	5	6	0	2	-	10, 1	0, 1	2, 1	III^*, I_1	2 : 2
C2(E)	0	0	0	-35	62	0	2	+	11, 2	0, 2	1, 2	II^*, I_2	2 : 1
D1(A)	0	0	0	-55	-157	0	1	-	4, 1	0, 1	2, 1	III, I_1	

185 **185**
 $N = 185 = 5 \cdot 37$ (3 isogeny classes)

A1(D)	0	1	1	-156	700	1	1	+	4, 1	4, 1	2, 1	I_4, I_1	
B1(A)	0	-1	1	-5	6	1	1	+	2, 1	2, 1	2, 1	I_2, I_1	
C1(B)	1	0	1	-4	-3	1	2	+	1, 1	1, 1	1, 1	I_1, I_1	2 : 2
C2(C)	1	0	1	1	-9	1	2	-	2, 2	2, 2	2, 2	I_2, I_2	2 : 1

186 **186**
 $N = 186 = 2 \cdot 3 \cdot 31$ (3 isogeny classes)

A1(D)	1	1	0	-83	-369	0	1	-	1, 11, 1	1, 11, 1	1, 1, 1	I_1, I_{11}, I_1	
B1(B)	1	0	0	15	9	0	5	-	5, 5, 1	5, 5, 1	5, 5, 1	I_5, I_5, I_1	5 : 2
B2(C)	1	0	0	-1395	-20181	0	1	-	1, 1, 5	1, 1, 5	1, 1, 5	I_1, I_1, I_5	5 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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187 $N = 187 = 11 \cdot 17$ (2 isogeny classes)**187**

A1(A)	0	1	1	11	30	0	3	−	3, 2	3, 2	3, 2	I_3, I_2	3 : 2
A2(B)	0	1	1	−99	−905	0	1	−	1, 6	1, 6	1, 2	I_1, I_6	3 : 1
B1(C)	0	0	1	7	1	0	1	−	3, 1	3, 1	1, 1	I_3, I_1	

189 $N = 189 = 3^3 \cdot 7$ (4 isogeny classes)**189**

A1(A)	0	0	1	−3	0	1	1	+	5, 1	0, 1	3, 1	IV, I_1	
B1(C)	0	0	1	−24	45	1	3	+	3, 1	0, 1	1, 1	II, I_1	3 : 2
B2(D)	0	0	1	−54	−88	1	3	+	9, 3	0, 3	3, 3	IV^*, I_3	3 : 1, 3
B3(E)	0	0	1	−3834	−91375	1	1	+	11, 1	0, 1	1, 1	II^*, I_1	3 : 2
C1(F)	0	0	1	−6	3	0	3	+	3, 3	0, 3	1, 3	II, I_3	3 : 2, 3
C2(G)	0	0	1	−216	−1222	0	1	+	9, 1	0, 1	1, 1	IV^*, I_1	3 : 1
C3(H)	0	0	1	−426	3384	0	3	+	5, 1	0, 1	3, 1	IV, I_1	3 : 1
D1(B)	0	0	1	−27	−7	0	1	+	11, 1	0, 1	1, 1	II^*, I_1	

190 $N = 190 = 2 \cdot 5 \cdot 19$ (3 isogeny classes)**190**

A1(D)	1	−1	1	−48	147	1	1	−	11, 2, 1	11, 2, 1	11, 2, 1	I_{11}, I_2, I_1	
B1(C)	1	1	0	2	2	1	1	−	1, 2, 1	1, 2, 1	1, 2, 1	I_1, I_2, I_1	
C1(A)	1	0	0	−30	−100	0	3	−	3, 6, 1	3, 6, 1	3, 6, 1	I_3, I_6, I_1	3 : 2
C2(B)	1	0	0	−2780	−56650	0	1	−	1, 2, 3	1, 2, 3	1, 2, 3	I_1, I_2, I_3	3 : 1

192 $N = 192 = 2^6 \cdot 3$ (4 isogeny classes)**192**

A1(Q)	0	−1	0	−4	−2	1	2	+	6, 1	0, 1	1, 1	II, I_1	2 : 2
A2(R)	0	−1	0	−9	9	1	4	+	12, 2	0, 2	4, 2	I_2^*, I_2	2 : 1, 3, 4
A3(T)	0	−1	0	−129	609	1	4	+	15, 1	0, 1	4, 1	I_5^*, I_1	2 : 2
A4(S)	0	−1	0	31	33	1	2	−	15, 4	0, 4	4, 2	I_5^*, I_4	2 : 2
B1(A)	0	1	0	−4	2	0	2	+	6, 1	0, 1	1, 1	II, I_1	2 : 2
B2(B)	0	1	0	−9	−9	0	4	+	12, 2	0, 2	4, 2	I_2^*, I_2	2 : 1, 3, 4
B3(D)	0	1	0	−129	−609	0	2	+	15, 1	0, 1	4, 1	I_5^*, I_1	2 : 2
B4(C)	0	1	0	31	−33	0	4	−	15, 4	0, 4	4, 4	I_5^*, I_4	2 : 2
C1(K)	0	1	0	3	3	0	2	−	10, 1	0, 1	2, 1	I_0^*, I_1	2 : 2
C2(L)	0	1	0	−17	15	0	4	+	14, 2	0, 2	4, 2	I_4^*, I_2	2 : 1, 3, 4
C3(M)	0	1	0	−97	−385	0	4	+	16, 4	0, 4	4, 4	I_6^*, I_4	2 : 2, 5, 6
C4(N)	0	1	0	−257	1503	0	2	+	16, 1	0, 1	2, 1	I_6^*, I_1	2 : 2
C5(P)	0	1	0	−1537	−23713	0	2	+	17, 2	0, 2	4, 2	I_7^*, I_2	2 : 3
C6(O)	0	1	0	63	−1377	0	4	−	17, 8	0, 8	4, 8	I_7^*, I_8	2 : 3
D1(E)	0	−1	0	3	−3	0	2	−	10, 1	0, 1	2, 1	I_0^*, I_1	2 : 2
D2(F)	0	−1	0	−17	−15	0	4	+	14, 2	0, 2	4, 2	I_4^*, I_2	2 : 1, 3, 4
D3(H)	0	−1	0	−257	−1503	0	2	+	16, 1	0, 1	4, 1	I_6^*, I_1	2 : 2
D4(G)	0	−1	0	−97	385	0	4	+	16, 4	0, 4	4, 2	I_6^*, I_4	2 : 2, 5, 6
D5(J)	0	−1	0	−1537	23713	0	4	+	17, 2	0, 2	4, 2	I_7^*, I_2	2 : 4
D6(I)	0	−1	0	63	1377	0	2	−	17, 8	0, 8	2, 2	I_7^*, I_8	2 : 4

194 $N = 194 = 2 \cdot 97$ (1 isogeny class)**194**

A1(A)	1	−1	1	−3	−1	0	2	+	2, 1	2, 1	2, 1	I_2, I_1	2 : 2
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	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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195

$N = 195 = 3 \cdot 5 \cdot 13$ (4 isogeny classes)

195

A1(A)	1	0	0	-110	435	0	4	+	4, 1, 1	4, 1, 1	4, 1, 1	I_4, I_1, I_1	2 : 2
A2(B)	1	0	0	-115	392	0	8	+	8, 2, 2	8, 2, 2	8, 2, 2	I_8, I_2, I_2	2 : 1, 3, 4
A3(D)	1	0	0	-520	-4225	0	8	+	4, 4, 4	4, 4, 4	4, 4, 4	I_4, I_4, I_4	2 : 2, 5, 6
A4(C)	1	0	0	210	2277	0	4	-	16, 1, 1	16, 1, 1	16, 1, 1	I_{16}, I_1, I_1	2 : 2
A5(E)	1	0	0	-8125	-282568	0	4	+	2, 8, 2	2, 8, 2	2, 8, 2	I_2, I_8, I_2	2 : 3, 7, 8
A6(F)	1	0	0	605	-19750	0	4	-	2, 2, 8	2, 2, 8	2, 2, 8	I_2, I_2, I_8	2 : 3
A7(H)	1	0	0	-130000	-18051943	0	2	+	1, 4, 1	1, 4, 1	1, 4, 1	I_1, I_4, I_1	2 : 5
A8(G)	1	0	0	-7930	-296725	0	2	-	1, 16, 1	1, 16, 1	1, 16, 1	I_1, I_{16}, I_1	2 : 5
B1(I)	0	1	1	0	-1	0	1	-	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	
C1(K)	0	1	1	-66	-349	0	1	-	3, 7, 1	3, 7, 1	3, 1, 1	I_3, I_7, I_1	
D1(J)	0	-1	1	-190	1101	0	1	-	7, 1, 3	7, 1, 3	1, 1, 1	I_7, I_1, I_3	

196

$N = 196 = 2^2 \cdot 7^2$ (2 isogeny classes)

196

A1(A)	0	-1	0	-2	1	1	1	+	4, 2	0, 0	3, 1	IV, II	3 : 2
A2(B)	0	-1	0	-142	701	1	1	+	4, 2	0, 0	1, 1	IV, II	3 : 1
B1(C)	0	1	0	-114	-127	0	3	+	4, 8	0, 0	3, 3	IV, IV*	3 : 2
B2(D)	0	1	0	-6974	-226507	0	1	+	4, 8	0, 0	1, 3	IV, IV*	3 : 1

197

$N = 197 = 197$ (1 isogeny class)

197

A1(A)	0	0	1	-5	4	1	1	+	1	1	1	I_1	
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198

$N = 198 = 2 \cdot 3^2 \cdot 11$ (5 isogeny classes)

198

A1(I)	1	-1	0	-18	4	1	2	+	4, 7, 1	4, 1, 1	2, 4, 1	I_4, I_1^*, I_1	2 : 2
A2(J)	1	-1	0	-198	1120	1	4	+	2, 8, 2	2, 2, 2	2, 4, 2	I_2, I_2^*, I_2	2 : 1, 3, 4
A3(L)	1	-1	0	-3168	69430	1	2	+	1, 7, 1	1, 1, 1	1, 2, 1	I_1, I_1^*, I_1	2 : 2
A4(K)	1	-1	0	-108	2074	1	2	-	1, 10, 4	1, 4, 4	1, 4, 4	I_1, I_4^*, I_4	2 : 2
B1(E)	1	-1	1	-50	-115	0	2	+	2, 9, 1	2, 3, 1	2, 2, 1	I_2, I_3^*, I_1	2 : 2; 3 : 3
B2(F)	1	-1	1	40	-547	0	2	-	1, 12, 2	1, 6, 2	1, 4, 2	I_1, I_6^*, I_2	2 : 1; 3 : 4
B3(G)	1	-1	1	-725	7661	0	6	+	6, 7, 3	6, 1, 3	6, 2, 3	I_6, I_1^*, I_3	2 : 4; 3 : 1
B4(H)	1	-1	1	-365	15005	0	6	-	3, 8, 6	3, 2, 6	3, 4, 6	I_3, I_2^*, I_6	2 : 3; 3 : 2
C1(M)	1	-1	1	-65	209	0	6	+	12, 3, 1	12, 0, 1	12, 2, 1	I_{12}, III, I_1	2 : 2; 3 : 3
C2(N)	1	-1	1	-1025	12881	0	6	+	6, 3, 2	6, 0, 2	6, 2, 2	I_6, III, I_2	2 : 1; 3 : 4
C3(O)	1	-1	1	-785	-8207	0	2	+	4, 9, 3	4, 0, 3	4, 2, 1	I_4, III^*, I_3	2 : 4; 3 : 1
C4(P)	1	-1	1	-1325	4969	0	2	+	2, 9, 6	2, 0, 6	2, 2, 2	I_2, III^*, I_6	2 : 3; 3 : 2
D1(A)	1	-1	0	-87	333	0	6	+	4, 3, 3	4, 0, 3	2, 2, 3	I_4, III, I_3	2 : 2; 3 : 3
D2(B)	1	-1	0	-147	-135	0	6	+	2, 3, 6	2, 0, 6	2, 2, 6	I_2, III, I_6	2 : 1; 3 : 4
D3(C)	1	-1	0	-582	-5068	0	2	+	12, 9, 1	12, 0, 1	2, 2, 1	I_{12}, III^*, I_1	2 : 4; 3 : 1
D4(D)	1	-1	0	-9222	-338572	0	2	+	6, 9, 2	6, 0, 2	2, 2, 2	I_6, III^*, I_2	2 : 3; 3 : 2
E1(Q)	1	-1	0	-405	-2187	0	2	+	10, 11, 1	10, 5, 1	2, 2, 1	I_{10}, I_5^*, I_1	2 : 2; 5 : 3
E2(R)	1	-1	0	1035	-15147	0	2	-	5, 16, 2	5, 10, 2	1, 4, 2	I_5, I_{10}^*, I_2	2 : 1; 5 : 4
E3(S)	1	-1	0	-90585	10516473	0	2	+	2, 7, 5	2, 1, 5	2, 2, 1	I_2, I_1^*, I_5	2 : 4; 5 : 1
E4(T)	1	-1	0	-90495	10538343	0	2	-	1, 8, 10	1, 2, 10	1, 4, 2	I_1, I_2^*, I_{10}	2 : 3; 5 : 2

200

$N = 200 = 2^3 \cdot 5^2$ (5 isogeny classes)

200

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
200	$N = 200 = 2^3 \cdot 5^2$ (continued)											200	
B1(C)	0	1	0	-3	-2	1	2	+	4, 3	0, 0	2, 2	III, III	2 : 2
B2(D)	0	1	0	-28	48	1	2	+	8, 3	0, 0	4, 2	I ₁ [*] , III	2 : 1
C1(G)	0	0	0	-50	125	0	4	+	4, 7	0, 1	2, 4	III, I ₁ [*]	2 : 2
C2(H)	0	0	0	-175	-750	0	4	+	8, 8	0, 2	4, 4	I ₁ [*] , I ₂ [*]	2 : 1, 3, 4
C3(J)	0	0	0	-2675	-53250	0	2	+	10, 7	0, 1	2, 4	III [*] , I ₁ [*]	2 : 2
C4(I)	0	0	0	325	-4250	0	2	-	10, 10	0, 4	2, 4	III [*] , I ₄ [*]	2 : 2
D1(E)	0	-1	0	-83	-88	0	2	+	4, 9	0, 0	2, 2	III, III [*]	2 : 2
D2(F)	0	-1	0	-708	7412	0	2	+	8, 9	0, 0	2, 2	I ₁ [*] , III [*]	2 : 1
E1(A)	0	0	0	5	-10	0	1	-	11, 2	0, 0	1, 1	II [*] , II	
201	$N = 201 = 3 \cdot 67$ (3 isogeny classes)											201	
A1	0	-1	1	2	0	1	1	-	2, 1	2, 1	2, 1	I ₂ , I ₁	
B1	1	0	0	-1	2	1	1	-	3, 1	3, 1	3, 1	I ₃ , I ₁	
C1	1	1	0	-794	8289	1	1	-	5, 1	5, 1	1, 1	I ₅ , I ₁	
202	$N = 202 = 2 \cdot 101$ (1 isogeny class)											202	
A1	1	-1	0	4	-176	0	1	-	17, 1	17, 1	1, 1	I ₁₇ , I ₁	
203	$N = 203 = 7 \cdot 29$ (3 isogeny classes)											203	
A1	0	-1	1	20	-8	0	5	-	5, 1	5, 1	5, 1	I ₅ , I ₁	5 : 2
A2	0	-1	1	-2150	-37668	0	1	-	1, 5	1, 5	1, 1	I ₁ , I ₅	5 : 1
B1	1	1	1	0	-2	1	1	-	2, 1	2, 1	2, 1	I ₂ , I ₁	
C1	1	1	0	-9	8	0	2	-	1, 2	1, 2	1, 2	I ₁ , I ₂	2 : 2
C2	1	1	0	-154	675	0	2	+	2, 1	2, 1	2, 1	I ₂ , I ₁	2 : 1
204	$N = 204 = 2^2 \cdot 3 \cdot 17$ (2 isogeny classes)											204	
A1	0	-1	0	-1621	-24623	0	1	-	8, 11, 1	0, 11, 1	3, 1, 1	IV [*] , I ₁₁ , I ₁	
B1	0	1	0	-5	-9	0	1	-	8, 1, 1	0, 1, 1	1, 1, 1	IV [*] , I ₁ , I ₁	
205	$N = 205 = 5 \cdot 41$ (3 isogeny classes)											205	
A1	1	-1	1	-22	44	1	4	+	2, 1	2, 1	2, 1	I ₂ , I ₁	2 : 2
A2	1	-1	1	-27	26	1	4	+	4, 2	4, 2	4, 2	I ₄ , I ₂	2 : 1, 3, 4
A3	1	-1	1	-232	-1286	1	2	+	8, 1	8, 1	8, 1	I ₈ , I ₁	2 : 2
A4	1	-1	1	98	126	1	4	-	2, 4	2, 4	2, 4	I ₂ , I ₄	2 : 2
B1	1	1	1	-21	-46	0	2	+	2, 1	2, 1	2, 1	I ₂ , I ₁	2 : 2
B2	1	1	1	-16	-62	0	2	-	4, 2	4, 2	2, 2	I ₄ , I ₂	2 : 1
C1	1	1	0	-2	-1	0	2	+	2, 1	2, 1	2, 1	I ₂ , I ₁	2 : 2
C2	1	1	0	-27	44	0	2	+	1, 2	1, 2	1, 2	I ₁ , I ₂	2 : 1
206	$N = 206 = 2 \cdot 103$ (1 isogeny class)											206	
A1	1	1	0	2	0	0	2	-	2, 1	2, 1	2, 1	I ₂ , I ₁	2 : 2
A2	1	1	0	-8	-10	0	2	+	1, 2	1, 2	1, 2	I ₁ , I ₂	2 : 1
207	$N = 207 = 3^2 \cdot 23$ (1 isogeny class)											207	
A1	1	-1	1	-5	20	1	2	-	8, 1	2, 1	4, 1	I [*] , I ₁	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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208

$N = 208 = 2^4 \cdot 13$ (4 isogeny classes)

208

A1	0	-1	0	8	-16	1	1	-	13, 1	1, 1	4, 1	I_5^*, I_1	3 : 2
A2	0	-1	0	-72	496	1	1	-	15, 3	3, 3	4, 3	I_7^*, I_3	3 : 1, 3
A3	0	-1	0	-7352	245104	1	1	-	21, 1	9, 1	4, 1	I_{13}^*, I_1	3 : 2
B1	0	-1	0	-16	32	1	1	-	11, 1	0, 1	4, 1	I_3^*, I_1	
C1	0	0	0	1	10	0	2	-	8, 2	0, 2	1, 2	I_0^*, I_2	2 : 2
C2	0	0	0	-4	3	0	2	+	4, 1	0, 1	1, 1	II, I_1	2 : 1
D1	0	0	0	-43	-166	0	1	-	19, 1	7, 1	2, 1	I_{11}^*, I_1	7 : 2
D2	0	0	0	-3403	83834	0	1	-	13, 7	1, 7	2, 1	I_5^*, I_7	7 : 1

209

$N = 209 = 11 \cdot 19$ (1 isogeny class)

209

A1	0	1	1	-27	55	1	3	-	3, 2	3, 2	3, 2	I_3, I_2	3 : 2
A2	0	1	1	193	-308	1	1	-	1, 6	1, 6	1, 6	I_1, I_6	3 : 1

210

$N = 210 = 2 \cdot 3 \cdot 5 \cdot 7$ (5 isogeny classes)

210

A1	1	0	0	-41	-39	0	6	+	12, 3, 1, 1	12, 3, 1, 1	12, 3, 1, 1	I_{12}, I_3, I_1, I_1	2 : 2; 3 : 3
A2	1	0	0	-361	2585	0	12	+	6, 6, 2, 2	6, 6, 2, 2	6, 6, 2, 2	I_6, I_6, I_2, I_2	2 : 1, 4, 5; 3 : 6
A3	1	0	0	-2681	-53655	0	2	+	4, 1, 3, 3	4, 1, 3, 3	4, 1, 1, 3	I_4, I_1, I_3, I_3	2 : 6; 3 : 1
A4	1	0	0	-5761	167825	0	6	+	3, 3, 1, 4	3, 3, 1, 4	3, 3, 1, 4	I_3, I_3, I_1, I_4	2 : 2; 3 : 7
A5	1	0	0	-81	6561	0	6	-	3, 12, 4, 1	3, 12, 4, 1	3, 12, 2, 1	I_3, I_{12}, I_4, I_1	2 : 2; 3 : 8
A6	1	0	0	-2701	-52819	0	4	+	2, 2, 6, 6	2, 2, 6, 6	2, 2, 2, 6	I_2, I_2, I_6, I_6	2 : 3, 7, 8; 3 : 2
A7	1	0	0	-6451	124931	0	2	+	1, 1, 3, 12	1, 1, 3, 12	1, 1, 1, 12	I_1, I_1, I_3, I_{12}	2 : 6; 3 : 4
A8	1	0	0	729	-176985	0	2	-	1, 4, 12, 3	1, 4, 12, 3	1, 4, 2, 3	I_1, I_4, I_{12}, I_3	2 : 6; 3 : 5
B1	1	0	1	-498	4228	0	6	+	8, 3, 3, 1	8, 3, 3, 1	2, 3, 3, 1	I_8, I_3, I_3, I_1	2 : 2; 3 : 3
B2	1	0	1	-578	2756	0	12	+	4, 6, 6, 2	4, 6, 6, 2	2, 6, 6, 2	I_4, I_6, I_6, I_2	2 : 1, 4, 5; 3 : 6
B3	1	0	1	-1473	-16652	0	2	+	24, 1, 1, 3	24, 1, 1, 3	2, 1, 1, 3	I_{24}, I_1, I_1, I_3	2 : 6; 3 : 1
B4	1	0	1	-4358	-109132	0	6	+	2, 3, 12, 1	2, 3, 12, 1	2, 3, 12, 1	I_2, I_3, I_{12}, I_1	2 : 2; 3 : 7
B5	1	0	1	1922	20756	0	12	-	2, 12, 3, 4	2, 12, 3, 4	2, 12, 3, 4	I_2, I_{12}, I_3, I_4	2 : 2; 3 : 8
B6	1	0	1	-21953	-1253644	0	4	+	12, 2, 2, 6	12, 2, 2, 6	2, 2, 2, 6	I_{12}, I_2, I_2, I_6	2 : 3, 7, 8; 3 : 2
B7	1	0	1	-351233	-80149132	0	2	+	6, 1, 4, 3	6, 1, 4, 3	2, 1, 4, 3	I_6, I_1, I_4, I_3	2 : 6; 3 : 4
B8	1	0	1	-20353	-1443724	0	4	-	6, 4, 1, 12	6, 4, 1, 12	2, 4, 1, 12	I_6, I_4, I_1, I_{12}	2 : 6; 3 : 5
C1	1	1	1	10	-13	0	4	-	8, 1, 1, 2	8, 1, 1, 2	8, 1, 1, 2	I_8, I_1, I_1, I_2	2 : 2
C2	1	1	1	-70	-205	0	8	+	4, 2, 2, 4	4, 2, 2, 4	4, 2, 2, 4	I_4, I_2, I_2, I_4	2 : 1, 3, 4
C3	1	1	1	-1050	-13533	0	4	+	2, 4, 4, 2	2, 4, 4, 2	2, 2, 4, 2	I_2, I_4, I_4, I_2	2 : 2, 5, 6
C4	1	1	1	-370	2435	0	4	+	2, 1, 1, 8	2, 1, 1, 8	2, 1, 1, 8	I_2, I_1, I_1, I_8	2 : 2
C5	1	1	1	-16800	-845133	0	2	+	1, 2, 2, 1	1, 2, 2, 1	1, 2, 2, 1	I_1, I_2, I_2, I_1	2 : 3
C6	1	1	1	-980	-15325	0	2	-	1, 8, 8, 1	1, 8, 8, 1	1, 2, 8, 1	I_1, I_8, I_8, I_1	2 : 3
D1	1	1	0	-3	-3	1	2	+	4, 1, 1, 1	4, 1, 1, 1	2, 1, 1, 1	I_4, I_1, I_1, I_1	2 : 2
D2	1	1	0	-23	33	1	4	+	2, 2, 2, 2	2, 2, 2, 2	2, 2, 2, 2	I_2, I_2, I_2, I_2	2 : 1, 3, 4
D3	1	1	0	-373	2623	1	2	+	1, 4, 1, 1	1, 4, 1, 1	1, 2, 1, 1	I_1, I_4, I_1, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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210 $N = 210 = 2 \cdot 3 \cdot 5 \cdot 7$ (continued)**210**

E1	1	0	0	210	900	0	8	−	16, 4, 2, 1	16, 4, 2, 1	16, 4, 2, 1	I_{16}, I_4, I_2, I_1	$2 : 2$
E2	1	0	0	−1070	7812	0	16	+	8, 8, 4, 2	8, 8, 4, 2	8, 8, 4, 2	I_8, I_8, I_4, I_2	$2 : 1, 3, 4$
E3	1	0	0	−7550	−247500	0	8	+	4, 4, 8, 4	4, 4, 8, 4	4, 4, 8, 2	I_4, I_4, I_8, I_4	$2 : 2, 5, 6$
E4	1	0	0	−15070	710612	0	8	+	4, 16, 2, 1	4, 16, 2, 1	4, 16, 2, 1	I_4, I_{16}, I_2, I_1	$2 : 2$
E5	1	0	0	−120050	−16020000	0	4	+	2, 2, 4, 8	2, 2, 4, 8	2, 2, 4, 2	I_2, I_2, I_4, I_8	$2 : 3, 7, 8$
E6	1	0	0	1270	−789048	0	4	−	2, 2, 16, 2	2, 2, 16, 2	2, 2, 16, 2	I_2, I_2, I_{16}, I_2	$2 : 3$
E7	1	0	0	−1920800	−1024800150	0	2	+	1, 1, 2, 4	1, 1, 2, 4	1, 1, 2, 2	I_1, I_1, I_2, I_4	$2 : 5$
E8	1	0	0	−119300	−16229850	0	2	−	1, 1, 2, 16	1, 1, 2, 16	1, 1, 2, 2	I_1, I_1, I_2, I_{16}	$2 : 5$

212 $N = 212 = 2^2 \cdot 53$ (2 isogeny classes)**212**

A1	0	−1	0	−4	8	1	1	−	8, 1	0, 1	3, 1	IV^*, I_1	
B1	0	−1	0	−12	−40	0	2	−	8, 2	0, 2	3, 2	IV^*, I_2	$2 : 2$
B2	0	−1	0	−17	−22	0	2	+	4, 1	0, 1	3, 1	IV, I_1	$2 : 1$

213 $N = 213 = 3 \cdot 71$ (1 isogeny class)**213**

A1	1	0	1	0	1	0	2	−	2, 1	2, 1	2, 1	I_2, I_1	$2 : 2$
A2	1	0	1	−15	19	0	2	+	1, 2	1, 2	1, 2	I_1, I_2	$2 : 1$

214 $N = 214 = 2 \cdot 107$ (4 isogeny classes)**214**

A1	1	0	0	−12	16	1	1	−	7, 1	7, 1	7, 1	I_7, I_1	
B1	1	0	1	1	0	1	1	−	1, 1	1, 1	1, 1	I_1, I_1	
C1	1	0	1	−193	1012	1	1	−	10, 1	10, 1	2, 1	I_{10}, I_1	
D1	1	0	0	2	4	0	3	−	6, 1	6, 1	6, 1	I_6, I_1	$3 : 2$
D2	1	0	0	−18	−112	0	1	−	2, 3	2, 3	2, 1	I_2, I_3	$3 : 1$

215 $N = 215 = 5 \cdot 43$ (1 isogeny class)**215**

A1	0	0	1	−8	−12	1	1	−	4, 1	4, 1	2, 1	I_4, I_1	
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216 $N = 216 = 2^3 \cdot 3^3$ (4 isogeny classes)**216**

A1	0	0	0	−12	20	1	1	−	8, 5	0, 0	4, 3	I_1^*, IV	
B1	0	0	0	−3	−34	0	1	−	11, 5	0, 0	1, 1	II^*, IV	
C1	0	0	0	−27	918	0	1	−	11, 11	0, 0	1, 1	II^*, II^*	
D1	0	0	0	−108	−540	0	1	−	8, 11	0, 0	2, 1	I_1^*, II^*	

218 $N = 218 = 2 \cdot 109$ (1 isogeny class)**218**

A1	1	0	0	−2	4	1	3	−	6, 1	6, 1	6, 1	I_6, I_1	$3 : 2$
A2	1	0	0	18	−104	1	1	−	2, 3	2, 3	2, 3	I_2, I_3	$3 : 1$

219 $N = 219 = 3 \cdot 73$ (3 isogeny classes)**219**

A1	0	−1	1	−6	8	1	1	−	1, 1	1, 1	1, 1	I_1, I_1	
B1	0	1	1	3	2	1	3	−	3, 1	3, 1	3, 1	I_3, I_1	$3 : 2$
B2	0	1	1	−27	−85	1	1	−	1, 3	1, 3	1, 3	I_1, I_3	$3 : 1$
C1	1	1	0	−82	−305	1	2	+	10, 1	10, 1	2, 1	I_{10}, I_1	$2 : 2$

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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220 $N = 220 = 2^2 \cdot 5 \cdot 11$ (2 isogeny classes) **220**

A1	0	1	0	-45	100	1	6	+	4, 3, 2	0, 3, 2	3, 3, 2	IV, I ₃ , I ₂	2 : 2; 3 : 3
A2	0	1	0	-100	-252	1	6	+	8, 6, 1	0, 6, 1	3, 6, 1	IV*, I ₆ , I ₁	2 : 1; 3 : 4
A3	0	1	0	-445	-3720	1	2	+	4, 1, 6	0, 1, 6	1, 1, 2	IV, I ₁ , I ₆	2 : 4; 3 : 1
A4	0	1	0	-7100	-232652	1	2	+	8, 2, 3	0, 2, 3	1, 2, 1	IV*, I ₂ , I ₃	2 : 3; 3 : 2
B1	0	-1	0	-5	2	0	2	+	4, 1, 2	0, 1, 2	1, 1, 2	IV, I ₁ , I ₂	2 : 2
B2	0	-1	0	-60	200	0	2	+	8, 2, 1	0, 2, 1	1, 2, 1	IV*, I ₂ , I ₁	2 : 1

221 $N = 221 = 13 \cdot 17$ (2 isogeny classes) **221**

A1	1	-1	1	-733	7804	0	2	+	6, 1	6, 1	2, 1	I ₆ , I ₁	2 : 2
A2	1	-1	1	-11718	491144	0	2	+	3, 2	3, 2	1, 2	I ₃ , I ₂	2 : 1
B1	1	1	0	-59	152	0	2	+	2, 1	2, 1	2, 1	I ₂ , I ₁	2 : 2
B2	1	1	0	-54	185	0	2	-	4, 2	4, 2	2, 2	I ₄ , I ₂	2 : 1

222 $N = 222 = 2 \cdot 3 \cdot 37$ (5 isogeny classes) **222**

A1	1	0	0	2	-4	0	3	-	3, 3, 1	3, 3, 1	3, 3, 1	I ₃ , I ₃ , I ₁	3 : 2
A2	1	0	0	-148	-706	0	1	-	1, 1, 3	1, 1, 3	1, 1, 3	I ₁ , I ₁ , I ₃	3 : 1
B1	1	1	1	17	179	0	1	-	1, 11, 1	1, 11, 1	1, 1, 1	I ₁ , I ₁₁ , I ₁	
C1	1	1	0	16	0	0	2	-	8, 3, 1	8, 3, 1	2, 1, 1	I ₈ , I ₃ , I ₁	2 : 2
C2	1	1	0	-64	-80	0	4	+	4, 6, 2	4, 6, 2	2, 2, 2	I ₄ , I ₆ , I ₂	2 : 1, 3, 4
C3	1	1	0	-804	-9108	0	2	+	2, 12, 1	2, 12, 1	2, 2, 1	I ₂ , I ₁₂ , I ₁	2 : 2
C4	1	1	0	-604	5428	0	4	+	2, 3, 4	2, 3, 4	2, 1, 4	I ₂ , I ₃ , I ₄	2 : 2
D1	1	0	1	1	-46	0	1	-	13, 1, 1	13, 1, 1	1, 1, 1	I ₁₃ , I ₁ , I ₁	
E1	1	1	0	-182317	29887645	0	1	-	23, 9, 1	23, 9, 1	1, 1, 1	I ₂₃ , I ₉ , I ₁	

224 $N = 224 = 2^5 \cdot 7$ (2 isogeny classes) **224**

A1	0	1	0	2	0	1	2	-	6, 1	0, 1	2, 1	III, I ₁	2 : 2
A2	0	1	0	-8	-8	1	2	+	9, 2	0, 2	2, 2	I ₀ *, I ₂	2 : 1
B1	0	-1	0	2	0	0	2	-	6, 1	0, 1	2, 1	III, I ₁	2 : 2
B2	0	-1	0	-8	8	0	2	+	9, 2	0, 2	1, 2	I ₀ *, I ₂	2 : 1

225 $N = 225 = 3^2 \cdot 5^2$ (5 isogeny classes) **225**

A1	0	0	1	0	1	1	1	-	3, 2	0, 0	2, 1	III, II	3 : 2
A2	0	0	1	0	-34	1	1	-	9, 2	0, 0	2, 1	III*, II	3 : 1
B1	0	0	1	0	156	0	3	-	3, 8	0, 0	2, 3	III, IV*	3 : 2
B2	0	0	1	0	-4219	0	1	-	9, 8	0, 0	2, 1	III*, IV*	3 : 1
C1	1	-1	1	-5	-628	0	4	-	7, 7	1, 1	4, 4	I ₁ *, I ₁ *	2 : 2
C2	1	-1	1	-1130	-14128	0	4	+	8, 8	2, 2	4, 4	I ₂ *, I ₂ *	2 : 1, 3, 4
C3	1	-1	1	-18005	-925378	0	2	+	7, 7	1, 1	2, 4	I ₁ *, I ₁ *	2 : 2
C4	1	-1	1	-2255	19622	0	4	+	10, 10	4, 4	4, 4	I ₄ *, I ₄ *	2 : 2, 5, 6
C5	1	-1	1	-30380	2044622	0	4	+	14, 8	8, 2	4, 4	I ₈ *, I ₂ *	2 : 4, 7, 8
C6	1	-1	1	7870	141122	0	2	-	8, 14	2, 8	2, 4	I ₂ *, I ₈ *	2 : 4
C7	1	-1	1	-486005	130530872	0	2	+	10, 7	4, 1	2, 2	I ₄ *, I ₁ *	2 : 5
C8	1	-1	1	-24755	2820872	0	2	-	22, 7	16, 1	4, 2	I ₁₆ *, I ₁ *	2 : 5
D1	0	0	1	15	-99	0	1	-	11, 2	5, 0	2, 1	I* II	5 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
225	$N = 225 = 3^2 \cdot 5^2$ (continued)											225	
E1	0	0	1	-75	256	1	1	-	7, 4	1, 0	4, 3	$I_{1,IV}^*$	5 : 2
E2	0	0	1	375	-12344	1	1	-	11, 8	5, 0	4, 3	$I_{5,IV}^*$	5 : 1
226	$N = 226 = 2 \cdot 113$ (1 isogeny class)											226	
A1	1	0	0	-5	1	1	2	+	6, 1	6, 1	6, 1	I_{6,I_1}	2 : 2
A2	1	0	0	-45	-119	1	2	+	3, 2	3, 2	3, 2	I_{3,I_2}	2 : 1
228	$N = 228 = 2^2 \cdot 3 \cdot 19$ (2 isogeny classes)											228	
A1	0	-1	0	3	18	0	2	-	4, 3, 2	0, 3, 2	1, 1, 2	IV, I_{3,I_2}	2 : 2
A2	0	-1	0	-92	360	0	2	+	8, 6, 1	0, 6, 1	1, 2, 1	IV^*, I_{6,I_1}	2 : 1
B1	0	-1	0	3	9	1	1	-	8, 2, 1	0, 2, 1	3, 2, 1	IV^*, I_{2,I_1}	
229	$N = 229 = 229$ (1 isogeny class)											229	
A1	1	0	0	-2	-1	1	1	+	1	1	1	I_1	
231	$N = 231 = 3 \cdot 7 \cdot 11$ (1 isogeny class)											231	
A1	1	1	1	-34	62	0	4	+	1, 2, 1	1, 2, 1	1, 2, 1	I_{1,I_2,I_1}	2 : 2
A2	1	1	1	-39	36	0	8	+	2, 4, 2	2, 4, 2	2, 4, 2	I_{2,I_4,I_2}	2 : 1, 3, 4
A3	1	1	1	-284	-1924	0	4	+	4, 2, 4	4, 2, 4	2, 2, 2	I_{4,I_2,I_4}	2 : 2, 5, 6
A4	1	1	1	126	432	0	4	-	1, 8, 1	1, 8, 1	1, 8, 1	I_{1,I_8,I_1}	2 : 2
A5	1	1	1	-4519	-118810	0	2	+	8, 1, 2	8, 1, 2	2, 1, 2	I_{8,I_1,I_2}	2 : 3
A6	1	1	1	31	-5578	0	2	-	2, 1, 8	2, 1, 8	2, 1, 2	I_{2,I_1,I_8}	2 : 3
232	$N = 232 = 2^3 \cdot 29$ (2 isogeny classes)											232	
A1	0	-1	0	8	-4	1	1	-	10, 1	0, 1	2, 1	III^*, I_1	
B1	0	1	0	-80	-304	0	1	-	10, 1	0, 1	2, 1	III^*, I_1	
233	$N = 233 = 233$ (1 isogeny class)											233	
A1	1	0	1	0	11	0	2	-	2	2	2	I_2	2 : 2
A2	1	0	1	-5	3	0	2	+	1	1	1	I_1	2 : 1
234	$N = 234 = 2 \cdot 3^2 \cdot 13$ (5 isogeny classes)											234	
A1	1	-1	0	-24	-64	0	1	-	7, 6, 1	7, 0, 1	1, 1, 1	I_{7,I_0^*,I_1}	7 : 2
A2	1	-1	0	-1914	35846	0	1	-	1, 6, 7	1, 0, 7	1, 1, 1	I_{1,I_0^*,I_7}	7 : 1
B1	1	-1	1	-29	-107	0	2	-	4, 9, 1	4, 0, 1	4, 2, 1	I_{4,III^*,I_1}	2 : 2
B2	1	-1	1	-569	-5075	0	2	+	2, 9, 2	2, 0, 2	2, 2, 2	I_{2,III^*,I_2}	2 : 1
C1	1	-1	0	-3	5	1	2	-	4, 3, 1	4, 0, 1	2, 2, 1	I_{4,III,I_1}	2 : 2
C2	1	-1	0	-63	209	1	2	+	2, 3, 2	2, 0, 2	2, 2, 2	I_{2,III,I_2}	2 : 1
D1	1	-1	1	-176	-18669	0	4	-	16, 11, 1	16, 5, 1	16, 4, 1	I_{16,I_5^*,I_1}	2 : 2
D2	1	-1	1	-11696	-479469	0	4	+	8, 16, 2	8, 10, 2	8, 4, 2	I_{8,I_{10}^*,I_2}	2 : 1, 3, 4
D3	1	-1	1	-186656	-30992493	0	2	+	4, 11, 4	4, 5, 4	4, 2, 4	I_{4,I_5^*,I_4}	2 : 2
D4	1	-1	1	-21056	404115	0	2	+	4, 26, 1	4, 20, 1	4, 4, 1	I_{4,I_{20}^*,I_1}	2 : 2
E1	1	-1	1	4	-7	0	1	-	1, 6, 1	1, 0, 1	1, 1, 1	I_{1,I_0^*,I_1}	3 : 2
E2	1	-1	1	-41	209	0	3	-	3, 6, 3	3, 0, 3	3, 1, 3	I_{3,I_1^*,I_2}	3 : 1, 3

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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235 $N = 235 = 5 \cdot 47$ (3 isogeny classes) **235**

A1	1	1	1	-5	0	1	1	+	3, 1	3, 1	3, 1	I_3, I_1	
B1	1	1	1	-3551	-82926	0	1	+	9, 1	9, 1	1, 1	I_9, I_1	
C1	0	-1	1	4	1	0	1	-	3, 1	3, 1	1, 1	I_3, I_1	

236 $N = 236 = 2^2 \cdot 59$ (2 isogeny classes) **236**

A1	0	-1	0	-1	2	1	1	-	4, 1	0, 1	3, 1	IV, I_1	
B1	0	1	0	-9	8	0	3	-	4, 1	0, 1	3, 1	IV, I_1	3 : 2
B2	0	1	0	31	68	0	1	-	4, 3	0, 3	1, 1	IV, I_3	3 : 1

238 $N = 238 = 2 \cdot 7 \cdot 17$ (5 isogeny classes) **238**

A1	1	0	0	-60	16	1	2	+	14, 2, 1	14, 2, 1	14, 2, 1	I_{14}, I_2, I_1	2 : 2
A2	1	0	0	-700	7056	1	2	+	7, 4, 2	7, 4, 2	7, 4, 2	I_7, I_4, I_2	2 : 1
B1	1	-1	0	2	0	1	2	-	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	2 : 2
B2	1	-1	0	-8	6	1	2	+	1, 2, 2	1, 2, 2	1, 2, 2	I_1, I_2, I_2	2 : 1
C1	1	-1	1	-19	35	0	4	+	4, 2, 1	4, 2, 1	4, 2, 1	I_4, I_2, I_1	2 : 2
C2	1	-1	1	-39	-37	0	4	+	2, 4, 2	2, 4, 2	2, 4, 2	I_2, I_4, I_2	2 : 1, 3, 4
C3	1	-1	1	-529	-4545	0	2	+	1, 2, 4	1, 2, 4	1, 2, 4	I_1, I_2, I_4	2 : 2
C4	1	-1	1	131	-377	0	2	-	1, 8, 1	1, 8, 1	1, 8, 1	I_1, I_8, I_1	2 : 2
D1	1	1	1	-18	-37	0	2	+	2, 2, 1	2, 2, 1	2, 2, 1	I_2, I_2, I_1	2 : 2
D2	1	1	1	-28	-5	0	2	+	1, 4, 2	1, 4, 2	1, 2, 2	I_1, I_4, I_2	2 : 1
E1	1	1	0	32	0	0	2	-	10, 1, 2	10, 1, 2	2, 1, 2	I_{10}, I_1, I_2	2 : 2
E2	1	1	0	-128	-160	0	2	+	5, 2, 4	5, 2, 4	1, 2, 2	I_5, I_2, I_4	2 : 1

240 $N = 240 = 2^4 \cdot 3 \cdot 5$ (4 isogeny classes) **240**

A1	0	-1	0	-15	-18	0	2	+	4, 2, 1	0, 2, 1	1, 2, 1	II, I_2, I_1	2 : 2
A2	0	-1	0	-20	0	0	4	+	8, 4, 2	0, 4, 2	2, 2, 2	I_0^*, I_4, I_2	2 : 1, 3, 4
A3	0	-1	0	-200	1152	0	8	+	10, 2, 4	0, 2, 4	4, 2, 4	I_2^*, I_2, I_4	2 : 2, 5, 6
A4	0	-1	0	80	-80	0	2	-	10, 8, 1	0, 8, 1	2, 2, 1	I_2^*, I_8, I_1	2 : 2
A5	0	-1	0	-3200	70752	0	4	+	11, 1, 2	0, 1, 2	4, 1, 2	I_3^*, I_1, I_2	2 : 3
A6	0	-1	0	-80	2400	0	4	-	11, 1, 8	0, 1, 8	2, 1, 8	I_3^*, I_1, I_8	2 : 3
B1	0	-1	0	24	-144	0	2	-	16, 3, 1	4, 3, 1	4, 1, 1	I_8^*, I_3, I_1	2 : 2; 3 : 3
B2	0	-1	0	-296	-1680	0	4	+	14, 6, 2	2, 6, 2	4, 2, 2	I_6^*, I_6, I_2	2 : 1, 4, 5; 3 : 6
B3	0	-1	0	-216	4080	0	2	-	24, 1, 3	12, 1, 3	4, 1, 1	I_{16}^*, I_1, I_3	2 : 6; 3 : 1
B4	0	-1	0	-4616	-119184	0	2	+	13, 3, 4	1, 3, 4	4, 1, 2	I_5^*, I_3, I_4	2 : 2; 3 : 7
B5	0	-1	0	-1096	12400	0	2	+	13, 12, 1	1, 12, 1	2, 2, 1	I_5^*, I_{12}, I_1	2 : 2; 3 : 8
B6	0	-1	0	-5336	151536	0	4	+	18, 2, 6	6, 2, 6	4, 2, 2	I_{10}^*, I_2, I_6	2 : 3, 7, 8; 3 : 2
B7	0	-1	0	-7256	34800	0	2	+	15, 1, 12	3, 1, 12	4, 1, 2	I_7^*, I_1, I_{12}	2 : 6; 3 : 4
B8	0	-1	0	-85336	9623536	0	2	+	15, 4, 3	3, 4, 3	2, 2, 1	I_7^*, I_4, I_3	2 : 6; 3 : 5
C1	0	-1	0	4	0	1	2	-	8, 1, 1	0, 1, 1	2, 1, 1	I_0^*, I_1, I_1	2 : 2
C2	0	-1	0	-16	16	1	4	+	10, 2, 2	0, 2, 2	4, 2, 2	I_2^*, I_2, I_2	2 : 1, 3, 4
C3	0	-1	0	-136	-560	1	2	+	11, 1, 4	0, 1, 4	2, 1, 2	I_0^*, I_1, I_4	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
240	$N = 240 = 2^4 \cdot 3 \cdot 5$ (continued)											240	
D1	0	1	0	0	-12	0	2	-	12, 1, 1	0, 1, 1	4, 1, 1	I_4^*, I_1, I_1	2 : 2
D2	0	1	0	-80	-300	0	4	+	12, 2, 2	0, 2, 2	4, 2, 2	I_4^*, I_2, I_2	2 : 1, 3, 4
D3	0	1	0	-1280	-18060	0	2	+	12, 1, 1	0, 1, 1	2, 1, 1	I_4^*, I_1, I_1	2 : 2
D4	0	1	0	-160	308	0	8	+	12, 4, 4	0, 4, 4	4, 4, 4	I_4^*, I_4, I_4	2 : 2, 5, 6
D5	0	1	0	-2160	37908	0	8	+	12, 8, 2	0, 8, 2	4, 8, 2	I_4^*, I_8, I_2	2 : 4, 7, 8
D6	0	1	0	560	2900	0	4	-	12, 2, 8	0, 2, 8	2, 2, 8	I_4^*, I_2, I_8	2 : 4
D7	0	1	0	-34560	2461428	0	4	+	12, 4, 1	0, 4, 1	4, 4, 1	I_4^*, I_4, I_1	2 : 5
D8	0	1	0	-1760	52788	0	4	-	12, 16, 1	0, 16, 1	2, 16, 1	I_4^*, I_{16}, I_1	2 : 5
242	$N = 242 = 2 \cdot 11^2$ (2 isogeny classes)											242	
A1	1	0	0	3	1	1	1	-	4, 2	4, 0	4, 1	I_4, II	3 : 2
A2	1	0	0	-52	144	1	1	-	12, 2	12, 0	12, 1	I_{12}, II	3 : 1
B1	1	0	1	360	-970	0	3	-	4, 8	4, 0	2, 3	I_4, IV^*	3 : 2
B2	1	0	1	-6295	-197958	0	1	-	12, 8	12, 0	2, 1	I_{12}, IV^*	3 : 1
243	$N = 243 = 3^5$ (2 isogeny classes)											243	
A1	0	0	1	0	-1	1	1	-	5	0	1	II	3 : 2
A2	0	0	1	0	20	1	3	-	11	0	3	IV^*	3 : 1
B1	0	0	1	0	2	0	3	-	7	0	3	IV	3 : 2
B2	0	0	1	0	-61	0	1	-	13	0	1	II^*	3 : 1
244	$N = 244 = 2^2 \cdot 61$ (1 isogeny class)											244	
A1	0	0	0	1	6	1	1	-	8, 1	0, 1	3, 1	IV^*, I_1	
245	$N = 245 = 5 \cdot 7^2$ (3 isogeny classes)											245	
A1	0	0	1	-7	12	1	1	-	3, 3	3, 0	3, 2	I_3, III	
B1	0	0	1	-343	-4202	0	1	-	3, 9	3, 0	1, 2	I_3, III^*	
C1	0	-1	1	-65	-204	1	1	-	1, 7	1, 1	1, 4	I_1, I_1^*	3 : 2
C2	0	-1	1	425	433	1	1	-	3, 9	3, 3	3, 4	I_3, I_3^*	3 : 1, 3
C3	0	-1	1	-6435	210006	1	1	-	9, 7	9, 1	9, 4	I_9, I_1^*	3 : 2
246	$N = 246 = 2 \cdot 3 \cdot 41$ (7 isogeny classes)											246	
A1	1	1	1	-270	-1821	0	1	-	3, 7, 1	3, 7, 1	3, 1, 1	I_3, I_7, I_1	
B1	1	0	0	-175	-27847	0	5	-	25, 5, 1	25, 5, 1	25, 5, 1	I_{25}, I_5, I_1	5 : 2
B2	1	0	0	-579535	-169860007	0	1	-	5, 1, 5	5, 1, 5	5, 1, 5	I_5, I_1, I_5	5 : 1
C1	1	0	1	-453897	-117739700	0	2	+	14, 12, 1	14, 12, 1	2, 12, 1	I_{14}, I_{12}, I_1	2 : 2
C2	1	0	1	-453257	-118088116	0	2	-	7, 24, 2	7, 24, 2	1, 24, 2	I_7, I_{24}, I_2	2 : 1
D1	1	1	0	-66	180	1	2	+	6, 4, 1	6, 4, 1	2, 2, 1	I_6, I_4, I_1	2 : 2
D2	1	1	0	-26	444	1	2	-	3, 8, 2	3, 8, 2	1, 2, 2	I_3, I_8, I_2	2 : 1
E1	1	0	0	-9	9	0	4	+	4, 2, 1	4, 2, 1	4, 2, 1	I_4, I_2, I_1	2 : 2
E2	1	0	0	-29	-51	0	4	+	2, 4, 2	2, 4, 2	2, 4, 2	I_2, I_4, I_2	2 : 1, 3, 4
E3	1	0	0	-439	-3577	0	2	+	1, 8, 1	1, 8, 1	1, 8, 1	I_1, I_8, I_1	2 : 2
E4	1	0	0	61	-285	0	2	-	1, 2, 4	1, 2, 4	1, 2, 4	I_1, I_2, I_4	2 : 2
F1	1	0	1	-2	2	0	3	-	1, 3, 1	1, 3, 1	1, 3, 1	I_1, I_3, I_1	3 : 2
F2	1	0	1	13	-58	0	1	-	3, 1, 3	3, 1, 3	1, 1, 1	I_3, I_1, I_3	3 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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248 $N = 248 = 2^3 \cdot 31$ (3 isogeny classes) **248**

A1	0	1	0	0	1	1	1	−	4, 1	0, 1	2, 1	III, I ₁	
B1	0	1	0	8	0	0	2	−	10, 1	0, 1	2, 1	III*, I ₁	2 : 2
B2	0	1	0	−32	−32	0	2	+	11, 2	0, 2	1, 2	II*, I ₂	2 : 1
C1	0	0	0	1	−1	1	1	−	4, 1	0, 1	2, 1	III, I ₁	

249 $N = 249 = 3 \cdot 83$ (2 isogeny classes) **249**

A1	1	1	1	−55	134	1	1	−	3, 1	3, 1	1, 1	I ₃ , I ₁	
B1	1	1	0	2	1	1	1	−	1, 1	1, 1	1, 1	I ₁ , I ₁	

252 $N = 252 = 2^2 \cdot 3^2 \cdot 7$ (2 isogeny classes) **252**

A1	0	0	0	60	61	0	2	−	4, 9, 2	0, 3, 2	1, 2, 2	IV, I ₃ *, I ₂	2 : 2; 3 : 3
A2	0	0	0	−255	502	0	2	+	8, 12, 1	0, 6, 1	1, 4, 1	IV*, I ₆ *, I ₁	2 : 1; 3 : 4
A3	0	0	0	−1020	12913	0	6	−	4, 7, 6	0, 1, 6	3, 2, 6	IV, I ₁ *, I ₆	2 : 4; 3 : 1
A4	0	0	0	−16455	812446	0	6	+	8, 8, 3	0, 2, 3	3, 4, 3	IV*, I ₂ *, I ₃	2 : 3; 3 : 2
B1	0	0	0	−12	65	1	2	−	4, 7, 2	0, 1, 2	3, 4, 2	IV, I ₁ *, I ₂	2 : 2
B2	0	0	0	−327	2270	1	2	+	8, 8, 1	0, 2, 1	3, 4, 1	IV*, I ₂ *, I ₁	2 : 1

254 $N = 254 = 2 \cdot 127$ (4 isogeny classes) **254**

A1	1	0	0	−22	36	1	3	+	9, 1	9, 1	9, 1	I ₉ , I ₁	3 : 2
A2	1	0	0	−302	−2036	1	3	+	3, 3	3, 3	3, 3	I ₃ , I ₃	3 : 1, 3
A3	1	0	0	−24432	−1471934	1	1	+	1, 1	1, 1	1, 1	I ₁ , I ₁	3 : 2
B1	1	0	0	2	0	0	2	−	2, 1	2, 1	2, 1	I ₂ , I ₁	2 : 2
B2	1	0	0	−8	−2	0	2	+	1, 2	1, 2	1, 2	I ₁ , I ₂	2 : 1
C1	1	−1	0	−5	−3	1	1	+	3, 1	3, 1	1, 1	I ₃ , I ₁	
D1	1	−1	1	−19	51	0	4	−	12, 1	12, 1	12, 1	I ₁₂ , I ₁	2 : 2
D2	1	−1	1	−339	2483	0	4	+	6, 2	6, 2	6, 2	I ₆ , I ₂	2 : 1, 3, 4
D3	1	−1	1	−379	1891	0	2	+	3, 4	3, 4	3, 2	I ₃ , I ₄	2 : 2
D4	1	−1	1	−5419	154883	0	2	+	3, 1	3, 1	3, 1	I ₃ , I ₁	2 : 2

256 $N = 256 = 2^8$ (4 isogeny classes) **256**

A1	0	1	0	−3	1	1	2	+	9	0	2	III	2 : 2
A2	0	1	0	−13	−21	1	2	+	15	0	2	III*	2 : 1
B1	0	0	0	−2	0	1	2	+	9	0	2	III	2 : 2
B2	0	0	0	8	0	1	2	−	15	0	2	III*	2 : 1
C1	0	0	0	2	0	0	2	−	9	0	2	III	2 : 2
C2	0	0	0	−8	0	0	2	+	15	0	2	III*	2 : 1
D1	0	−1	0	−3	−1	0	2	+	9	0	2	III	2 : 2
D2	0	−1	0	−13	21	0	2	+	15	0	2	III*	2 : 1

258 $N = 258 = 2 \cdot 3 \cdot 43$ (7 isogeny classes) **258**

A1	1	1	0	3	−3	1	1	−	6, 1, 1	6, 1, 1	2, 1, 1	I ₆ , I ₁ , I ₁	
B1	1	1	0	−1916	31440	0	2	+	14, 7, 1	14, 7, 1	2, 1, 1	I ₁₄ , I ₇ , I ₁	2 : 2
B2	1	1	0	−1276	53584	0	2	−	7, 14, 2	7, 14, 2	1, 2, 2	I ₇ , I ₁₄ , I ₂	2 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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258 $N = 258 = 2 \cdot 3 \cdot 43$ (continued)**258**

D1	1	1	1	-24	-39	0	4	+	12, 1, 1	12, 1, 1	12, 1, 1	I_{12}, I_1, I_1	2 : 2
D2	1	1	1	-344	-2599	0	4	+	6, 2, 2	6, 2, 2	6, 2, 2	I_6, I_2, I_2	2 : 1, 3, 4
D3	1	1	1	-5504	-159463	0	2	+	3, 1, 1	3, 1, 1	3, 1, 1	I_3, I_1, I_1	2 : 2
D4	1	1	1	-304	-3175	0	2	-	3, 4, 4	3, 4, 4	3, 2, 2	I_3, I_4, I_4	2 : 2
E1	1	1	1	-44124	3549153	0	1	-	2, 19, 1	2, 19, 1	2, 1, 1	I_2, I_{19}, I_1	
F1	1	0	0	159	1737	0	7	-	14, 7, 1	14, 7, 1	14, 7, 1	I_{14}, I_7, I_1	7 : 2
F2	1	0	0	-59901	-5648523	0	1	-	2, 1, 7	2, 1, 7	2, 1, 7	I_2, I_1, I_7	7 : 1
G1	1	0	0	-2	0	0	2	+	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	2 : 2
G2	1	0	0	8	2	0	2	-	1, 2, 2	1, 2, 2	1, 2, 2	I_1, I_2, I_2	2 : 1

259 $N = 259 = 7 \cdot 37$ (1 isogeny class)**259**

A1	1	-1	0	-5	-32	0	2	-	3, 2	3, 2	3, 2	I_3, I_2	2 : 2
A2	1	-1	0	-190	-957	0	2	+	6, 1	6, 1	6, 1	I_6, I_1	2 : 1

260 $N = 260 = 2^2 \cdot 5 \cdot 13$ (1 isogeny class)**260**

A1	0	-1	0	-281	1910	0	2	+	4, 1, 2	0, 1, 2	1, 1, 2	IV, I_1, I_2	2 : 2
A2	0	-1	0	-276	1976	0	2	-	8, 2, 4	0, 2, 4	1, 2, 2	IV^*, I_2, I_4	2 : 1

262 $N = 262 = 2 \cdot 131$ (2 isogeny classes)**262**

A1	1	0	0	1	25	1	1	-	11, 1	11, 1	11, 1	I_{11}, I_1	
B1	1	-1	0	-2	2	1	1	-	1, 1	1, 1	1, 1	I_1, I_1	

264 $N = 264 = 2^3 \cdot 3 \cdot 11$ (4 isogeny classes)**264**

A1	0	1	0	-8	0	0	2	+	10, 1, 1	0, 1, 1	2, 1, 1	III^*, I_1, I_1	2 : 2
A2	0	1	0	32	32	0	2	-	11, 2, 2	0, 2, 2	1, 2, 2	II^*, I_2, I_2	2 : 1
B1	0	-1	0	-12	-12	0	2	+	8, 1, 1	0, 1, 1	2, 1, 1	I_1^*, I_1, I_1	2 : 2
B2	0	-1	0	-32	60	0	4	+	10, 2, 2	0, 2, 2	2, 2, 2	III^*, I_2, I_2	2 : 1, 3, 4
B3	0	-1	0	-472	4108	0	2	+	11, 4, 1	0, 4, 1	1, 2, 1	II^*, I_4, I_1	2 : 2
B4	0	-1	0	88	300	0	2	-	11, 1, 4	0, 1, 4	1, 1, 4	II^*, I_1, I_4	2 : 2
C1	0	1	0	1	6	0	4	-	4, 4, 1	0, 4, 1	2, 4, 1	III, I_4, I_1	2 : 2
C2	0	1	0	-44	96	0	4	+	8, 2, 2	0, 2, 2	2, 2, 2	I_1^*, I_2, I_2	2 : 1, 3, 4
C3	0	1	0	-104	-288	0	2	+	10, 1, 4	0, 1, 4	2, 1, 2	III^*, I_1, I_4	2 : 2
C4	0	1	0	-704	6960	0	2	+	10, 1, 1	0, 1, 1	2, 1, 1	III^*, I_1, I_1	2 : 2
D1	0	1	0	-8016	-278928	0	2	+	10, 7, 1	0, 7, 1	2, 7, 1	III^*, I_7, I_1	2 : 2
D2	0	1	0	-7976	-281808	0	2	-	11, 14, 2	0, 14, 2	1, 14, 2	II^*, I_{14}, I_2	2 : 1

265 $N = 265 = 5 \cdot 53$ (1 isogeny class)**265**

A1	1	-1	1	-138	656	1	2	+	3, 1	3, 1	1, 1	I_3, I_1	2 : 2
A2	1	-1	1	-133	702	1	2	-	6, 2	6, 2	2, 2	I_6, I_2	2 : 1

267 $N = 267 = 3 \cdot 89$ (2 isogeny classes)**267**

A1	0	1	1	-3	2	0	3	-	3, 1	3, 1	3, 1	I_3, I_1	3 : 2
A2	0	1	1	27	-37	0	1	-	1, 3	1, 3	1, 1	I_1, I_3	3 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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268 $N = 268 = 2^2 \cdot 67$ (1 isogeny class) **268**

A1	0	-1	0	3	-7	0	1	-	8, 1	0, 1	1, 1	IV^*, I_1	
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269 $N = 269 = 269$ (1 isogeny class) **269**

A1	0	0	1	-2	-1	1	1	+	1	1	1	I_1	
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270 $N = 270 = 2 \cdot 3^3 \cdot 5$ (4 isogeny classes) **270**

A1	1	-1	0	-15	35	0	3	-	1, 9, 1	1, 0, 1	1, 3, 1	I_1, IV^*, I_1	3 : 2
A2	1	-1	0	120	-424	0	1	-	3, 11, 3	3, 0, 3	1, 1, 1	I_3, II^*, I_3	3 : 1
B1	1	-1	1	7	-103	0	3	-	15, 3, 1	15, 0, 1	15, 1, 1	I_{15}, II, I_1	3 : 2
B2	1	-1	1	-1433	-20519	0	1	-	5, 9, 3	5, 0, 3	5, 1, 1	I_5, IV^*, I_3	3 : 1
C1	1	-1	1	-2	-1	0	1	-	1, 3, 1	1, 0, 1	1, 1, 1	I_1, II, I_1	3 : 2
C2	1	-1	1	13	11	0	3	-	3, 5, 3	3, 0, 3	3, 1, 3	I_3, IV, I_3	3 : 1
D1	1	-1	0	-159	813	0	3	-	5, 3, 3	5, 0, 3	1, 1, 3	I_5, II, I_3	3 : 2
D2	1	-1	0	66	2708	0	1	-	15, 9, 1	15, 0, 1	1, 1, 1	I_{15}, IV^*, I_1	3 : 1

272 $N = 272 = 2^4 \cdot 17$ (4 isogeny classes) **272**

A1	0	1	0	-8	4	1	2	+	10, 1	0, 1	4, 1	I_2^*, I_1	2 : 2
A2	0	1	0	-48	-140	1	2	+	11, 2	0, 2	2, 2	I_3^*, I_2	2 : 1
B1	0	0	0	-11	-6	1	2	+	12, 1	0, 1	4, 1	I_4^*, I_1	2 : 2
B2	0	0	0	-91	330	1	4	+	12, 2	0, 2	4, 2	I_4^*, I_2	2 : 1, 3, 4
B3	0	0	0	-1451	21274	1	4	+	12, 1	0, 1	4, 1	I_4^*, I_1	2 : 2
B4	0	0	0	-11	890	1	4	-	12, 4	0, 4	2, 4	I_4^*, I_4	2 : 2
C1	0	-1	0	-4	0	0	2	+	8, 1	0, 1	2, 1	I_0^*, I_1	2 : 2
C2	0	-1	0	16	-16	0	2	-	10, 2	0, 2	2, 2	I_2^*, I_2	2 : 1
D1	0	-1	0	-48	-64	0	2	+	18, 1	6, 1	4, 1	I_{10}^*, I_1	2 : 2; 3 : 3
D2	0	-1	0	-688	-6720	0	2	+	15, 2	3, 2	4, 2	I_7^*, I_2	2 : 1; 3 : 4
D3	0	-1	0	-1648	26304	0	2	+	14, 3	2, 3	4, 1	I_6^*, I_3	2 : 4; 3 : 1
D4	0	-1	0	-1808	21056	0	2	+	13, 6	1, 6	4, 2	I_5^*, I_6	2 : 3; 3 : 2

273 $N = 273 = 3 \cdot 7 \cdot 13$ (2 isogeny classes) **273**

A1	0	-1	1	-26	68	1	1	-	4, 3, 1	4, 3, 1	2, 3, 1	I_4, I_3, I_1	
B1	0	1	1	2540	-157433	0	1	-	8, 7, 3	8, 7, 3	8, 1, 1	I_8, I_7, I_3	

274 $N = 274 = 2 \cdot 137$ (3 isogeny classes) **274**

A1	1	0	0	-7	9	1	1	-	7, 1	7, 1	7, 1	I_7, I_1	
B1	1	-1	0	-2846	59156	1	1	-	11, 1	11, 1	1, 1	I_{11}, I_1	
C1	1	-1	0	-2	0	1	2	+	2, 1	2, 1	2, 1	I_2, I_1	2 : 2
C2	1	-1	0	8	-6	1	2	-	1, 2	1, 2	1, 2	I_1, I_2	2 : 1

275 $N = 275 = 5^2 \cdot 11$ (2 isogeny classes) **275**

A1	1	-1	1	20	22	1	4	-	7, 1	1, 1	4, 1	I_1^*, I_1	2 : 2
A2	1	-1	1	-105	272	1	4	+	8, 2	2, 2	4, 2	I_2^*, I_2	2 : 1, 3, 4
A3	1	-1	1	-730	-7228	1	2	+	7, 4	1, 4	4, 2	I_4^*, I_4	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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275 $N = 275 = 5^2 \cdot 11$ (continued)**275**

B1	0	1	1	-8	19	0	1	-	6, 1	0, 1	1, 1	I_0^*, I_1	5 : 2
B2	0	1	1	-258	-2981	0	1	-	6, 5	0, 5	1, 5	I_0^*, I_5	5 : 1, 3
B3	0	1	1	-195508	-33338481	0	1	-	6, 1	0, 1	1, 1	I_0^*, I_1	5 : 2

277 $N = 277 = 277$ (1 isogeny class)**277**

A1	1	0	1	0	-1	1	1	-	1	1	1	I_1	
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278 $N = 278 = 2 \cdot 139$ (2 isogeny classes)**278**

A1	1	0	0	-1	9	1	1	-	8, 1	8, 1	8, 1	I_8, I_1	
B1	1	0	1	-537	6908	0	3	-	12, 3	12, 3	2, 3	I_{12}, I_3	3 : 2, 3
B2	1	0	1	4328	-100122	0	1	-	36, 1	36, 1	2, 1	I_{36}, I_1	3 : 1
B3	1	0	1	-602	5628	0	3	-	4, 1	4, 1	2, 1	I_4, I_1	3 : 1

280 $N = 280 = 2^3 \cdot 5 \cdot 7$ (2 isogeny classes)**280**

A1	0	-1	0	-1	5	1	1	-	8, 1, 1	0, 1, 1	4, 1, 1	I_1^*, I_1, I_1	
B1	0	0	0	-412	3316	1	1	-	8, 5, 3	0, 5, 3	4, 5, 3	I_1^*, I_5, I_3	

282 $N = 282 = 2 \cdot 3 \cdot 47$ (2 isogeny classes)**282**

A1	1	1	1	58	-61	0	4	-	12, 4, 1	12, 4, 1	12, 2, 1	I_{12}, I_4, I_1	2 : 2
A2	1	1	1	-262	-829	0	4	+	6, 8, 2	6, 8, 2	6, 2, 2	I_6, I_8, I_2	2 : 1, 3, 4
A3	1	1	1	-3502	-81181	0	2	+	3, 4, 4	3, 4, 4	3, 2, 2	I_3, I_4, I_4	2 : 2
A4	1	1	1	-2142	36771	0	2	+	3, 16, 1	3, 16, 1	3, 2, 1	I_3, I_{16}, I_1	2 : 2
B1	1	1	1	-15	21	1	2	-	8, 2, 1	8, 2, 1	8, 2, 1	I_8, I_2, I_1	2 : 2
B2	1	1	1	-255	1461	1	2	+	4, 1, 2	4, 1, 2	4, 1, 2	I_4, I_1, I_2	2 : 1

285 $N = 285 = 3 \cdot 5 \cdot 19$ (3 isogeny classes)**285**

A1	1	0	0	19	0	1	2	-	5, 1, 2	5, 1, 2	5, 1, 2	I_5, I_1, I_2	2 : 2
A2	1	0	0	-76	-19	1	2	+	10, 2, 1	10, 2, 1	10, 2, 1	I_{10}, I_2, I_1	2 : 1
B1	1	1	0	2	-17	1	2	-	1, 3, 2	1, 3, 2	1, 1, 2	I_1, I_3, I_2	2 : 2
B2	1	1	0	-93	-378	1	2	+	2, 6, 1	2, 6, 1	2, 2, 1	I_2, I_6, I_1	2 : 1
C1	1	1	0	23	-176	0	2	-	8, 3, 1	8, 3, 1	2, 3, 1	I_8, I_3, I_1	2 : 2
C2	1	1	0	-382	-2849	0	4	+	4, 6, 2	4, 6, 2	2, 6, 2	I_4, I_6, I_2	2 : 1, 3, 4
C3	1	1	0	-6007	-181724	0	2	+	2, 3, 4	2, 3, 4	2, 3, 2	I_2, I_3, I_4	2 : 2
C4	1	1	0	-1237	13054	0	4	+	2, 12, 1	2, 12, 1	2, 12, 1	I_2, I_{12}, I_1	2 : 2

286 $N = 286 = 2 \cdot 11 \cdot 13$ (6 isogeny classes)**286**

A1	1	0	1	-7	42	0	3	-	5, 1, 3	5, 1, 3	1, 1, 3	I_5, I_1, I_3	3 : 2
A2	1	0	1	58	-1128	0	1	-	15, 3, 1	15, 3, 1	1, 1, 1	I_{15}, I_3, I_1	3 : 1
B1	1	1	1	13	177	1	1	-	13, 2, 1	13, 2, 1	13, 2, 1	I_{13}, I_2, I_1	
C1	1	1	0	-33	61	1	1	-	3, 2, 1	3, 2, 1	1, 2, 1	I_3, I_2, I_1	
D1	1	1	1	280	393	0	5	-	5, 2, 5	5, 2, 5	5, 2, 5	I_5, I_2, I_5	5 : 2
D2	1	1	1	-27930	-1808687	0	1	-	1, 10, 1	1, 10, 1	1, 10, 1	I_1, I_{10}, I_1	5 : 1
E1	1	1	1	-66	-313	0	1	-	3, 5, 1	3, 5, 1	3, 1, 1	I_3, I_5, I_1	

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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288 $N = 288 = 2^5 \cdot 3^2$ (5 isogeny classes) **288**

A1	0	0	0	3	0	1	2	−	6, 3	0, 0	2, 2	III, III	2 : 2
A2	0	0	0	−12	0	1	2	+	12, 3	0, 0	4, 2	I_3^*, III	2 : 1
B1	0	0	0	−21	−20	1	4	+	6, 8	0, 2	2, 4	III, I_2^*	2 : 2, 3, 4
B2	0	0	0	−291	−1910	1	2	+	9, 7	0, 1	1, 2	I_0^*, I_1^*	2 : 1
B3	0	0	0	−156	736	1	4	+	12, 7	0, 1	4, 4	I_3^*, I_1^*	2 : 1
B4	0	0	0	69	−146	1	2	−	9, 10	0, 4	2, 4	I_0^*, I_4^*	2 : 1
C1	0	0	0	−21	20	0	4	+	6, 8	0, 2	2, 4	III, I_2^*	2 : 2, 3, 4
C2	0	0	0	−156	−736	0	2	+	12, 7	0, 1	2, 2	I_3^*, I_1^*	2 : 1
C3	0	0	0	−291	1910	0	4	+	9, 7	0, 1	2, 4	I_0^*, I_1^*	2 : 1
C4	0	0	0	69	146	0	2	−	9, 10	0, 4	1, 4	I_0^*, I_4^*	2 : 1
D1	0	0	0	−9	0	0	4	+	6, 6	0, 0	2, 4	III, I_0^*	2 : 2, 3, 4
D2	0	0	0	−99	−378	0	2	+	9, 6	0, 0	2, 2	I_0^*, I_0^*	2 : 1
D3	0	0	0	−99	378	0	2	+	9, 6	0, 0	1, 2	I_0^*, I_0^*	2 : 1
D4	0	0	0	36	0	0	2	−	12, 6	0, 0	2, 2	I_3^*, I_0^*	2 : 1
E1	0	0	0	27	0	0	2	−	6, 9	0, 0	2, 2	III, III*	2 : 2
E2	0	0	0	−108	0	0	2	+	12, 9	0, 0	2, 2	I_3^*, III^*	2 : 1

289 $N = 289 = 17^2$ (1 isogeny class) **289**

A1	1	−1	1	−199	510	1	4	+	7	1	4	I_1^*	2 : 2
A2	1	−1	1	−1644	−24922	1	4	+	8	2	4	I_2^*	2 : 1, 3, 4
A3	1	−1	1	−26209	−1626560	1	2	+	7	1	4	I_1^*	2 : 2
A4	1	−1	1	−199	−68272	1	2	−	10	4	4	I_4^*	2 : 2

290 $N = 290 = 2 \cdot 5 \cdot 29$ (1 isogeny class) **290**

A1	1	−1	0	−70	−204	1	2	+	8, 3, 1	8, 3, 1	2, 1, 1	I_8, I_3, I_1	2 : 2
A2	1	−1	0	10	−700	1	2	−	4, 6, 2	4, 6, 2	2, 2, 2	I_4, I_6, I_2	2 : 1

291 $N = 291 = 3 \cdot 97$ (4 isogeny classes) **291**

A1	0	−1	1	−2174	151262	0	1	−	23, 1	23, 1	1, 1	I_{23}, I_1	
B1	1	1	1	−169	686	0	4	+	8, 2	8, 2	2, 2	I_8, I_2	2 : 2, 3, 4
B2	1	1	1	−654	−5910	0	2	+	16, 1	16, 1	2, 1	I_{16}, I_1	2 : 1
B3	1	1	1	−164	740	0	4	+	4, 1	4, 1	2, 1	I_4, I_1	2 : 1
B4	1	1	1	236	3926	0	4	−	4, 4	4, 4	2, 4	I_4, I_4	2 : 1
C1	1	1	1	−3	0	1	2	+	2, 1	2, 1	2, 1	I_2, I_1	2 : 2
C2	1	1	1	−18	−36	1	2	+	1, 2	1, 2	1, 2	I_1, I_2	2 : 1
D1	0	−1	1	0	−1	0	1	−	1, 1	1, 1	1, 1	I_1, I_1	

294 $N = 294 = 2 \cdot 3 \cdot 7^2$ (7 isogeny classes) **294**

A1	1	1	1	−50	293	0	1	−	1, 1, 8	1, 1, 0	1, 1, 1	I_1, I_1, IV^*	7 : 2
A2	1	1	1	−6910	−232261	0	1	−	7, 7, 8	7, 7, 0	7, 1, 1	I_7, I_7, IV^*	7 : 1
B1	1	0	0	−1	−1	0	1	−	1, 1, 2	1, 1, 0	1, 1, 1	I_1, I_1, II	7 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
294 $N = 294 = 2 \cdot 3 \cdot 7^2$ (continued) 294													
C1	1	0	0	-197	-2367	0	4	-	8, 2, 7	8, 2, 1	8, 2, 4	I_8, I_2, I_1^*	2 : 2
C2	1	0	0	-4117	-101935	0	4	+	4, 4, 8	4, 4, 2	4, 4, 4	I_4, I_4, I_2^*	2 : 1, 3, 4
C3	1	0	0	-65857	-6510547	0	2	+	2, 2, 7	2, 2, 1	2, 2, 2	I_2, I_2, I_1^*	2 : 2
C4	1	0	0	-5097	-49995	0	4	+	2, 8, 10	2, 8, 4	2, 8, 4	I_2, I_8, I_4^*	2 : 2, 5, 6
C5	1	0	0	-44787	3609423	0	2	+	1, 4, 14	1, 4, 8	1, 4, 4	I_1, I_4, I_8^*	2 : 4
C6	1	0	0	18913	-381333	0	2	-	1, 16, 8	1, 16, 2	1, 16, 2	I_1, I_{16}, I_2^*	2 : 4
D1	1	0	1	23	-52	0	3	-	5, 3, 4	5, 3, 0	1, 3, 3	I_5, I_3, IV	3 : 2
D2	1	0	1	-712	-7402	0	1	-	15, 1, 4	15, 1, 0	1, 1, 3	I_{15}, I_1, IV	3 : 1
E1	1	1	0	1151	18901	0	1	-	5, 3, 10	5, 3, 0	1, 1, 1	I_5, I_3, II^*	3 : 2
E2	1	1	0	-34864	2503936	0	1	-	15, 1, 10	15, 1, 0	1, 1, 1	I_{15}, I_1, II^*	3 : 1
F1	1	1	0	122	-10940	0	2	-	4, 4, 9	4, 4, 0	2, 2, 2	I_4, I_4, III^*	2 : 2
F2	1	1	0	-6738	-209880	0	2	+	2, 8, 9	2, 8, 0	2, 2, 2	I_2, I_8, III^*	2 : 1
G1	1	0	1	2	32	1	2	-	4, 4, 3	4, 4, 0	2, 4, 2	I_4, I_4, III	2 : 2
G2	1	0	1	-138	592	1	2	+	2, 8, 3	2, 8, 0	2, 8, 2	I_2, I_8, III	2 : 1
296 $N = 296 = 2^3 \cdot 37$ (2 isogeny classes) 296													
A1	0	-1	0	-9	13	1	1	+	8, 1	0, 1	4, 1	I_1^*, I_1	
B1	0	-1	0	-33	85	1	1	+	8, 1	0, 1	2, 1	I_1^*, I_1	
297 $N = 297 = 3^3 \cdot 11$ (4 isogeny classes) 297													
A1	0	0	1	-81	290	1	1	-	9, 2	0, 2	3, 2	IV^*, I_2	
B1	1	-1	1	1	0	1	1	-	3, 1	0, 1	1, 1	II, I_1	
C1	1	-1	0	12	-19	1	1	-	9, 1	0, 1	3, 1	IV^*, I_1	
D1	0	0	1	-9	-11	0	1	-	3, 2	0, 2	1, 2	II, I_2	
298 $N = 298 = 2 \cdot 149$ (2 isogeny classes) 298													
A1	1	0	0	-19	33	1	1	-	9, 1	9, 1	9, 1	I_9, I_1	
B1	1	-1	0	1	-1	1	1	-	1, 1	1, 1	1, 1	I_1, I_1	
300 $N = 300 = 2^2 \cdot 3 \cdot 5^2$ (4 isogeny classes) 300													
A1	0	-1	0	-13	-23	0	1	-	8, 3, 2	0, 3, 0	1, 1, 1	IV^*, I_3, II	3 : 2
A2	0	-1	0	-1213	-15863	0	1	-	8, 1, 2	0, 1, 0	3, 1, 1	IV^*, I_1, II	3 : 1
B1	0	1	0	-333	-3537	0	3	-	8, 3, 8	0, 3, 0	3, 3, 3	IV^*, I_3, IV^*	3 : 2
B2	0	1	0	-30333	-2043537	0	1	-	8, 1, 8	0, 1, 0	1, 1, 1	IV^*, I_1, IV^*	3 : 1
C1	0	1	0	-333	2088	0	2	+	4, 2, 9	0, 2, 0	1, 2, 2	IV, I_2, III^*	2 : 2
C2	0	1	0	292	9588	0	2	-	8, 4, 9	0, 4, 0	1, 4, 2	IV^*, I_4, III^*	2 : 1
D1	0	-1	0	-13	22	1	2	+	4, 2, 3	0, 2, 0	3, 2, 2	IV, I_2, III	2 : 2
D2	0	-1	0	12	72	1	2	-	8, 4, 3	0, 4, 0	3, 2, 2	IV^*, I_4, III	2 : 1
302 $N = 302 = 2 \cdot 151$ (3 isogeny classes) 302													
A1	1	1	1	-230	1251	1	5	-	15, 1	15, 1	15, 1	I_{15}, I_1	5 : 2
A2	1	1	1	1650	-27389	1	1	-	3, 5	3, 5	3, 5	I_3, I_5	5 : 1
B1	1	1	0	1	5	0	2	-	6, 1	6, 1	2, 1	I_6, I_1	2 : 2
B2	1	1	0	-39	77	0	2	+	3, 2	3, 2	1, 2	I_3, I_2	2 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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303 **303**
 $N = 303 = 3 \cdot 101$ (2 isogeny classes)

A1	0	1	1	-197	-208	1	1	+	14, 1	14, 1	14, 1	I_{14}, I_1	
B1	0	1	1	-6	2	1	1	+	4, 1	4, 1	4, 1	I_4, I_1	

304 **304**
 $N = 304 = 2^4 \cdot 19$ (6 isogeny classes)

A1	0	1	0	0	-76	1	1	-	17, 1	5, 1	4, 1	I_9^*, I_1	5 : 2
A2	0	1	0	-1120	15604	1	1	-	13, 5	1, 5	4, 5	I_5^*, I_5	5 : 1
B1	0	-1	0	-248	-1424	0	1	-	15, 1	3, 1	2, 1	I_7^*, I_1	3 : 2
B2	0	-1	0	152	-5776	0	1	-	21, 3	9, 3	2, 1	I_{13}^*, I_3	3 : 1, 3
B3	0	-1	0	-1368	157168	0	1	-	39, 1	27, 1	2, 1	I_{31}^*, I_1	3 : 2
C1	0	-1	0	-8	16	1	1	-	11, 1	0, 1	4, 1	I_3^*, I_1	
D1	0	-1	0	-1	-3	0	1	-	8, 1	0, 1	1, 1	I_0^*, I_1	
E1	0	-1	0	11	-3	0	1	-	12, 1	0, 1	1, 1	II^*, I_1	3 : 2
E2	0	-1	0	-149	797	0	1	-	12, 3	0, 3	1, 1	II^*, I_3	3 : 1, 3
E3	0	-1	0	-12309	529757	0	1	-	12, 1	0, 1	1, 1	II^*, I_1	3 : 2
F1	0	1	0	-21	31	1	1	-	8, 1	0, 1	2, 1	I_0^*, I_1	

306 **306**
 $N = 306 = 2 \cdot 3^2 \cdot 17$ (4 isogeny classes)

A1	1	-1	1	-2300	-41857	0	2	+	6, 12, 1	6, 6, 1	6, 2, 1	I_6, I_6^*, I_1	2 : 2; 3 : 3
A2	1	-1	1	-1940	-55681	0	2	-	3, 18, 2	3, 12, 2	3, 4, 2	I_3, I_{12}^*, I_2	2 : 1; 3 : 4
A3	1	-1	1	-6755	163235	0	6	+	18, 8, 3	18, 2, 3	18, 2, 3	I_{18}, I_2^*, I_3	2 : 4; 3 : 1
A4	1	-1	1	16285	1020323	0	6	-	9, 10, 6	9, 4, 6	9, 4, 6	I_9, I_4^*, I_6	2 : 3; 3 : 2
B1	1	-1	0	-27	-27	1	2	+	6, 6, 1	6, 0, 1	2, 2, 1	I_6, I_0^*, I_1	2 : 2; 3 : 3
B2	1	-1	0	-387	-2835	1	2	+	3, 6, 2	3, 0, 2	1, 2, 2	I_3, I_0^*, I_2	2 : 1; 3 : 4
B3	1	-1	0	-927	11097	1	6	+	2, 6, 3	2, 0, 3	2, 2, 3	I_2, I_0^*, I_3	2 : 4; 3 : 1
B4	1	-1	0	-1017	8883	1	6	+	1, 6, 6	1, 0, 6	1, 2, 6	I_1, I_0^*, I_6	2 : 3; 3 : 2
C1	1	-1	0	-306	-1836	0	2	+	8, 10, 1	8, 4, 1	2, 2, 1	I_8, I_4^*, I_1	2 : 2
C2	1	-1	0	-1026	10692	0	4	+	4, 14, 2	4, 8, 2	2, 4, 2	I_4, I_8^*, I_2	2 : 1, 3, 4
C3	1	-1	0	-15606	754272	0	4	+	2, 10, 4	2, 4, 4	2, 4, 2	I_2, I_4^*, I_4	2 : 2, 5, 6
C4	1	-1	0	2034	60264	0	2	-	2, 22, 1	2, 16, 1	2, 4, 1	I_2, I_{16}^*, I_1	2 : 2
C5	1	-1	0	-249696	48087270	0	2	+	1, 8, 2	1, 2, 2	1, 2, 2	I_1, I_2^*, I_2	2 : 3
C6	1	-1	0	-14796	835434	0	2	-	1, 8, 8	1, 2, 8	1, 4, 2	I_1, I_2^*, I_8	2 : 3
D1	1	-1	1	-23	-21	0	2	+	2, 8, 1	2, 2, 1	2, 2, 1	I_2, I_2^*, I_1	2 : 2
D2	1	-1	1	67	-201	0	2	-	1, 10, 2	1, 4, 2	1, 4, 2	I_1, I_4^*, I_2	2 : 1

307 **307**
 $N = 307 = 307$ (4 isogeny classes)

A1	0	0	1	-8	-9	0	1	-	1	1	1	I_1	
B1	1	1	0	0	-1	0	1	-	1	1	1	I_1	
C1	0	0	1	1	-1	0	1	-	1	1	1	I_1	
D1	0	-1	1	2	-1	0	1	-	1	1	1	I_1	

308 **308**
 $N = 308 = 2^2 \cdot 7 \cdot 11$ (1 isogeny class)

A1	0	-1	0	-21	49	1	1	-	8, 2, 1	0, 2, 1	3, 2, 1	IV^*, I_2, I_1	
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309 **309**
 $N = 309 = 3 \cdot 103$ (1 isogeny class)

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
310	$N = 310 = 2 \cdot 5 \cdot 31$ (2 isogeny classes)											310	
A1	1	1	1	-66	-241	0	2	-	6, 4, 1	6, 4, 1	6, 2, 1	I_6, I_4, I_1	2 : 2
A2	1	1	1	-1066	-13841	0	2	+	3, 2, 2	3, 2, 2	3, 2, 2	I_3, I_2, I_2	2 : 1
B1	1	0	0	-106	420	1	6	-	12, 2, 1	12, 2, 1	12, 2, 1	I_{12}, I_2, I_1	2 : 2; 3 : 3
B2	1	0	0	-1706	26980	1	6	+	6, 1, 2	6, 1, 2	6, 1, 2	I_6, I_1, I_2	2 : 1; 3 : 4
B3	1	0	0	454	1876	1	2	-	4, 6, 3	4, 6, 3	4, 2, 3	I_4, I_6, I_3	2 : 4; 3 : 1
B4	1	0	0	-2046	15376	1	2	+	2, 3, 6	2, 3, 6	2, 1, 6	I_2, I_3, I_6	2 : 3; 3 : 2
312	$N = 312 = 2^3 \cdot 3 \cdot 13$ (6 isogeny classes)											312	
A1	0	1	0	-3	-6	0	2	-	4, 1, 2	0, 1, 2	2, 1, 2	III, I_1, I_2	2 : 2
A2	0	1	0	-68	-240	0	2	+	8, 2, 1	0, 2, 1	2, 2, 1	I_1^*, I_2, I_1	2 : 1
B1	0	-1	0	-3	0	1	2	+	4, 2, 1	0, 2, 1	2, 2, 1	III, I_2, I_1	2 : 2
B2	0	-1	0	12	-12	1	2	-	8, 1, 2	0, 1, 2	2, 1, 2	I_1^*, I_1, I_2	2 : 1
C1	0	1	0	-7	2	0	4	+	4, 4, 1	0, 4, 1	2, 4, 1	III, I_4, I_1	2 : 2
C2	0	1	0	-52	-160	0	4	+	8, 2, 2	0, 2, 2	4, 2, 2	I_1^*, I_2, I_2	2 : 1, 3, 4
C3	0	1	0	-832	-9520	0	2	+	10, 1, 1	0, 1, 1	2, 1, 1	III^*, I_1, I_1	2 : 2
C4	0	1	0	8	-448	0	2	-	10, 1, 4	0, 1, 4	2, 1, 4	III^*, I_1, I_4	2 : 2
D1	0	-1	0	-39	108	0	4	+	4, 2, 1	0, 2, 1	2, 2, 1	III, I_2, I_1	2 : 2
D2	0	-1	0	-44	84	0	4	+	8, 4, 2	0, 4, 2	2, 2, 2	I_1^*, I_4, I_2	2 : 1, 3, 4
D3	0	-1	0	-304	-1892	0	2	+	10, 8, 1	0, 8, 1	2, 2, 1	III^*, I_8, I_1	2 : 2
D4	0	-1	0	136	444	0	4	-	10, 2, 4	0, 2, 4	2, 2, 4	III^*, I_2, I_4	2 : 2
E1	0	-1	0	-651	6228	0	2	+	4, 10, 3	0, 10, 3	2, 2, 1	III, I_{10}, I_3	2 : 2
E2	0	-1	0	564	25668	0	2	-	8, 5, 6	0, 5, 6	4, 1, 2	I_1^*, I_5, I_6	2 : 1
F1	0	1	0	5	14	1	2	-	4, 3, 2	0, 3, 2	2, 3, 2	III, I_3, I_2	2 : 2
F2	0	1	0	-60	144	1	2	+	8, 6, 1	0, 6, 1	4, 6, 1	I_1^*, I_6, I_1	2 : 1
314	$N = 314 = 2 \cdot 157$ (1 isogeny class)											314	
A1	1	-1	0	13	-11	1	1	-	10, 1	10, 1	2, 1	I_{10}, I_1	
315	$N = 315 = 3^2 \cdot 5 \cdot 7$ (2 isogeny classes)											315	
A1	0	0	1	-12	-18	0	1	-	6, 1, 1	0, 1, 1	1, 1, 1	I_0^*, I_1, I_1	3 : 2
A2	0	0	1	78	45	0	3	-	6, 3, 3	0, 3, 3	1, 3, 3	I_0^*, I_3, I_3	3 : 1, 3
A3	0	0	1	-1182	16362	0	3	-	6, 9, 1	0, 9, 1	1, 9, 1	I_0^*, I_9, I_1	3 : 2
B1	1	-1	1	-23	-34	1	2	+	7, 1, 1	1, 1, 1	2, 1, 1	I_1^*, I_1, I_1	2 : 2
B2	1	-1	1	-68	182	1	4	+	8, 2, 2	2, 2, 2	4, 2, 2	I_2^*, I_2, I_2	2 : 1, 3, 4
B3	1	-1	1	-1013	12656	1	2	+	7, 4, 1	1, 4, 1	4, 2, 1	I_1^*, I_4, I_1	2 : 2
B4	1	-1	1	157	992	1	2	-	10, 1, 4	4, 1, 4	4, 1, 4	I_4^*, I_1, I_4	2 : 2
316	$N = 316 = 2^2 \cdot 79$ (2 isogeny classes)											316	
A1	0	-1	0	-180	-872	0	1	+	8, 1	0, 1	1, 1	IV^*, I_1	
B1	0	0	0	-7	-2	1	1	+	8, 1	0, 1	3, 1	IV^*, I_1	
318	$N = 318 = 2 \cdot 3 \cdot 53$ (5 isogeny classes)											318	
A1	1	1	1	2	-7	0	1	-	1, 5, 1	1, 5, 1	1, 1, 1	I_1, I_5, I_1	
B1	1	0	1	-61	176	0	3	-	3, 3, 1	3, 3, 1	1, 3, 1	I_3, I_3, I_1	3 : 2
B2	1	0	1	44	722	0	1	-	9, 1, 3	9, 1, 3	1, 1, 1	I_9, I_1, I_3	3 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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318 $N = 318 = 2 \cdot 3 \cdot 53$ (continued) **318**

D1	1	1	1	-12	45	1	1	-	11, 2, 1	11, 2, 1	11, 2, 1	I_{11}, I_2, I_1	
E1	1	1	0	142	180	0	1	-	17, 3, 1	17, 3, 1	1, 1, 1	I_{17}, I_3, I_1	

319 $N = 319 = 11 \cdot 29$ (1 isogeny class) **319**

A1	0	0	1	-37	-87	0	1	-	1, 2	1, 2	1, 2	I_1, I_2	
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320 $N = 320 = 2^6 \cdot 5$ (6 isogeny classes) **320**

A1	0	0	0	-8	-8	0	2	+	10, 1	0, 1	2, 1	I_0^*, I_1	2 : 2
A2	0	0	0	-28	48	0	4	+	14, 2	0, 2	4, 2	I_4^*, I_2	2 : 1, 3, 4
A3	0	0	0	-428	3408	0	2	+	16, 1	0, 1	2, 1	I_6^*, I_1	2 : 2
A4	0	0	0	52	272	0	2	-	16, 4	0, 4	2, 2	I_6^*, I_4	2 : 2
B1	0	0	0	-8	8	1	2	+	10, 1	0, 1	2, 1	I_0^*, I_1	2 : 2
B2	0	0	0	-28	-48	1	4	+	14, 2	0, 2	4, 2	I_4^*, I_2	2 : 1, 3, 4
B3	0	0	0	-428	-3408	1	2	+	16, 1	0, 1	2, 1	I_6^*, I_1	2 : 2
B4	0	0	0	52	-272	1	2	-	16, 4	0, 4	4, 2	I_6^*, I_4	2 : 2
C1	0	-1	0	-5	5	0	2	+	10, 1	0, 1	2, 1	I_0^*, I_1	2 : 2; 3 : 3
C2	0	-1	0	15	17	0	2	-	14, 2	0, 2	2, 2	I_4^*, I_2	2 : 1; 3 : 4
C3	0	-1	0	-165	-763	0	2	+	10, 3	0, 3	2, 3	I_0^*, I_3	2 : 4; 3 : 1
C4	0	-1	0	-145	-975	0	2	-	14, 6	0, 6	2, 6	I_4^*, I_6	2 : 3; 3 : 2
D1	0	-1	0	0	2	0	2	-	6, 2	0, 2	1, 2	II, I_2	2 : 2
D2	0	-1	0	-25	57	0	2	+	12, 1	0, 1	2, 1	I_2^*, I_1	2 : 1
E1	0	1	0	0	-2	0	2	-	6, 2	0, 2	1, 2	II, I_2	2 : 2
E2	0	1	0	-25	-57	0	2	+	12, 1	0, 1	2, 1	I_2^*, I_1	2 : 1
F1	0	1	0	-5	-5	1	2	+	10, 1	0, 1	2, 1	I_0^*, I_1	2 : 2; 3 : 3
F2	0	1	0	15	-17	1	2	-	14, 2	0, 2	4, 2	I_4^*, I_2	2 : 1; 3 : 4
F3	0	1	0	-165	763	1	2	+	10, 3	0, 3	2, 3	I_0^*, I_3	2 : 4; 3 : 1
F4	0	1	0	-145	975	1	2	-	14, 6	0, 6	4, 6	I_4^*, I_6	2 : 3; 3 : 2

322 $N = 322 = 2 \cdot 7 \cdot 23$ (4 isogeny classes) **322**

A1	1	-1	0	-8	44	1	2	-	2, 3, 2	2, 3, 2	2, 3, 2	I_2, I_3, I_2	2 : 2
A2	1	-1	0	-238	1470	1	2	+	1, 6, 1	1, 6, 1	1, 6, 1	I_1, I_6, I_1	2 : 1
B1	1	1	0	35	381	0	2	-	14, 1, 2	14, 1, 2	2, 1, 2	I_{14}, I_1, I_2	2 : 2
B2	1	1	0	-605	5117	0	2	+	7, 2, 4	7, 2, 4	1, 2, 2	I_7, I_2, I_4	2 : 1
C1	1	1	1	-4	1	0	2	+	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	2 : 2
C2	1	1	1	-14	-23	0	2	+	1, 2, 2	1, 2, 2	1, 2, 2	I_1, I_2, I_2	2 : 1
D1	1	0	0	-14	4	1	2	+	10, 1, 1	10, 1, 1	10, 1, 1	I_{10}, I_1, I_1	2 : 2
D2	1	0	0	-174	868	1	2	+	5, 2, 2	5, 2, 2	5, 2, 2	I_5, I_2, I_2	2 : 1

323 $N = 323 = 17 \cdot 19$ (1 isogeny class) **323**

A1	0	0	1	-46	277	0	1	-	5, 1	5, 1	1, 1	I_5, I_1	
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324 $N = 324 = 2^2 \cdot 3^4$ (4 isogeny classes) **324**

A1	0	0	0	-21	37	0	3	+	4, 4	0, 0	3, 1	IV, II	3 : 2
A2	0	0	0	-81	-243	0	1	+	4, 12	0, 0	1, 1	IV, II^*	3 : 1
B1	0	0	0	9	-18	0	3	-	8, 6	0, 0	3, 3	IV^*, IV	3 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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324 **324**
 $N = 324 = 2^2 \cdot 3^4$ (continued)

C1	0	0	0	-9	9	1	3	+	4, 6	0, 0	3, 3	IV, IV	3 : 2
C2	0	0	0	-189	-999	1	1	+	4, 10	0, 0	1, 1	IV, IV*	3 : 1
D1	0	0	0	-39	94	0	3	-	8, 4	0, 0	3, 1	IV*, II	3 : 2
D2	0	0	0	81	486	0	1	-	8, 12	0, 0	1, 1	IV*, II*	3 : 1

325 **325**
 $N = 325 = 5^2 \cdot 13$ (5 isogeny classes)

A1	0	1	1	-83	244	1	3	+	8, 1	0, 1	3, 1	IV*, I ₁	3 : 2
A2	0	1	1	-1333	-19131	1	1	+	8, 3	0, 3	1, 3	IV*, I ₃	3 : 1
B1	0	-1	1	-3	3	1	1	+	2, 1	0, 1	1, 1	II, I ₁	3 : 2
B2	0	-1	1	-53	-132	1	1	+	2, 3	0, 3	1, 1	II, I ₃	3 : 1
C1	1	1	0	-25	0	0	2	+	7, 1	1, 1	4, 1	I ₁ *, I ₁	2 : 2
C2	1	1	0	100	125	0	2	-	8, 2	2, 2	4, 2	I ₂ *, I ₂	2 : 1
D1	0	1	1	-508	-4581	0	1	+	4, 1	0, 1	3, 1	IV, I ₁	5 : 2
D2	0	1	1	-2458	42369	0	1	+	8, 5	0, 5	3, 1	IV*, I ₅	5 : 1
E1	0	-1	1	-98	378	0	5	+	2, 5	0, 5	1, 5	II, I ₅	5 : 2
E2	0	-1	1	-12708	-547182	0	1	+	10, 1	0, 1	1, 1	II*, I ₁	5 : 1

326 **326**
 $N = 326 = 2 \cdot 163$ (3 isogeny classes)

A1	1	-1	0	-80	-256	1	1	+	9, 1	9, 1	1, 1	I ₉ , I ₁	
B1	1	0	0	-6	4	1	1	+	5, 1	5, 1	5, 1	I ₅ , I ₁	
C1	1	0	1	-355	1182	0	3	+	9, 3	9, 3	1, 3	I ₉ , I ₃	3 : 2, 3
C2	1	0	1	-14210	-653100	0	1	+	27, 1	27, 1	1, 1	I ₂₇ , I ₁	3 : 1
C3	1	0	1	-300	1970	0	3	+	3, 1	3, 1	1, 1	I ₃ , I ₁	3 : 1

327 **327**
 $N = 327 = 3 \cdot 109$ (1 isogeny class)

A1	1	0	0	4	-3	1	1	-	4, 1	4, 1	4, 1	I ₄ , I ₁	
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328 **328**
 $N = 328 = 2^3 \cdot 41$ (2 isogeny classes)

A1	0	0	0	-11	-10	1	2	+	10, 1	0, 1	2, 1	III*, I ₁	2 : 2
A2	0	0	0	29	-66	1	2	-	11, 2	0, 2	1, 2	II*, I ₂	2 : 1
B1	0	-1	0	-12	20	0	2	+	8, 1	0, 1	2, 1	I ₁ *, I ₁	2 : 2
B2	0	-1	0	8	60	0	2	-	10, 2	0, 2	2, 2	III*, I ₂	2 : 1

329 **329**
 $N = 329 = 7 \cdot 47$ (1 isogeny class)

A1	1	1	1	246	-1376	0	1	-	9, 1	9, 1	1, 1	I ₉ , I ₁	
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330 **330**
 $N = 330 = 2 \cdot 3 \cdot 5 \cdot 11$ (5 isogeny classes)

A1	1	1	0	-1393	-20603	0	2	+	4, 5, 2, 1	4, 5, 2, 1	2, 1, 2, 1	I ₄ , I ₅ , I ₂ , I ₁	2 : 2
A2	1	1	0	-1413	-20007	0	4	+	2, 10, 4, 2	2, 10, 4, 2	2, 2, 2, 2	I ₂ , I ₁₀ , I ₄ , I ₂	2 : 1, 3, 4
A3	1	1	0	-4163	77343	0	2	+	1, 20, 2, 1	1, 20, 2, 1	1, 2, 2, 1	I ₁ , I ₂₀ , I ₂ , I ₁	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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330

$N = 330 = 2 \cdot 3 \cdot 5 \cdot 11$ (continued)

330

B1	1	0	0	5	17	0	4	−	8, 2, 1, 1	8, 2, 1, 1	8, 2, 1, 1	I_8, I_2, I_1, I_1	2 : 2
B2	1	0	0	−75	225	0	8	+	4, 4, 2, 2	4, 4, 2, 2	4, 4, 2, 2	I_4, I_4, I_2, I_2	2 : 1, 3, 4
B3	1	0	0	−255	−1323	0	4	+	2, 2, 4, 4	2, 2, 4, 4	2, 2, 4, 2	I_2, I_2, I_4, I_4	2 : 2, 5, 6
B4	1	0	0	−1175	15405	0	4	+	2, 8, 1, 1	2, 8, 1, 1	2, 8, 1, 1	I_2, I_8, I_1, I_1	2 : 2
B5	1	0	0	−3885	−93525	0	2	+	1, 1, 8, 2	1, 1, 8, 2	1, 1, 8, 2	I_1, I_1, I_8, I_2	2 : 3
B6	1	0	0	495	−7473	0	2	−	1, 1, 2, 8	1, 1, 2, 8	1, 1, 2, 2	I_1, I_1, I_2, I_8	2 : 3

C1	1	1	1	255	255	0	4	−	16, 3, 1, 2	16, 3, 1, 2	16, 1, 1, 2	I_{16}, I_3, I_1, I_2	2 : 2
C2	1	1	1	−1025	767	0	8	+	8, 6, 2, 4	8, 6, 2, 4	8, 2, 2, 4	I_8, I_6, I_2, I_4	2 : 1, 3, 4
C3	1	1	1	−10705	−429025	0	4	+	4, 12, 4, 2	4, 12, 4, 2	4, 2, 4, 2	I_4, I_{12}, I_4, I_2	2 : 2, 5, 6
C4	1	1	1	−11825	488927	0	4	+	4, 3, 1, 8	4, 3, 1, 8	4, 1, 1, 8	I_4, I_3, I_1, I_8	2 : 2
C5	1	1	1	−171085	−27308713	0	2	+	2, 6, 8, 1	2, 6, 8, 1	2, 2, 8, 1	I_2, I_6, I_8, I_1	2 : 3
C6	1	1	1	−5205	−862425	0	2	−	2, 24, 2, 1	2, 24, 2, 1	2, 2, 2, 1	I_2, I_{24}, I_2, I_1	2 : 3

D1	1	1	1	−40266	2921559	0	4	+	28, 5, 4, 1	28, 5, 4, 1	28, 1, 2, 1	I_{28}, I_5, I_4, I_1	2 : 2
D2	1	1	1	−122186	−12872617	0	4	+	14, 10, 8, 2	14, 10, 8, 2	14, 2, 2, 2	I_{14}, I_{10}, I_8, I_2	2 : 1, 3, 4
D3	1	1	1	−1832906	−955821481	0	2	+	7, 5, 16, 1	7, 5, 16, 1	7, 1, 2, 1	I_7, I_5, I_{16}, I_1	2 : 2
D4	1	1	1	277814	−79112617	0	2	−	7, 20, 4, 4	7, 20, 4, 4	7, 2, 2, 2	I_7, I_{20}, I_4, I_4	2 : 2

E1	1	1	0	−22	−44	1	2	+	8, 1, 2, 1	8, 1, 2, 1	2, 1, 2, 1	I_8, I_1, I_2, I_1	2 : 2
E2	1	1	0	−102	324	1	4	+	4, 2, 4, 2	4, 2, 4, 2	2, 2, 4, 2	I_4, I_2, I_4, I_2	2 : 1, 3, 4
E3	1	1	0	−1602	24024	1	4	+	2, 1, 2, 4	2, 1, 2, 4	2, 1, 2, 4	I_2, I_1, I_2, I_4	2 : 2
E4	1	1	0	118	1776	1	2	−	2, 4, 8, 1	2, 4, 8, 1	2, 2, 8, 1	I_2, I_4, I_8, I_1	2 : 2

331

$N = 331 = 331$ (1 isogeny class)

331

A1	1	0	0	−5	4	1	1	−	1	1	1	I_1	
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333

$N = 333 = 3^2 \cdot 37$ (4 isogeny classes)

333

A1	0	0	1	−30	−63	1	1	+	6, 1	0, 1	1, 1	I_0^*, I_1	3 : 2
A2	0	0	1	−210	1134	1	3	+	6, 3	0, 3	1, 3	I_0^*, I_3	3 : 1, 3
A3	0	0	1	−16860	842625	1	3	+	6, 1	0, 1	1, 1	I_0^*, I_1	3 : 2

B1	1	−1	0	12	35	1	2	−	9, 1	0, 1	2, 1	III^*, I_1	2 : 2
B2	1	−1	0	−123	494	1	2	+	9, 2	0, 2	2, 2	III^*, I_2	2 : 1

C1	1	−1	1	1	−2	1	2	−	3, 1	0, 1	2, 1	III, I_1	2 : 2
C2	1	−1	1	−14	−14	1	2	+	3, 2	0, 2	2, 2	III, I_2	2 : 1

D1	0	0	1	−9	−7	0	1	+	6, 1	0, 1	1, 1	I_0^*, I_1	

334

$N = 334 = 2 \cdot 167$ (1 isogeny class)

334

A1	1	−1	1	−1	−1	0	1	−	1, 1	1, 1	1, 1	I_1, I_1	
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335

$N = 335 = 5 \cdot 67$ (1 isogeny class)

335

A1	0	0	1	−2	2	1	1	−	2, 1	2, 1	2, 1	I_2, I_1	
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336

$N = 336 = 2^4 \cdot 3 \cdot 7$ (6 isogeny classes)

336

A1	0	−1	0	7	0	0	2	−	4, 3, 2	0, 3, 2	1, 1, 2	II, I_3, I_2	2 : 2; 3 : 3
A2	0	−1	0	−28	28	0	2	+	8, 6, 1	0, 6, 1	1, 2, 1	I_0^*, I_6, I_1	2 : 1; 3 : 4
A3	0	−1	0	−113	516	0	2	−	4, 1, 6	0, 1, 6	1, 1, 2	II, I, I_c	2 : 4; 3 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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336 $N = 336 = 2^4 \cdot 3 \cdot 7$ (continued)**336**

B1	0	-1	0	-7	10	0	2	+	4, 1, 1	0, 1, 1	1, 1, 1	II, I ₁ , I ₁	2 : 2
B2	0	-1	0	-12	0	0	4	+	8, 2, 2	0, 2, 2	2, 2, 2	I ₀ [*] , I ₂ , I ₂	2 : 1, 3, 4
B3	0	-1	0	-152	-672	0	2	+	10, 4, 1	0, 4, 1	2, 2, 1	I ₂ [*] , I ₄ , I ₁	2 : 2
B4	0	-1	0	48	-48	0	4	-	10, 1, 4	0, 1, 4	4, 1, 4	I ₂ [*] , I ₁ , I ₄	2 : 2
C1	0	1	0	-7	-52	0	2	-	4, 3, 4	0, 3, 4	1, 3, 2	II, I ₃ , I ₄	2 : 2
C2	0	1	0	-252	-1620	0	4	+	8, 6, 2	0, 6, 2	2, 6, 2	I ₀ [*] , I ₆ , I ₂	2 : 1, 3, 4
C3	0	1	0	-4032	-99900	0	2	+	10, 3, 1	0, 3, 1	4, 3, 1	I ₂ [*] , I ₃ , I ₁	2 : 2
C4	0	1	0	-392	228	0	4	+	10, 12, 1	0, 12, 1	2, 12, 1	I ₂ [*] , I ₁₂ , I ₁	2 : 2
D1	0	1	0	-64	-460	0	2	-	20, 2, 1	8, 2, 1	4, 2, 1	I ₁₂ [*] , I ₂ , I ₁	2 : 2
D2	0	1	0	-1344	-19404	0	4	+	16, 4, 2	4, 4, 2	4, 4, 2	I ₈ [*] , I ₄ , I ₂	2 : 1, 3, 4
D3	0	1	0	-21504	-1220940	0	2	+	14, 2, 1	2, 2, 1	2, 2, 1	I ₆ [*] , I ₂ , I ₁	2 : 2
D4	0	1	0	-1664	-9804	0	8	+	14, 8, 4	2, 8, 4	4, 8, 4	I ₆ [*] , I ₈ , I ₄	2 : 2, 5, 6
D5	0	1	0	-14624	669300	0	8	+	13, 4, 8	1, 4, 8	4, 4, 8	I ₅ [*] , I ₄ , I ₈	2 : 4
D6	0	1	0	6176	-69388	0	4	-	13, 16, 2	1, 16, 2	2, 16, 2	I ₅ [*] , I ₁₆ , I ₂	2 : 4
E1	0	-1	0	16	0	1	2	-	12, 2, 1	0, 2, 1	4, 2, 1	I ₄ [*] , I ₂ , I ₁	2 : 2
E2	0	-1	0	-64	64	1	4	+	12, 4, 2	0, 4, 2	4, 2, 2	I ₄ [*] , I ₄ , I ₂	2 : 1, 3, 4
E3	0	-1	0	-624	-5760	1	2	+	12, 8, 1	0, 8, 1	2, 2, 1	I ₄ [*] , I ₈ , I ₁	2 : 2
E4	0	-1	0	-784	8704	1	8	+	12, 2, 4	0, 2, 4	4, 2, 4	I ₄ [*] , I ₂ , I ₄	2 : 2, 5, 6
E5	0	-1	0	-12544	544960	1	4	+	12, 1, 2	0, 1, 2	2, 1, 2	I ₄ [*] , I ₁ , I ₂	2 : 4
E6	0	-1	0	-544	13888	1	4	-	12, 1, 8	0, 1, 8	4, 1, 8	I ₄ [*] , I ₁ , I ₈	2 : 4
F1	0	1	0	-1	2	0	2	-	4, 1, 2	0, 1, 2	1, 1, 2	II, I ₁ , I ₂	2 : 2
F2	0	1	0	-36	72	0	2	+	8, 2, 1	0, 2, 1	1, 2, 1	I ₀ [*] , I ₂ , I ₁	2 : 1

338 $N = 338 = 2 \cdot 13^2$ (6 isogeny classes)**338**

A1	1	-1	0	1	1	1	1	-	2, 2	2, 0	2, 1	I ₂ , II	7 : 2
A2	1	-1	0	-389	-2859	1	1	-	14, 2	14, 0	2, 1	I ₁₄ , II	7 : 1
B1	1	-1	1	137	2643	0	1	-	2, 8	2, 0	2, 1	I ₂ , IV [*]	7 : 2
B2	1	-1	1	-65773	-6478507	0	1	-	14, 8	14, 0	14, 1	I ₁₄ , IV [*]	7 : 1
C1	1	0	0	81	467	0	1	-	1, 7	1, 1	1, 2	I ₁ , I ₁ [*]	3 : 2
C2	1	0	0	-764	-16264	0	1	-	3, 9	3, 3	3, 2	I ₃ , I ₃ [*]	3 : 1, 3
C3	1	0	0	-77659	-8336303	0	1	-	9, 7	9, 1	9, 2	I ₉ , I ₁ [*]	3 : 2
D1	1	1	0	504	-13112	0	1	-	3, 9	3, 0	1, 2	I ₃ , III [*]	5 : 2
D2	1	1	0	-54421	4945517	0	1	-	15, 9	15, 0	1, 2	I ₁₅ , III [*]	5 : 1
E1	1	1	1	3	-5	1	1	-	3, 3	3, 0	3, 2	I ₃ , III	5 : 2
E2	1	1	1	-322	2127	1	1	-	15, 3	15, 0	15, 2	I ₁₅ , III	5 : 1
F1	1	-1	0	-454	5812	1	1	-	7, 7	7, 1	1, 4	I ₇ , I ₁ [*]	7 : 2
F2	1	-1	0	-35944	-2868878	1	1	-	1, 13	1, 7	1, 4	I ₁ , I ₇ [*]	7 : 1

339 $N = 339 = 3 \cdot 113$ (3 isogeny classes)**339**

A1	0	1	1	-441	3422	1	1	-	9, 1	9, 1	9, 1	I ₉ , I ₁	
B1	0	-1	1	-112	501	0	1	-	9, 1	9, 1	1, 1	I ₉ , I ₁	
C1	0	1	1	-2	2	1	1	-	3, 1	3, 1	3, 1	I ₃ , I ₁	

340 $N = 340 = 2^2 \cdot 5 \cdot 17$ (1 isogeny class)**340**

A1	0	0	0	-28	57	1	2	+	4, 1, 1	0, 1, 1	3, 1, 1	IV, I, I ₁	2 : 2
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	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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342

$N = 342 = 2 \cdot 3^2 \cdot 19$ (7 isogeny classes)

342

A1	1	-1	1	-140	-601	0	1	-	3, 6, 1	3, 0, 1	3, 1, 1	I_3, I_0^*, I_1	3 : 2
A2	1	-1	1	85	-2437	0	3	-	9, 6, 3	9, 0, 3	9, 1, 3	I_9, I_0^*, I_3	3 : 1, 3
A3	1	-1	1	-770	66305	0	3	-	27, 6, 1	27, 0, 1	27, 1, 1	I_{27}, I_0^*, I_1	3 : 2
B1	1	-1	1	-860	9915	0	2	+	2, 11, 1	2, 5, 1	2, 2, 1	I_2, I_5^*, I_1	2 : 2
B2	1	-1	1	-770	12003	0	2	-	1, 16, 2	1, 10, 2	1, 4, 2	I_1, I_{10}^*, I_2	2 : 1
C1	1	-1	0	-72	0	1	2	+	6, 9, 1	6, 3, 1	2, 4, 1	I_6, I_3^*, I_1	2 : 2; 3 : 3
C2	1	-1	0	288	-216	1	2	-	3, 12, 2	3, 6, 2	1, 4, 2	I_3, I_6^*, I_2	2 : 1; 3 : 4
C3	1	-1	0	-3852	92988	1	6	+	2, 7, 3	2, 1, 3	2, 4, 3	I_2, I_1^*, I_3	2 : 4; 3 : 1
C4	1	-1	0	-3762	97470	1	6	-	1, 8, 6	1, 2, 6	1, 4, 6	I_1, I_2^*, I_6	2 : 3; 3 : 2
D1	1	-1	1	-29	1	0	2	+	2, 9, 1	2, 0, 1	2, 2, 1	I_2, III^*, I_1	2 : 2
D2	1	-1	1	-299	2053	0	2	+	1, 9, 2	1, 0, 2	1, 2, 2	I_1, III^*, I_2	2 : 1
E1	1	-1	0	-3	1	1	2	+	2, 3, 1	2, 0, 1	2, 2, 1	I_2, III, I_1	2 : 2
E2	1	-1	0	-33	-65	1	2	+	1, 3, 2	1, 0, 2	1, 2, 2	I_1, III, I_2	2 : 1
F1	1	-1	0	-3168	62464	0	2	+	20, 9, 1	20, 3, 1	2, 2, 1	I_{20}, I_3^*, I_1	2 : 2
F2	1	-1	0	-49248	4218880	0	4	+	10, 12, 2	10, 6, 2	2, 4, 2	I_{10}, I_6^*, I_2	2 : 1, 3, 4
F3	1	-1	0	-787968	269419360	0	2	+	5, 9, 1	5, 3, 1	1, 4, 1	I_5, I_3^*, I_1	2 : 2
F4	1	-1	0	-47808	4476064	0	2	-	5, 18, 4	5, 12, 4	1, 4, 2	I_5, I_{12}^*, I_4	2 : 2
G1	1	-1	0	0	-32	0	1	-	5, 6, 1	5, 0, 1	1, 1, 1	I_5, I_0^*, I_1	5 : 2
G2	1	-1	0	-630	6898	0	1	-	1, 6, 5	1, 0, 5	1, 1, 1	I_1, I_0^*, I_5	5 : 1

344

$N = 344 = 2^3 \cdot 43$ (1 isogeny class)

344

A1	0	0	0	4	4	1	1	-	8, 1	0, 1	2, 1	I_1^*, I_1	
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345

$N = 345 = 3 \cdot 5 \cdot 23$ (6 isogeny classes)

345

A1	0	-1	1	-731	-7369	0	1	-	2, 5, 1	2, 5, 1	2, 1, 1	I_2, I_5, I_1	
B1	0	1	1	-1	1	1	1	-	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	
C1	1	0	1	456	2401	0	2	-	5, 3, 4	5, 3, 4	5, 1, 2	I_5, I_3, I_4	2 : 2
C2	1	0	1	-2189	20387	0	4	+	10, 6, 2	10, 6, 2	10, 2, 2	I_{10}, I_6, I_2	2 : 1, 3, 4
C3	1	0	1	-16564	-807613	0	2	+	20, 3, 1	20, 3, 1	20, 1, 1	I_{20}, I_3, I_1	2 : 2
C4	1	0	1	-30134	2010071	0	2	+	5, 12, 1	5, 12, 1	5, 2, 1	I_5, I_{12}, I_1	2 : 2
D1	1	0	0	9	0	0	4	-	4, 2, 1	4, 2, 1	4, 2, 1	I_4, I_2, I_1	2 : 2
D2	1	0	0	-36	-9	0	4	+	2, 4, 2	2, 4, 2	2, 2, 2	I_2, I_4, I_2	2 : 1, 3, 4
D3	1	0	0	-411	-3234	0	2	+	1, 2, 4	1, 2, 4	1, 2, 2	I_1, I_2, I_4	2 : 2
D4	1	0	0	-381	2820	0	2	+	1, 8, 1	1, 8, 1	1, 2, 1	I_1, I_8, I_1	2 : 2
E1	0	-1	1	30	-97	0	1	-	4, 1, 3	4, 1, 3	2, 1, 1	I_4, I_1, I_3	
F1	0	1	1	-100	406	1	1	-	8, 3, 1	8, 3, 1	8, 3, 1	I_8, I_3, I_1	

346

$N = 346 = 2 \cdot 173$ (2 isogeny classes)

346

A1	1	0	0	-16	-26	0	1	+	1, 1	1, 1	1, 1	I_1, I_1	
B1	1	1	1	-7	-3	1	1	+	7, 1	7, 1	7, 1	I_7, I_1	

347

$N = 347 = 347$ (1 isogeny class)

347

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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348 $N = 348 = 2^2 \cdot 3 \cdot 29$ (4 isogeny classes) **348**

A1	0	-1	0	2	1	1	1	-	4, 1, 1	0, 1, 1	3, 1, 1	IV, I ₁ , I ₁	
B1	0	1	0	-2	-3	0	1	-	4, 1, 1	0, 1, 1	1, 1, 1	IV, I ₁ , I ₁	
C1	0	-1	0	-94	3973	0	1	-	4, 15, 1	0, 15, 1	1, 1, 1	IV, I ₁₅ , I ₁	
D1	0	1	0	-50	129	1	1	-	4, 7, 1	0, 7, 1	3, 7, 1	IV, I ₇ , I ₁	

350 $N = 350 = 2 \cdot 5^2 \cdot 7$ (6 isogeny classes) **350**

A1	1	-1	0	58	-284	0	2	-	4, 8, 1	4, 2, 1	2, 2, 1	I ₄ , I ₂ [*] , I ₁	2 : 2
A2	1	-1	0	-442	-2784	0	4	+	2, 10, 2	2, 4, 2	2, 4, 2	I ₂ , I ₄ [*] , I ₂	2 : 1, 3, 4
A3	1	-1	0	-6692	-209034	0	2	+	1, 8, 4	1, 2, 4	1, 2, 4	I ₁ , I ₂ [*] , I ₄	2 : 2
A4	1	-1	0	-2192	37466	0	2	+	1, 14, 1	1, 8, 1	1, 4, 1	I ₁ , I ₈ [*] , I ₁	2 : 2
B1	1	0	0	112	392	0	3	-	3, 8, 2	3, 0, 2	3, 3, 2	I ₃ , IV [*] , I ₂	3 : 2
B2	1	0	0	-1138	-20858	0	1	-	1, 8, 6	1, 0, 6	1, 1, 6	I ₁ , IV [*] , I ₆	3 : 1
C1	1	1	0	5	5	1	1	-	3, 2, 2	3, 0, 2	1, 1, 2	I ₃ , II, I ₂	3 : 2
C2	1	1	0	-45	-185	1	1	-	1, 2, 6	1, 0, 6	1, 1, 2	I ₁ , II, I ₆	3 : 1
D1	1	1	1	-13	31	0	2	-	2, 6, 1	2, 0, 1	2, 2, 1	I ₂ , I ₀ [*] , I ₁	2 : 2; 3 : 3
D2	1	1	1	-263	1531	0	2	+	1, 6, 2	1, 0, 2	1, 2, 2	I ₁ , I ₀ [*] , I ₂	2 : 1; 3 : 4
D3	1	1	1	112	-719	0	2	-	6, 6, 3	6, 0, 3	6, 2, 1	I ₆ , I ₀ [*] , I ₃	2 : 4; 3 : 1, 5
D4	1	1	1	-888	-8719	0	2	+	3, 6, 6	3, 0, 6	3, 2, 2	I ₃ , I ₀ [*] , I ₆	2 : 3; 3 : 2, 6
D5	1	1	1	-4263	-109219	0	2	-	18, 6, 1	18, 0, 1	18, 2, 1	I ₁₈ , I ₀ [*] , I ₁	2 : 6; 3 : 3
D6	1	1	1	-68263	-6893219	0	2	+	9, 6, 2	9, 0, 2	9, 2, 2	I ₉ , I ₀ [*] , I ₂	2 : 5; 3 : 4
E1	1	-1	0	-4492	126416	0	1	-	11, 10, 2	11, 0, 2	1, 1, 2	I ₁₁ , II [*] , I ₂	
F1	1	-1	1	-180	1047	1	1	-	11, 4, 2	11, 0, 2	11, 3, 2	I ₁₁ , IV, I ₂	

352 $N = 352 = 2^5 \cdot 11$ (6 isogeny classes) **352**

A1	0	1	0	-45	-133	0	1	-	12, 1	0, 1	2, 1	III [*] , I ₁	
B1	0	1	0	3	11	1	1	-	12, 1	0, 1	2, 1	III [*] , I ₁	
C1	0	-1	0	-45	133	1	1	-	12, 1	0, 1	2, 1	III [*] , I ₁	
D1	0	-1	0	3	-11	1	1	-	12, 1	0, 1	2, 1	III [*] , I ₁	
E1	0	0	0	8	-112	0	1	-	12, 3	0, 3	2, 1	III [*] , I ₃	
F1	0	0	0	8	112	1	1	-	12, 3	0, 3	2, 3	III [*] , I ₃	

353 $N = 353 = 353$ (1 isogeny class) **353**

A1	1	1	1	-2	16	0	2	-	2	2	2	I ₂	2 : 2
A2	1	1	1	-7	4	0	2	+	1	1	1	I ₁	2 : 1

354 $N = 354 = 2 \cdot 3 \cdot 59$ (6 isogeny classes) **354**

A1	1	1	1	-3	-3	0	2	+	2, 1, 1	2, 1, 1	2, 1, 1	I ₂ , I ₁ , I ₁	2 : 2
A2	1	1	1	7	-7	0	2	-	1, 2, 2	1, 2, 2	1, 2, 2	I ₁ , I ₂ , I ₂	2 : 1
B1	1	0	1	9	-8	0	3	-	1, 6, 1	1, 6, 1	1, 6, 1	I ₁ , I ₆ , I ₁	3 : 2
B2	1	0	1	-216	-1250	0	1	-	3, 2, 3	3, 2, 3	1, 2, 1	I ₃ , I ₂ , I ₃	3 : 1
C1	1	1	0	-715	7069	1	1	-	5, 6, 1	5, 6, 1	1, 2, 1	I ₅ , I ₆ , I ₁	
D1	1	1	0	-34	-92	0	2	+	4, 3, 1	4, 3, 1	2, 1, 1	I ₄ , I ₃ , I ₁	2 : 2
D2	1	1	0	-54	0	0	4	+	2, 6, 2	2, 6, 2	2, 2, 2	I ₂ , I ₆ , I ₂	2 : 1, 3, 4
D3	1	1	0	-644	6018	0	2	+	1, 12, 1	1, 12, 1	1, 2, 1	I ₁ , I ₁₂ , I ₁	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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354 $N = 354 = 2 \cdot 3 \cdot 59$ (continued) **354**

E1	1	1	1	-23511	-1393299	0	2	+	22, 9, 1	22, 9, 1	22, 1, 1	I_{22}, I_9, I_1	2 : 2
E2	1	1	1	-13271	-2601619	0	2	-	11, 18, 2	11, 18, 2	11, 2, 2	I_{11}, I_{18}, I_2	2 : 1
F1	1	1	1	-5	11	1	1	-	7, 2, 1	7, 2, 1	7, 2, 1	I_7, I_2, I_1	

355 $N = 355 = 5 \cdot 71$ (1 isogeny class) **355**

A1	0	1	1	5	-1	0	3	-	3, 1	3, 1	3, 1	I_3, I_1	3 : 2
A2	0	1	1	-95	-396	0	1	-	1, 3	1, 3	1, 1	I_1, I_3	3 : 1

356 $N = 356 = 2^2 \cdot 89$ (1 isogeny class) **356**

A1	0	-1	0	4	-8	1	1	-	8, 1	0, 1	3, 1	IV^*, I_1	
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357 $N = 357 = 3 \cdot 7 \cdot 17$ (4 isogeny classes) **357**

A1	0	-1	1	3565	72914	0	1	-	17, 4, 1	17, 4, 1	1, 2, 1	I_{17}, I_4, I_1	
B1	0	-1	1	-5	-16	1	1	-	1, 4, 1	1, 4, 1	1, 4, 1	I_1, I_4, I_1	
C1	0	1	1	20	-17	0	1	-	1, 2, 3	1, 2, 3	1, 2, 1	I_1, I_2, I_3	
D1	0	1	1	-42	110	1	1	-	7, 2, 1	7, 2, 1	7, 2, 1	I_7, I_2, I_1	

358 $N = 358 = 2 \cdot 179$ (2 isogeny classes) **358**

A1	1	1	0	55	197	0	1	-	17, 1	17, 1	1, 1	I_{17}, I_1	
B1	1	0	0	-18	28	0	3	-	3, 1	3, 1	3, 1	I_3, I_1	3 : 2
B2	1	0	0	32	150	0	1	-	1, 3	1, 3	1, 1	I_1, I_3	3 : 1

359 $N = 359 = 359$ (2 isogeny classes) **359**

A1	1	0	1	-23	39	1	1	+	1	1	1	I_1	
B1	1	-1	1	-7	8	1	1	+	1	1	1	I_1	

360 $N = 360 = 2^3 \cdot 3^2 \cdot 5$ (5 isogeny classes) **360**

A1	0	0	0	-138	-623	0	2	+	4, 8, 1	0, 2, 1	2, 2, 1	III, I_2^*, I_1	2 : 2
A2	0	0	0	-183	-182	0	4	+	8, 10, 2	0, 4, 2	2, 4, 2	I_1^*, I_4^*, I_2	2 : 1, 3, 4
A3	0	0	0	-1803	29302	0	4	+	10, 8, 4	0, 2, 4	2, 4, 2	III^*, I_2^*, I_4	2 : 2, 5, 6
A4	0	0	0	717	-1442	0	2	-	10, 14, 1	0, 8, 1	2, 4, 1	III^*, I_8^*, I_1	2 : 2
A5	0	0	0	-28803	1881502	0	2	+	11, 7, 2	0, 1, 2	1, 2, 2	II^*, I_1^*, I_2	2 : 3
A6	0	0	0	-723	64078	0	2	-	11, 7, 8	0, 1, 8	1, 4, 2	II^*, I_1^*, I_8	2 : 3
B1	0	0	0	-3	-18	0	2	-	10, 3, 1	0, 0, 1	2, 2, 1	III^*, III, I_1	2 : 2
B2	0	0	0	-123	-522	0	2	+	11, 3, 2	0, 0, 2	1, 2, 2	II^*, III, I_2	2 : 1
C1	0	0	0	-27	486	0	2	-	10, 9, 1	0, 0, 1	2, 2, 1	III^*, III^*, I_1	2 : 2
C2	0	0	0	-1107	14094	0	2	+	11, 9, 2	0, 0, 2	1, 2, 2	II^*, III^*, I_2	2 : 1
D1	0	0	0	33	34	0	4	-	8, 7, 1	0, 1, 1	4, 4, 1	I_1^*, I_1^*, I_1	2 : 2
D2	0	0	0	-147	286	0	4	+	10, 8, 2	0, 2, 2	2, 4, 2	III^*, I_2^*, I_2	2 : 1, 3, 4
D3	0	0	0	-1227	-16346	0	2	+	11, 7, 4	0, 1, 4	1, 2, 4	II^*, I_1^*, I_4	2 : 2
D4	0	0	0	-1947	33046	0	2	+	11, 10, 1	0, 4, 1	1, 4, 1	II^*, I_4^*, I_1	2 : 2
E1	0	0	0	-18	-27	1	2	+	4, 6, 1	0, 0, 1	2, 2, 1	III, I_0^*, I_1	2 : 2
E2	0	0	0	-63	162	1	4	+	8, 6, 2	0, 0, 2	4, 4, 2	I_1^*, I_0^*, I_2	2 : 1, 3, 4
E3	0	0	0	-963	11502	1	2	+	10, 6, 1	0, 0, 1	2, 2, 1	III^*, I_1^*, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
361	$N = 361 = 19^2$ (2 isogeny classes)											361	
A1	0	0	1	-38	90	1	1	-	3	0	2	III	19 : 2
A2	0	0	1	-13718	-619025	1	1	-	9	0	2	III*	19 : 1
B1	0	-1	1	241	-17	0	1	-	7	1	2	I ₁ *	3 : 2
B2	0	-1	1	-3369	81208	0	1	-	9	3	2	I ₃ *	3 : 1, 3
B3	0	-1	1	-277729	56427893	0	1	-	7	1	2	I ₁ *	3 : 2
362	$N = 362 = 2 \cdot 181$ (2 isogeny classes)											362	
A1	1	1	0	-4	2	1	1	-	1, 1	1, 1	1, 1	I ₁ , I ₁	
B1	1	1	1	6	7	1	1	-	7, 1	7, 1	7, 1	I ₇ , I ₁	
363	$N = 363 = 3 \cdot 11^2$ (3 isogeny classes)											363	
A1	1	1	1	-789	8130	0	4	+	3, 7	3, 1	1, 4	I ₃ , I ₁ *	2 : 2
A2	1	1	1	-1394	-6874	0	4	+	6, 8	6, 2	2, 4	I ₆ , I ₂ *	2 : 1, 3, 4
A3	1	1	1	-17729	-915100	0	2	+	3, 10	3, 4	1, 4	I ₃ , I ₄ *	2 : 2
A4	1	1	1	5261	-46804	0	2	-	12, 7	12, 1	2, 2	I ₁₂ , I ₁ *	2 : 2
B1	0	-1	1	4	-1	0	1	-	3, 2	3, 0	1, 1	I ₃ , II	
C1	0	-1	1	444	-826	0	1	-	3, 8	3, 0	1, 1	I ₃ , IV*	
364	$N = 364 = 2^2 \cdot 7 \cdot 13$ (2 isogeny classes)											364	
A1	0	0	0	-584	5444	1	1	-	8, 5, 1	0, 5, 1	3, 5, 1	IV*, I ₅ , I ₁	
B1	0	1	0	-5	7	1	1	-	8, 1, 1	0, 1, 1	3, 1, 1	IV*, I ₁ , I ₁	
366	$N = 366 = 2 \cdot 3 \cdot 61$ (7 isogeny classes)											366	
A1	1	0	0	-205	-1147	0	1	-	2, 2, 1	2, 2, 1	2, 2, 1	I ₂ , I ₂ , I ₁	
B1	1	0	0	-5	33	0	5	-	5, 5, 1	5, 5, 1	5, 5, 1	I ₅ , I ₅ , I ₁	5 : 2
B2	1	0	0	-515	-5697	0	1	-	1, 1, 5	1, 1, 5	1, 1, 5	I ₁ , I ₁ , I ₅	5 : 1
C1	1	0	1	-913	-10780	0	1	-	19, 3, 1	19, 3, 1	1, 3, 1	I ₁₉ , I ₃ , I ₁	
D1	1	1	1	-7096	-233095	0	1	-	7, 13, 1	7, 13, 1	7, 1, 1	I ₇ , I ₁₃ , I ₁	
E1	1	1	0	-1	-11	0	2	-	8, 1, 1	8, 1, 1	2, 1, 1	I ₈ , I ₁ , I ₁	2 : 2
E2	1	1	0	-81	-315	0	4	+	4, 2, 2	4, 2, 2	2, 2, 2	I ₄ , I ₂ , I ₂	2 : 1, 3, 4
E3	1	1	0	-1301	-18615	0	2	+	2, 4, 1	2, 4, 1	2, 2, 1	I ₂ , I ₄ , I ₁	2 : 2
E4	1	1	0	-141	129	0	4	+	2, 1, 4	2, 1, 4	2, 1, 4	I ₂ , I ₁ , I ₄	2 : 2
F1	1	0	1	-5	20	1	3	-	2, 6, 1	2, 6, 1	2, 6, 1	I ₂ , I ₆ , I ₁	3 : 2
F2	1	0	1	40	-538	1	1	-	6, 2, 3	6, 2, 3	2, 2, 3	I ₆ , I ₂ , I ₃	3 : 1
G1	1	1	1	-32	65	1	1	-	10, 2, 1	10, 2, 1	10, 2, 1	I ₁₀ , I ₂ , I ₁	
368	$N = 368 = 2^4 \cdot 23$ (7 isogeny classes)											368	
A1	0	0	0	5	-6	1	2	-	10, 1	0, 1	4, 1	I ₂ , I ₁	2 : 2
A2	0	0	0	-35	-62	1	2	+	11, 2	0, 2	4, 2	I ₃ , I ₂	2 : 1
B1	0	0	0	-163	930	0	2	-	22, 1	10, 1	4, 1	I ₁₄ , I ₁	2 : 2
B2	0	0	0	-2723	54690	0	2	+	17, 2	5, 2	2, 2	I ₉ , I ₂	2 : 1
C1	0	1	0	-4	-5	0	1	-	4, 1	0, 1	1, 1	II, I ₁	
D1	0	1	0	0	-1	1	1	-	4, 1	0, 1	1, 1	II, I ₁	
E1	0	-1	0	2	-1	1	1	-	4, 1	0, 1	1, 1	II, I ₁	3 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
368 368 $N = 368 = 2^4 \cdot 23$ (continued)													
F1	0	0	0	-1	-1	0	1	-	4, 1	0, 1	1, 1	II, I ₁	
G1	0	0	0	-55	157	1	1	-	4, 1	0, 1	1, 1	II, I ₁	
369 369 $N = 369 = 3^2 \cdot 41$ (2 isogeny classes)													
A1	0	0	1	6	13	1	1	-	7, 1	1, 1	2, 1	I ₁ [*] , I ₁	
B1	0	0	1	-93	-369	0	1	-	11, 1	5, 1	4, 1	I ₅ [*] , I ₁	5 : 2
B2	0	0	1	177	24201	0	1	-	7, 5	1, 5	4, 1	I ₁ [*] , I ₅	5 : 1
370 370 $N = 370 = 2 \cdot 5 \cdot 37$ (4 isogeny classes)													
A1	1	-1	0	-5	5	1	2	+	4, 1, 1	4, 1, 1	2, 1, 1	I ₄ , I ₁ , I ₁	2 : 2
A2	1	-1	0	-25	-39	1	4	+	2, 2, 2	2, 2, 2	2, 2, 2	I ₂ , I ₂ , I ₂	2 : 1, 3, 4
A3	1	-1	0	-395	-2925	1	2	+	1, 4, 1	1, 4, 1	1, 2, 1	I ₁ , I ₄ , I ₁	2 : 2
A4	1	-1	0	25	-209	1	2	-	1, 1, 4	1, 1, 4	1, 1, 2	I ₁ , I ₁ , I ₄	2 : 2
B1	1	1	0	13	-19	0	1	-	11, 1, 1	11, 1, 1	1, 1, 1	I ₁₁ , I ₁ , I ₁	
C1	1	0	1	-19	342	0	3	-	3, 3, 3	3, 3, 3	1, 1, 3	I ₃ , I ₃ , I ₃	3 : 2, 3
C2	1	0	1	166	-9204	0	1	-	9, 9, 1	9, 9, 1	1, 1, 1	I ₉ , I ₉ , I ₁	3 : 1
C3	1	0	1	-54	146	0	3	-	1, 1, 1	1, 1, 1	1, 1, 1	I ₁ , I ₁ , I ₁	3 : 1
D1	1	0	0	-75	-143	0	6	+	12, 3, 1	12, 3, 1	12, 3, 1	I ₁₂ , I ₃ , I ₁	2 : 2; 3 : 3
D2	1	0	0	245	-975	0	6	-	6, 6, 2	6, 6, 2	6, 6, 2	I ₆ , I ₆ , I ₂	2 : 1; 3 : 4
D3	1	0	0	-5275	-147903	0	2	+	4, 1, 3	4, 1, 3	4, 1, 3	I ₄ , I ₁ , I ₃	2 : 4; 3 : 1
D4	1	0	0	-5255	-149075	0	2	-	2, 2, 6	2, 2, 6	2, 2, 6	I ₂ , I ₂ , I ₆	2 : 3; 3 : 2
371 371 $N = 371 = 7 \cdot 53$ (2 isogeny classes)													
A1	1	1	0	-35	-98	1	1	-	4, 1	4, 1	2, 1	I ₄ , I ₁	
B1	0	0	1	-31	-67	0	1	-	3, 1	3, 1	3, 1	I ₃ , I ₁	
372 372 $N = 372 = 2^2 \cdot 3 \cdot 31$ (4 isogeny classes)													
A1	0	-1	0	-6	9	1	1	-	4, 2, 1	0, 2, 1	3, 2, 1	IV, I ₂ , I ₁	
B1	0	1	0	-9	12	0	2	-	4, 1, 2	0, 1, 2	1, 1, 2	IV, I ₁ , I ₂	2 : 2
B2	0	1	0	-164	756	0	2	+	8, 2, 1	0, 2, 1	1, 2, 1	IV [*] , I ₂ , I ₁	2 : 1
C1	0	1	0	-3054	-69327	0	3	-	4, 18, 1	0, 18, 1	3, 18, 1	IV, I ₁₈ , I ₁	3 : 2
C2	0	1	0	-250914	-48460347	0	1	-	4, 6, 3	0, 6, 3	1, 6, 3	IV, I ₆ , I ₃	3 : 1
D1	0	1	0	-2	9	1	1	-	4, 4, 1	0, 4, 1	3, 4, 1	IV, I ₄ , I ₁	
373 373 $N = 373 = 373$ (1 isogeny class)													
A1	0	1	1	-2	-2	1	1	+	1	1	1	I ₁	
374 374 $N = 374 = 2 \cdot 11 \cdot 17$ (1 isogeny class)													
A1	1	-1	0	-32	0	1	2	+	10, 2, 1	10, 2, 1	2, 2, 1	I ₁₀ , I ₂ , I ₁	2 : 2
A2	1	-1	0	128	-96	1	2	-	5, 4, 2	5, 4, 2	1, 2, 2	I ₅ , I ₄ , I ₂	2 : 1
377 377 $N = 377 = 13 \cdot 29$ (1 isogeny class)													
A1	1	-1	0	-8	11	1	2	+	1, 1	1, 1	1, 1	I ₁ , I ₁	2 : 2
A2	1	-1	0	-13	0	1	4	+	2, 2	2, 2	2, 2	I ₂ , I ₂	2 : 1, 3, 4
A3	1	-1	0	-158	-725	1	2	+	4, 1	4, 1	4, 1	I ₄ , I ₁	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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378 $N = 378 = 2 \cdot 3^3 \cdot 7$ (8 isogeny classes)**378**

A1	1	-1	1	10	5	0	3	-	9, 3, 1	9, 0, 1	9, 1, 1	I_9, II, I_1	3 : 2
A2	1	-1	1	-110	-539	0	3	-	3, 9, 3	3, 0, 3	3, 3, 3	I_3, IV^*, I_3	3 : 1, 3
A3	1	-1	1	-9560	-357371	0	1	-	1, 11, 1	1, 0, 1	1, 1, 1	I_1, II^*, I_1	3 : 2
B1	1	-1	0	-12	24	0	3	-	3, 3, 3	3, 0, 3	1, 1, 3	I_3, II, I_3	3 : 2, 3
B2	1	-1	0	93	-235	0	1	-	9, 9, 1	9, 0, 1	1, 1, 1	I_9, IV^*, I_1	3 : 1
B3	1	-1	0	-1062	13590	0	3	-	1, 5, 1	1, 0, 1	1, 3, 1	I_1, IV, I_1	3 : 1
C1	1	-1	1	-2	-107	0	1	-	2, 11, 1	2, 0, 1	2, 1, 1	I_2, II^*, I_1	
D1	1	-1	0	0	4	1	1	-	2, 5, 1	2, 0, 1	2, 3, 1	I_2, IV, I_1	
E1	1	-1	1	-11	-37	0	3	-	6, 3, 3	6, 0, 3	6, 1, 3	I_6, II, I_3	3 : 2, 3
E2	1	-1	1	-1271	-17117	0	1	-	2, 9, 1	2, 0, 1	2, 3, 1	I_2, IV^*, I_1	3 : 1
E3	1	-1	1	94	929	0	3	-	18, 5, 1	18, 0, 1	18, 1, 1	I_{18}, IV, I_1	3 : 1
F1	1	-1	0	-141	681	1	3	-	2, 3, 1	2, 0, 1	2, 1, 1	I_2, II, I_1	3 : 2
F2	1	-1	0	-96	1088	1	3	-	6, 9, 3	6, 0, 3	2, 3, 3	I_6, IV^*, I_3	3 : 1, 3
F3	1	-1	0	849	-25939	1	1	-	18, 11, 1	18, 0, 1	2, 1, 1	I_{18}, II^*, I_1	3 : 2
G1	1	-1	1	3967	38449	0	1	-	5, 11, 7	5, 0, 7	5, 1, 1	I_5, II^*, I_7	
H1	1	-1	0	441	-1571	0	1	-	5, 5, 7	5, 0, 7	1, 1, 1	I_5, IV, I_7	

380 $N = 380 = 2^2 \cdot 5 \cdot 19$ (2 isogeny classes)**380**

A1	0	0	0	-8	-3	1	2	+	4, 1, 2	0, 1, 2	1, 1, 2	IV, I_1, I_2	2 : 2
A2	0	0	0	-103	-402	1	2	+	8, 2, 1	0, 2, 1	1, 2, 1	IV^*, I_2, I_1	2 : 1
B1	0	-1	0	-921	10346	0	2	+	4, 5, 4	0, 5, 4	3, 1, 2	IV, I_5, I_4	2 : 2
B2	0	-1	0	884	44280	0	2	-	8, 10, 2	0, 10, 2	3, 2, 2	IV^*, I_{10}, I_2	2 : 1

381 $N = 381 = 3 \cdot 127$ (2 isogeny classes)**381**

A1	0	1	1	-11	-16	1	1	+	5, 1	5, 1	5, 1	I_5, I_1	
B1	0	1	1	-4	-5	0	1	+	1, 1	1, 1	1, 1	I_1, I_1	

384 $N = 384 = 2^7 \cdot 3$ (8 isogeny classes)**384**

A1	0	1	0	-3	-3	0	2	+	8, 1	0, 1	2, 1	III, I_1	2 : 2
A2	0	1	0	7	-9	0	2	-	13, 2	0, 2	2, 2	I_2^*, I_2	2 : 1
B1	0	-1	0	2	-2	0	2	-	7, 2	0, 2	1, 2	II, I_2	2 : 2
B2	0	-1	0	-13	-11	0	2	+	14, 1	0, 1	2, 1	III^*, I_1	2 : 1
C1	0	1	0	2	2	0	2	-	7, 2	0, 2	1, 2	II, I_2	2 : 2
C2	0	1	0	-13	11	0	2	+	14, 1	0, 1	2, 1	III^*, I_1	2 : 1
D1	0	-1	0	-3	3	1	2	+	8, 1	0, 1	2, 1	III, I_1	2 : 2
D2	0	-1	0	7	9	1	2	-	13, 2	0, 2	4, 2	I_2^*, I_2	2 : 1
E1	0	1	0	-6	-18	0	2	-	7, 6	0, 6	1, 6	II, I_6	2 : 2
E2	0	1	0	-141	-693	0	2	+	14, 3	0, 3	2, 3	III^*, I_3	2 : 1
F1	0	-1	0	-6	18	0	2	-	7, 6	0, 6	1, 2	II, I_6	2 : 2
F2	0	-1	0	-141	693	0	2	+	14, 3	0, 3	2, 1	III^*, I_3	2 : 1
G1	0	-1	0	-35	-69	0	2	+	8, 3	0, 3	2, 1	III, I_3	2 : 2
G2	0	-1	0	-25	-119	0	2	-	13, 6	0, 6	2, 2	I_2^*, I_6	2 : 1
H1	0	1	0	-35	69	1	2	+	8, 3	0, 3	2, 3	III, I_3	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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385 $N = 385 = 5 \cdot 7 \cdot 11$ (2 isogeny classes) **385**

A1	1	-1	1	-37	124	1	4	-	2, 1, 4	2, 1, 4	2, 1, 4	I_2, I_1, I_4	2 : 2
A2	1	-1	1	-642	6416	1	4	+	4, 2, 2	4, 2, 2	4, 2, 2	I_4, I_2, I_2	2 : 1, 3, 4
A3	1	-1	1	-697	5294	1	2	+	8, 4, 1	8, 4, 1	8, 2, 1	I_8, I_4, I_1	2 : 2
A4	1	-1	1	-10267	402966	1	2	+	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	2 : 2
B1	1	0	0	0	7	1	2	-	2, 1, 2	2, 1, 2	2, 1, 2	I_2, I_1, I_2	2 : 2
B2	1	0	0	-55	150	1	2	+	4, 2, 1	4, 2, 1	4, 2, 1	I_4, I_2, I_1	2 : 1

387 $N = 387 = 3^2 \cdot 43$ (5 isogeny classes) **387**

A1	0	0	1	-174	-887	0	1	-	10, 1	4, 1	2, 1	I_4^*, I_1	
B1	1	-1	0	-15	-46	1	1	-	9, 1	0, 1	2, 1	III^*, I_1	
C1	1	-1	1	-2	2	1	1	-	3, 1	0, 1	2, 1	III, I_1	
D1	1	-1	1	-221	1316	0	4	+	9, 1	3, 1	4, 1	I_3^*, I_1	2 : 2
D2	1	-1	1	-266	776	0	4	+	12, 2	6, 2	4, 2	I_6^*, I_2	2 : 1, 3, 4
D3	1	-1	1	-2201	-38698	0	2	+	18, 1	12, 1	4, 1	I_{12}^*, I_1	2 : 2
D4	1	-1	1	949	5150	0	2	-	9, 4	3, 4	2, 2	I_3^*, I_4	2 : 2
E1	0	0	1	-3	-9	0	1	-	6, 1	0, 1	2, 1	I_0^*, I_1	

389 $N = 389 = 389$ (1 isogeny class) **389**

A1	0	1	1	-2	0	2	1	+	1	1	1	I_1	
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390 $N = 390 = 2 \cdot 3 \cdot 5 \cdot 13$ (7 isogeny classes) **390**

A1	1	1	0	-13	13	1	2	+	4, 2, 1, 1	4, 2, 1, 1	2, 2, 1, 1	I_4, I_2, I_1, I_1	2 : 2
A2	1	1	0	-33	-63	1	4	+	2, 4, 2, 2	2, 4, 2, 2	2, 2, 2, 2	I_2, I_4, I_2, I_2	2 : 1, 3, 4
A3	1	1	0	-483	-4293	1	2	+	1, 2, 1, 4	1, 2, 1, 4	1, 2, 1, 2	I_1, I_2, I_1, I_4	2 : 2
A4	1	1	0	97	-297	1	2	-	1, 8, 4, 1	1, 8, 4, 1	1, 2, 2, 1	I_1, I_8, I_4, I_1	2 : 2
B1	1	1	1	15	15	0	4	-	8, 1, 2, 1	8, 1, 2, 1	8, 1, 2, 1	I_8, I_1, I_2, I_1	2 : 2
B2	1	1	1	-65	47	0	8	+	4, 2, 4, 2	4, 2, 4, 2	4, 2, 4, 2	I_4, I_2, I_4, I_2	2 : 1, 3, 4
B3	1	1	1	-565	-5353	0	4	+	2, 4, 2, 4	2, 4, 2, 4	2, 2, 2, 4	I_2, I_4, I_2, I_4	2 : 2, 5, 6
B4	1	1	1	-845	9095	0	4	+	2, 1, 8, 1	2, 1, 8, 1	2, 1, 8, 1	I_2, I_1, I_8, I_1	2 : 2
B5	1	1	1	-9015	-333213	0	2	+	1, 8, 1, 2	1, 8, 1, 2	1, 2, 1, 2	I_1, I_8, I_1, I_2	2 : 3
B6	1	1	1	-115	-13093	0	2	-	1, 2, 1, 8	1, 2, 1, 8	1, 2, 1, 8	I_1, I_2, I_1, I_8	2 : 3
C1	1	0	0	-6	36	0	6	-	6, 3, 2, 1	6, 3, 2, 1	6, 3, 2, 1	I_6, I_3, I_2, I_1	2 : 2; 3 : 3
C2	1	0	0	-206	1116	0	6	+	3, 6, 1, 2	3, 6, 1, 2	3, 6, 1, 2	I_3, I_6, I_1, I_2	2 : 1; 3 : 4
C3	1	0	0	54	-960	0	2	-	2, 1, 6, 3	2, 1, 6, 3	2, 1, 2, 3	I_2, I_1, I_6, I_3	2 : 4; 3 : 1
C4	1	0	0	-1196	-15210	0	2	+	1, 2, 3, 6	1, 2, 3, 6	1, 2, 1, 6	I_1, I_2, I_3, I_6	2 : 3; 3 : 2
D1	1	0	1	3997	3998	0	6	-	10, 9, 6, 1	10, 9, 6, 1	2, 9, 6, 1	I_{10}, I_9, I_6, I_1	2 : 2; 3 : 3
D2	1	0	1	-16003	27998	0	6	+	5, 18, 3, 2	5, 18, 3, 2	1, 18, 3, 2	I_5, I_{18}, I_3, I_2	2 : 1; 3 : 4
D3	1	0	1	-53378	-5124652	0	2	-	30, 3, 2, 3	30, 3, 2, 3	2, 3, 2, 3	I_{30}, I_3, I_2, I_3	2 : 4; 3 : 1
D4	1	0	1	-872578	-313799212	0	2	+	15, 6, 1, 6	15, 6, 1, 6	1, 6, 1, 6	I_{15}, I_6, I_1, I_6	2 : 3; 3 : 2
E1	1	1	1	4	-7	0	2	-	2, 3, 2, 1	2, 3, 2, 1	2, 1, 2, 1	I_2, I_3, I_2, I_1	2 : 2
E2	1	1	1	-46	-127	0	2	+	1, 6, 1, 2	1, 6, 1, 2	1, 2, 1, 2	I_1, I_6, I_1, I_2	2 : 1
F1	1	1	0	-52	-176	0	2	-	10, 1, 2, 1	10, 1, 2, 1	2, 1, 2, 1	I_{10}, I_1, I_2, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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390 **390**
 $N = 390 = 2 \cdot 3 \cdot 5 \cdot 13$ (continued)

G1	1	0	1	-289	3092	0	2	-	20, 1, 1, 2	20, 1, 1, 2	2, 1, 1, 2	I_{20}, I_1, I_1, I_2	2 : 2
G2	1	0	1	-5409	152596	0	4	+	10, 2, 2, 4	10, 2, 2, 4	2, 2, 2, 2	I_{10}, I_2, I_2, I_4	2 : 1, 3, 4
G3	1	0	1	-6209	104276	0	2	+	5, 4, 1, 8	5, 4, 1, 8	1, 4, 1, 2	I_5, I_4, I_1, I_8	2 : 2
G4	1	0	1	-86529	9789652	0	2	+	5, 1, 4, 2	5, 1, 4, 2	1, 1, 2, 2	I_5, I_1, I_4, I_2	2 : 2

392 **392**
 $N = 392 = 2^3 \cdot 7^2$ (6 isogeny classes)

A1	0	0	0	49	-686	1	4	-	8, 7	0, 1	4, 4	I_1^*, I_1^*	2 : 2
A2	0	0	0	-931	-10290	1	4	+	10, 8	0, 2	2, 4	III^*, I_2^*	2 : 1, 3, 4
A3	0	0	0	-14651	-682570	1	2	+	11, 7	0, 1	1, 2	II^*, I_1^*	2 : 2
A4	0	0	0	-2891	47334	1	2	+	11, 10	0, 4	1, 4	II^*, I_4^*	2 : 2
B1	0	1	0	-800	-8359	0	1	+	4, 10	0, 0	2, 1	III, II^*	
C1	0	-1	0	-16	29	1	1	+	4, 4	0, 0	2, 3	III, IV	
D1	0	1	0	-16	1392	0	2	-	10, 7	0, 1	2, 2	III^*, I_1^*	2 : 2
D2	0	1	0	-1976	32752	0	2	+	11, 8	0, 2	1, 4	II^*, I_2^*	2 : 1
E1	0	0	0	-343	-2401	0	1	+	4, 8	0, 0	2, 1	III, IV^*	
F1	0	0	0	-7	7	1	1	+	4, 2	0, 0	2, 1	III, II	

395 **395**
 $N = 395 = 5 \cdot 79$ (3 isogeny classes)

A1	1	-1	1	-7	14	0	4	-	4, 1	4, 1	4, 1	I_4, I_1	2 : 2
A2	1	-1	1	-132	614	0	4	+	2, 2	2, 2	2, 2	I_2, I_2	2 : 1, 3, 4
A3	1	-1	1	-157	384	0	2	+	1, 4	1, 4	1, 2	I_1, I_4	2 : 2
A4	1	-1	1	-2107	37744	0	2	+	1, 1	1, 1	1, 1	I_1, I_1	2 : 2
B1	1	1	1	-40	-128	0	2	-	6, 1	6, 1	6, 1	I_6, I_1	2 : 2
B2	1	1	1	-665	-6878	0	2	+	3, 2	3, 2	3, 2	I_3, I_2	2 : 1
C1	0	-1	1	-50	156	0	5	-	5, 1	5, 1	5, 1	I_5, I_1	5 : 2
C2	0	-1	1	300	-5724	0	1	-	1, 5	1, 5	1, 1	I_1, I_5	5 : 1

396 **396**
 $N = 396 = 2^2 \cdot 3^2 \cdot 11$ (3 isogeny classes)

A1	0	0	0	-696	-8215	0	2	-	4, 16, 1	0, 10, 1	3, 4, 1	IV, I_{10}^*, I_1	2 : 2
A2	0	0	0	-11631	-482794	0	2	+	8, 11, 2	0, 5, 2	3, 2, 2	IV^*, I_5^*, I_2	2 : 1
B1	0	0	0	24	25	1	2	-	4, 8, 1	0, 2, 1	3, 4, 1	IV, I_2^*, I_1	2 : 2
B2	0	0	0	-111	214	1	2	+	8, 7, 2	0, 1, 2	3, 4, 2	IV^*, I_1^*, I_2	2 : 1
C1	0	0	0	24	52	0	1	-	8, 6, 1	0, 0, 1	1, 1, 1	IV^*, I_0^*, I_1	3 : 2
C2	0	0	0	-696	7108	0	3	-	8, 6, 3	0, 0, 3	3, 1, 3	IV^*, I_0^*, I_3	3 : 1

398 **398**
 $N = 398 = 2 \cdot 199$ (1 isogeny class)

A1	1	1	0	-6	20	0	2	-	10, 1	10, 1	2, 1	I_{10}, I_1	2 : 2
A2	1	1	0	-166	756	0	2	+	5, 2	5, 2	1, 2	I_5, I_2	2 : 1

399 **399**
 $N = 399 = 3 \cdot 7 \cdot 19$ (3 isogeny classes)

A1	1	1	0	-210	-441	1	2	+	5, 6, 1	5, 6, 1	1, 2, 1	I_5, I_6, I_1	2 : 2
A2	1	1	0	-1925	31458	1	2	+	10, 3, 2	10, 3, 2	2, 1, 2	I_{10}, I_3, I_2	2 : 1
B1	1	1	1	-13	-22	1	2	+	3, 2, 1	3, 2, 1	1, 2, 1	I_3, I_2, I_1	2 : 2
B2	1	1	1	-48	90	1	2	+	6, 1, 2	6, 1, 2	2, 1, 2	I_6, I_1, I_2	2 : 1
C1	1	0	0	-431	3408	0	2	+	1, 2, 3	1, 2, 3	1, 2, 1	I_1, I_2, I_3	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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400 $N = 400 = 2^4 \cdot 5^2$ (8 isogeny classes) **400**

A1	0	0	0	-50	-125	1	2	+	4, 7	0, 1	1, 4	II, I ₁ *	2 : 2
A2	0	0	0	-175	750	1	4	+	8, 8	0, 2	2, 4	I ₀ *, I ₂ *	2 : 1, 3, 4
A3	0	0	0	-2675	53250	1	4	+	10, 7	0, 1	4, 4	I ₂ *, I ₁ *	2 : 2
A4	0	0	0	325	4250	1	2	-	10, 10	0, 4	2, 4	I ₂ *, I ₄ *	2 : 2
B1	0	1	0	-48	-172	0	1	-	17, 2	5, 0	2, 1	I ₉ *, II	3 : 2; 5 : 3
B2	0	1	0	352	1268	0	1	-	27, 2	15, 0	2, 1	I ₁₉ *, II	3 : 1; 5 : 4
B3	0	1	0	-208	13588	0	1	-	13, 10	1, 0	2, 1	I ₅ *, II*	3 : 4; 5 : 1
B4	0	1	0	-50208	4313588	0	1	-	15, 10	3, 0	2, 1	I ₇ *, II*	3 : 3; 5 : 2
C1	0	-1	0	-8	112	1	1	-	13, 4	1, 0	4, 3	I ₅ *, IV	3 : 2; 5 : 3
C2	0	-1	0	-2008	35312	1	1	-	15, 4	3, 0	4, 1	I ₇ *, IV	3 : 1; 5 : 4
C3	0	-1	0	-1208	-19088	1	1	-	17, 8	5, 0	4, 3	I ₉ *, IV*	3 : 4; 5 : 1
C4	0	-1	0	8792	140912	1	1	-	27, 8	15, 0	4, 1	I ₁₉ *, IV*	3 : 3; 5 : 2
D1	0	-1	0	-3	2	0	2	+	4, 3	0, 0	1, 2	II, III	2 : 2
D2	0	-1	0	-28	-48	0	2	+	8, 3	0, 0	2, 2	I ₀ *, III	2 : 1
E1	0	1	0	-33	-62	0	2	+	4, 7	0, 1	1, 2	II, I ₁ *	2 : 2; 3 : 3
E2	0	1	0	92	-312	0	2	-	8, 8	0, 2	1, 4	I ₀ *, I ₂ *	2 : 1; 3 : 4
E3	0	1	0	-1033	12438	0	2	+	4, 9	0, 3	1, 2	II, I ₃ *	2 : 4; 3 : 1
E4	0	1	0	-908	15688	0	2	-	8, 12	0, 6	1, 4	I ₀ *, I ₆ *	2 : 3; 3 : 2
F1	0	1	0	-83	88	0	2	+	4, 9	0, 0	1, 2	II, III*	2 : 2
F2	0	1	0	-708	-7412	0	2	+	8, 9	0, 0	2, 2	I ₀ *, III*	2 : 1
G1	0	0	0	125	1250	0	1	-	11, 8	0, 0	2, 1	I ₃ *, IV*	
H1	0	0	0	5	10	1	1	-	11, 2	0, 0	4, 1	I ₃ *, II	

402 $N = 402 = 2 \cdot 3 \cdot 67$ (4 isogeny classes) **402**

A1	1	1	0	-2	-12	1	1	-	8, 1, 1	8, 1, 1	2, 1, 1	I ₈ , I ₁ , I ₁	
B1	1	0	1	-10	-4	0	2	+	8, 1, 1	8, 1, 1	2, 1, 1	I ₈ , I ₁ , I ₁	2 : 2
B2	1	0	1	-90	316	0	4	+	4, 2, 2	4, 2, 2	2, 2, 2	I ₄ , I ₂ , I ₂	2 : 1, 3, 4
B3	1	0	1	-1430	20684	0	4	+	2, 4, 1	2, 4, 1	2, 4, 1	I ₂ , I ₄ , I ₁	2 : 2
B4	1	0	1	-30	748	0	2	-	2, 1, 4	2, 1, 4	2, 1, 2	I ₂ , I ₁ , I ₄	2 : 2
C1	1	1	1	-37	71	0	2	+	2, 3, 1	2, 3, 1	2, 1, 1	I ₂ , I ₃ , I ₁	2 : 2
C2	1	1	1	-27	123	0	2	-	1, 6, 2	1, 6, 2	1, 2, 2	I ₁ , I ₆ , I ₂	2 : 1
D1	1	0	1	-145	692	1	3	-	4, 9, 1	4, 9, 1	2, 9, 1	I ₄ , I ₉ , I ₁	3 : 2
D2	1	0	1	800	1070	1	3	-	12, 3, 3	12, 3, 3	2, 3, 3	I ₁₂ , I ₃ , I ₃	3 : 1, 3
D3	1	0	1	-10255	-438718	1	1	-	36, 1, 1	36, 1, 1	2, 1, 1	I ₃₆ , I ₁ , I ₁	3 : 2

404 $N = 404 = 2^2 \cdot 101$ (2 isogeny classes) **404**

A1	0	0	0	-8	4	1	1	+	8, 1	0, 1	3, 1	IV*, I ₁	
B1	0	1	0	-69	199	0	3	+	8, 1	0, 1	3, 1	IV*, I ₁	3 : 2
B2	0	1	0	-229	-1161	0	1	+	8, 3	0, 3	1, 1	IV*, I ₃	3 : 1

405 $N = 405 = 3^4 \cdot 5$ (6 isogeny classes) **405**

A1	0	0	1	-12	15	0	3	+	4, 3	0, 3	1, 3	II, I ₃	3 : 2
A2	0	0	1	-162	-790	0	1	+	12, 1	0, 1	1, 1	II*, I ₁	3 : 1
B1	0	0	1	-18	29	1	3	+	6, 1	0, 1	3, 1	IV, I ₁	3 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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405 $N = 405 = 3^4 \cdot 5$ (continued)**405**

C1	1	-1	0	0	1	1	1	-	4, 1	0, 1	1, 1	II, I ₁	7 : 2
C2	1	-1	0	-225	-1250	1	1	-	4, 7	0, 7	1, 1	II, I ₇	7 : 1
D1	1	-1	1	-2	-26	1	1	-	10, 1	0, 1	3, 1	IV*, I ₁	7 : 2
D2	1	-1	1	-2027	35776	1	1	-	10, 7	0, 7	3, 7	IV*, I ₇	7 : 1
E1	0	0	1	-27	47	0	1	+	10, 1	0, 1	1, 1	IV*, I ₁	
F1	0	0	1	-3	-2	1	1	+	4, 1	0, 1	1, 1	II, I ₁	

406 $N = 406 = 2 \cdot 7 \cdot 29$ (4 isogeny classes)**406**

A1	1	-1	0	-302	2260	1	2	-	10, 3, 2	10, 3, 2	2, 1, 2	I ₁₀ , I ₃ , I ₂	2 : 2
A2	1	-1	0	-4942	134964	1	2	+	5, 6, 1	5, 6, 1	1, 2, 1	I ₅ , I ₆ , I ₁	2 : 1
B1	1	0	1	-15	210	1	3	-	4, 2, 3	4, 2, 3	2, 2, 3	I ₄ , I ₂ , I ₃	3 : 2
B2	1	0	1	130	-5648	1	1	-	12, 6, 1	12, 6, 1	2, 6, 1	I ₁₂ , I ₆ , I ₁	3 : 1
C1	1	1	1	-102	355	1	1	-	8, 2, 1	8, 2, 1	8, 2, 1	I ₈ , I ₂ , I ₁	
D1	1	1	0	-2124	-60592	0	2	-	16, 5, 2	16, 5, 2	2, 5, 2	I ₁₆ , I ₅ , I ₂	2 : 2
D2	1	1	0	-39244	-3007920	0	2	+	8, 10, 1	8, 10, 1	2, 10, 1	I ₈ , I ₁₀ , I ₁	2 : 1

408 $N = 408 = 2^3 \cdot 3 \cdot 17$ (4 isogeny classes)**408**

A1	0	1	0	-48	-144	0	2	+	10, 2, 1	0, 2, 1	2, 2, 1	III*, I ₂ , I ₁	2 : 2
A2	0	1	0	-8	-336	0	2	-	11, 4, 2	0, 4, 2	1, 4, 2	II*, I ₄ , I ₂	2 : 1
B1	0	1	0	-52	128	0	4	+	8, 2, 1	0, 2, 1	4, 2, 1	I ₁ *, I ₂ , I ₁	2 : 2
B2	0	1	0	-72	0	0	4	+	10, 4, 2	0, 4, 2	2, 4, 2	III*, I ₄ , I ₂	2 : 1, 3, 4
B3	0	1	0	-752	-8160	0	2	+	11, 8, 1	0, 8, 1	1, 8, 1	II*, I ₈ , I ₁	2 : 2
B4	0	1	0	288	288	0	2	-	11, 2, 4	0, 2, 4	1, 2, 4	II*, I ₂ , I ₄	2 : 2
C1	0	-1	0	511	-1899	0	1	-	8, 3, 5	0, 3, 5	2, 1, 1	I ₁ *, I ₃ , I ₅	
D1	0	1	0	-17	51	1	1	-	8, 5, 1	0, 5, 1	4, 5, 1	I ₁ *, I ₅ , I ₁	

410 $N = 410 = 2 \cdot 5 \cdot 41$ (4 isogeny classes)**410**

A1	1	-1	0	-14	20	1	2	+	6, 2, 1	6, 2, 1	2, 2, 1	I ₆ , I ₂ , I ₁	2 : 2
A2	1	-1	0	-214	1260	1	2	+	3, 1, 2	3, 1, 2	1, 1, 2	I ₃ , I ₁ , I ₂	2 : 1
B1	1	-1	1	-1387	-18501	0	4	+	24, 2, 1	24, 2, 1	24, 2, 1	I ₂₄ , I ₂ , I ₁	2 : 2
B2	1	-1	1	-21867	-1239109	0	4	+	12, 4, 2	12, 4, 2	12, 4, 2	I ₁₂ , I ₄ , I ₂	2 : 1, 3, 4
B3	1	-1	1	-349867	-79565509	0	2	+	6, 2, 1	6, 2, 1	6, 2, 1	I ₆ , I ₂ , I ₁	2 : 2
B4	1	-1	1	-21547	-1277381	0	4	-	6, 8, 4	6, 8, 4	6, 8, 4	I ₆ , I ₈ , I ₄	2 : 2
C1	1	0	1	-168	806	0	6	+	4, 6, 1	4, 6, 1	2, 6, 1	I ₄ , I ₆ , I ₁	2 : 2; 3 : 3
C2	1	0	1	-2668	52806	0	6	+	2, 3, 2	2, 3, 2	2, 3, 2	I ₂ , I ₃ , I ₂	2 : 1; 3 : 4
C3	1	0	1	-1543	-23094	0	2	+	12, 2, 3	12, 2, 3	2, 2, 1	I ₁₂ , I ₂ , I ₃	2 : 4; 3 : 1
C4	1	0	1	-3143	32586	0	2	+	6, 1, 6	6, 1, 6	2, 1, 2	I ₆ , I ₁ , I ₆	2 : 3; 3 : 2
D1	1	0	0	-16	0	1	2	+	8, 2, 1	8, 2, 1	8, 2, 1	I ₈ , I ₂ , I ₁	2 : 2
D2	1	0	0	64	16	1	2	-	4, 4, 2	4, 4, 2	4, 2, 2	I ₄ , I ₄ , I ₂	2 : 1

414 $N = 414 = 2 \cdot 3^2 \cdot 23$ (4 isogeny classes)**414**

A1	1	-1	1	-320	-2221	0	2	-	4, 12, 1	4, 6, 1	4, 4, 1	I ₄ , I ₆ *, I ₁	2 : 2; 3 : 3
A2	1	-1	1	-5180	-142189	0	2	+	2, 9, 2	2, 3, 2	2, 4, 2	I ₂ , I ₃ *, I ₂	2 : 1; 3 : 4
A3	1	-1	1	1705	-5137	0	6	-	12, 8, 3	12, 2, 3	12, 4, 3	I ₁₂ , I ₃ *, I ₂	2 : 4; 3 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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414 $N = 414 = 2 \cdot 3^2 \cdot 23$ (continued) **414**

B1	1	-1	1	-14	-39	0	2	-	2, 8, 1	2, 2, 1	2, 4, 1	I_2, I_2^*, I_1	2 : 2
B2	1	-1	1	-284	-1767	0	2	+	1, 7, 2	1, 1, 2	1, 4, 2	I_1, I_1^*, I_2	2 : 1
C1	1	-1	0	27	-59	1	2	-	4, 8, 1	4, 2, 1	2, 4, 1	I_4, I_2^*, I_1	2 : 2
C2	1	-1	0	-153	-455	1	4	+	2, 10, 2	2, 4, 2	2, 4, 2	I_2, I_4^*, I_2	2 : 1, 3, 4
C3	1	-1	0	-2223	-39785	1	2	+	1, 14, 1	1, 8, 1	1, 4, 1	I_1, I_8^*, I_1	2 : 2
C4	1	-1	0	-963	11371	1	2	+	1, 8, 4	1, 2, 4	1, 2, 4	I_1, I_2^*, I_4	2 : 2
D1	1	-1	1	-92	415	1	2	-	10, 6, 1	10, 0, 1	10, 4, 1	I_{10}, I_0^*, I_1	2 : 2
D2	1	-1	1	-1532	23455	1	2	+	5, 6, 2	5, 0, 2	5, 2, 2	I_5, I_0^*, I_2	2 : 1

415 $N = 415 = 5 \cdot 83$ (1 isogeny class) **415**

A1	1	-1	0	-109	-412	0	1	-	4, 1	4, 1	4, 1	I_4, I_1	
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416 $N = 416 = 2^5 \cdot 13$ (2 isogeny classes) **416**

A1	0	1	0	0	-4	0	1	-	9, 1	0, 1	1, 1	I_0^*, I_1	
B1	0	-1	0	0	4	1	1	-	9, 1	0, 1	2, 1	I_0^*, I_1	

417 $N = 417 = 3 \cdot 139$ (1 isogeny class) **417**

A1	1	1	0	26	73	0	1	-	9, 1	9, 1	1, 1	I_9, I_1	
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418 $N = 418 = 2 \cdot 11 \cdot 19$ (3 isogeny classes) **418**

A1	1	-1	1	-4	3	0	2	+	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	2 : 2
A2	1	-1	1	6	11	0	2	-	1, 2, 2	1, 2, 2	1, 2, 2	I_1, I_2, I_2	2 : 1
B1	1	1	1	66	-5	1	1	-	13, 2, 1	13, 2, 1	13, 2, 1	I_{13}, I_2, I_1	
C1	1	-1	1	-6	-5	0	1	-	1, 2, 1	1, 2, 1	1, 2, 1	I_1, I_2, I_1	

420 $N = 420 = 2^2 \cdot 3 \cdot 5 \cdot 7$ (4 isogeny classes) **420**

A1	0	-1	0	-4061	67590	0	2	+	4, 7, 10, 1	0, 7, 10, 1	3, 1, 2, 1	IV, I_7, I_{10}, I_1	2 : 2
A2	0	-1	0	11564	448840	0	2	-	8, 14, 5, 2	0, 14, 5, 2	3, 2, 1, 2	IV^*, I_{14}, I_5, I_2	2 : 1
B1	0	-1	0	-565	5362	0	2	+	4, 5, 2, 1	0, 5, 2, 1	1, 1, 2, 1	IV, I_5, I_2, I_1	2 : 2
B2	0	-1	0	-540	5832	0	2	-	8, 10, 1, 2	0, 10, 1, 2	1, 2, 1, 2	IV^*, I_{10}, I_1, I_2	2 : 1
C1	0	1	0	-61	164	0	6	+	4, 3, 2, 1	0, 3, 2, 1	3, 3, 2, 1	IV, I_3, I_2, I_1	2 : 2; 3 : 3
C2	0	1	0	-36	324	0	6	-	8, 6, 1, 2	0, 6, 1, 2	3, 6, 1, 2	IV^*, I_6, I_1, I_2	2 : 1; 3 : 4
C3	0	1	0	-301	-1960	0	2	+	4, 1, 6, 3	0, 1, 6, 3	1, 1, 2, 3	IV, I_1, I_6, I_3	2 : 4; 3 : 1
C4	0	1	0	324	-8460	0	2	-	8, 2, 3, 6	0, 2, 3, 6	1, 2, 1, 6	IV^*, I_2, I_3, I_6	2 : 3; 3 : 2
D1	0	1	0	-5	0	0	2	+	4, 1, 2, 1	0, 1, 2, 1	1, 1, 2, 1	IV, I_1, I_2, I_1	2 : 2
D2	0	1	0	20	20	0	2	-	8, 2, 1, 2	0, 2, 1, 2	1, 2, 1, 2	IV^*, I_2, I_1, I_2	2 : 1

422 $N = 422 = 2 \cdot 211$ (1 isogeny class) **422**

A1	1	-1	0	1	-3	1	1	-	4, 1	4, 1	2, 1	I_4, I_1	
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423 $N = 423 = 3^2 \cdot 47$ (7 isogeny classes) **423**

A1	0	0	1	-12	4	1	1	+	7, 1	1, 1	4, 1	I_1^*, I_1	
B1	1	-1	0	-72	355	0	2	-	12, 1	6, 1	4, 1	I_1^*, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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423 $N = 423 = 3^2 \cdot 47$ (continued)**423**

C1	1	-1	0	-18	-81	1	2	-	10, 1	4, 1	4, 1	I_4^*, I_1	2 : 2
C2	1	-1	0	-423	-3240	1	4	+	8, 2	2, 2	4, 2	I_2^*, I_2	2 : 1, 3, 4
C3	1	-1	0	-6768	-212625	1	2	+	7, 1	1, 1	2, 1	I_1^*, I_1	2 : 2
C4	1	-1	0	-558	-891	1	4	+	7, 4	1, 4	4, 4	I_1^*, I_4	2 : 2
D1	0	0	1	-81	-277	0	1	+	9, 1	0, 1	2, 1	III^*, I_1	
E1	0	0	1	-111	-171	0	1	+	13, 1	7, 1	2, 1	I_7^*, I_1	
F1	0	0	1	-237	1404	1	1	+	7, 1	1, 1	2, 1	I_1^*, I_1	
G1	0	0	1	-9	10	1	1	+	3, 1	0, 1	2, 1	III, I_1	

425 $N = 425 = 5^2 \cdot 17$ (4 isogeny classes)**425**

A1	1	-1	0	-17	16	1	2	+	6, 1	0, 1	2, 1	I_0^*, I_1	2 : 2
A2	1	-1	0	-142	-609	1	4	+	6, 2	0, 2	4, 2	I_0^*, I_2	2 : 1, 3, 4
A3	1	-1	0	-2267	-40984	1	2	+	6, 1	0, 1	2, 1	I_0^*, I_1	2 : 2
A4	1	-1	0	-17	-1734	1	2	-	6, 4	0, 4	4, 2	I_0^*, I_4	2 : 2
B1	1	1	0	-75	250	1	1	-	8, 1	0, 1	3, 1	IV^*, I_1	
C1	1	0	0	-3	2	1	1	-	2, 1	0, 1	1, 1	II, I_1	
D1	1	0	0	-213	-1208	1	2	+	8, 1	2, 1	2, 1	I_2^*, I_1	2 : 2
D2	1	0	0	-88	-2583	1	2	-	10, 2	4, 2	4, 2	I_4^*, I_2	2 : 1

426 $N = 426 = 2 \cdot 3 \cdot 71$ (3 isogeny classes)**426**

A1	1	0	0	-20	48	0	5	-	5, 5, 1	5, 5, 1	5, 5, 1	I_5, I_5, I_1	5 : 2
A2	1	0	0	-230	-5202	0	1	-	1, 1, 5	1, 1, 5	1, 1, 5	I_1, I_1, I_5	5 : 1
B1	1	1	0	-286	1780	1	2	-	10, 6, 1	10, 6, 1	2, 2, 1	I_{10}, I_6, I_1	2 : 2
B2	1	1	0	-4606	118420	1	2	+	5, 3, 2	5, 3, 2	1, 1, 2	I_5, I_3, I_2	2 : 1
C1	1	0	1	-23007	1341682	0	3	-	9, 15, 1	9, 15, 1	1, 15, 1	I_9, I_{15}, I_1	3 : 2
C2	1	0	1	14658	5154352	0	1	-	27, 5, 3	27, 5, 3	1, 5, 1	I_{27}, I_5, I_3	3 : 1

427 $N = 427 = 7 \cdot 61$ (3 isogeny classes)**427**

A1	0	-1	1	-1	-1	0	1	-	1, 1	1, 1	1, 1	I_1, I_1	
B1	1	0	1	-8	7	1	1	+	1, 1	1, 1	1, 1	I_1, I_1	
C1	1	0	0	-28	-59	1	1	+	3, 1	3, 1	1, 1	I_3, I_1	

428 $N = 428 = 2^2 \cdot 107$ (2 isogeny classes)**428**

A1	0	1	0	-157	-812	0	1	-	4, 1	0, 1	3, 1	IV, I_1	
B1	0	-1	0	3	-2	1	1	-	4, 1	0, 1	3, 1	IV, I_1	

429 $N = 429 = 3 \cdot 11 \cdot 13$ (2 isogeny classes)**429**

A1	1	1	1	2	2	1	2	-	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	2 : 2
A2	1	1	1	-13	8	1	2	+	1, 2, 2	1, 2, 2	1, 2, 2	I_1, I_2, I_2	2 : 1
B1	1	0	0	-24	63	1	4	-	8, 1, 1	8, 1, 1	8, 1, 1	I_8, I_1, I_1	2 : 2
B2	1	0	0	-429	3384	1	8	+	4, 2, 2	4, 2, 2	4, 2, 2	I_4, I_2, I_2	2 : 1, 3, 4
B3	1	0	0	-474	2619	1	4	+	2, 4, 4	2, 4, 4	2, 2, 4	I_2, I_4, I_4	2 : 2, 5, 6
B4	1	0	0	-6864	218313	1	4	+	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	2 : 2
B5	1	0	0	-3009	-61770	1	2	+	1, 8, 2	1, 8, 2	1, 2, 2	I_1, I_8, I_2	2 : 3

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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430 $N = 430 = 2 \cdot 5 \cdot 43$ (4 isogeny classes) **430**

A1	1	-1	0	-20	40	1	1	-	3, 1, 1	3, 1, 1	1, 1, 1	I_3, I_1, I_1	
B1	1	-1	0	16	-10	1	1	-	1, 5, 1	1, 5, 1	1, 5, 1	I_1, I_5, I_1	
C1	1	0	0	4	16	1	3	-	9, 1, 1	9, 1, 1	9, 1, 1	I_9, I_1, I_1	3 : 2
C2	1	0	0	-36	-440	1	3	-	3, 3, 3	3, 3, 3	3, 1, 3	I_3, I_3, I_3	3 : 1, 3
C3	1	0	0	-5626	-162894	1	1	-	1, 9, 1	1, 9, 1	1, 1, 1	I_1, I_9, I_1	3 : 2
D1	1	0	0	-1415	20617	1	1	-	15, 5, 1	15, 5, 1	15, 5, 1	I_{15}, I_5, I_1	

431 $N = 431 = 431$ (2 isogeny classes) **431**

A1	1	0	0	0	-1	1	1	-	1	1	1	I_1	
B1	1	-1	1	-9	-8	0	1	-	1	1	1	I_1	

432 $N = 432 = 2^4 \cdot 3^3$ (8 isogeny classes) **432**

A1	0	0	0	0	-16	0	1	-	12, 3	0, 0	1, 1	II^*, II	3 : 2, 3
A2	0	0	0	-480	-4048	0	1	-	12, 5	0, 0	1, 3	II^*, IV	3 : 1
A3	0	0	0	0	432	0	1	-	12, 9	0, 0	1, 1	II^*, IV^*	3 : 1, 4
A4	0	0	0	-4320	109296	0	1	-	12, 11	0, 0	1, 1	II^*, II^*	3 : 3
B1	0	0	0	0	-4	1	1	-	8, 3	0, 0	2, 1	I_0^*, II	3 : 2
B2	0	0	0	0	108	1	1	-	8, 9	0, 0	2, 3	I_0^*, IV^*	3 : 1
C1	0	0	0	-27	-918	0	1	-	11, 11	0, 0	2, 1	I_3^*, II^*	
D1	0	0	0	-3	34	1	1	-	11, 5	0, 0	4, 3	I_3^*, IV	
E1	0	0	0	-51	-142	0	1	-	13, 3	1, 0	2, 1	I_5^*, II	3 : 2
E2	0	0	0	189	-702	0	1	-	15, 9	3, 0	2, 1	I_7^*, IV^*	3 : 1, 3
E3	0	0	0	-1971	44658	0	1	-	21, 11	9, 0	2, 1	I_{13}^*, II^*	3 : 2
F1	0	0	0	21	26	1	1	-	15, 3	3, 0	4, 1	I_7^*, II	3 : 2, 3
F2	0	0	0	-219	-1654	1	1	-	21, 5	9, 0	4, 1	I_{13}^*, IV	3 : 1
F3	0	0	0	-459	3834	1	1	-	13, 9	1, 0	4, 3	I_5^*, IV^*	3 : 1
G1	0	0	0	-108	540	0	1	-	8, 11	0, 0	1, 1	I_0^*, II^*	
H1	0	0	0	-12	-20	0	1	-	8, 5	0, 0	1, 1	I_0^*, IV	

433 $N = 433 = 433$ (1 isogeny class) **433**

A1	1	0	0	0	1	2	1	-	1	1	1	I_1	
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434 $N = 434 = 2 \cdot 7 \cdot 31$ (5 isogeny classes) **434**

A1	1	-1	0	-7	-3	1	2	+	6, 1, 1	6, 1, 1	2, 1, 1	I_6, I_1, I_1	2 : 2
A2	1	-1	0	-47	133	1	2	+	3, 2, 2	3, 2, 2	1, 2, 2	I_3, I_2, I_2	2 : 1
B1	1	0	0	-4	16	0	3	-	9, 1, 1	9, 1, 1	9, 1, 1	I_9, I_1, I_1	3 : 2
B2	1	0	0	36	-424	0	3	-	3, 3, 3	3, 3, 3	3, 3, 3	I_3, I_3, I_3	3 : 1, 3
B3	1	0	0	-3374	-75754	0	1	-	1, 9, 1	1, 9, 1	1, 9, 1	I_1, I_9, I_1	3 : 2
C1	1	1	1	-32	61	0	2	-	2, 4, 1	2, 4, 1	2, 2, 1	I_2, I_4, I_1	2 : 2
C2	1	1	1	-522	4373	0	2	+	1, 2, 2	1, 2, 2	1, 2, 2	I_1, I_2, I_2	2 : 1
D1	1	0	0	21	49	1	2	-	10, 2, 1	10, 2, 1	10, 2, 1	I_{10}, I_2, I_1	2 : 2
D2	1	0	0	-139	465	1	2	+	5, 4, 2	5, 4, 2	5, 4, 2	I_5, I_4, I_2	2 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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435 $N = 435 = 3 \cdot 5 \cdot 29$ (4 isogeny classes)**435**

A1	0	1	1	-11	11	0	3	-	3, 1, 1	3, 1, 1	3, 1, 1	I_3, I_1, I_1	3 : 2
A2	0	1	1	49	80	0	1	-	1, 3, 3	1, 3, 3	1, 1, 1	I_1, I_3, I_3	3 : 1
B1	0	-1	1	79	-1123	0	1	-	5, 7, 1	5, 7, 1	1, 1, 1	I_5, I_7, I_1	
C1	1	0	1	-28	53	0	2	+	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	2 : 2
C2	1	0	1	-33	31	0	4	+	4, 2, 2	4, 2, 2	4, 2, 2	I_4, I_2, I_2	2 : 1, 3, 4
C3	1	0	1	-258	-1589	0	2	+	2, 1, 4	2, 1, 4	2, 1, 4	I_2, I_1, I_4	2 : 2
C4	1	0	1	112	263	0	4	-	8, 4, 1	8, 4, 1	8, 4, 1	I_8, I_4, I_1	2 : 2
D1	1	0	0	-30	-45	0	4	+	8, 1, 1	8, 1, 1	8, 1, 1	I_8, I_1, I_1	2 : 2
D2	1	0	0	-435	-3528	0	4	+	4, 2, 2	4, 2, 2	4, 2, 2	I_4, I_2, I_2	2 : 1, 3, 4
D3	1	0	0	-6960	-224073	0	2	+	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	2 : 2
D4	1	0	0	-390	-4275	0	4	-	2, 4, 4	2, 4, 4	2, 4, 4	I_2, I_4, I_4	2 : 2

437 $N = 437 = 19 \cdot 23$ (2 isogeny classes)**437**

A1	0	-1	1	19	100	1	1	-	1, 4	1, 4	1, 4	I_1, I_4	
B1	0	-1	1	0	-5	0	1	-	1, 2	1, 2	1, 2	I_1, I_2	

438 $N = 438 = 2 \cdot 3 \cdot 73$ (7 isogeny classes)**438**

A1	1	0	0	-938	-9564	0	6	+	18, 6, 1	18, 6, 1	18, 6, 1	I_{18}, I_6, I_1	2 : 2; 3 : 3
A2	1	0	0	1622	-52060	0	6	-	9, 12, 2	9, 12, 2	9, 12, 2	I_9, I_{12}, I_2	2 : 1; 3 : 4
A3	1	0	0	-72938	-7587996	0	2	+	6, 2, 3	6, 2, 3	6, 2, 3	I_6, I_2, I_3	2 : 4; 3 : 1
A4	1	0	0	-72898	-7596724	0	2	-	3, 4, 6	3, 4, 6	3, 4, 6	I_3, I_4, I_6	2 : 3; 3 : 2
B1	1	0	0	-13	-19	0	2	+	2, 2, 1	2, 2, 1	2, 2, 1	I_2, I_2, I_1	2 : 2
B2	1	0	0	-3	-45	0	2	-	1, 4, 2	1, 4, 2	1, 4, 2	I_1, I_4, I_2	2 : 1
C1	1	1	0	-5	-3	1	2	+	4, 2, 1	4, 2, 1	2, 2, 1	I_4, I_2, I_1	2 : 2
C2	1	1	0	-65	-231	1	2	+	2, 1, 2	2, 1, 2	2, 1, 2	I_2, I_1, I_2	2 : 1
D1	1	0	1	-1946	32780	1	6	+	6, 12, 1	6, 12, 1	2, 12, 1	I_6, I_{12}, I_1	2 : 2; 3 : 3
D2	1	0	1	-31106	2108972	1	6	+	3, 6, 2	3, 6, 2	1, 6, 2	I_3, I_6, I_2	2 : 1; 3 : 4
D3	1	0	1	-9641	-337876	1	2	+	18, 4, 3	18, 4, 3	2, 4, 3	I_{18}, I_4, I_3	2 : 4; 3 : 1
D4	1	0	1	-32681	1883180	1	2	+	9, 2, 6	9, 2, 6	1, 2, 6	I_9, I_2, I_6	2 : 3; 3 : 2
E1	1	0	1	-130	-556	0	2	+	14, 2, 1	14, 2, 1	2, 2, 1	I_{14}, I_2, I_1	2 : 2
E2	1	0	1	-2050	-35884	0	2	+	7, 1, 2	7, 1, 2	1, 1, 2	I_7, I_1, I_2	2 : 1
F1	1	1	1	-19	17	1	4	+	8, 2, 1	8, 2, 1	8, 2, 1	I_8, I_2, I_1	2 : 2
F2	1	1	1	-99	-399	1	4	+	4, 4, 2	4, 4, 2	4, 2, 2	I_4, I_4, I_2	2 : 1, 3, 4
F3	1	1	1	-1559	-24343	1	2	+	2, 8, 1	2, 8, 1	2, 2, 1	I_2, I_8, I_1	2 : 2
F4	1	1	1	81	-1479	1	4	-	2, 2, 4	2, 2, 4	2, 2, 4	I_2, I_2, I_4	2 : 2
G1	1	0	1	-8	2	1	2	+	2, 4, 1	2, 4, 1	2, 4, 1	I_2, I_4, I_1	2 : 2
G2	1	0	1	-98	362	1	2	+	1, 2, 2	1, 2, 2	1, 2, 2	I_1, I_2, I_2	2 : 1

440 $N = 440 = 2^3 \cdot 5 \cdot 11$ (4 isogeny classes)**440**

A1	0	0	0	-38	-87	1	2	+	4, 3, 2	0, 3, 2	2, 1, 2	III, I_3, I_2	2 : 2
A2	0	0	0	17	-318	1	2	-	8, 6, 1	0, 6, 1	2, 2, 1	I_1^*, I_6, I_1	2 : 1
B1	0	0	0	2	-3	1	2	-	4, 2, 1	0, 2, 1	2, 2, 1	III, I_2, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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440 $N = 440 = 2^3 \cdot 5 \cdot 11$ (continued) **440**

C1	0	0	0	-5042	137801	0	4	+	4, 3, 2	0, 3, 2	2, 3, 2	III, I ₃ , I ₂	2 : 2
C2	0	0	0	-5047	137514	0	4	+	8, 6, 4	0, 6, 4	2, 6, 2	I ₁ [*] , I ₆ , I ₄	2 : 1, 3, 4
C3	0	0	0	-7547	-12986	0	2	+	10, 3, 8	0, 3, 8	2, 3, 2	III [*] , I ₃ , I ₈	2 : 2
C4	0	0	0	-2627	269646	0	4	-	10, 12, 2	0, 12, 2	2, 12, 2	III [*] , I ₁₂ , I ₂	2 : 2
D1	0	0	0	-67	-226	0	1	-	11, 3, 1	0, 3, 1	1, 3, 1	II [*] , I ₃ , I ₁	

441 $N = 441 = 3^2 \cdot 7^2$ (6 isogeny classes) **441**

A1	0	0	1	0	-4202	0	1	-	3, 10	0, 0	2, 1	III, II [*]	3 : 2
A2	0	0	1	0	113447	0	1	-	9, 10	0, 0	2, 1	III [*] , II [*]	3 : 1
B1	0	0	1	0	12	1	3	-	3, 4	0, 0	2, 3	III, IV	3 : 2
B2	0	0	1	0	-331	1	1	-	9, 4	0, 0	2, 3	III [*] , IV	3 : 1
C1	1	-1	0	432	-869	1	2	-	8, 7	2, 1	2, 4	I ₂ [*] , I ₁ [*]	2 : 2
C2	1	-1	0	-1773	-5720	1	4	+	10, 8	4, 2	4, 4	I ₄ [*] , I ₂ [*]	2 : 1, 3, 4
C3	1	-1	0	-21618	-1216265	1	4	+	8, 10	2, 4	4, 4	I ₂ [*] , I ₄ [*]	2 : 2, 5, 6
C4	1	-1	0	-17208	867901	1	2	+	14, 7	8, 1	4, 2	I ₈ [*] , I ₁ [*]	2 : 2
C5	1	-1	0	-345753	-78165914	1	2	+	7, 8	1, 2	2, 2	I ₁ [*] , I ₂ [*]	2 : 3
C6	1	-1	0	-15003	-1979636	1	2	-	7, 14	1, 8	4, 4	I ₁ [*] , I ₈ [*]	2 : 3
D1	1	-1	1	-20	46	1	2	-	6, 3	0, 0	2, 2	I ₀ [*] , III	2 : 2; 7 : 3
D2	1	-1	1	-335	2440	1	2	+	6, 3	0, 0	4, 2	I ₀ [*] , III	2 : 1; 7 : 4
D3	1	-1	1	-965	-13940	1	2	-	6, 9	0, 0	2, 2	I ₀ [*] , III [*]	2 : 4; 7 : 1
D4	1	-1	1	-16400	-804212	1	2	+	6, 9	0, 0	4, 2	I ₀ [*] , III [*]	2 : 3; 7 : 2
E1	0	0	1	-1029	-13806	0	1	-	7, 8	1, 0	2, 1	I ₁ [*] , IV [*]	13 : 2
E2	0	0	1	-402339	98307144	0	1	-	19, 8	13, 0	2, 1	I ₁₃ [*] , IV [*]	13 : 1
F1	0	0	1	-21	40	1	1	-	7, 2	1, 0	4, 1	I ₁ [*] , II	13 : 2
F2	0	0	1	-8211	-286610	1	1	-	19, 2	13, 0	4, 1	I ₁₃ [*] , II	13 : 1

442 $N = 442 = 2 \cdot 13 \cdot 17$ (5 isogeny classes) **442**

A1	1	-1	1	-94	361	0	2	+	2, 2, 3	2, 2, 3	2, 2, 1	I ₂ , I ₂ , I ₃	2 : 2
A2	1	-1	1	36	1193	0	2	-	1, 1, 6	1, 1, 6	1, 1, 2	I ₁ , I ₁ , I ₆	2 : 1
B1	1	-1	1	-172	-465	1	2	+	8, 2, 3	8, 2, 3	8, 2, 3	I ₈ , I ₂ , I ₃	2 : 2
B2	1	-1	1	-1212	16175	1	2	+	4, 1, 6	4, 1, 6	4, 1, 6	I ₄ , I ₁ , I ₆	2 : 1
C1	1	1	0	-54	-172	0	2	+	8, 2, 1	8, 2, 1	2, 2, 1	I ₈ , I ₂ , I ₁	2 : 2
C2	1	1	0	26	-540	0	2	-	4, 4, 2	4, 4, 2	2, 2, 2	I ₄ , I ₄ , I ₂	2 : 1
D1	1	1	1	-9	-13	0	2	+	2, 2, 1	2, 2, 1	2, 2, 1	I ₂ , I ₂ , I ₁	2 : 2
D2	1	1	1	-139	-689	0	2	+	1, 1, 2	1, 1, 2	1, 1, 2	I ₁ , I ₁ , I ₂	2 : 1
E1	1	1	1	-144951	7520141	0	2	+	22, 4, 5	22, 4, 5	22, 2, 1	I ₂₂ , I ₄ , I ₅	2 : 2
E2	1	1	1	-1875511	987017101	0	2	+	11, 2, 10	11, 2, 10	11, 2, 2	I ₁₁ , I ₂ , I ₁₀	2 : 1

443 $N = 443 = 443$ (3 isogeny classes) **443**

A1	0	1	1	1	1	1	1	-	1	1	1	I ₁	
B1	1	0	0	-3	-2	1	1	+	1	1	1	I ₁	

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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444 $N = 444 = 2^2 \cdot 3 \cdot 37$ (2 isogeny classes) **444**

A1	0	-1	0	-13	-14	0	2	+	4, 2, 1	0, 2, 1	1, 2, 1	IV, I ₂ , I ₁	2 : 2
A2	0	-1	0	-28	40	0	2	+	8, 1, 2	0, 1, 2	1, 1, 2	IV*, I ₁ , I ₂	2 : 1
B1	0	1	0	-9	0	1	2	+	4, 4, 1	0, 4, 1	3, 4, 1	IV, I ₄ , I ₁	2 : 2
B2	0	1	0	36	36	1	2	-	8, 2, 2	0, 2, 2	3, 2, 2	IV*, I ₂ , I ₂	2 : 1

446 $N = 446 = 2 \cdot 223$ (4 isogeny classes) **446**

A1	1	1	0	-30	52	1	1	+	6, 1	6, 1	2, 1	I ₆ , I ₁	
B1	1	1	1	-39	-35	1	1	+	14, 1	14, 1	14, 1	I ₁₄ , I ₁	
C1	1	1	1	2	-5	0	2	-	6, 1	6, 1	6, 1	I ₆ , I ₁	2 : 2
C2	1	1	1	-38	-101	0	2	+	3, 2	3, 2	3, 2	I ₃ , I ₂	2 : 1
D1	1	-1	0	-4	4	2	1	+	2, 1	2, 1	2, 1	I ₂ , I ₁	

448 $N = 448 = 2^6 \cdot 7$ (8 isogeny classes) **448**

A1	0	0	0	4	16	1	2	-	14, 1	0, 1	4, 1	I ₄ *, I ₁	2 : 2
A2	0	0	0	-76	240	1	4	+	16, 2	0, 2	4, 2	I ₆ *, I ₂	2 : 1, 3, 4
A3	0	0	0	-236	-1104	1	2	+	17, 4	0, 4	4, 2	I ₇ *, I ₄	2 : 2
A4	0	0	0	-1196	15920	1	4	+	17, 1	0, 1	4, 1	I ₇ *, I ₁	2 : 2
B1	0	0	0	4	-16	1	2	-	14, 1	0, 1	4, 1	I ₄ *, I ₁	2 : 2
B2	0	0	0	-76	-240	1	4	+	16, 2	0, 2	4, 2	I ₆ *, I ₂	2 : 1, 3, 4
B3	0	0	0	-1196	-15920	1	2	+	17, 1	0, 1	2, 1	I ₇ *, I ₁	2 : 2
B4	0	0	0	-236	1104	1	4	+	17, 4	0, 4	4, 4	I ₇ *, I ₄	2 : 2
C1	0	-1	0	-33	161	0	2	-	20, 1	2, 1	4, 1	I ₁₀ *, I ₁	2 : 2; 3 : 3
C2	0	-1	0	-673	6945	0	2	+	19, 2	1, 2	2, 2	I ₉ *, I ₂	2 : 1; 3 : 4
C3	0	-1	0	287	-3231	0	2	-	24, 3	6, 3	4, 3	I ₁₄ *, I ₃	2 : 4; 3 : 1, 5
C4	0	-1	0	-2273	-33439	0	2	+	21, 6	3, 6	2, 6	I ₁₁ *, I ₆	2 : 3; 3 : 2, 6
C5	0	-1	0	-10913	-436447	0	2	-	36, 1	18, 1	4, 1	I ₂₆ *, I ₁	2 : 6; 3 : 3
C6	0	-1	0	-174753	-28059871	0	2	+	27, 2	9, 2	2, 2	I ₁₇ *, I ₂	2 : 5; 3 : 4
D1	0	-1	0	7	-7	0	2	-	12, 1	0, 1	4, 1	I ₂ *, I ₁	2 : 2
D2	0	-1	0	-33	-31	0	2	+	15, 2	0, 2	2, 2	I ₅ *, I ₂	2 : 1
E1	0	-1	0	-1	33	0	2	-	16, 1	0, 1	4, 1	I ₆ *, I ₁	2 : 2
E2	0	-1	0	-161	833	0	2	+	17, 2	0, 2	2, 2	I ₇ *, I ₂	2 : 1
F1	0	1	0	-33	-161	0	2	-	20, 1	2, 1	4, 1	I ₁₀ *, I ₁	2 : 2; 3 : 3
F2	0	1	0	-673	-6945	0	2	+	19, 2	1, 2	2, 2	I ₉ *, I ₂	2 : 1; 3 : 4
F3	0	1	0	287	3231	0	2	-	24, 3	6, 3	4, 1	I ₁₄ *, I ₃	2 : 4; 3 : 1, 5
F4	0	1	0	-2273	33439	0	2	+	21, 6	3, 6	2, 2	I ₁₁ *, I ₆	2 : 3; 3 : 2, 6
F5	0	1	0	-10913	436447	0	2	-	36, 1	18, 1	4, 1	I ₂₆ *, I ₁	2 : 6; 3 : 3
F6	0	1	0	-174753	28059871	0	2	+	27, 2	9, 2	2, 2	I ₁₇ *, I ₂	2 : 5; 3 : 4
G1	0	1	0	7	7	1	2	-	12, 1	0, 1	4, 1	I ₂ *, I ₁	2 : 2
G2	0	1	0	-33	31	1	2	+	15, 2	0, 2	4, 2	I ₅ *, I ₂	2 : 1
H1	0	1	0	-1	-33	0	2	-	16, 1	0, 1	4, 1	I ₆ *, I ₁	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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450

$N = 450 = 2 \cdot 3^2 \cdot 5^2$ (7 isogeny classes)

450

A1	1	-1	1	-680	9447	0	2	-	2, 7, 9	2, 1, 0	2, 2, 2	I_2, I_1^*, III^*	2 : 2; 5 : 3
A2	1	-1	1	-11930	504447	0	2	+	1, 8, 9	1, 2, 0	1, 4, 2	I_1, I_2^*, III^*	2 : 1; 5 : 4
A3	1	-1	1	-6305	-924303	0	2	-	10, 11, 9	10, 5, 0	10, 2, 2	I_{10}, I_5^*, III^*	2 : 4; 5 : 1
A4	1	-1	1	-186305	-30804303	0	2	+	5, 16, 9	5, 10, 0	5, 4, 2	I_5, I_{10}^*, III^*	2 : 3; 5 : 2
B1	1	-1	1	-5	47	0	1	-	1, 6, 4	1, 0, 0	1, 1, 1	I_1, I_0^*, IV	3 : 2; 5 : 3
B2	1	-1	1	-1130	14897	0	3	-	3, 6, 4	3, 0, 0	3, 1, 3	I_3, I_0^*, IV	3 : 1; 5 : 4
B3	1	-1	1	-680	-8053	0	1	-	5, 6, 8	5, 0, 0	5, 1, 1	I_5, I_0^*, IV^*	3 : 4; 5 : 1
B4	1	-1	1	4945	59447	0	3	-	15, 6, 8	15, 0, 0	15, 1, 3	I_{15}, I_0^*, IV^*	3 : 3; 5 : 2
C1	1	-1	0	-27	81	1	2	-	2, 7, 3	2, 1, 0	2, 4, 2	I_2, I_1^*, III	2 : 2; 5 : 3
C2	1	-1	0	-477	4131	1	2	+	1, 8, 3	1, 2, 0	1, 4, 2	I_1, I_2^*, III	2 : 1; 5 : 4
C3	1	-1	0	-252	-7344	1	2	-	10, 11, 3	10, 5, 0	2, 4, 2	I_{10}, I_5^*, III	2 : 4; 5 : 1
C4	1	-1	0	-7452	-244944	1	2	+	5, 16, 3	5, 10, 0	1, 4, 2	I_5, I_{10}^*, III	2 : 3; 5 : 2
D1	1	-1	0	-27	-59	0	1	-	5, 6, 2	5, 0, 0	1, 1, 1	I_5, I_0^*, II	3 : 2; 5 : 3
D2	1	-1	0	198	436	0	1	-	15, 6, 2	15, 0, 0	1, 1, 1	I_{15}, I_0^*, II	3 : 1; 5 : 4
D3	1	-1	0	-117	5791	0	1	-	1, 6, 10	1, 0, 0	1, 1, 1	I_1, I_0^*, II^*	3 : 4; 5 : 1
D4	1	-1	0	-28242	1833916	0	1	-	3, 6, 10	3, 0, 0	1, 1, 1	I_3, I_0^*, II^*	3 : 3; 5 : 2
E1	1	-1	1	145	147	0	2	-	2, 3, 9	2, 0, 3	2, 2, 2	I_2, III, I_3^*	2 : 2; 3 : 3
E2	1	-1	1	-605	1647	0	2	+	1, 3, 12	1, 0, 6	1, 2, 4	I_1, III, I_6^*	2 : 1; 3 : 4
E3	1	-1	1	-1730	-31103	0	2	-	6, 9, 7	6, 0, 1	6, 2, 2	I_6, III^*, I_1^*	2 : 4; 3 : 1
E4	1	-1	1	-28730	-1867103	0	2	+	3, 9, 8	3, 0, 2	3, 2, 4	I_3, III^*, I_2^*	2 : 3; 3 : 2
F1	1	-1	0	-192	1216	1	2	-	6, 3, 7	6, 0, 1	2, 2, 4	I_6, III, I_1^*	2 : 2; 3 : 3
F2	1	-1	0	-3192	70216	1	2	+	3, 3, 8	3, 0, 2	1, 2, 4	I_3, III, I_2^*	2 : 1; 3 : 4
F3	1	-1	0	1308	-5284	1	2	-	2, 9, 9	2, 0, 3	2, 2, 4	I_2, III^*, I_3^*	2 : 4; 3 : 1
F4	1	-1	0	-5442	-39034	1	2	+	1, 9, 12	1, 0, 6	1, 2, 4	I_1, III^*, I_6^*	2 : 3; 3 : 2
G1	1	-1	0	333	-7259	0	2	-	4, 9, 7	4, 3, 1	2, 2, 2	I_4, I_3^*, I_1^*	2 : 2; 3 : 3
G2	1	-1	0	-4167	-92759	0	4	+	2, 12, 8	2, 6, 2	2, 4, 4	I_2, I_6^*, I_2^*	2 : 1, 4, 5; 3 : 6
G3	1	-1	0	-3042	212116	0	2	-	12, 7, 9	12, 1, 3	2, 2, 2	I_{12}, I_1^*, I_3^*	2 : 6; 3 : 1
G4	1	-1	0	-64917	-6350009	0	2	+	1, 9, 10	1, 3, 4	1, 4, 4	I_1, I_3^*, I_4^*	2 : 2; 3 : 7
G5	1	-1	0	-15417	638491	0	2	+	1, 18, 7	1, 12, 1	1, 4, 2	I_1, I_{12}^*, I_1^*	2 : 2; 3 : 8
G6	1	-1	0	-75042	7916116	0	4	+	6, 8, 12	6, 2, 6	2, 4, 4	I_6, I_2^*, I_6^*	2 : 3, 7, 8; 3 : 2
G7	1	-1	0	-102042	1733116	0	2	+	3, 7, 18	3, 1, 12	1, 4, 4	I_3, I_1^*, I_{12}^*	2 : 6; 3 : 4
G8	1	-1	0	-120042	506291116	0	2	+	3, 10, 9	3, 4, 3	1, 4, 2	I_3, I_4^*, I_3^*	2 : 6; 3 : 5

451

$N = 451 = 11 \cdot 41$ (1 isogeny class)

451

A1	0	1	1	3	7	1	1	-	1, 2	1, 2	1, 2	I_1, I_2	
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455

$N = 455 = 5 \cdot 7 \cdot 13$ (2 isogeny classes)

455

A1	1	-1	0	-50	111	1	2	+	3, 4, 1	3, 4, 1	1, 2, 1	I_3, I_4, I_1	2 : 2
A2	1	-1	0	-295	-1800	1	4	+	6, 2, 2	6, 2, 2	2, 2, 2	I_6, I_2, I_2	2 : 1, 3, 4
A3	1	-1	0	-4670	-121675	1	2	+	3, 1, 4	3, 1, 4	1, 1, 2	I_3, I_1, I_4	2 : 2
A4	1	-1	0	160	-7169	1	2	-	12, 1, 1	12, 1, 1	2, 1, 1	I_{12}, I_1, I_1	2 : 2
B1	1	-1	1	-67	226	1	4	+	1, 2, 1	1, 2, 1	1, 2, 1	I_1, I_2, I_1	2 : 2
B2	1	-1	1	-72	194	1	4	+	2, 4, 2	2, 4, 2	2, 2, 2	I_2, I_4, I_2	2 : 1, 3, 4
B3	1	-1	1	-397	-2796	1	2	+	1, 8, 1	1, 8, 1	1, 2, 1	I_1, I_8, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
456	$N = 456 = 2^3 \cdot 3 \cdot 19$ (4 isogeny classes)											456	
A1	0	-1	0	-16	28	0	2	+	10, 1, 1	0, 1, 1	2, 1, 1	III*, I ₁ , I ₁	2 : 2
A2	0	-1	0	24	108	0	2	-	11, 2, 2	0, 2, 2	1, 2, 2	II*, I ₂ , I ₂	2 : 1
B1	0	1	0	-172	-928	0	2	+	8, 3, 1	0, 3, 1	2, 3, 1	I ₁ *, I ₃ , I ₁	2 : 2
B2	0	1	0	-192	-720	0	4	+	10, 6, 2	0, 6, 2	2, 6, 2	III*, I ₆ , I ₂	2 : 1, 3, 4
B3	0	1	0	-1272	16560	0	2	+	11, 3, 4	0, 3, 4	1, 3, 2	II*, I ₃ , I ₄	2 : 2
B4	0	1	0	568	-4368	0	2	-	11, 12, 1	0, 12, 1	1, 12, 1	II*, I ₁₂ , I ₁	2 : 2
C1	0	1	0	-57	171	1	1	-	8, 6, 1	0, 6, 1	4, 6, 1	I ₁ *, I ₆ , I ₁	
D1	0	-1	0	55	93	1	1	-	8, 2, 3	0, 2, 3	2, 2, 3	I ₁ *, I ₂ , I ₃	
458	$N = 458 = 2 \cdot 229$ (2 isogeny classes)											458	
A1	1	-1	0	-19	37	1	1	+	4, 1	4, 1	2, 1	I ₄ , I ₁	
B1	1	1	1	-16	-15	1	1	+	10, 1	10, 1	10, 1	I ₁₀ , I ₁	
459	$N = 459 = 3^3 \cdot 17$ (8 isogeny classes)											459	
A1	1	-1	0	0	-1	1	1	-	3, 1	0, 1	1, 1	II, I ₁	
B1	0	0	1	3	-4	1	1	-	3, 2	0, 2	1, 2	II, I ₂	
C1	0	0	1	-6	-6	0	1	-	3, 1	0, 1	1, 1	II, I ₁	3 : 2
C2	0	0	1	24	-27	0	3	-	5, 3	0, 3	3, 3	IV, I ₃	3 : 1
D1	0	0	1	-351	2531	0	1	-	9, 1	0, 1	1, 1	IV*, I ₁	
E1	0	0	1	27	101	0	1	-	9, 2	0, 2	1, 2	IV*, I ₂	
F1	0	0	1	-54	155	0	3	-	9, 1	0, 1	3, 1	IV*, I ₁	3 : 2
F2	0	0	1	216	722	0	1	-	11, 3	0, 3	1, 1	II*, I ₃	3 : 1
G1	0	0	1	-39	-94	0	1	-	3, 1	0, 1	1, 1	II, I ₁	
H1	1	-1	1	-2	28	1	1	-	9, 1	0, 1	3, 1	IV*, I ₁	
460	$N = 460 = 2^2 \cdot 5 \cdot 23$ (4 isogeny classes)											460	
A1	0	0	0	-8	-12	0	1	-	8, 1, 1	0, 1, 1	1, 1, 1	IV*, I ₁ , I ₁	
B1	0	0	0	-73	2453	0	1	-	4, 2, 5	0, 2, 5	1, 2, 1	IV, I ₂ , I ₅	
C1	0	1	0	-46	529	1	3	-	4, 4, 3	0, 4, 3	3, 2, 3	IV, I ₄ , I ₃	3 : 2
C2	0	1	0	414	-13915	1	1	-	4, 12, 1	0, 12, 1	1, 2, 1	IV, I ₁₂ , I ₁	3 : 1
D1	0	-1	0	-10	17	1	1	-	4, 2, 1	0, 2, 1	3, 2, 1	IV, I ₂ , I ₁	
462	$N = 462 = 2 \cdot 3 \cdot 7 \cdot 11$ (7 isogeny classes)											462	
A1	1	1	0	5	-23	1	2	-	2, 4, 1, 2	2, 4, 1, 2	2, 2, 1, 2	I ₂ , I ₄ , I ₁ , I ₂	2 : 2
A2	1	1	0	-105	-441	1	2	+	1, 8, 2, 1	1, 8, 2, 1	1, 2, 2, 1	I ₁ , I ₈ , I ₂ , I ₁	2 : 1
B1	1	1	0	-644	-2352	0	2	+	20, 3, 2, 1	20, 3, 2, 1	2, 1, 2, 1	I ₂₀ , I ₃ , I ₂ , I ₁	2 : 2
B2	1	1	0	-5764	164560	0	4	+	10, 6, 4, 2	10, 6, 4, 2	2, 2, 2, 2	I ₁₀ , I ₆ , I ₄ , I ₂	2 : 1, 3, 4
B3	1	1	0	-92004	10703088	0	2	+	5, 12, 2, 1	5, 12, 2, 1	1, 2, 2, 1	I ₅ , I ₁₂ , I ₂ , I ₁	2 : 2
B4	1	1	0	-1444	410800	0	2	-	5, 3, 8, 4	5, 3, 8, 4	1, 1, 2, 4	I ₅ , I ₃ , I ₈ , I ₄	2 : 2
C1	1	1	0	4	0	1	2	-	4, 1, 1, 1	4, 1, 1, 1	2, 1, 1, 1	I ₄ , I ₁ , I ₁ , I ₁	2 : 2
C2	1	1	0	-16	-20	1	4	+	2, 2, 2, 2	2, 2, 2, 2	2, 2, 2, 2	I ₂ , I ₂ , I ₂ , I ₂	2 : 1, 3, 4
C3	1	1	0	-226	-1406	1	2	+	1, 1, 1, 4	1, 1, 1, 4	1, 1, 1, 4	I ₁ , I ₁ , I ₁ , I ₄	2 : 2
C4	1	1	0	-126	486	1	2	+	1, 4, 4, 1	1, 4, 4, 1	1, 2, 4, 1	I ₁ , I ₄ , I ₄ , I ₁	2 : 2
D1	1	0	1	-1676	5058506	0	2	-	26, 4, 5, 2	26, 4, 5, 2	2, 4, 1, 2	I ₂₆ , I ₄ , I ₅ , I ₂	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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462

$N = 462 = 2 \cdot 3 \cdot 7 \cdot 11$ (continued)

462

E1	1	1	1	-405	4731	1	2	-	14, 2, 3, 2	14, 2, 3, 2	14, 2, 3, 2	I_{14}, I_2, I_3, I_2	2 : 2
E2	1	1	1	-7445	244091	1	2	+	7, 4, 6, 1	7, 4, 6, 1	7, 2, 6, 1	I_7, I_4, I_6, I_1	2 : 1
F1	1	0	0	-97	1337	0	4	-	4, 2, 3, 4	4, 2, 3, 4	4, 2, 1, 4	I_4, I_2, I_3, I_4	2 : 2
F2	1	0	0	-2517	48285	0	4	+	2, 4, 6, 2	2, 4, 6, 2	2, 4, 2, 2	I_2, I_4, I_6, I_2	2 : 1, 3, 4
F3	1	0	0	-3507	6507	0	2	+	1, 2, 12, 1	1, 2, 12, 1	1, 2, 2, 1	I_1, I_2, I_{12}, I_1	2 : 2
F4	1	0	0	-40247	3104415	0	2	+	1, 8, 3, 1	1, 8, 3, 1	1, 8, 1, 1	I_1, I_8, I_3, I_1	2 : 2
G1	1	0	0	77	161	0	6	-	6, 6, 1, 2	6, 6, 1, 2	6, 6, 1, 2	I_6, I_6, I_1, I_2	2 : 2; 3 : 3
G2	1	0	0	-363	1305	0	6	+	3, 12, 2, 1	3, 12, 2, 1	3, 12, 2, 1	I_3, I_{12}, I_2, I_1	2 : 1; 3 : 4
G3	1	0	0	-823	-11611	0	2	-	2, 2, 3, 6	2, 2, 3, 6	2, 2, 3, 2	I_2, I_2, I_3, I_6	2 : 4; 3 : 1
G4	1	0	0	-14133	-647829	0	2	+	1, 4, 6, 3	1, 4, 6, 3	1, 4, 6, 1	I_1, I_4, I_6, I_3	2 : 3; 3 : 2

464

$N = 464 = 2^4 \cdot 29$ (7 isogeny classes)

464

A1	0	1	0	8	4	1	1	-	10, 1	0, 1	2, 1	I_2^*, I_1	
B1	0	-1	0	-80	304	1	1	-	10, 1	0, 1	2, 1	I_2^*, I_1	
C1	0	1	0	80	-428	0	1	-	22, 1	10, 1	2, 1	I_{14}^*, I_1	5 : 2
C2	0	1	0	-7280	238292	0	1	-	14, 5	2, 5	2, 1	I_6^*, I_5	5 : 1
D1	0	-1	0	-4	-4	0	1	-	8, 1	0, 1	1, 1	I_0^*, I_1	3 : 2
D2	0	-1	0	36	76	0	1	-	8, 3	0, 3	1, 1	I_0^*, I_3	3 : 1
E1	0	1	0	-4	-24	0	2	-	8, 2	0, 2	1, 2	I_0^*, I_2	2 : 2
E2	0	1	0	-9	-14	0	2	+	4, 1	0, 1	1, 1	II, I_1	2 : 1
F1	0	0	0	-4831	129242	0	1	-	8, 1	0, 1	1, 1	I_0^*, I_1	
G1	0	0	0	-19	-46	0	1	-	14, 1	2, 1	2, 1	I_6^*, I_1	

465

$N = 465 = 3 \cdot 5 \cdot 31$ (2 isogeny classes)

465

A1	1	1	0	-7	16	1	2	-	3, 1, 2	3, 1, 2	1, 1, 2	I_3, I_1, I_2	2 : 2
A2	1	1	0	-162	729	1	2	+	6, 2, 1	6, 2, 1	2, 2, 1	I_6, I_2, I_1	2 : 1
B1	1	0	0	-10	-13	1	2	+	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	2 : 2
B2	1	0	0	-15	0	1	4	+	2, 2, 2	2, 2, 2	2, 2, 2	I_2, I_2, I_2	2 : 1, 3, 4
B3	1	0	0	-170	837	1	4	+	4, 4, 1	4, 4, 1	4, 4, 1	I_4, I_4, I_1	2 : 2
B4	1	0	0	60	15	1	2	-	1, 1, 4	1, 1, 4	1, 1, 2	I_1, I_1, I_4	2 : 2

466

$N = 466 = 2 \cdot 233$ (2 isogeny classes)

466

A1	1	1	0	-5	-7	0	2	+	2, 1	2, 1	2, 1	I_2, I_1	2 : 2
A2	1	1	0	-15	11	0	2	+	1, 2	1, 2	1, 2	I_1, I_2	2 : 1
B1	1	0	0	-23	41	0	3	-	6, 1	6, 1	6, 1	I_6, I_1	3 : 2
B2	1	0	0	77	229	0	1	-	2, 3	2, 3	2, 1	I_2, I_3	3 : 1

467

$N = 467 = 467$ (1 isogeny class)

467

A1	0	0	1	-4	3	1	1	-	1	1	1	I_1	
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468

$N = 468 = 2^2 \cdot 3^2 \cdot 13$ (5 isogeny classes)

468

A1	0	0	0	-168	-855	0	2	-	4, 3, 4	0, 0, 4	3, 2, 2	IV, III, I_4	2 : 2
A2	0	0	0	-2703	-54090	0	2	+	8, 3, 2	0, 0, 2	3, 2, 2	IV^*, III, I_2	2 : 1
B1	0	0	0	-1512	23085	0	2	-	4, 9, 4	0, 0, 4	1, 2, 2	IV, III^*, I_4	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
468	$N = 468 = 2^2 \cdot 3^2 \cdot 13$ (continued)											468	
C1	0	0	0	-36	81	1	2	+	4, 6, 1	0, 0, 1	3, 4, 1	IV, I_0^*, I_1	2 : 2
C2	0	0	0	9	270	1	2	-	8, 6, 2	0, 0, 2	3, 2, 2	IV^*, I_0^*, I_2	2 : 1
D1	0	0	0	-120	-11	0	2	+	4, 12, 1	0, 6, 1	1, 4, 1	IV, I_6^*, I_1	2 : 2; 3 : 3
D2	0	0	0	-1335	-18722	0	2	+	8, 9, 2	0, 3, 2	1, 4, 2	IV^*, I_3^*, I_2	2 : 1; 3 : 4
D3	0	0	0	-6600	206377	0	6	+	4, 8, 3	0, 2, 3	3, 4, 3	IV, I_2^*, I_3	2 : 4; 3 : 1
D4	0	0	0	-6735	197494	0	6	+	8, 7, 6	0, 1, 6	3, 4, 6	IV^*, I_1^*, I_6	2 : 3; 3 : 2
E1	0	0	0	-48	-115	0	2	+	4, 8, 1	0, 2, 1	1, 4, 1	IV, I_2^*, I_1	2 : 2
E2	0	0	0	-183	830	0	2	+	8, 7, 2	0, 1, 2	1, 2, 2	IV^*, I_1^*, I_2	2 : 1
469	$N = 469 = 7 \cdot 67$ (2 isogeny classes)											469	
A1	1	0	1	-80	-275	1	1	+	5, 1	5, 1	1, 1	I_5, I_1	
B1	1	-1	1	-12	18	1	1	+	1, 1	1, 1	1, 1	I_1, I_1	
470	$N = 470 = 2 \cdot 5 \cdot 47$ (6 isogeny classes)											470	
A1	1	0	1	-44	106	1	1	+	8, 1, 1	8, 1, 1	2, 1, 1	I_8, I_1, I_1	
B1	1	0	1	-5773	168328	0	3	+	8, 3, 1	8, 3, 1	2, 3, 1	I_8, I_3, I_1	3 : 2
B2	1	0	1	-6348	132618	0	1	+	24, 1, 3	24, 1, 3	2, 1, 1	I_{24}, I_1, I_3	3 : 1
C1	1	1	0	-97	281	1	1	+	2, 7, 1	2, 7, 1	2, 7, 1	I_2, I_7, I_1	
D1	1	0	0	-36	80	0	3	+	6, 1, 1	6, 1, 1	6, 1, 1	I_6, I_1, I_1	3 : 2
D2	1	0	0	-176	-844	0	1	+	2, 3, 3	2, 3, 3	2, 1, 1	I_2, I_3, I_3	3 : 1
E1	1	1	1	-11	9	1	1	+	4, 1, 1	4, 1, 1	4, 1, 1	I_4, I_1, I_1	
F1	1	-1	1	-117	141	1	1	+	14, 3, 1	14, 3, 1	14, 3, 1	I_{14}, I_3, I_1	
471	$N = 471 = 3 \cdot 157$ (1 isogeny class)											471	
A1	1	1	1	1	2	1	1	-	2, 1	2, 1	2, 1	I_2, I_1	
472	$N = 472 = 2^3 \cdot 59$ (5 isogeny classes)											472	
A1	0	0	0	2	1	1	1	-	4, 1	0, 1	2, 1	III, I_1	
B1	0	-1	0	-276	-1676	0	1	-	8, 1	0, 1	2, 1	I_1^*, I_1	
C1	0	-1	0	8	12	0	1	-	11, 1	0, 1	1, 1	II^*, I_1	
D1	0	0	0	-19	-34	0	1	-	10, 1	0, 1	2, 1	III^*, I_1	
E1	0	-1	0	4	4	1	1	-	8, 1	0, 1	4, 1	I_1^*, I_1	
473	$N = 473 = 11 \cdot 43$ (1 isogeny class)											473	
A1	0	1	1	-1006	11952	1	1	-	3, 2	3, 2	1, 2	I_3, I_2	
474	$N = 474 = 2 \cdot 3 \cdot 79$ (2 isogeny classes)											474	
A1	1	1	0	81	-27	1	1	-	14, 3, 1	14, 3, 1	2, 1, 1	I_{14}, I_3, I_1	
B1	1	0	1	-7	14	1	1	-	2, 5, 1	2, 5, 1	2, 5, 1	I_2, I_5, I_1	
475	$N = 475 = 5^2 \cdot 19$ (3 isogeny classes)											475	
A1	0	-1	1	17	-7	0	1	-	6, 1	0, 1	1, 1	I_0^*, I_1	3 : 2
A2	0	-1	1	-233	-1382	0	1	-	6, 3	0, 3	1, 3	I_1^*, I_2	3 : 1, 3

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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475

$N = 475 = 5^2 \cdot 19$ (continued)

475

B1	1	-1	0	8	291	1	2	-	9, 1	0, 1	2, 1	III*, I ₁	2 : 2
B2	1	-1	0	-617	5916	1	2	+	9, 2	0, 2	2, 2	III*, I ₂	2 : 1
C1	1	-1	1	0	2	1	2	-	3, 1	0, 1	2, 1	III, I ₁	2 : 2
C2	1	-1	1	-25	52	1	2	+	3, 2	0, 2	2, 2	III, I ₂	2 : 1

477

$N = 477 = 3^2 \cdot 53$ (1 isogeny class)

477

A1	1	-1	0	3	-10	1	1	-	6, 1	0, 1	1, 1	I ₀ *, I ₁	
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480

$N = 480 = 2^5 \cdot 3 \cdot 5$ (8 isogeny classes)

480

A1	0	-1	0	-6	0	1	4	+	6, 2, 2	0, 2, 2	2, 2, 2	III, I ₂ , I ₂	2 : 2, 3, 4
A2	0	-1	0	-81	-255	1	2	+	12, 1, 1	0, 1, 1	2, 1, 1	I ₃ *, I ₁ , I ₁	2 : 1
A3	0	-1	0	-56	180	1	2	+	9, 4, 1	0, 4, 1	2, 2, 1	I ₀ *, I ₄ , I ₁	2 : 1
A4	0	-1	0	24	-24	1	2	-	9, 1, 4	0, 1, 4	1, 1, 2	I ₀ *, I ₁ , I ₄	2 : 1
B1	0	-1	0	-10	-8	0	4	+	6, 2, 2	0, 2, 2	2, 2, 2	III, I ₂ , I ₂	2 : 2, 3, 4
B2	0	-1	0	-160	-728	0	2	+	9, 1, 1	0, 1, 1	1, 1, 1	I ₀ *, I ₁ , I ₁	2 : 1
B3	0	-1	0	-40	100	0	4	+	9, 1, 4	0, 1, 4	2, 1, 4	I ₀ *, I ₁ , I ₄	2 : 1
B4	0	-1	0	15	-63	0	2	-	12, 4, 1	0, 4, 1	2, 2, 1	I ₃ *, I ₄ , I ₁	2 : 1
C1	0	1	0	-6	0	0	4	+	6, 2, 2	0, 2, 2	2, 2, 2	III, I ₂ , I ₂	2 : 2, 3, 4
C2	0	1	0	-56	-180	0	2	+	9, 4, 1	0, 4, 1	1, 4, 1	I ₀ *, I ₄ , I ₁	2 : 1
C3	0	1	0	-81	255	0	2	+	12, 1, 1	0, 1, 1	2, 1, 1	I ₃ *, I ₁ , I ₁	2 : 1
C4	0	1	0	24	24	0	2	-	9, 1, 4	0, 1, 4	2, 1, 2	I ₀ *, I ₁ , I ₄	2 : 1
D1	0	1	0	-226	-1360	0	4	+	6, 6, 4	0, 6, 4	2, 6, 2	III, I ₆ , I ₄	2 : 2, 3, 4
D2	0	1	0	-3601	-84385	0	2	+	12, 3, 2	0, 3, 2	2, 3, 2	I ₃ *, I ₃ , I ₂	2 : 1
D3	0	1	0	-496	2204	0	2	+	9, 3, 8	0, 3, 8	1, 3, 2	I ₀ *, I ₃ , I ₈	2 : 1
D4	0	1	0	24	-3960	0	4	-	9, 12, 2	0, 12, 2	2, 12, 2	I ₀ *, I ₁₂ , I ₂	2 : 1
E1	0	-1	0	-226	1360	0	4	+	6, 6, 4	0, 6, 4	2, 2, 2	III, I ₆ , I ₄	2 : 2, 3, 4
E2	0	-1	0	-496	-2204	0	2	+	9, 3, 8	0, 3, 8	2, 1, 2	I ₀ *, I ₃ , I ₈	2 : 1
E3	0	-1	0	-3601	84385	0	4	+	12, 3, 2	0, 3, 2	4, 1, 2	I ₃ *, I ₃ , I ₂	2 : 1
E4	0	-1	0	24	3960	0	2	-	9, 12, 2	0, 12, 2	1, 2, 2	I ₀ *, I ₁₂ , I ₂	2 : 1
F1	0	-1	0	-30	72	1	4	+	6, 4, 2	0, 4, 2	2, 2, 2	III, I ₄ , I ₂	2 : 2, 3, 4
F2	0	-1	0	-80	-168	1	2	+	9, 8, 1	0, 8, 1	1, 2, 1	I ₀ *, I ₈ , I ₁	2 : 1
F3	0	-1	0	-480	4212	1	4	+	9, 2, 1	0, 2, 1	2, 2, 1	I ₀ *, I ₂ , I ₁	2 : 1
F4	0	-1	0	15	225	1	4	-	12, 2, 4	0, 2, 4	4, 2, 4	I ₃ *, I ₂ , I ₄	2 : 1
G1	0	1	0	-10	8	0	4	+	6, 2, 2	0, 2, 2	2, 2, 2	III, I ₂ , I ₂	2 : 2, 3, 4
G2	0	1	0	-40	-100	0	2	+	9, 1, 4	0, 1, 4	1, 1, 4	I ₀ *, I ₁ , I ₄	2 : 1
G3	0	1	0	-160	728	0	2	+	9, 1, 1	0, 1, 1	2, 1, 1	I ₀ *, I ₁ , I ₁	2 : 1
G4	0	1	0	15	63	0	4	-	12, 4, 1	0, 4, 1	4, 4, 1	I ₃ *, I ₄ , I ₁	2 : 1
H1	0	1	0	-30	-72	0	4	+	6, 4, 2	0, 4, 2	2, 4, 2	III, I ₄ , I ₂	2 : 2, 3, 4
H2	0	1	0	-480	-4212	0	2	+	9, 2, 1	0, 2, 1	1, 2, 1	I ₀ *, I ₂ , I ₁	2 : 1
H3	0	1	0	-80	168	0	4	+	9, 8, 1	0, 8, 1	2, 8, 1	I ₀ *, I ₈ , I ₁	2 : 1
H4	0	1	0	15	-225	0	4	-	12, 2, 4	0, 2, 4	4, 2, 4	I ₃ *, I ₂ , I ₄	2 : 1

481

$N = 481 = 13 \cdot 37$ (1 isogeny class)

481

A1	1	-1	0	-1693	27240	1	2	+	3, 1	3, 1	1, 1	I ₃ , I ₁	2 : 2
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	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
482	$N = 482 = 2 \cdot 241$ (1 isogeny class)											482	
A1	1	0	1	-44	-150	1	1	-	14, 1	14, 1	2, 1	I_{14}, I_1	
483	$N = 483 = 3 \cdot 7 \cdot 23$ (2 isogeny classes)											483	
A1	0	1	1	-96	-457	0	1	-	5, 1, 3	5, 1, 3	5, 1, 1	I_5, I_1, I_3	
B1	0	1	1	2	1	0	1	-	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	
484	$N = 484 = 2^2 \cdot 11^2$ (1 isogeny class)											484	
A1	0	1	0	323	2671	1	1	-	8, 7	0, 1	1, 4	IV^*, I_1^*	3 : 2
A2	0	1	0	-9357	347279	1	1	-	8, 9	0, 3	3, 4	IV^*, I_3^*	3 : 1
485	$N = 485 = 5 \cdot 97$ (2 isogeny classes)											485	
A1	0	1	1	-121	-64	0	3	+	3, 3	3, 3	1, 3	I_3, I_3	3 : 2, 3
A2	0	1	1	-6911	-223455	0	1	+	9, 1	9, 1	1, 1	I_9, I_1	3 : 1
A3	0	1	1	-81	255	0	3	+	1, 1	1, 1	1, 1	I_1, I_1	3 : 1
B1	0	0	1	-2	0	1	1	+	1, 1	1, 1	1, 1	I_1, I_1	
486	$N = 486 = 2 \cdot 3^5$ (6 isogeny classes)											486	
A1	1	-1	0	3	5	1	1	-	6, 5	6, 0	2, 1	I_6, II	3 : 2
A2	1	-1	0	-177	953	1	3	-	2, 11	2, 0	2, 3	I_2, IV^*	3 : 1
B1	1	-1	0	-6	-4	1	1	+	3, 5	3, 0	1, 1	I_3, II	3 : 2
B2	1	-1	0	-96	386	1	3	+	1, 11	1, 0	1, 3	I_1, IV^*	3 : 1
C1	1	-1	0	-123	557	0	3	+	3, 7	3, 0	1, 3	I_3, IV	3 : 2
C2	1	-1	0	-258	-748	0	1	+	9, 13	9, 0	1, 1	I_9, II^*	3 : 1
D1	1	-1	1	-20	-29	0	1	-	2, 5	2, 0	2, 1	I_2, II	3 : 2
D2	1	-1	1	25	-161	0	3	-	6, 11	6, 0	6, 3	I_6, IV^*	3 : 1
E1	1	-1	1	-11	-11	0	1	+	1, 5	1, 0	1, 1	I_1, II	3 : 2
E2	1	-1	1	-56	163	0	3	+	3, 11	3, 0	3, 3	I_3, IV^*	3 : 1
F1	1	-1	1	-29	37	1	3	+	9, 7	9, 0	9, 3	I_9, IV	3 : 2
F2	1	-1	1	-1109	-13931	1	1	+	3, 13	3, 0	3, 1	I_3, II^*	3 : 1
490	$N = 490 = 2 \cdot 5 \cdot 7^2$ (11 isogeny classes)											490	
A1	1	0	1	121	46	1	3	-	2, 1, 8	2, 1, 0	2, 1, 3	I_2, I_1, IV^*	3 : 2
A2	1	0	1	-1594	-26708	1	1	-	6, 3, 8	6, 3, 0	2, 1, 3	I_6, I_3, IV^*	3 : 1
B1	1	1	0	17	-27	0	1	-	7, 3, 2	7, 3, 0	1, 1, 1	I_7, I_3, II	3 : 2
B2	1	1	0	-158	1268	0	1	-	21, 1, 2	21, 1, 0	1, 1, 1	I_{21}, I_1, II	3 : 1
C1	1	0	1	807	11708	0	3	-	7, 3, 8	7, 3, 0	1, 3, 3	I_7, I_3, IV^*	3 : 2
C2	1	0	1	-7768	-458202	0	1	-	21, 1, 8	21, 1, 0	1, 1, 3	I_{21}, I_1, IV^*	3 : 1
D1	1	1	0	3	1	1	1	-	2, 1, 2	2, 1, 0	2, 1, 1	I_2, I_1, II	3 : 2
D2	1	1	0	-32	64	1	1	-	6, 3, 2	6, 3, 0	2, 3, 1	I_6, I_3, II	3 : 1
E1	1	0	0	-1	-15	0	3	-	3, 1, 4	3, 1, 0	3, 1, 3	I_3, I_1, IV	3 : 2
E2	1	0	0	-491	-4229	0	1	-	1, 3, 4	1, 3, 0	1, 1, 3	I_1, I_3, IV	3 : 1
F1	1	-1	1	-6453	201121	0	1	-	2, 1, 8	2, 1, 0	2, 1, 1	I_2, I_1, IV^*	7 : 2
F2	1	-1	1	44997	-1904213	0	1	-	14, 7, 8	14, 7, 0	14, 1, 1	I_{14}, I_7, IV^*	7 : 1
G1	1	0	0	-71	265	1	2	-	10, 2, 3	10, 2, 0	10, 2, 2	I_{10}, I_5, III	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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490 $N = 490 = 2 \cdot 5 \cdot 7^2$ (continued) **490**

H1	1	-1	1	113	711	0	4	-	4, 2, 7	4, 2, 1	4, 2, 4	I_4, I_2, I_1^*	2 : 2
H2	1	-1	1	-867	8159	0	4	+	2, 4, 8	2, 4, 2	2, 4, 4	I_2, I_4, I_2^*	2 : 1, 3, 4
H3	1	-1	1	-4297	-100229	0	2	+	1, 8, 7	1, 8, 1	1, 8, 2	I_1, I_8, I_1^*	2 : 2
H4	1	-1	1	-13117	581459	0	2	+	1, 2, 10	1, 2, 4	1, 2, 4	I_1, I_2, I_4^*	2 : 2
I1	1	1	1	-50	5095	0	1	-	3, 1, 10	3, 1, 0	3, 1, 1	I_3, I_1, II^*	3 : 2
I2	1	1	1	-24060	1426487	0	1	-	1, 3, 10	1, 3, 0	1, 3, 1	I_1, I_3, II^*	3 : 1
J1	1	1	1	-3480	-94375	0	2	-	10, 2, 9	10, 2, 0	10, 2, 2	I_{10}, I_2, III^*	2 : 2
J2	1	1	1	-58360	-5450663	0	2	+	5, 4, 9	5, 4, 0	5, 4, 2	I_5, I_4, III^*	2 : 1
K1	1	-1	1	-132	-549	0	1	-	2, 1, 2	2, 1, 0	2, 1, 1	I_2, I_1, II	7 : 2
K2	1	-1	1	918	5289	0	7	-	14, 7, 2	14, 7, 0	14, 7, 1	I_{14}, I_7, II	7 : 1

492 $N = 492 = 2^2 \cdot 3 \cdot 41$ (2 isogeny classes) **492**

A1	0	-1	0	-13	25	1	1	-	8, 1, 1	0, 1, 1	3, 1, 1	IV^*, I_1, I_1	
B1	0	1	0	11	695	1	1	-	8, 9, 1	0, 9, 1	3, 9, 1	IV^*, I_9, I_1	

493 $N = 493 = 17 \cdot 29$ (2 isogeny classes) **493**

A1	1	-1	1	-7741	801682	0	1	-	1, 9	1, 9	1, 1	I_1, I_9	
B1	1	-1	1	-57	222	1	1	-	2, 3	2, 3	2, 3	I_2, I_3	

494 $N = 494 = 2 \cdot 13 \cdot 19$ (4 isogeny classes) **494**

A1	1	1	0	13	13	1	1	-	5, 1, 2	5, 1, 2	1, 1, 2	I_5, I_1, I_2	
B1	1	-1	0	4	0	0	2	-	4, 1, 1	4, 1, 1	2, 1, 1	I_4, I_1, I_1	2 : 2
B2	1	-1	0	-16	12	0	4	+	2, 2, 2	2, 2, 2	2, 2, 2	I_2, I_2, I_2	2 : 1, 3, 4
B3	1	-1	0	-146	-638	0	2	+	1, 1, 4	1, 1, 4	1, 1, 4	I_1, I_1, I_4	2 : 2
B4	1	-1	0	-206	1190	0	2	+	1, 4, 1	1, 4, 1	1, 2, 1	I_1, I_4, I_1	2 : 2
C1	1	-1	0	-61	-169	0	1	-	1, 1, 2	1, 1, 2	1, 1, 2	I_1, I_1, I_2	
D1	1	1	1	-1001	12375	1	1	-	13, 3, 2	13, 3, 2	13, 3, 2	I_{13}, I_3, I_2	

495 $N = 495 = 3^2 \cdot 5 \cdot 11$ (1 isogeny class) **495**

A1	1	-1	1	7	-8	1	2	-	6, 1, 1	0, 1, 1	2, 1, 1	I_0^*, I_1, I_1	2 : 2
A2	1	-1	1	-38	-44	1	4	+	6, 2, 2	0, 2, 2	4, 2, 2	I_0^*, I_2, I_2	2 : 1, 3, 4
A3	1	-1	1	-533	-4598	1	2	+	6, 4, 1	0, 4, 1	2, 2, 1	I_0^*, I_4, I_1	2 : 2
A4	1	-1	1	-263	1666	1	2	+	6, 1, 4	0, 1, 4	2, 1, 4	I_0^*, I_1, I_4	2 : 2

496 $N = 496 = 2^4 \cdot 31$ (6 isogeny classes) **496**

A1	0	0	0	1	1	1	1	-	4, 1	0, 1	1, 1	II, I_1	
B1	0	-1	0	0	-1	0	1	-	4, 1	0, 1	1, 1	II, I_1	
C1	0	-1	0	8	0	0	2	-	10, 1	0, 1	4, 1	I_2^*, I_1	2 : 2
C2	0	-1	0	-32	32	0	2	+	11, 2	0, 2	2, 2	I_3^*, I_2	2 : 1
D1	0	-1	0	-2	-1	0	1	-	4, 1	0, 1	1, 1	II, I_1	3 : 2
D2	0	-1	0	18	11	0	1	-	4, 3	0, 3	1, 1	II, I_3	3 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
496	$N = 496 = 2^4 \cdot 31$ (continued)											496	
F1	0	0	0	-11	-70	1	2	-	16, 1	4, 1	4, 1	I_{8, I_1}^*	2 : 2
F2	0	0	0	-331	-2310	1	4	+	14, 2	2, 2	4, 2	I_{6, I_2}^*	2 : 1, 3, 4
F3	0	0	0	-5291	-148134	1	2	+	13, 1	1, 1	2, 1	I_{5, I_1}^*	2 : 2
F4	0	0	0	-491	154	1	4	+	13, 4	1, 4	4, 4	I_{5, I_4}^*	2 : 2
497	$N = 497 = 7 \cdot 71$ (1 isogeny class)											497	
A1	1	1	0	25	-14	1	1	-	5, 1	5, 1	5, 1	I_{5, I_1}	
498	$N = 498 = 2 \cdot 3 \cdot 83$ (2 isogeny classes)											498	
A1	1	0	1	-5	-4	0	2	+	2, 1, 1	2, 1, 1	2, 1, 1	I_{2, I_1, I_1}	2 : 2
A2	1	0	1	5	-16	0	2	-	1, 2, 2	1, 2, 2	1, 2, 2	I_{1, I_2, I_2}	2 : 1
B1	1	0	1	-9	28	1	1	-	4, 5, 1	4, 5, 1	2, 5, 1	I_{4, I_5, I_1}	
501	$N = 501 = 3 \cdot 167$ (1 isogeny class)											501	
A1	1	1	0	3	0	0	2	-	2, 1	2, 1	2, 1	I_{2, I_1}	2 : 2
A2	1	1	0	-12	-15	0	2	+	1, 2	1, 2	1, 2	I_{1, I_2}	2 : 1
503	$N = 503 = 503$ (3 isogeny classes)											503	
A1	1	0	1	-32	-71	1	1	-	1	1	1	I_1	
B1	1	-1	0	2	-1	0	1	-	1	1	1	I_1	
C1	1	0	0	-210	-1189	0	1	-	1	1	1	I_1	
504	$N = 504 = 2^3 \cdot 3^2 \cdot 7$ (8 isogeny classes)											504	
A1	0	0	0	-6	9	1	2	-	4, 3, 2	0, 0, 2	2, 2, 2	III, III, I_2	2 : 2
A2	0	0	0	-111	450	1	2	+	8, 3, 1	0, 0, 1	2, 2, 1	I_1^*, III, I_1	2 : 1
B1	0	0	0	-54	-135	0	2	+	4, 9, 1	0, 0, 1	2, 2, 1	III, III^*, I_1	2 : 2
B2	0	0	0	81	-702	0	2	-	8, 9, 2	0, 0, 2	2, 2, 2	I_1^*, III^*, I_2	2 : 1
C1	0	0	0	9	-54	0	2	-	8, 6, 1	0, 0, 1	2, 2, 1	I_1^*, I_0^*, I_1	2 : 2
C2	0	0	0	-171	-810	0	4	+	10, 6, 2	0, 0, 2	2, 4, 2	III^*, I_0^*, I_2	2 : 1, 3, 4
C3	0	0	0	-2691	-53730	0	2	+	11, 6, 1	0, 0, 1	1, 2, 1	II^*, I_0^*, I_1	2 : 2
C4	0	0	0	-531	3726	0	2	+	11, 6, 4	0, 0, 4	1, 2, 2	II^*, I_0^*, I_4	2 : 2
D1	0	0	0	-54	-243	0	2	-	4, 9, 2	0, 0, 2	2, 2, 2	III, III^*, I_2	2 : 2
D2	0	0	0	-999	-12150	0	2	+	8, 9, 1	0, 0, 1	4, 2, 1	I_1^*, III^*, I_1	2 : 1
E1	0	0	0	-6	5	1	2	+	4, 3, 1	0, 0, 1	2, 2, 1	III, III, I_1	2 : 2
E2	0	0	0	9	26	1	2	-	8, 3, 2	0, 0, 2	4, 2, 2	I_1^*, III, I_2	2 : 1
F1	0	0	0	-66	205	1	4	+	4, 7, 1	0, 1, 1	2, 4, 1	III, I_1^*, I_1	2 : 2
F2	0	0	0	-111	-110	1	4	+	8, 8, 2	0, 2, 2	4, 4, 2	I_1^*, I_2^*, I_2	2 : 1, 3, 4
F3	0	0	0	-1371	-19514	1	2	+	10, 10, 1	0, 4, 1	2, 4, 1	III^*, I_4^*, I_1	2 : 2
F4	0	0	0	429	-866	1	2	-	10, 7, 4	0, 1, 4	2, 2, 2	III^*, I_1^*, I_4	2 : 2
G1	0	0	0	-66	-1339	0	4	-	4, 9, 4	0, 3, 4	2, 4, 4	III, I_3^*, I_4	2 : 2
G2	0	0	0	-2271	-41470	0	4	+	8, 12, 2	0, 6, 2	4, 4, 2	I_1^*, I_6^*, I_2	2 : 1, 3, 4
G3	0	0	0	-36291	-2661010	0	2	+	10, 9, 1	0, 3, 1	2, 2, 1	III^*, I_3^*, I_1	2 : 2
G4	0	0	0	-3531	9686	0	2	+	10, 18, 1	0, 12, 1	2, 4, 1	III^*, I_{12}^*, I_1	2 : 2
H1	0	0	0	-3	110	0	2	-	10, 6, 1	0, 0, 1	2, 2, 1	III^*, I_1^*, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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505 $N = 505 = 5 \cdot 101$ (1 isogeny class) **505**

A1	1	-1	0	-10	15	1	2	+	1, 1	1, 1	1, 1	I_1, I_1	2 : 2
A2	1	-1	0	-5	26	1	2	-	2, 2	2, 2	2, 2	I_2, I_2	2 : 1

506 $N = 506 = 2 \cdot 11 \cdot 23$ (6 isogeny classes) **506**

A1	1	0	1	-48	-130	1	1	+	7, 1, 1	7, 1, 1	1, 1, 1	I_7, I_1, I_1	
B1	1	-1	0	-290561	60356981	0	1	+	3, 7, 1	3, 7, 1	1, 1, 1	I_3, I_7, I_1	
C1	1	0	1	-12	8	0	3	+	1, 3, 1	1, 3, 1	1, 3, 1	I_1, I_3, I_1	3 : 2
C2	1	0	1	-397	-3072	0	1	+	3, 1, 3	3, 1, 3	1, 1, 1	I_3, I_1, I_3	3 : 1
D1	1	-1	0	-935	11229	1	1	+	5, 5, 1	5, 5, 1	1, 5, 1	I_5, I_5, I_1	
E1	1	-1	1	-4	-1	1	1	+	3, 1, 1	3, 1, 1	3, 1, 1	I_3, I_1, I_1	
F1	1	0	0	-86	292	1	1	+	13, 1, 1	13, 1, 1	13, 1, 1	I_{13}, I_1, I_1	

507 $N = 507 = 3 \cdot 13^2$ (3 isogeny classes) **507**

A1	1	1	0	-1693	26434	1	1	-	2, 8	2, 0	2, 3	I_2, IV^*	7 : 2
A2	1	1	0	-12678	-3060351	1	1	-	14, 8	14, 0	2, 3	I_{14}, IV^*	7 : 1
B1	1	1	1	-10	8	1	1	-	2, 2	2, 0	2, 1	I_2, II	7 : 2
B2	1	1	1	-75	-1422	1	1	-	14, 2	14, 0	2, 1	I_{14}, II	7 : 1
C1	1	1	1	81	-564	1	4	-	1, 7	1, 1	1, 4	I_1, I_1^*	2 : 2
C2	1	1	1	-764	-7324	1	4	+	2, 8	2, 2	2, 4	I_2, I_2^*	2 : 1, 3, 4
C3	1	1	1	-11749	-495058	1	2	+	4, 7	4, 1	2, 4	I_4, I_1^*	2 : 2
C4	1	1	1	-3299	64670	1	2	+	1, 10	1, 4	1, 4	I_1, I_4^*	2 : 2

510 $N = 510 = 2 \cdot 3 \cdot 5 \cdot 17$ (7 isogeny classes) **510**

A1	1	1	0	-2673	67797	0	2	-	18, 7, 1, 2	18, 7, 1, 2	2, 1, 1, 2	I_{18}, I_7, I_1, I_2	2 : 2
A2	1	1	0	-46193	3801813	0	2	+	9, 14, 2, 1	9, 14, 2, 1	1, 2, 2, 1	I_9, I_{14}, I_2, I_1	2 : 1
B1	1	0	1	-723	-7634	0	2	-	14, 3, 1, 2	14, 3, 1, 2	2, 3, 1, 2	I_{14}, I_3, I_1, I_2	2 : 2
B2	1	0	1	-11603	-482002	0	2	+	7, 6, 2, 1	7, 6, 2, 1	1, 6, 2, 1	I_7, I_6, I_2, I_1	2 : 1
C1	1	1	1	14	59	0	2	-	2, 5, 1, 2	2, 5, 1, 2	2, 1, 1, 2	I_2, I_5, I_1, I_2	2 : 2
C2	1	1	1	-156	603	0	2	+	1, 10, 2, 1	1, 10, 2, 1	1, 2, 2, 1	I_1, I_{10}, I_2, I_1	2 : 1
D1	1	1	1	-101	299	1	4	+	12, 2, 2, 1	12, 2, 2, 1	12, 2, 2, 1	I_{12}, I_2, I_2, I_1	2 : 2
D2	1	1	1	-421	-3157	1	4	+	6, 4, 4, 2	6, 4, 4, 2	6, 2, 2, 2	I_6, I_4, I_4, I_2	2 : 1, 3, 4
D3	1	1	1	-6541	-206341	1	2	+	3, 2, 8, 1	3, 2, 8, 1	3, 2, 2, 1	I_3, I_2, I_8, I_1	2 : 2
D4	1	1	1	579	-14757	1	2	-	3, 8, 2, 4	3, 8, 2, 4	3, 2, 2, 4	I_3, I_8, I_2, I_4	2 : 2
E1	1	1	1	-80	305	0	4	-	16, 1, 1, 1	16, 1, 1, 1	16, 1, 1, 1	I_{16}, I_1, I_1, I_1	2 : 2
E2	1	1	1	-1360	18737	0	8	+	8, 2, 2, 2	8, 2, 2, 2	8, 2, 2, 2	I_8, I_2, I_2, I_2	2 : 1, 3, 4
E3	1	1	1	-1440	16305	0	8	+	4, 4, 4, 4	4, 4, 4, 4	4, 2, 4, 4	I_4, I_4, I_4, I_4	2 : 2, 5, 6
E4	1	1	1	-21760	1226417	0	4	+	4, 1, 1, 1	4, 1, 1, 1	4, 1, 1, 1	I_4, I_1, I_1, I_1	2 : 2
E5	1	1	1	-7220	-224143	0	4	+	2, 8, 8, 2	2, 8, 8, 2	2, 2, 8, 2	I_2, I_8, I_8, I_2	2 : 3, 7, 8
E6	1	1	1	3060	102705	0	4	-	2, 2, 2, 8	2, 2, 2, 8	2, 2, 2, 8	I_2, I_2, I_2, I_8	2 : 3
E7	1	1	1	-113470	-14759143	0	2	+	1, 16, 4, 1	1, 16, 4, 1	1, 2, 4, 1	I_1, I_{16}, I_4, I_1	2 : 5
E8	1	1	1	6550	-962215	0	2	-	1, 4, 16, 1	1, 4, 16, 1	1, 2, 16, 1	I_1, I_4, I_{16}, I_1	2 : 5
F1	1	0	0	4	0	0	2	-	4, 1, 1, 1	4, 1, 1, 1	4, 1, 1, 1	I_4, I_1, I_1, I_1	2 : 2
F2	1	0	0	-16	-4	0	4	+	2, 2, 2, 2	2, 2, 2, 2	2, 2, 2, 2	I_2, I_2, I_2, I_2	2 : 1, 3, 4
F3	1	0	0	-186	-990	0	2	+	1, 4, 4, 1	1, 4, 4, 1	1, 4, 2, 1	I_1, I_4, I_4, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
510	$N = 510 = 2 \cdot 3 \cdot 5 \cdot 17$ (continued)											510	
G1	1	0	0	25	-375	0	6	-	6, 3, 3, 2	6, 3, 3, 2	6, 3, 3, 2	I_6, I_3, I_3, I_2	2 : 2; 3 : 3
G2	1	0	0	-655	-6223	0	6	+	3, 6, 6, 1	3, 6, 6, 1	3, 6, 6, 1	I_3, I_6, I_6, I_1	2 : 1; 3 : 4
G3	1	0	0	-3275	-72435	0	2	-	2, 1, 1, 6	2, 1, 1, 6	2, 1, 1, 2	I_2, I_1, I_1, I_6	2 : 4; 3 : 1
G4	1	0	0	-52405	-4621873	0	2	+	1, 2, 2, 3	1, 2, 2, 3	1, 2, 2, 1	I_1, I_2, I_2, I_3	2 : 3; 3 : 2
513	$N = 513 = 3^3 \cdot 19$ (2 isogeny classes)											513	
A1	1	-1	0	-42	-127	1	1	-	11, 1	0, 1	1, 1	II^*, I_1	
B1	1	-1	1	-5	6	1	1	-	5, 1	0, 1	3, 1	IV, I_1	
514	$N = 514 = 2 \cdot 257$ (2 isogeny classes)											514	
A1	1	-1	1	-91	-245	1	4	+	16, 1	16, 1	16, 1	I_{16}, I_1	2 : 2
A2	1	-1	1	-1371	-19189	1	4	+	8, 2	8, 2	8, 2	I_8, I_2	2 : 1, 3, 4
A3	1	-1	1	-21931	-1244565	1	2	+	4, 1	4, 1	4, 1	I_4, I_1	2 : 2
A4	1	-1	1	-1291	-21589	1	4	-	4, 4	4, 4	4, 4	I_4, I_4	2 : 2
B1	1	0	0	-4	0	1	2	+	4, 1	4, 1	4, 1	I_4, I_1	2 : 2
B2	1	0	0	16	4	1	2	-	2, 2	2, 2	2, 2	I_2, I_2	2 : 1
516	$N = 516 = 2^2 \cdot 3 \cdot 43$ (4 isogeny classes)											516	
A1	0	-1	0	-4	-8	0	1	-	8, 1, 1	0, 1, 1	1, 1, 1	IV^*, I_1, I_1	
B1	0	-1	0	11	-47	1	1	-	8, 4, 1	0, 4, 1	3, 2, 1	IV^*, I_4, I_1	
C1	0	1	0	-13	-28	0	2	-	4, 1, 2	0, 1, 2	3, 1, 2	IV, I_1, I_2	2 : 2
C2	0	1	0	-228	-1404	0	2	+	8, 2, 1	0, 2, 1	3, 2, 1	IV^*, I_2, I_1	2 : 1
D1	0	1	0	-44	-732	0	3	-	8, 9, 1	0, 9, 1	3, 9, 1	IV^*, I_9, I_1	3 : 2
D2	0	1	0	-7604	-257772	0	1	-	8, 3, 3	0, 3, 3	1, 3, 3	IV^*, I_3, I_3	3 : 1
517	$N = 517 = 11 \cdot 47$ (3 isogeny classes)											517	
A1	0	-1	1	36	-3	0	1	-	3, 2	3, 2	1, 2	I_3, I_2	
B1	0	0	1	-16	-26	0	1	-	1, 2	1, 2	1, 2	I_1, I_2	
C1	0	-1	1	-52	-3863	1	1	-	3, 4	3, 4	3, 4	I_3, I_4	
520	$N = 520 = 2^3 \cdot 5 \cdot 13$ (2 isogeny classes)											520	
A1	0	0	0	-23	42	1	2	+	8, 1, 1	0, 1, 1	2, 1, 1	I_1^*, I_1, I_1	2 : 2
A2	0	0	0	-43	-42	1	4	+	10, 2, 2	0, 2, 2	2, 2, 2	III^*, I_2, I_2	2 : 1, 3, 4
A3	0	0	0	-563	-5138	1	2	+	11, 4, 1	0, 4, 1	1, 2, 1	II^*, I_4, I_1	2 : 2
A4	0	0	0	157	-322	1	2	-	11, 1, 4	0, 1, 4	1, 1, 2	II^*, I_1, I_4	2 : 2
B1	0	-1	0	-20	-28	0	2	+	8, 1, 1	0, 1, 1	4, 1, 1	I_1^*, I_1, I_1	2 : 2
B2	0	-1	0	0	-100	0	2	-	10, 2, 2	0, 2, 2	2, 2, 2	III^*, I_2, I_2	2 : 1
522	$N = 522 = 2 \cdot 3^2 \cdot 29$ (13 isogeny classes)											522	
A1	1	-1	0	12	-208	1	1	-	5, 9, 1	5, 0, 1	1, 2, 1	I_5, III^*, I_1	
B1	1	-1	0	-2046	36244	0	2	-	22, 3, 1	22, 0, 1	2, 2, 1	I_{22}, III, I_1	2 : 2
B2	1	-1	0	-32766	2291092	0	2	+	11, 3, 2	11, 0, 2	1, 2, 2	I_{11}, III, I_2	2 : 1
C1	1	-1	0	-6	-54	0	3	-	1, 3, 3	1, 0, 3	1, 2, 3	I_1, III, I_3	3 : 2
C2	1	-1	0	-1311	-17947	0	1	-	3, 9, 1	3, 0, 1	1, 2, 1	I_3, III^*, I_1	3 : 1
D1	1	-1	0	-9	-3699	0	1	-	7, 13, 1	7, 7, 1	1, 2, 1	I_7, I_1^*, I_1	7 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
522	$N = 522 = 2 \cdot 3^2 \cdot 29$ (continued)												522
E1	1	-1	0	-45	139	1	1	-	1, 9, 1	1, 3, 1	1, 4, 1	I_1, I_3^*, I_1	
F1	1	-1	0	45	-203	1	1	-	10, 6, 1	10, 0, 1	2, 1, 1	I_{10}, I_0^*, I_1	5 : 2
F2	1	-1	0	-4095	102577	1	1	-	2, 6, 5	2, 0, 5	2, 1, 5	I_2, I_0^*, I_5	5 : 1
G1	1	-1	1	-18416	-960173	0	2	-	22, 9, 1	22, 0, 1	22, 2, 1	I_{22}, III^*, I_1	2 : 2
G2	1	-1	1	-294896	-61564589	0	2	+	11, 9, 2	11, 0, 2	11, 2, 2	I_{11}, III^*, I_2	2 : 1
H1	1	-1	1	-146	713	0	3	-	3, 3, 1	3, 0, 1	3, 2, 1	I_3, III, I_1	3 : 2
H2	1	-1	1	-56	1513	0	1	-	1, 9, 3	1, 0, 3	1, 2, 1	I_1, III^*, I_3	3 : 1
I1	1	-1	1	1	7	1	1	-	5, 3, 1	5, 0, 1	5, 2, 1	I_5, III, I_1	
J1	1	-1	1	-509	4677	1	1	-	13, 7, 1	13, 1, 1	13, 4, 1	I_{13}, I_1^*, I_1	
K1	1	-1	1	4	47	0	4	-	4, 7, 1	4, 1, 1	4, 4, 1	I_4, I_1^*, I_1	2 : 2
K2	1	-1	1	-176	911	0	4	+	2, 8, 2	2, 2, 2	2, 4, 2	I_2, I_2^*, I_2	2 : 1, 3, 4
K3	1	-1	1	-446	-2329	0	2	+	1, 7, 4	1, 1, 4	1, 2, 4	I_1, I_1^*, I_4	2 : 2
K4	1	-1	1	-2786	57287	0	2	+	1, 10, 1	1, 4, 1	1, 4, 1	I_1, I_4^*, I_1	2 : 2
L1	1	-1	1	-11	-17	0	1	-	2, 6, 1	2, 0, 1	2, 1, 1	I_2, I_0^*, I_1	
M1	1	-1	1	-69341	-33115291	0	1	-	11, 27, 1	11, 21, 1	11, 2, 1	I_{11}, I_{21}^*, I_1	3 : 2
M2	1	-1	1	619564	858878903	0	3	-	33, 13, 3	33, 7, 3	33, 2, 3	I_{33}, I_7^*, I_3	3 : 1

524	$N = 524 = 2^2 \cdot 131$ (1 isogeny class)												524
A1	0	1	0	-309	1991	1	1	-	8, 1	0, 1	1, 1	IV^*, I_1	

525	$N = 525 = 3 \cdot 5^2 \cdot 7$ (4 isogeny classes)												525
A1	1	1	1	-63	156	1	4	+	1, 7, 1	1, 1, 1	1, 4, 1	I_1, I_1^*, I_1	2 : 2
A2	1	1	1	-188	-844	1	4	+	2, 8, 2	2, 2, 2	2, 4, 2	I_2, I_2^*, I_2	2 : 1, 3, 4
A3	1	1	1	-2813	-58594	1	2	+	1, 10, 1	1, 4, 1	1, 4, 1	I_1, I_4^*, I_1	2 : 2
A4	1	1	1	437	-4594	1	2	-	4, 7, 4	4, 1, 4	2, 4, 2	I_4, I_1^*, I_4	2 : 2
B1	1	1	0	25	0	0	2	-	2, 6, 1	2, 0, 1	2, 2, 1	I_2, I_0^*, I_1	2 : 2
B2	1	1	0	-100	-125	0	4	+	4, 6, 2	4, 0, 2	2, 4, 2	I_4, I_0^*, I_2	2 : 1, 3, 4
B3	1	1	0	-1225	-17000	0	4	+	2, 6, 4	2, 0, 4	2, 4, 4	I_2, I_0^*, I_4	2 : 2, 5, 6
B4	1	1	0	-975	11250	0	2	+	8, 6, 1	8, 0, 1	2, 2, 1	I_8, I_0^*, I_1	2 : 2
B5	1	1	0	-19600	-1064375	0	2	+	1, 6, 2	1, 0, 2	1, 2, 2	I_1, I_0^*, I_2	2 : 3
B6	1	1	0	-850	-27125	0	2	-	1, 6, 8	1, 0, 8	1, 2, 8	I_1, I_0^*, I_8	2 : 3
C1	1	1	0	-450	3375	1	2	+	3, 9, 1	3, 0, 1	1, 2, 1	I_3, III^*, I_1	2 : 2
C2	1	1	0	175	12750	1	2	-	6, 9, 2	6, 0, 2	2, 2, 2	I_6, III^*, I_2	2 : 1
D1	1	0	0	-18	27	1	2	+	3, 3, 1	3, 0, 1	3, 2, 1	I_3, III, I_1	2 : 2
D2	1	0	0	7	102	1	2	-	6, 3, 2	6, 0, 2	6, 2, 2	I_6, III, I_2	2 : 1

528	$N = 528 = 2^4 \cdot 3 \cdot 11$ (10 isogeny classes)												528
A1	0	-1	0	-8	0	1	2	+	10, 1, 1	0, 1, 1	4, 1, 1	I_2^*, I_1, I_1	2 : 2
A2	0	-1	0	32	-32	1	2	-	11, 2, 2	0, 2, 2	4, 2, 2	I_3^*, I_2, I_2	2 : 1
B1	0	-1	0	1	-6	0	2	-	4, 4, 1	0, 4, 1	1, 2, 1	II, I_4, I_1	2 : 2
B2	0	-1	0	-44	-96	0	4	+	8, 2, 2	0, 2, 2	2, 2, 2	I_0^*, I_2, I_2	2 : 1, 3, 4
B3	0	-1	0	-704	-6960	0	2	+	10, 1, 1	0, 1, 1	4, 1, 1	I_2^*, I_1, I_1	2 : 2
B4	0	-1	0	-104	288	0	4	+	10, 1, 4	0, 1, 4	2, 1, 4	I_2^*, I_1, I_4	2 : 2
C1	0	-1	0	-8016	278928	0	2	+	10, 7, 1	0, 7, 1	4, 1, 1	I_7^*, I_7, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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528 $N = 528 = 2^4 \cdot 3 \cdot 11$ (continued)**528**

D1	0	1	0	-12	12	0	2	+	8, 1, 1	0, 1, 1	2, 1, 1	I_0^*, I_1, I_1	2 : 2
D2	0	1	0	-32	-60	0	4	+	10, 2, 2	0, 2, 2	4, 2, 2	I_2^*, I_2, I_2	2 : 1, 3, 4
D3	0	1	0	-472	-4108	0	2	+	11, 4, 1	0, 4, 1	2, 4, 1	I_3^*, I_4, I_1	2 : 2
D4	0	1	0	88	-300	0	2	-	11, 1, 4	0, 1, 4	4, 1, 2	I_3^*, I_1, I_4	2 : 2
E1	0	-1	0	3	0	0	2	-	4, 2, 1	0, 2, 1	1, 2, 1	II, I_2, I_1	2 : 2
E2	0	-1	0	-12	12	0	2	+	8, 1, 2	0, 1, 2	1, 1, 2	I_0^*, I_1, I_2	2 : 1
F1	0	-1	0	-720	-5184	0	2	+	22, 5, 1	10, 5, 1	4, 1, 1	I_{14}^*, I_5, I_1	2 : 2; 5 : 3
F2	0	-1	0	1840	-35904	0	2	-	17, 10, 2	5, 10, 2	2, 2, 2	I_9^*, I_{10}, I_2	2 : 1; 5 : 4
F3	0	-1	0	-161040	24927936	0	2	+	14, 1, 5	2, 1, 5	4, 1, 1	I_6^*, I_1, I_5	2 : 4; 5 : 1
F4	0	-1	0	-160880	24979776	0	2	-	13, 2, 10	1, 2, 10	2, 2, 2	I_5^*, I_2, I_{10}	2 : 3; 5 : 2
G1	0	-1	0	-88	-272	1	2	+	14, 3, 1	2, 3, 1	4, 1, 1	I_6^*, I_3, I_1	2 : 2; 3 : 3
G2	0	-1	0	72	-1296	1	2	-	13, 6, 2	1, 6, 2	4, 2, 2	I_5^*, I_6, I_2	2 : 1; 3 : 4
G3	0	-1	0	-1288	18160	1	2	+	18, 1, 3	6, 1, 3	4, 1, 3	I_{10}^*, I_1, I_3	2 : 4; 3 : 1
G4	0	-1	0	-648	35568	1	2	-	15, 2, 6	3, 2, 6	4, 2, 6	I_7^*, I_2, I_6	2 : 3; 3 : 2
H1	0	1	0	-104	372	1	2	+	12, 3, 1	0, 3, 1	4, 3, 1	I_4^*, I_3, I_1	2 : 2
H2	0	1	0	-184	-364	1	4	+	12, 6, 2	0, 6, 2	4, 6, 2	I_4^*, I_6, I_2	2 : 1, 3, 4
H3	0	1	0	-2344	-44428	1	2	+	12, 3, 4	0, 3, 4	2, 3, 2	I_4^*, I_3, I_4	2 : 2
H4	0	1	0	696	-2124	1	4	-	12, 12, 1	0, 12, 1	4, 12, 1	I_4^*, I_{12}, I_1	2 : 2
I1	0	1	0	-77	-330	0	2	-	4, 10, 1	0, 10, 1	1, 10, 1	II, I_{10}, I_1	2 : 2
I2	0	1	0	-1292	-18312	0	2	+	8, 5, 2	0, 5, 2	1, 5, 2	I_0^*, I_5, I_2	2 : 1
J1	0	1	0	-32	-12	0	2	+	16, 1, 1	4, 1, 1	4, 1, 1	I_8^*, I_1, I_1	2 : 2
J2	0	1	0	-352	2420	0	4	+	14, 2, 2	2, 2, 2	4, 2, 2	I_6^*, I_2, I_2	2 : 1, 3, 4
J3	0	1	0	-5632	160820	0	2	+	13, 1, 1	1, 1, 1	4, 1, 1	I_5^*, I_1, I_1	2 : 2
J4	0	1	0	-192	4788	0	4	-	13, 4, 4	1, 4, 4	2, 4, 4	I_5^*, I_4, I_4	2 : 2

530 $N = 530 = 2 \cdot 5 \cdot 53$ (4 isogeny classes)**530**

A1	1	0	1	-14	-188	0	3	-	2, 2, 3	2, 2, 3	2, 2, 3	I_2, I_2, I_3	3 : 2
A2	1	0	1	-2929	-61244	0	1	-	6, 6, 1	6, 6, 1	2, 2, 1	I_6, I_6, I_1	3 : 1
B1	1	-1	0	-4	0	1	2	+	4, 1, 1	4, 1, 1	2, 1, 1	I_4, I_1, I_1	2 : 2
B2	1	-1	0	16	-12	1	2	-	2, 2, 2	2, 2, 2	2, 2, 2	I_2, I_2, I_2	2 : 1
C1	1	-1	0	1226	30580	1	1	-	10, 10, 1	10, 10, 1	2, 10, 1	I_{10}, I_{10}, I_1	
D1	1	1	1	9	13	1	1	-	6, 2, 1	6, 2, 1	6, 2, 1	I_6, I_2, I_1	

532 $N = 532 = 2^2 \cdot 7 \cdot 19$ (1 isogeny class)**532**

A1	0	0	0	4	5	0	2	-	4, 2, 1	0, 2, 1	1, 2, 1	IV, I_2, I_1	2 : 2
A2	0	0	0	-31	54	0	2	+	8, 1, 2	0, 1, 2	1, 1, 2	IV^*, I_1, I_2	2 : 1

534 $N = 534 = 2 \cdot 3 \cdot 89$ (1 isogeny class)**534**

A1	1	1	1	-14	11	1	2	+	6, 2, 1	6, 2, 1	6, 2, 1	I_6, I_2, I_1	2 : 2
A2	1	1	1	26	107	1	2	-	3, 4, 2	3, 4, 2	3, 2, 2	I_3, I_4, I_2	2 : 1

537 $N = 537 = 3 \cdot 179$ (5 isogeny classes)**537**

A1	1	1	0	-120	909	0	1	-	13, 1	13, 1	1, 1	I_{13}, I_1	
B1	0	1	1	-75	-277	0	1	-	2, 1	2, 1	2, 1	I_2, I_1	
C1	0	1	1	13	5	0	3	-	6, 1	6, 1	6, 1	I_6, I_1	3 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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537 $N = 537 = 3 \cdot 179$ (continued) **537**

D1	1	0	1	1	-1	0	1	-	1, 1	1, 1	1, 1	I_{1, I_1}	
E1	0	1	1	-340	2308	0	5	-	10, 1	10, 1	10, 1	I_{10, I_1}	5 : 2
E2	0	1	1	2450	-39812	0	1	-	2, 5	2, 5	2, 1	I_{2, I_5}	5 : 1

539 $N = 539 = 7^2 \cdot 11$ (4 isogeny classes) **539**

A1	0	-1	1	-4377	-110013	0	1	-	8, 1	2, 1	2, 1	I_{2, I_1}^*	3 : 2
A2	0	-1	1	-2417	-210708	0	1	-	12, 3	6, 3	2, 1	I_{6, I_3}^*	3 : 1, 3
A3	0	-1	1	21593	5467657	0	1	-	8, 9	2, 9	2, 1	I_{2, I_9}^*	3 : 2
B1	0	0	1	98	-86	0	1	-	8, 1	2, 1	2, 1	I_{2, I_1}^*	
C1	1	0	1	170	-3237	1	2	-	9, 2	3, 2	4, 2	I_{3, I_2}^*	2 : 2
C2	1	0	1	-2525	-45279	1	2	+	12, 1	6, 1	4, 1	I_{6, I_1}^*	2 : 1
D1	0	1	1	-16	-66	1	1	-	6, 1	0, 1	2, 1	I_{0, I_1}^*	5 : 2
D2	0	1	1	-506	7774	1	1	-	6, 5	0, 5	2, 5	I_{0, I_5}^*	5 : 1, 3
D3	0	1	1	-383196	91174234	1	1	-	6, 1	0, 1	2, 1	I_{0, I_1}^*	5 : 2

540 $N = 540 = 2^2 \cdot 3^3 \cdot 5$ (6 isogeny classes) **540**

A1	0	0	0	-33	73	0	3	-	4, 3, 1	0, 0, 1	3, 1, 1	IV, II, I_1	3 : 2
A2	0	0	0	27	297	0	1	-	4, 9, 3	0, 0, 3	1, 1, 1	IV, IV^*, I_3	3 : 1
B1	0	0	0	3	1	1	1	-	4, 3, 1	0, 0, 1	1, 1, 1	IV, II, I_1	3 : 2
B2	0	0	0	-57	169	1	3	-	4, 5, 3	0, 0, 3	3, 3, 3	IV, IV, I_3	3 : 1
C1	0	0	0	-648	6372	1	3	-	8, 9, 2	0, 0, 2	3, 3, 2	IV^*, IV^*, I_2	3 : 2
C2	0	0	0	1512	33588	1	1	-	8, 11, 6	0, 0, 6	1, 1, 2	IV^*, II^*, I_6	3 : 1
D1	0	0	0	27	-27	1	3	-	4, 9, 1	0, 0, 1	3, 3, 1	IV, IV^*, I_1	3 : 2
D2	0	0	0	-513	-4563	1	1	-	4, 11, 3	0, 0, 3	1, 1, 1	IV, II^*, I_3	3 : 1
E1	0	0	0	-72	-236	0	1	-	8, 3, 2	0, 0, 2	1, 1, 2	IV^*, II, I_2	3 : 2
E2	0	0	0	168	-1244	0	3	-	8, 5, 6	0, 0, 6	3, 1, 6	IV^*, IV, I_6	3 : 1
F1	0	0	0	3	-11	0	3	-	4, 3, 3	0, 0, 3	3, 1, 3	IV, II, I_3	3 : 2
F2	0	0	0	-297	-1971	0	1	-	4, 9, 1	0, 0, 1	1, 3, 1	IV, IV^*, I_1	3 : 1

542 $N = 542 = 2 \cdot 271$ (2 isogeny classes) **542**

A1	1	1	1	-37	-149	0	2	-	14, 1	14, 1	14, 1	I_{14, I_1}	2 : 2
A2	1	1	1	-677	-7061	0	2	+	7, 2	7, 2	7, 2	I_{7, I_2}	2 : 1
B1	1	1	1	-8	9	1	1	-	7, 1	7, 1	7, 1	I_{7, I_1}	

544 $N = 544 = 2^5 \cdot 17$ (6 isogeny classes) **544**

A1	0	0	0	-5	4	1	2	+	6, 1	0, 1	2, 1	III, I_1	2 : 2
A2	0	0	0	5	18	1	2	-	9, 2	0, 2	1, 2	I_{0, I_2}^*	2 : 1
B1	0	-1	0	-22	48	0	2	+	6, 1	0, 1	2, 1	III, I_1	2 : 2
B2	0	-1	0	-17	65	0	2	-	12, 2	0, 2	2, 2	I_{3, I_2}^*	2 : 1
C1	0	1	0	-22	-48	0	2	+	6, 1	0, 1	2, 1	III, I_1	2 : 2
C2	0	1	0	-17	-65	0	2	-	12, 2	0, 2	2, 2	I_{3, I_2}^*	2 : 1
D1	0	0	0	-5	-4	0	2	+	6, 1	0, 1	2, 1	III, I_1	2 : 2
D2	0	0	0	5	-18	0	2	-	9, 2	0, 2	2, 2	I_{0, I_2}^*	2 : 1
E1	0	-1	0	-6	8	0	2	+	6, 1	0, 1	2, 1	III, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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544 $N = 544 = 2^5 \cdot 17$ (continued)**544**

F1	0	1	0	-6	-8	0	2	+	6, 1	0, 1	2, 1	III, I ₁	2 : 2
F2	0	1	0	-16	12	0	2	+	9, 2	0, 2	1, 2	I ₀ [*] , I ₂	2 : 1

545 $N = 545 = 5 \cdot 109$ (1 isogeny class)**545**

A1	1	-1	0	-284	1915	1	2	+	3, 1	3, 1	3, 1	I ₃ , I ₁	2 : 2
A2	1	-1	0	-289	1848	1	4	+	6, 2	6, 2	6, 2	I ₆ , I ₂	2 : 1, 3, 4
A3	1	-1	0	-914	-8277	1	2	+	3, 4	3, 4	3, 4	I ₃ , I ₄	2 : 2
A4	1	-1	0	256	7625	1	4	-	12, 1	12, 1	12, 1	I ₁₂ , I ₁	2 : 2

546 $N = 546 = 2 \cdot 3 \cdot 7 \cdot 13$ (7 isogeny classes)**546**

A1	1	1	0	-108	-486	0	1	-	1, 5, 3, 1	1, 5, 3, 1	1, 1, 1, 1	I ₁ , I ₅ , I ₃ , I ₁	
B1	1	0	1	-8	-10	0	1	-	5, 1, 1, 1	5, 1, 1, 1	1, 1, 1, 1	I ₅ , I ₁ , I ₁ , I ₁	
C1	1	0	1	-57	-164	1	2	+	8, 3, 1, 1	8, 3, 1, 1	2, 3, 1, 1	I ₈ , I ₃ , I ₁ , I ₁	2 : 2
C2	1	0	1	-137	380	1	4	+	4, 6, 2, 2	4, 6, 2, 2	2, 6, 2, 2	I ₄ , I ₆ , I ₂ , I ₂	2 : 1, 3, 4
C3	1	0	1	-1957	33140	1	4	+	2, 12, 1, 1	2, 12, 1, 1	2, 12, 1, 1	I ₂ , I ₁₂ , I ₁ , I ₁	2 : 2
C4	1	0	1	403	2756	1	2	-	2, 3, 4, 4	2, 3, 4, 4	2, 3, 2, 4	I ₂ , I ₃ , I ₄ , I ₄	2 : 2
D1	1	0	1	13	182	0	3	-	3, 9, 1, 1	3, 9, 1, 1	1, 9, 1, 1	I ₃ , I ₉ , I ₁ , I ₁	3 : 2
D2	1	0	1	-122	-4948	0	3	-	9, 3, 3, 3	9, 3, 3, 3	1, 3, 3, 3	I ₉ , I ₃ , I ₃ , I ₃	3 : 1, 3
D3	1	0	1	-26057	-1621108	0	1	-	27, 1, 1, 1	27, 1, 1, 1	1, 1, 1, 1	I ₂₇ , I ₁ , I ₁ , I ₁	3 : 2
E1	1	1	1	-100484	-12372091	0	1	-	17, 7, 1, 5	17, 7, 1, 5	17, 1, 1, 1	I ₁₇ , I ₇ , I ₁ , I ₅	
F1	1	0	0	714	-82908	0	7	-	7, 7, 7, 1	7, 7, 7, 1	7, 7, 7, 1	I ₇ , I ₇ , I ₇ , I ₁	7 : 2
F2	1	0	0	-3674496	-2711401518	0	1	-	1, 1, 1, 7	1, 1, 1, 7	1, 1, 1, 1	I ₁ , I ₁ , I ₁ , I ₇	7 : 1
G1	1	0	0	-7	-7	0	2	+	4, 1, 1, 1	4, 1, 1, 1	4, 1, 1, 1	I ₄ , I ₁ , I ₁ , I ₁	2 : 2
G2	1	0	0	-27	45	0	4	+	2, 2, 2, 2	2, 2, 2, 2	2, 2, 2, 2	I ₂ , I ₂ , I ₂ , I ₂	2 : 1, 3, 4
G3	1	0	0	-417	3243	0	2	+	1, 1, 4, 1	1, 1, 4, 1	1, 1, 4, 1	I ₁ , I ₁ , I ₄ , I ₁	2 : 2
G4	1	0	0	43	255	0	2	-	1, 4, 1, 4	1, 4, 1, 4	1, 4, 1, 2	I ₁ , I ₄ , I ₁ , I ₄	2 : 2

549 $N = 549 = 3^2 \cdot 61$ (3 isogeny classes)**549**

A1	1	-1	0	3	0	1	2	-	3, 1	0, 1	2, 1	III, I ₁	2 : 2
A2	1	-1	0	-12	9	1	2	+	3, 2	0, 2	2, 2	III, I ₂	2 : 1
B1	1	-1	1	25	-26	1	2	-	9, 1	0, 1	2, 1	III [*] , I ₁	2 : 2
B2	1	-1	1	-110	-134	1	2	+	9, 2	0, 2	2, 2	III [*] , I ₂	2 : 1
C1	1	-1	0	-18	-27	0	1	-	6, 1	0, 1	2, 1	I ₀ [*] , I ₁	

550 $N = 550 = 2 \cdot 5^2 \cdot 11$ (13 isogeny classes)**550**

A1	1	1	0	-25	125	1	1	-	3, 7, 1	3, 1, 1	1, 4, 1	I ₃ , I ₁ [*] , I ₁	3 : 2
A2	1	1	0	225	-3125	1	1	-	1, 9, 3	1, 3, 3	1, 4, 1	I ₁ , I ₃ [*] , I ₃	3 : 1
B1	1	0	1	249	-6102	0	1	-	5, 11, 1	5, 5, 1	1, 2, 1	I ₅ , I ₅ [*] , I ₁	5 : 2
B2	1	0	1	-148501	-22038602	0	1	-	1, 7, 5	1, 1, 5	1, 2, 5	I ₁ , I ₁ [*] , I ₅	5 : 1
C1	1	0	1	-206	-1152	0	1	-	11, 2, 1	11, 0, 1	1, 1, 1	I ₁₁ , II, I ₁	
D1	1	0	1	49	48	0	3	-	1, 8, 1	1, 0, 1	1, 3, 1	I ₁ , IV [*] , I ₁	3 : 2
D2	1	0	1	-576	-6202	0	1	-	3, 8, 3	3, 0, 3	1, 1, 1	I ₃ , IV [*] , I ₃	3 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
550 550													
$N = 550 = 2 \cdot 5^2 \cdot 11$ (continued)													
F1	1	0	1	-701	-7202	1	1	-	1, 9, 1	1, 0, 1	1, 2, 1	I_{1,III^*,I_1}	5 : 2
F2	1	0	1	4924	75298	1	1	-	5, 9, 5	5, 0, 5	1, 2, 5	I_{5,III^*,I_5}	5 : 1, 3
F3	1	0	1	-758201	254051548	1	1	-	25, 9, 1	25, 0, 1	1, 2, 1	I_{25,III^*,I_1}	5 : 2
G1	1	0	1	-6	8	1	2	-	4, 3, 1	4, 0, 1	2, 2, 1	I_{4,III,I_1}	2 : 2
G2	1	0	1	-106	408	1	2	+	2, 3, 2	2, 0, 2	2, 2, 2	I_{2,III,I_2}	2 : 1
H1	1	1	1	2	1	0	1	-	1, 2, 1	1, 0, 1	1, 1, 1	I_{1,II,I_1}	3 : 2
H2	1	1	1	-23	-59	0	1	-	3, 2, 3	3, 0, 3	3, 1, 1	I_{3,II,I_3}	3 : 1
I1	1	1	1	-2213	39531	1	1	-	7, 7, 3	7, 1, 3	7, 4, 3	I_{7,I_1^*,I_3}	3 : 2
I2	1	1	1	7412	212781	1	1	-	21, 9, 1	21, 3, 1	21, 4, 1	I_{21,I_3^*,I_1}	3 : 1
J1	1	-1	1	-15	87	1	1	-	11, 3, 1	11, 0, 1	11, 2, 1	I_{11,III,I_1}	
K1	1	1	1	-28	-69	0	1	-	1, 3, 1	1, 0, 1	1, 2, 1	I_{1,III,I_1}	5 : 2
K2	1	1	1	197	681	0	5	-	5, 3, 5	5, 0, 5	5, 2, 5	I_{5,III,I_5}	5 : 1, 3
K3	1	1	1	-30328	2020281	0	5	-	25, 3, 1	25, 0, 1	25, 2, 1	I_{25,III,I_1}	5 : 2
L1	1	1	1	-138	1031	0	2	-	4, 9, 1	4, 0, 1	4, 2, 1	I_{4,III^*,I_1}	2 : 2
L2	1	1	1	-2638	51031	0	2	+	2, 9, 2	2, 0, 2	2, 2, 2	I_{2,III^*,I_2}	2 : 1
M1	1	1	1	-5138	-143969	0	1	-	11, 8, 1	11, 0, 1	11, 1, 1	I_{11,IV^*,I_1}	

551 551													
$N = 551 = 19 \cdot 29$ (4 isogeny classes)													
A1	1	0	1	1	-5	1	1	-	2, 1	2, 1	2, 1	I_{2,I_1}	
B1	1	0	0	-11	14	1	1	-	2, 1	2, 1	2, 1	I_{2,I_1}	
C1	0	1	1	-2376	-61851	1	1	-	7, 2	7, 2	7, 2	I_{7,I_2}	
D1	0	1	1	-116	444	1	1	-	1, 2	1, 2	1, 2	I_{1,I_2}	

552 552													
$N = 552 = 2^3 \cdot 3 \cdot 23$ (5 isogeny classes)													
A1	0	-1	0	-64	-260	1	2	-	10, 6, 1	0, 6, 1	2, 2, 1	III^*,I_6,I_1	2 : 2
A2	0	-1	0	-1144	-14516	1	2	+	11, 3, 2	0, 3, 2	1, 1, 2	II^*,I_3,I_2	2 : 1
B1	0	-1	0	-2908	61876	0	2	-	8, 14, 1	0, 14, 1	2, 2, 1	I_{1^*,I_{14},I_1}	2 : 2
B2	0	-1	0	-46648	3893500	0	2	+	10, 7, 2	0, 7, 2	2, 1, 2	III^*,I_7,I_2	2 : 1
C1	0	-1	0	4	-12	0	2	-	8, 2, 1	0, 2, 1	2, 2, 1	I_{1^*,I_2,I_1}	2 : 2
C2	0	-1	0	-56	-132	0	2	+	10, 1, 2	0, 1, 2	2, 1, 2	III^*,I_1,I_2	2 : 1
D1	0	-1	0	-207	-1080	1	2	+	4, 3, 1	0, 3, 1	2, 1, 1	III,I_3,I_1	2 : 2
D2	0	-1	0	-212	-1020	1	4	+	8, 6, 2	0, 6, 2	4, 2, 2	I_{1^*,I_6,I_2}	2 : 1, 3, 4
D3	0	-1	0	-752	6972	1	4	+	10, 3, 4	0, 3, 4	2, 1, 4	III^*,I_3,I_4	2 : 2
D4	0	-1	0	248	-5252	1	2	-	10, 12, 1	0, 12, 1	2, 2, 1	III^*,I_{12},I_1	2 : 2
E1	0	1	0	-4	32	1	4	-	8, 4, 1	0, 4, 1	4, 4, 1	I_{1^*,I_4,I_1}	2 : 2
E2	0	1	0	-184	896	1	4	+	10, 2, 2	0, 2, 2	2, 2, 2	III^*,I_2,I_2	2 : 1, 3, 4
E3	0	1	0	-304	-544	1	2	+	11, 1, 4	0, 1, 4	1, 1, 2	II^*,I_1,I_4	2 : 2
E4	0	1	0	-2944	60512	1	2	+	11, 1, 1	0, 1, 1	1, 1, 1	II^*,I_1,I_1	2 : 2

555 555													
$N = 555 = 3 \cdot 5 \cdot 37$ (2 isogeny classes)													
A1	0	1	1	-1	-29	0	1	-	1, 5, 1	1, 5, 1	1, 1, 1	I_{1,I_5,I_1}	
B1	0	1	1	-2405	-47869	0	3	-	15, 3, 1	15, 3, 1	15, 3, 1	I_{15,I_3,I_1}	3 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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556 $N = 556 = 2^2 \cdot 139$ (1 isogeny class) **556**

A1	0	0	0	-8	9	1	1	-	4, 1	0, 1	3, 1	IV, I ₁	
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557 $N = 557 = 557$ (2 isogeny classes) **557**

A1	1	1	0	0	1	1	1	-	1	1	1	I ₁	
B1	0	-1	1	-268	1781	0	1	+	1	1	1	I ₁	

558 $N = 558 = 2 \cdot 3^2 \cdot 31$ (8 isogeny classes) **558**

A1	1	-1	0	0	2	1	1	-	1, 3, 1	1, 0, 1	1, 2, 1	I ₁ , III, I ₁	
B1	1	-1	0	-48	288	0	3	-	5, 3, 3	5, 0, 3	1, 2, 3	I ₅ , III, I ₃	3 : 2
B2	1	-1	0	417	-6067	0	1	-	15, 9, 1	15, 0, 1	1, 2, 1	I ₁₅ , III*, I ₁	3 : 1
C1	1	-1	0	-6	-28	0	2	-	4, 6, 1	4, 0, 1	2, 2, 1	I ₄ , I ₀ *	2 : 2
C2	1	-1	0	-186	-928	0	4	+	2, 6, 2	2, 0, 2	2, 4, 2	I ₂ , I ₀ *	2 : 1, 3, 4
C3	1	-1	0	-2976	-61750	0	2	+	1, 6, 1	1, 0, 1	1, 2, 1	I ₁ , I ₀ *	2 : 2
C4	1	-1	0	-276	134	0	2	+	1, 6, 4	1, 0, 4	1, 2, 2	I ₁ , I ₀ *	2 : 2
D1	1	-1	0	135	-243	1	1	-	5, 11, 1	5, 5, 1	1, 4, 1	I ₅ , I ₅ *	5 : 2
D2	1	-1	0	-12555	544887	1	1	-	1, 7, 5	1, 1, 5	1, 4, 5	I ₁ , I ₁ *	5 : 1
E1	1	-1	1	-2	-53	0	1	-	1, 9, 1	1, 0, 1	1, 2, 1	I ₁ , III*, I ₁	
F1	1	-1	1	46	209	1	3	-	15, 3, 1	15, 0, 1	15, 2, 1	I ₁₅ , III, I ₁	3 : 2
F2	1	-1	1	-434	-7343	1	1	-	5, 9, 3	5, 0, 3	5, 2, 3	I ₅ , III*, I ₃	3 : 1
G1	1	-1	1	-149	749	1	1	-	7, 7, 1	7, 1, 1	7, 4, 1	I ₇ , I ₁ *	
H1	1	-1	1	-752	9213	0	1	-	1, 17, 1	1, 11, 1	1, 2, 1	I ₁ , I ₁₁ *	

560 $N = 560 = 2^4 \cdot 5 \cdot 7$ (6 isogeny classes) **560**

A1	0	1	0	-1	-5	0	1	-	8, 1, 1	0, 1, 1	1, 1, 1	I ₀ *	I ₁ , I ₁	
B1	0	0	0	-412	-3316	0	1	-	8, 5, 3	0, 5, 3	1, 5, 1	I ₀ *	I ₅ , I ₃	
C1	0	-1	0	-21	-35	0	1	-	12, 1, 1	0, 1, 1	1, 1, 1	II*	I ₁ , I ₁	3 : 2
C2	0	-1	0	139	61	0	1	-	12, 3, 3	0, 3, 3	1, 1, 1	II*	I ₃ , I ₃	3 : 1, 3
C3	0	-1	0	-2101	39485	0	1	-	12, 9, 1	0, 9, 1	1, 1, 1	II*	I ₉ , I ₁	3 : 2
D1	0	0	0	37	138	1	2	-	16, 2, 1	4, 2, 1	4, 2, 1	I ₈ *	I ₂ , I ₁	2 : 2
D2	0	0	0	-283	1482	1	4	+	14, 4, 2	2, 4, 2	4, 2, 2	I ₆ *	I ₄ , I ₂	2 : 1, 3, 4
D3	0	0	0	-1403	-18902	1	2	+	13, 8, 1	1, 8, 1	2, 2, 1	I ₅ *	I ₈ , I ₁	2 : 2
D4	0	0	0	-4283	107882	1	4	+	13, 2, 4	1, 2, 4	4, 2, 4	I ₅ *	I ₂ , I ₄	2 : 2
E1	0	0	0	32	-212	1	1	-	8, 1, 5	0, 1, 5	2, 1, 5	I ₀ *	I ₁ , I ₅	
F1	0	-1	0	-5	25	1	1	-	8, 3, 1	0, 3, 1	2, 3, 1	I ₀ *	I ₃ , I ₁	3 : 2
F2	0	-1	0	-805	9065	1	1	-	8, 1, 3	0, 1, 3	2, 1, 1	I ₀ *	I ₁ , I ₃	3 : 1

561 $N = 561 = 3 \cdot 11 \cdot 17$ (4 isogeny classes) **561**

A1	0	-1	1	-3729	-86416	0	1	-	10, 1, 1	10, 1, 1	2, 1, 1	I ₁₀	I ₁ , I ₁	
B1	0	1	1	-269	1628	1	1	-	2, 5, 1	2, 5, 1	2, 5, 1	I ₂	I ₅ , I ₁	
C1	0	1	1	-8	8	1	1	-	4, 1, 1	4, 1, 1	4, 1, 1	I ₄	I ₁ , I ₁	
D1	1	0	0	-12	15	0	2	+	1, 1, 1	1, 1, 1	1, 1, 1	I ₁	I ₁ , I ₁	2 : 2
D2	1	0	0	-17	0	0	4	+	2, 2, 2	2, 2, 2	2, 2, 2	I ₂	I ₂ , I ₂	2 : 1, 3, 4
D3	1	0	0	-182	-957	0	2	+	1, 1, 4	1, 1, 4	1, 1, 4	I ₁	I ₁ , I ₁	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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562 $N = 562 = 2 \cdot 281$ (1 isogeny class) **562**

A1	1	1	0	-4	0	0	2	+	4, 1	4, 1	2, 1	I_4, I_1	2 : 2
A2	1	1	0	16	20	0	2	-	2, 2	2, 2	2, 2	I_2, I_2	2 : 1

563 $N = 563 = 563$ (1 isogeny class) **563**

A1	1	1	1	-15	16	2	1	-	1	1	1	I_1	
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564 $N = 564 = 2^2 \cdot 3 \cdot 47$ (2 isogeny classes) **564**

A1	0	-1	0	-221	-1191	1	1	+	8, 5, 1	0, 5, 1	1, 1, 1	IV^*, I_5, I_1	
B1	0	1	0	-37	71	1	3	+	8, 3, 1	0, 3, 1	3, 3, 1	IV^*, I_3, I_1	3 : 2
B2	0	1	0	-517	-4681	1	1	+	8, 1, 3	0, 1, 3	1, 1, 1	IV^*, I_1, I_3	3 : 1

565 $N = 565 = 5 \cdot 113$ (1 isogeny class) **565**

A1	1	0	1	-19	-33	0	1	-	3, 1	3, 1	1, 1	I_3, I_1	
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566 $N = 566 = 2 \cdot 283$ (2 isogeny classes) **566**

A1	1	-1	0	-2	4	1	1	-	4, 1	4, 1	2, 1	I_4, I_1	
B1	1	0	0	1	-1	0	1	-	1, 1	1, 1	1, 1	I_1, I_1	

567 $N = 567 = 3^4 \cdot 7$ (2 isogeny classes) **567**

A1	1	-1	0	0	-3	1	1	-	4, 2	0, 2	1, 2	II, I_2	
B1	1	-1	1	-2	82	1	1	-	10, 2	0, 2	3, 2	IV^*, I_2	

568 $N = 568 = 2^3 \cdot 71$ (1 isogeny class) **568**

A1	0	-1	0	-72	-212	0	1	+	11, 1	0, 1	1, 1	II^*, I_1	
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570 $N = 570 = 2 \cdot 3 \cdot 5 \cdot 19$ (13 isogeny classes) **570**

A1	1	1	0	-98	372	1	2	-	8, 3, 1, 2	8, 3, 1, 2	2, 1, 1, 2	I_8, I_3, I_1, I_2	2 : 2
A2	1	1	0	-1618	24388	1	2	+	4, 6, 2, 1	4, 6, 2, 1	2, 2, 2, 1	I_4, I_6, I_2, I_1	2 : 1
B1	1	1	0	-78	-972	0	2	-	14, 2, 3, 1	14, 2, 3, 1	2, 2, 1, 1	I_{14}, I_2, I_3, I_1	2 : 2
B2	1	1	0	-1998	-35148	0	2	+	7, 1, 6, 2	7, 1, 6, 2	1, 1, 2, 2	I_7, I_1, I_6, I_2	2 : 1
C1	1	1	0	-17	69	1	2	-	4, 1, 3, 2	4, 1, 3, 2	2, 1, 3, 2	I_4, I_1, I_3, I_2	2 : 2
C2	1	1	0	-397	2881	1	2	+	2, 2, 6, 1	2, 2, 6, 1	2, 2, 6, 1	I_2, I_2, I_6, I_1	2 : 1
D1	1	0	1	3676	-514654	0	2	-	28, 5, 1, 2	28, 5, 1, 2	2, 5, 1, 2	I_{28}, I_5, I_1, I_2	2 : 2
D2	1	0	1	-78244	-7985758	0	4	+	14, 10, 2, 4	14, 10, 2, 4	2, 10, 2, 2	I_{14}, I_{10}, I_2, I_4	2 : 1, 3, 4
D3	1	0	1	-1233444	-527363678	0	2	+	7, 20, 1, 2	7, 20, 1, 2	1, 20, 1, 2	I_7, I_{20}, I_1, I_2	2 : 2
D4	1	0	1	-233764	33569186	0	2	+	7, 5, 4, 8	7, 5, 4, 8	1, 5, 2, 2	I_7, I_5, I_4, I_8	2 : 2
E1	1	0	1	12	-14	1	2	-	8, 2, 1, 1	8, 2, 1, 1	2, 2, 1, 1	I_8, I_2, I_1, I_1	2 : 2
E2	1	0	1	-68	-142	1	4	+	4, 4, 2, 2	4, 4, 2, 2	2, 4, 2, 2	I_4, I_4, I_2, I_2	2 : 1, 3, 4
E3	1	0	1	-968	-11662	1	2	+	2, 2, 1, 4	2, 2, 1, 4	2, 2, 1, 2	I_2, I_2, I_1, I_4	2 : 2
E4	1	0	1	-448	3506	1	4	+	2, 8, 4, 1	2, 8, 4, 1	2, 8, 4, 1	I_2, I_8, I_4, I_1	2 : 2
F1	1	0	1	-23	506	0	6	-	6, 6, 3, 1	6, 6, 3, 1	2, 6, 3, 1	I_6, I_6, I_3, I_1	2 : 2; 3 : 3
F2	1	0	1	-1103	13898	0	6	+	3, 3, 6, 2	3, 3, 6, 2	1, 3, 6, 2	I_3, I_3, I_6, I_2	2 : 1; 3 : 4
F3	1	0	1	202	-13624	0	2	-	18, 2, 1, 3	18, 2, 1, 3	2, 2, 1, 3	I_{18}, I_2, I_1, I_2	2 : 4; 3 : 1

TABLE 1: ELLIPTIC CURVES 570G–574C

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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570 **570**
 $N = 570 = 2 \cdot 3 \cdot 5 \cdot 19$ (continued)

G1	1	1	1	-31	53	0	4	+	4, 1, 2, 1	4, 1, 2, 1	4, 1, 2, 1	I_4, I_1, I_2, I_1	2 : 2
G2	1	1	1	-51	-51	0	4	+	2, 2, 4, 2	2, 2, 4, 2	2, 2, 2, 2	I_2, I_2, I_4, I_2	2 : 1, 3, 4
G3	1	1	1	-621	-6207	0	2	+	1, 1, 8, 1	1, 1, 8, 1	1, 1, 2, 1	I_1, I_1, I_8, I_1	2 : 2
G4	1	1	1	199	-151	0	2	-	1, 4, 2, 4	1, 4, 2, 4	1, 2, 2, 2	I_1, I_4, I_2, I_4	2 : 2
H1	1	1	1	0	-3	0	2	-	2, 2, 1, 1	2, 2, 1, 1	2, 2, 1, 1	I_2, I_2, I_1, I_1	2 : 2
H2	1	1	1	-30	-75	0	2	+	1, 1, 2, 2	1, 1, 2, 2	1, 1, 2, 2	I_1, I_1, I_2, I_2	2 : 1
I1	1	1	1	-1900	32525	0	4	-	8, 5, 1, 4	8, 5, 1, 4	8, 1, 1, 4	I_8, I_5, I_1, I_4	2 : 2
I2	1	1	1	-30780	2065677	0	4	+	4, 10, 2, 2	4, 10, 2, 2	4, 2, 2, 2	I_4, I_{10}, I_2, I_2	2 : 1, 3, 4
I3	1	1	1	-31160	2011565	0	2	+	2, 20, 4, 1	2, 20, 4, 1	2, 2, 4, 1	I_2, I_{20}, I_4, I_1	2 : 2
I4	1	1	1	-492480	132819117	0	2	+	2, 5, 1, 1	2, 5, 1, 1	2, 1, 1, 1	I_2, I_5, I_1, I_1	2 : 2
J1	1	0	0	-1456	-21604	0	2	-	2, 14, 1, 1	2, 14, 1, 1	2, 14, 1, 1	I_2, I_{14}, I_1, I_1	2 : 2
J2	1	0	0	-23326	-1373170	0	2	+	1, 7, 2, 2	1, 7, 2, 2	1, 7, 2, 2	I_1, I_7, I_2, I_2	2 : 1
K1	1	0	0	-25871	1614201	0	6	-	24, 3, 3, 2	24, 3, 3, 2	24, 3, 1, 2	I_{24}, I_3, I_3, I_2	2 : 2; 3 : 3
K2	1	0	0	-414991	102863225	0	6	+	12, 6, 6, 1	12, 6, 6, 1	12, 6, 2, 1	I_{12}, I_6, I_6, I_1	2 : 1; 3 : 4
K3	1	0	0	85489	8420985	0	2	-	8, 1, 9, 6	8, 1, 9, 6	8, 1, 1, 6	I_8, I_1, I_9, I_6	2 : 4; 3 : 1
K4	1	0	0	-463231	77449961	0	2	+	4, 2, 18, 3	4, 2, 18, 3	4, 2, 2, 3	I_4, I_2, I_{18}, I_3	2 : 3; 3 : 2
L1	1	0	0	9335	-737383	0	10	-	20, 5, 5, 2	20, 5, 5, 2	20, 5, 5, 2	I_{20}, I_5, I_5, I_2	2 : 2; 5 : 3
L2	1	0	0	-87945	-8655975	0	10	+	10, 10, 10, 1	10, 10, 10, 1	10, 10, 10, 1	$I_{10}, I_{10}, I_{10}, I_1$	2 : 1; 5 : 4
L3	1	0	0	-3301465	-2309192023	0	2	-	4, 1, 1, 10	4, 1, 1, 10	4, 1, 1, 2	I_4, I_1, I_1, I_{10}	2 : 4; 5 : 1
L4	1	0	0	-52823445	-147775056075	0	2	+	2, 2, 2, 5	2, 2, 2, 5	2, 2, 2, 1	I_2, I_2, I_2, I_5	2 : 3; 5 : 2
M1	1	0	0	-10	20	0	4	-	4, 4, 1, 1	4, 4, 1, 1	4, 4, 1, 1	I_4, I_4, I_1, I_1	2 : 2
M2	1	0	0	-190	992	0	4	+	2, 2, 2, 2	2, 2, 2, 2	2, 2, 2, 2	I_2, I_2, I_2, I_2	2 : 1, 3, 4
M3	1	0	0	-220	650	0	2	+	1, 1, 4, 4	1, 1, 4, 4	1, 1, 4, 2	I_1, I_1, I_4, I_4	2 : 2
M4	1	0	0	-3040	64262	0	2	+	1, 1, 1, 1	1, 1, 1, 1	1, 1, 1, 1	I_1, I_1, I_1, I_1	2 : 2

571 **571**
 $N = 571 = 571$ (2 isogeny classes)

A1	0	-1	1	-929	-10595	0	1	-	1	1	1	I_1	
B1	0	1	1	-4	2	2	1	-	1	1	1	I_1	

572 **572**
 $N = 572 = 2^2 \cdot 11 \cdot 13$ (1 isogeny class)

A1	0	1	0	91	-121	0	3	-	8, 3, 2	0, 3, 2	3, 3, 2	IV^*, I_3, I_2	3 : 2
A2	0	1	0	-1669	-27401	0	1	-	8, 1, 6	0, 1, 6	1, 1, 6	IV^*, I_1, I_6	3 : 1

573 **573**
 $N = 573 = 3 \cdot 191$ (3 isogeny classes)

A1	1	0	0	3	0	0	2	-	2, 1	2, 1	2, 1	I_2, I_1	2 : 2
A2	1	0	0	-12	-3	0	2	+	1, 2	1, 2	1, 2	I_1, I_2	2 : 1
B1	0	1	1	-1422	-21121	0	1	+	5, 1	5, 1	5, 1	I_5, I_1	
C1	0	1	1	-4	-2	1	1	+	3, 1	3, 1	3, 1	I_3, I_1	

574 **574**
 $N = 574 = 2 \cdot 7 \cdot 41$ (10 isogeny classes)

A1	1	1	0	-2	-2	1	1	+	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	
B1	1	1	0	-2061	35165	1	2	+	10, 4, 1	10, 4, 1	2, 2, 1	I_{10}, I_4, I_1	2 : 2
B2	1	1	0	-2221	29181	1	2	+	5, 8, 2	5, 8, 2	1, 2, 2	I_5, I_8, I_2	2 : 1
C1	1	1	0	-84	80	0	2	+	14, 2, 1	14, 2, 1	2, 2, 1	I_{14}, I_2, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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574

$N = 574 = 2 \cdot 7 \cdot 41$ (continued)

574

D1	1	0	1	-31679	5254674	0	2	-	34, 3, 2	34, 3, 2	2, 3, 2	I_{34}, I_3, I_2	2 : 2
D2	1	0	1	-687039	218902034	0	2	+	17, 6, 4	17, 6, 4	1, 6, 2	I_{17}, I_6, I_4	2 : 1
E1	1	-1	0	-40	-88	0	1	+	3, 1, 1	3, 1, 1	1, 1, 1	I_3, I_1, I_1	
F1	1	0	1	-80	190	1	3	+	5, 1, 3	5, 1, 3	1, 1, 3	I_5, I_1, I_3	3 : 2
F2	1	0	1	-2335	-43598	1	1	+	15, 3, 1	15, 3, 1	1, 3, 1	I_{15}, I_3, I_1	3 : 1
G1	1	1	1	-21	-5	1	1	+	11, 1, 1	11, 1, 1	11, 1, 1	I_{11}, I_1, I_1	
H1	1	-1	1	3	5	1	2	-	6, 1, 1	6, 1, 1	6, 1, 1	I_6, I_1, I_1	2 : 2
H2	1	-1	1	-37	85	1	2	+	3, 2, 2	3, 2, 2	3, 2, 2	I_3, I_2, I_2	2 : 1
I1	1	-1	1	-19353	958713	1	7	+	21, 7, 1	21, 7, 1	21, 7, 1	I_{21}, I_7, I_1	7 : 2
I2	1	-1	1	-9611313	-11466507927	1	1	+	3, 1, 7	3, 1, 7	3, 1, 1	I_3, I_1, I_7	7 : 1
J1	1	1	1	-175	789	0	5	+	5, 5, 1	5, 5, 1	5, 5, 1	I_5, I_5, I_1	5 : 2
J2	1	1	1	-15785	-769911	0	1	+	1, 1, 5	1, 1, 5	1, 1, 5	I_1, I_1, I_5	5 : 1

575

$N = 575 = 5^2 \cdot 23$ (5 isogeny classes)

575

A1	1	-1	0	-2	1	1	1	+	2, 1	0, 1	1, 1	II, I_1	
B1	0	0	1	175	-1344	1	1	-	11, 1	5, 1	4, 1	I_5^*, I_1	
C1	0	-1	1	-458	3943	0	1	-	9, 1	0, 1	2, 1	III^*, I_1	
D1	1	-1	1	-55	72	1	1	+	8, 1	0, 1	3, 1	IV^*, I_1	
E1	0	1	1	-18	24	1	1	-	3, 1	0, 1	2, 1	III, I_1	

576

$N = 576 = 2^6 \cdot 3^2$ (9 isogeny classes)

576

A1	0	0	0	0	8	1	2	-	10, 3	0, 0	2, 2	I_0^*, III	2 : 2; 3 : 3
A2	0	0	0	-60	176	1	2	+	14, 3	0, 0	4, 2	I_4^*, III	2 : 1; 3 : 4
A3	0	0	0	0	-216	1	2	-	10, 9	0, 0	2, 2	I_0^*, III^*	2 : 4; 3 : 1
A4	0	0	0	-540	-4752	1	2	+	14, 9	0, 0	4, 2	I_4^*, III^*	2 : 3; 3 : 2
B1	0	0	0	-39	-92	0	2	+	6, 7	0, 1	1, 4	II, I_1^*	2 : 2
B2	0	0	0	-84	160	0	4	+	12, 8	0, 2	4, 4	I_2^*, I_2^*	2 : 1, 3, 4
B3	0	0	0	-1164	15280	0	2	+	15, 7	0, 1	2, 2	I_5^*, I_1^*	2 : 2
B4	0	0	0	276	1168	0	2	-	15, 10	0, 4	2, 4	I_5^*, I_4^*	2 : 2
C1	0	0	0	-39	92	0	2	+	6, 7	0, 1	1, 2	II, I_1^*	2 : 2
C2	0	0	0	-84	-160	0	4	+	12, 8	0, 2	4, 4	I_2^*, I_2^*	2 : 1, 3, 4
C3	0	0	0	-1164	-15280	0	2	+	15, 7	0, 1	2, 4	I_5^*, I_1^*	2 : 2
C4	0	0	0	276	-1168	0	2	-	15, 10	0, 4	2, 4	I_5^*, I_4^*	2 : 2
D1	0	0	0	24	-56	0	2	-	10, 7	0, 1	2, 2	I_0^*, I_1^*	2 : 2
D2	0	0	0	-156	-560	0	4	+	14, 8	0, 2	4, 4	I_4^*, I_2^*	2 : 1, 3, 4
D3	0	0	0	-2316	-42896	0	2	+	16, 7	0, 1	2, 4	I_6^*, I_1^*	2 : 2
D4	0	0	0	-876	9520	0	4	+	16, 10	0, 4	4, 4	I_6^*, I_4^*	2 : 2, 5, 6
D5	0	0	0	-13836	626416	0	2	+	17, 8	0, 2	2, 2	I_7^*, I_2^*	2 : 4
D6	0	0	0	564	37744	0	2	-	17, 14	0, 8	2, 4	I_7^*, I_8^*	2 : 4
E1	0	0	0	0	-8	0	2	-	10, 3	0, 0	2, 2	I_0^*, III	2 : 2; 3 : 3
E2	0	0	0	-60	-176	0	2	+	14, 3	0, 0	2, 2	I_4^*, III	2 : 1; 3 : 4
E3	0	0	0	0	216	0	2	-	10, 9	0, 0	2, 2	I_0^*, III^*	2 : 4; 3 : 1
E4	0	0	0	-540	4752	0	2	+	14, 9	0, 0	2, 2	I_4^*, III^*	2 : 3; 3 : 2
F1	0	0	0	-3	0	0	2	+	6, 3	0, 0	1, 2	II, III	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
576 576													
$N = 576 = 2^6 \cdot 3^2$ (continued)													
G1	0	0	0	-27	0	0	2	+	6, 9	0, 0	1, 2	II, III*	2 : 2
G2	0	0	0	108	0	0	2	-	12, 9	0, 0	2, 2	I ₂ *, III*	2 : 1
<hr style="border-top: 1px dashed black;"/>													
H1	0	0	0	9	0	1	2	-	6, 6	0, 0	1, 2	II, I ₀ *	2 : 2
H2	0	0	0	-36	0	1	4	+	12, 6	0, 0	4, 4	I ₂ *, I ₀ *	2 : 1, 3, 4
H3	0	0	0	-396	-3024	1	2	+	15, 6	0, 0	2, 2	I ₅ *, I ₀ *	2 : 2
H4	0	0	0	-396	3024	1	2	+	15, 6	0, 0	4, 2	I ₅ *, I ₀ *	2 : 2
<hr style="border-top: 1px dashed black;"/>													
I1	0	0	0	24	56	1	2	-	10, 7	0, 1	2, 4	I ₀ *, I ₁ *	2 : 2
I2	0	0	0	-156	560	1	4	+	14, 8	0, 2	4, 4	I ₄ *, I ₂ *	2 : 1, 3, 4
I3	0	0	0	-876	-9520	1	4	+	16, 10	0, 4	4, 4	I ₆ *, I ₄ *	2 : 2, 5, 6
I4	0	0	0	-2316	42896	1	2	+	16, 7	0, 1	4, 2	I ₆ *, I ₁ *	2 : 2
I5	0	0	0	-13836	-626416	1	2	+	17, 8	0, 2	2, 2	I ₇ *, I ₂ *	2 : 3
I6	0	0	0	564	-37744	1	2	-	17, 14	0, 8	4, 4	I ₇ *, I ₈ *	2 : 3
<hr style="border-top: 1px dashed black;"/>													
578 578													
$N = 578 = 2 \cdot 17^2$ (1 isogeny class)													
A1	1	1	1	-873	5783	0	2	+	6, 7	6, 1	6, 2	I ₆ , I ₁ *	2 : 2; 3 : 3
A2	1	1	1	-12433	528295	0	2	+	3, 8	3, 2	3, 4	I ₃ , I ₂ *	2 : 1; 3 : 4
A3	1	1	1	-29773	-1989473	0	2	+	2, 9	2, 3	2, 2	I ₂ , I ₃ *	2 : 4; 3 : 1
A4	1	1	1	-32663	-1583717	0	2	+	1, 12	1, 6	1, 4	I ₁ , I ₆ *	2 : 3; 3 : 2
<hr style="border-top: 1px dashed black;"/>													
579 579													
$N = 579 = 3 \cdot 193$ (2 isogeny classes)													
A1	0	-1	1	-2	11	0	1	-	5, 1	5, 1	1, 1	I ₅ , I ₁	
<hr style="border-top: 1px dashed black;"/>													
B1	1	0	0	-3	0	1	2	+	2, 1	2, 1	2, 1	I ₂ , I ₁	2 : 2
B2	1	0	0	12	3	1	2	-	1, 2	1, 2	1, 2	I ₁ , I ₂	2 : 1
<hr style="border-top: 1px dashed black;"/>													
580 580													
$N = 580 = 2^2 \cdot 5 \cdot 29$ (2 isogeny classes)													
A1	0	0	0	-8	-7	1	2	+	4, 2, 1	0, 2, 1	3, 2, 1	IV, I ₂ , I ₁	2 : 2
A2	0	0	0	17	-42	1	2	-	8, 1, 2	0, 1, 2	3, 1, 2	IV*, I ₁ , I ₂	2 : 1
<hr style="border-top: 1px dashed black;"/>													
B1	0	0	0	-32	-31	1	2	+	4, 3, 2	0, 3, 2	3, 3, 2	IV, I ₃ , I ₂	2 : 2
B2	0	0	0	113	-234	1	2	-	8, 6, 1	0, 6, 1	3, 6, 1	IV*, I ₆ , I ₁	2 : 1
<hr style="border-top: 1px dashed black;"/>													
582 582													
$N = 582 = 2 \cdot 3 \cdot 97$ (4 isogeny classes)													
A1	1	1	0	-15	-27	1	2	+	6, 2, 1	6, 2, 1	2, 2, 1	I ₆ , I ₂ , I ₁	2 : 2
A2	1	1	0	25	-99	1	2	-	3, 4, 2	3, 4, 2	1, 2, 2	I ₃ , I ₄ , I ₂	2 : 1
<hr style="border-top: 1px dashed black;"/>													
B1	1	1	1	-46658	-3898033	0	2	+	12, 14, 1	12, 14, 1	12, 2, 1	I ₁₂ , I ₁₄ , I ₁	2 : 2
B2	1	1	1	-746498	-248562097	0	2	+	6, 7, 2	6, 7, 2	6, 1, 2	I ₆ , I ₇ , I ₂	2 : 1
<hr style="border-top: 1px dashed black;"/>													
C1	1	1	1	-34	47	1	2	+	10, 2, 1	10, 2, 1	10, 2, 1	I ₁₀ , I ₂ , I ₁	2 : 2
C2	1	1	1	-514	4271	1	2	+	5, 1, 2	5, 1, 2	5, 1, 2	I ₅ , I ₁ , I ₂	2 : 1
<hr style="border-top: 1px dashed black;"/>													
D1	1	0	0	-14	-12	0	4	+	4, 4, 1	4, 4, 1	4, 4, 1	I ₄ , I ₄ , I ₁	2 : 2
D2	1	0	0	-194	-1056	0	4	+	2, 2, 2	2, 2, 2	2, 2, 2	I ₂ , I ₂ , I ₂	2 : 1, 3, 4
D3	1	0	0	-3104	-66822	0	2	+	1, 1, 1	1, 1, 1	1, 1, 1	I ₁ , I ₁ , I ₁	2 : 2
D4	1	0	0	-164	-1386	0	2	-	1, 1, 4	1, 1, 4	1, 1, 4	I ₁ , I ₁ , I ₄	2 : 2
<hr style="border-top: 1px dashed black;"/>													
583 583													
$N = 583 = 11 \cdot 53$ (3 isogeny classes)													
A1	0	1	1	6	-5	0	1	-	1, 2	1, 2	1, 2	I ₁ , I ₂	
<hr style="border-top: 1px dashed black;"/>													
B1	1	1	0	-358	-3595	0	1	-	4, 3	4, 3	4, 1	I ₄ , I ₃	

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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585

$N = 585 = 3^2 \cdot 5 \cdot 13$ (9 isogeny classes)

585

A1	1	-1	1	-218	1432	1	2	-	9, 4, 1	0, 4, 1	2, 2, 1	III*, I ₄ , I ₁	2 : 2
A2	1	-1	1	-3593	83782	1	2	+	9, 2, 2	0, 2, 2	2, 2, 2	III*, I ₂ , I ₂	2 : 1
B1	0	0	1	12	-21	0	3	-	3, 1, 3	0, 1, 3	2, 1, 3	III, I ₁ , I ₃	3 : 2
B2	0	0	1	-378	-2842	0	1	-	9, 3, 1	0, 3, 1	2, 1, 1	III*, I ₃ , I ₁	3 : 1
C1	1	-1	0	-24	-45	0	2	-	3, 4, 1	0, 4, 1	2, 4, 1	III, I ₄ , I ₁	2 : 2
C2	1	-1	0	-399	-2970	0	2	+	3, 2, 2	0, 2, 2	2, 2, 2	III, I ₂ , I ₂	2 : 1
D1	0	0	1	-42	105	1	3	-	3, 3, 1	0, 3, 1	2, 3, 1	III, I ₃ , I ₁	3 : 2
D2	0	0	1	108	560	1	1	-	9, 1, 3	0, 1, 3	2, 1, 3	III*, I ₁ , I ₃	3 : 1
E1	0	0	1	-1713	-28022	0	1	-	13, 1, 3	7, 1, 3	2, 1, 1	I ₇ *, I ₁ , I ₃	
F1	1	-1	0	-990	-11745	1	2	+	10, 1, 1	4, 1, 1	2, 1, 1	I ₄ *, I ₁ , I ₁	2 : 2
F2	1	-1	0	-1035	-10584	1	4	+	14, 2, 2	8, 2, 2	4, 2, 2	I ₈ *, I ₂ , I ₂	2 : 1, 3, 4
F3	1	-1	0	-4680	114075	1	4	+	10, 4, 4	4, 4, 4	4, 2, 4	I ₄ *, I ₄ , I ₄	2 : 2, 5, 6
F4	1	-1	0	1890	-61479	1	2	-	22, 1, 1	16, 1, 1	4, 1, 1	I ₁₆ *, I ₁ , I ₁	2 : 2
F5	1	-1	0	-73125	7629336	1	4	+	8, 8, 2	2, 8, 2	4, 2, 2	I ₂ *, I ₈ , I ₂	2 : 3, 7, 8
F6	1	-1	0	5445	533250	1	2	-	8, 2, 8	2, 2, 8	2, 2, 8	I ₂ *, I ₂ , I ₈	2 : 3
F7	1	-1	0	-1170000	487402461	1	2	+	7, 4, 1	1, 4, 1	4, 2, 1	I ₁ *, I ₄ , I ₁	2 : 5
F8	1	-1	0	-71370	8011575	1	2	-	7, 16, 1	1, 16, 1	2, 2, 1	I ₁ *, I ₁₆ , I ₁	2 : 5
G1	0	0	1	-3	18	1	1	-	7, 1, 1	1, 1, 1	4, 1, 1	I ₁ *, I ₁ , I ₁	
H1	1	-1	0	-9	0	1	2	+	6, 1, 1	0, 1, 1	2, 1, 1	I ₀ *, I ₁ , I ₁	2 : 2
H2	1	-1	0	36	-27	1	2	-	6, 2, 2	0, 2, 2	2, 2, 2	I ₀ *, I ₂ , I ₂	2 : 1
I1	0	0	1	-597	8820	1	1	-	9, 7, 1	3, 7, 1	4, 7, 1	I ₃ *, I ₇ , I ₁	

586

$N = 586 = 2 \cdot 293$ (3 isogeny classes)

586

A1	1	1	0	-1	-3	0	1	-	3, 1	3, 1	1, 1	I ₃ , I ₁	
B1	1	1	1	-18	415	1	1	-	18, 1	18, 1	18, 1	I ₁₈ , I ₁	
C1	1	1	1	-9	7	1	1	-	4, 1	4, 1	4, 1	I ₄ , I ₁	

588

$N = 588 = 2^2 \cdot 3 \cdot 7^2$ (6 isogeny classes)

588

A1	0	-1	0	131	-167	0	1	-	8, 5, 4	0, 5, 0	1, 1, 1	IV*, I ₅ , IV	
B1	0	-1	0	327	666	1	2	-	4, 3, 8	0, 3, 2	3, 1, 4	IV, I ₃ , I ₂ *	2 : 2; 3 : 3
B2	0	-1	0	-1388	6840	1	2	+	8, 6, 7	0, 6, 1	3, 2, 4	IV*, I ₆ , I ₁ *	2 : 1; 3 : 4
B3	0	-1	0	-5553	165894	1	2	-	4, 1, 12	0, 1, 6	1, 1, 4	IV, I ₁ , I ₆ *	2 : 4; 3 : 1
B4	0	-1	0	-89588	10350936	1	2	+	8, 2, 9	0, 2, 3	1, 2, 4	IV*, I ₂ , I ₃ *	2 : 3; 3 : 2
C1	0	-1	0	-9	-6	1	2	+	4, 1, 3	0, 1, 0	3, 1, 2	IV, I ₁ , III	2 : 2
C2	0	-1	0	-44	120	1	2	+	8, 2, 3	0, 2, 0	3, 2, 2	IV*, I ₂ , III	2 : 1
D1	0	1	0	6403	44463	0	1	-	8, 5, 10	0, 5, 0	1, 5, 1	IV*, I ₅ , II*	
E1	0	1	0	-457	2960	0	2	+	4, 1, 9	0, 1, 0	3, 1, 2	IV, I ₁ , III*	2 : 2
E2	0	1	0	-2172	-36828	0	2	+	8, 2, 9	0, 2, 0	3, 2, 2	IV*, I ₂ , III*	2 : 1
F1	0	1	0	-65	804	0	2	-	4, 1, 8	0, 1, 2	1, 1, 4	IV, I ₁ , I ₂ *	2 : 2
F2	0	1	0	-1780	28244	0	2	+	8, 2, 7	0, 2, 1	1, 2, 2	IV*, I ₂ , I ₁ *	2 : 1

590

$N = 590 = 2 \cdot 5 \cdot 59$ (4 isogeny classes)

590

A1	1	0	1	156	176	0	3	-	1, 4, 3	1, 4, 3	1, 2, 3	I ₁ , I ₄ , I ₃	3 : 2
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	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
590	$N = 590 = 2 \cdot 5 \cdot 59$ (continued)											590	
B1	1	-1	0	1	13	0	2	-	8, 1, 1	8, 1, 1	2, 1, 1	I_8, I_1, I_1	2 : 2
B2	1	-1	0	-79	285	0	4	+	4, 2, 2	4, 2, 2	2, 2, 2	I_4, I_2, I_2	2 : 1, 3, 4
B3	1	-1	0	-179	-495	0	2	+	2, 1, 4	2, 1, 4	2, 1, 2	I_2, I_1, I_4	2 : 2
B4	1	-1	0	-1259	17513	0	4	+	2, 4, 1	2, 4, 1	2, 4, 1	I_2, I_4, I_1	2 : 2
C1	1	-1	0	1	5	1	1	-	3, 2, 1	3, 2, 1	1, 2, 1	I_3, I_2, I_1	
D1	1	0	0	-350	2500	1	1	-	9, 4, 1	9, 4, 1	9, 4, 1	I_9, I_4, I_1	
591	$N = 591 = 3 \cdot 197$ (1 isogeny class)											591	
A1	0	-1	1	-3	2	1	1	+	2, 1	2, 1	2, 1	I_2, I_1	
592	$N = 592 = 2^4 \cdot 37$ (5 isogeny classes)											592	
A1	0	1	0	-9	-13	1	1	+	8, 1	0, 1	1, 1	I_0^*, I_1	
B1	0	1	0	-33	-85	0	1	+	8, 1	0, 1	1, 1	I_0^*, I_1	
C1	0	0	0	-16	-16	0	1	+	12, 1	0, 1	1, 1	II^*, I_1	
D1	0	1	0	-5	-1	1	1	+	8, 1	0, 1	2, 1	I_0^*, I_1	
E1	0	-1	0	-53	-131	1	1	+	12, 1	0, 1	1, 1	II^*, I_1	3 : 2
E2	0	-1	0	-373	2813	1	1	+	12, 3	0, 3	1, 3	II^*, I_3	3 : 1, 3
E3	0	-1	0	-29973	2007325	1	1	+	12, 1	0, 1	1, 1	II^*, I_1	3 : 2
593	$N = 593 = 593$ (2 isogeny classes)											593	
A1	1	0	1	-2	1	1	1	-	1	1	1	I_1	
B1	1	0	0	-7	-30	0	2	-	2	2	2	I_2	2 : 2
B2	1	0	0	-12	-17	0	2	+	1	1	1	I_1	2 : 1
594	$N = 594 = 2 \cdot 3^3 \cdot 11$ (8 isogeny classes)											594	
A1	1	-1	0	-18	36	1	1	-	4, 5, 1	4, 0, 1	2, 3, 1	I_4, IV, I_1	
B1	1	-1	0	-9	-9	0	1	-	1, 5, 1	1, 0, 1	1, 1, 1	I_1, IV, I_1	
C1	1	-1	0	-4146	103796	0	3	-	5, 9, 1	5, 0, 1	1, 3, 1	I_5, IV^*, I_1	3 : 2
C2	1	-1	0	-3201	151613	0	1	-	15, 11, 3	15, 0, 3	1, 1, 1	I_{15}, II^*, I_3	3 : 1
D1	1	-1	0	-153	4909	1	1	-	8, 5, 5	8, 0, 5	2, 1, 5	I_8, IV, I_5	
E1	1	-1	1	-1379	-131165	0	1	-	8, 11, 5	8, 0, 5	8, 1, 1	I_8, II^*, I_5	
F1	1	-1	1	-83	325	0	1	-	1, 11, 1	1, 0, 1	1, 1, 1	I_1, II^*, I_1	
G1	1	-1	1	-164	-809	0	1	-	4, 11, 1	4, 0, 1	4, 1, 1	I_4, II^*, I_1	
H1	1	-1	1	-461	-3691	0	1	-	5, 3, 1	5, 0, 1	5, 1, 1	I_5, II, I_1	3 : 2
H2	1	-1	1	-356	-5497	0	3	-	15, 5, 3	15, 0, 3	15, 1, 3	I_{15}, IV, I_3	3 : 1
595	$N = 595 = 5 \cdot 7 \cdot 17$ (3 isogeny classes)											595	
A1	0	-1	1	-9996	388876	0	1	-	11, 3, 1	11, 3, 1	1, 1, 1	I_{11}, I_3, I_1	
B1	0	-1	1	434	-9589	0	1	-	5, 7, 1	5, 7, 1	1, 7, 1	I_5, I_7, I_1	
C1	0	-1	1	0	1	0	1	-	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	
598	$N = 598 = 2 \cdot 13 \cdot 23$ (4 isogeny classes)											598	
A1	1	-1	0	-112	492	1	2	-	2, 4, 1	2, 4, 1	2, 4, 1	I_2, I_4, I_1	2 : 2
A2	1	-1	0	-1802	29898	1	2	+	1, 2, 2	1, 2, 2	1, 2, 2	I_1, I_2, I_2	2 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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598

$N = 598 = 2 \cdot 13 \cdot 23$ (continued)

598

C1	1	1	1	-14	-27	0	1	-	1, 1, 2	1, 1, 2	1, 1, 2	I_1, I_1, I_2	
D1	1	1	1	4	-1443	1	1	-	17, 1, 2	17, 1, 2	17, 1, 2	I_{17}, I_1, I_2	

600

$N = 600 = 2^3 \cdot 3 \cdot 5^2$ (9 isogeny classes)

600

A1	0	-1	0	-383	3012	1	4	+	4, 2, 7	0, 2, 1	2, 2, 4	III, I_2, I_1^*	2 : 2
A2	0	-1	0	-508	1012	1	4	+	8, 4, 8	0, 4, 2	2, 2, 4	I_1^*, I_4, I_2^*	2 : 1, 3, 4
A3	0	-1	0	-5008	-133988	1	4	+	10, 2, 10	0, 2, 4	2, 2, 4	III^*, I_2, I_4^*	2 : 2, 5, 6
A4	0	-1	0	1992	6012	1	2	-	10, 8, 7	0, 8, 1	2, 2, 4	III^*, I_8, I_1^*	2 : 2
A5	0	-1	0	-80008	-8683988	1	2	+	11, 1, 8	0, 1, 2	1, 1, 4	II^*, I_1, I_2^*	2 : 3
A6	0	-1	0	-2008	-295988	1	2	-	11, 1, 14	0, 1, 8	1, 1, 4	II^*, I_1, I_8^*	2 : 3
B1	0	-1	0	7	-3	1	1	-	8, 1, 2	0, 1, 0	4, 1, 1	I_1^*, I_1, II	
C1	0	-1	0	32	-68	0	2	-	10, 3, 3	0, 3, 0	2, 1, 2	III^*, I_3, III	2 : 2
C2	0	-1	0	-168	-468	0	2	+	11, 6, 3	0, 6, 0	1, 2, 2	II^*, I_6, III	2 : 1
D1	0	1	0	17	38	0	2	-	4, 1, 6	0, 1, 0	2, 1, 2	III, I_1, I_0^*	2 : 2
D2	0	1	0	-108	288	0	4	+	8, 2, 6	0, 2, 0	2, 2, 4	I_1^*, I_2, I_0^*	2 : 1, 3, 4
D3	0	1	0	-608	-5712	0	4	+	10, 4, 6	0, 4, 0	2, 4, 4	III^*, I_4, I_0^*	2 : 2, 5, 6
D4	0	1	0	-1608	24288	0	2	+	10, 1, 6	0, 1, 0	2, 1, 2	III^*, I_1, I_0^*	2 : 2
D5	0	1	0	-9608	-365712	0	2	+	11, 2, 6	0, 2, 0	1, 2, 2	II^*, I_2, I_0^*	2 : 3
D6	0	1	0	392	-21712	0	2	-	11, 8, 6	0, 8, 0	1, 8, 2	II^*, I_8, I_0^*	2 : 3
E1	0	1	0	-233	1563	1	1	-	8, 7, 4	0, 7, 0	4, 7, 3	I_1^*, I_7, IV	
F1	0	-1	0	92	-188	0	4	-	8, 1, 7	0, 1, 1	4, 1, 4	I_1^*, I_1, I_1^*	2 : 2
F2	0	-1	0	-408	-1188	0	4	+	10, 2, 8	0, 2, 2	2, 2, 4	III^*, I_2, I_2^*	2 : 1, 3, 4
F3	0	-1	0	-5408	-151188	0	2	+	11, 4, 7	0, 4, 1	1, 2, 4	II^*, I_4, I_1^*	2 : 2
F4	0	-1	0	-3408	76812	0	2	+	11, 1, 10	0, 1, 4	1, 1, 4	II^*, I_1, I_4^*	2 : 2
G1	0	-1	0	-5833	207037	0	1	-	8, 7, 10	0, 7, 0	2, 1, 1	I_1^*, I_7, II^*	
H1	0	1	0	792	-6912	0	2	-	10, 3, 9	0, 3, 0	2, 3, 2	III^*, I_3, III^*	2 : 2
H2	0	1	0	-4208	-66912	0	2	+	11, 6, 9	0, 6, 0	1, 6, 2	II^*, I_6, III^*	2 : 1
I1	0	1	0	167	-37	0	1	-	8, 1, 8	0, 1, 0	2, 1, 1	I_1^*, I_1, IV^*	

602

$N = 602 = 2 \cdot 7 \cdot 43$ (3 isogeny classes)

602

A1	1	-1	0	121	-4291	0	2	-	8, 5, 2	8, 5, 2	2, 1, 2	I_8, I_5, I_2	2 : 2
A2	1	-1	0	-3319	-69651	0	2	+	4, 10, 1	4, 10, 1	2, 2, 1	I_4, I_{10}, I_1	2 : 1
B1	1	1	0	-22564	1295312	0	1	-	17, 5, 1	17, 5, 1	1, 1, 1	I_{17}, I_5, I_1	
C1	1	-1	0	-1	-1	0	1	-	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	

603

$N = 603 = 3^2 \cdot 67$ (6 isogeny classes)

603

A1	1	-1	0	-3	0	0	2	+	3, 1	0, 1	2, 1	III, I_1	2 : 2
A2	1	-1	0	12	-9	0	2	-	3, 2	0, 2	2, 2	III, I_2	2 : 1
B1	1	-1	1	-29	28	0	2	+	9, 1	0, 1	2, 1	III^*, I_1	2 : 2
B2	1	-1	1	106	136	0	2	-	9, 2	0, 2	2, 2	III^*, I_2	2 : 1
C1	1	-1	1	-7151	-230952	0	1	-	11, 1	5, 1	4, 1	I_5^*, I_1	
D1	0	0	1	15	-23	0	1	-	8, 1	2, 1	2, 1	I_2^*, I_1	
E1	1	-1	0	-9	-54	1	1	-	9, 1	3, 1	2, 1	I_3^*, I_1	

TABLE 1: ELLIPTIC CURVES 605A–610C

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
605 605													
$N = 605 = 5 \cdot 11^2$ (3 isogeny classes)													
A1	1	-1	0	-1414	-44027	1	1	-	5, 8	5, 0	5, 3	I_5, IV^*	
B1	1	-1	1	98	-316	1	4	-	1, 7	1, 1	1, 4	I_1, I_1^*	2 : 2
B2	1	-1	1	-507	-2494	1	4	+	2, 8	2, 2	2, 4	I_2, I_2^*	2 : 1, 3, 4
B3	1	-1	1	-7162	-231426	1	2	+	4, 7	4, 1	4, 2	I_4, I_1^*	2 : 2
B4	1	-1	1	-3532	79786	1	2	+	1, 10	1, 4	1, 4	I_1, I_4^*	2 : 2
C1	1	-1	1	-12	36	1	1	-	5, 2	5, 0	5, 1	I_5, II	
606 606													
$N = 606 = 2 \cdot 3 \cdot 101$ (6 isogeny classes)													
A1	1	0	1	35	-136	0	2	-	12, 3, 1	12, 3, 1	2, 3, 1	I_{12}, I_3, I_1	2 : 2
A2	1	0	1	-285	-1544	0	4	+	6, 6, 2	6, 6, 2	2, 6, 2	I_6, I_6, I_2	2 : 1, 3, 4
A3	1	0	1	-4325	-109816	0	2	+	3, 12, 1	3, 12, 1	1, 12, 1	I_3, I_{12}, I_1	2 : 2
A4	1	0	1	-1365	17896	0	2	+	3, 3, 4	3, 3, 4	1, 3, 2	I_3, I_3, I_4	2 : 2
B1	1	0	1	4	2	1	1	-	3, 2, 1	3, 2, 1	1, 2, 1	I_3, I_2, I_1	
C1	1	1	1	-33	-87	0	1	-	1, 2, 1	1, 2, 1	1, 2, 1	I_1, I_2, I_1	
D1	1	1	1	-1314	-65361	0	1	-	7, 17, 1	7, 17, 1	7, 1, 1	I_7, I_{17}, I_1	
E1	1	0	0	-120	576	1	1	-	9, 6, 1	9, 6, 1	9, 6, 1	I_9, I_6, I_1	
F1	1	0	0	-90	324	0	5	-	5, 5, 1	5, 5, 1	5, 5, 1	I_5, I_5, I_1	5 : 2
F2	1	0	0	600	-10626	0	1	-	1, 1, 5	1, 1, 5	1, 1, 5	I_1, I_1, I_5	5 : 1
608 608													
$N = 608 = 2^5 \cdot 19$ (6 isogeny classes)													
A1	0	0	0	-8	-16	1	1	-	12, 1	0, 1	2, 1	III^*, I_1	
B1	0	0	0	-56	4848	0	1	-	12, 5	0, 5	2, 1	III^*, I_5	
C1	0	0	0	5	2	0	1	-	9, 1	0, 1	1, 1	I_0^*, I_1	
D1	0	0	0	-8	16	1	1	-	12, 1	0, 1	2, 1	III^*, I_1	
E1	0	0	0	-56	-4848	1	1	-	12, 5	0, 5	2, 5	III^*, I_5	
F1	0	0	0	5	-2	1	1	-	9, 1	0, 1	2, 1	I_0^*, I_1	
609 609													
$N = 609 = 3 \cdot 7 \cdot 29$ (2 isogeny classes)													
A1	1	1	0	0	3	1	2	-	1, 2, 1	1, 2, 1	1, 2, 1	I_1, I_2, I_1	2 : 2
A2	1	1	0	-35	66	1	2	+	2, 1, 2	2, 1, 2	2, 1, 2	I_2, I_1, I_2	2 : 1
B1	1	1	1	-784	8720	1	4	-	3, 8, 1	3, 8, 1	1, 8, 1	I_3, I_8, I_1	2 : 2
B2	1	1	1	-12789	551346	1	8	+	6, 4, 2	6, 4, 2	2, 4, 2	I_6, I_4, I_2	2 : 1, 3, 4
B3	1	1	1	-13034	528806	1	4	+	12, 2, 4	12, 2, 4	2, 2, 4	I_{12}, I_2, I_4	2 : 2, 5, 6
B4	1	1	1	-204624	35542050	1	4	+	3, 2, 1	3, 2, 1	1, 2, 1	I_3, I_2, I_1	2 : 2
B5	1	1	1	-42469	-2756140	1	2	+	24, 1, 2	24, 1, 2	2, 1, 2	I_{24}, I_1, I_2	2 : 3
B6	1	1	1	12481	2376092	1	2	-	6, 1, 8	6, 1, 8	2, 1, 8	I_6, I_1, I_8	2 : 3
610 610													
$N = 610 = 2 \cdot 5 \cdot 61$ (3 isogeny classes)													
A1	1	-1	0	-35	-75	0	1	-	5, 3, 1	5, 3, 1	1, 1, 1	I_5, I_3, I_1	
B1	1	-1	0	-164	848	1	2	+	8, 3, 1	8, 3, 1	2, 3, 1	I_8, I_3, I_1	2 : 2
B2	1	-1	0	-244	0	1	4	+	4, 6, 2	4, 6, 2	2, 6, 2	I_4, I_6, I_2	2 : 1, 3, 4
B3	1	-1	0	-2744	-54500	1	2	+	2, 3, 4	2, 3, 4	2, 3, 4	I_2, I_3, I_4	2 : 2
B4	1	-1	0	976	-732	1	4	-	2, 12, 1	2, 12, 1	2, 12, 1	I_2, I_{12}, I_1	2 : 2
C1	1	1	1	-5	-5	0	2	+	4, 1, 1	4, 1, 1	4, 1, 1	I_4, I_1, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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611 $N = 611 = 13 \cdot 47$ (1 isogeny class) **611**

A1	0	0	1	-1	1	0	1	-	1, 1	1, 1	1, 1	I_1, I_1	
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612 $N = 612 = 2^2 \cdot 3^2 \cdot 17$ (4 isogeny classes) **612**

A1	0	0	0	-456	3748	0	3	-	8, 3, 1	0, 0, 1	3, 2, 1	IV^*, III, I_1	3 : 2
A2	0	0	0	-216	7668	0	1	-	8, 9, 3	0, 0, 3	1, 2, 1	IV^*, III^*, I_3	3 : 1
B1	0	0	0	-24	-284	1	3	-	8, 3, 3	0, 0, 3	3, 2, 3	IV^*, III, I_3	3 : 2
B2	0	0	0	-4104	-101196	1	1	-	8, 9, 1	0, 0, 1	1, 2, 1	IV^*, III^*, I_1	3 : 1
C1	0	0	0	-48	196	1	1	-	8, 7, 1	0, 1, 1	3, 4, 1	IV^*, I_1^*, I_1	
D1	0	0	0	-14592	679412	0	1	-	8, 17, 1	0, 11, 1	1, 2, 1	IV^*, I_{11}^*, I_1	

614 $N = 614 = 2 \cdot 307$ (2 isogeny classes) **614**

A1	1	-1	1	-61	197	1	1	-	6, 1	6, 1	6, 1	I_6, I_1	
B1	1	0	0	27	1	1	3	-	12, 1	12, 1	12, 1	I_{12}, I_1	3 : 2
B2	1	0	0	-373	-2991	1	1	-	4, 3	4, 3	4, 3	I_4, I_3	3 : 1

615 $N = 615 = 3 \cdot 5 \cdot 41$ (2 isogeny classes) **615**

A1	1	1	1	-6	-6	1	2	+	2, 2, 1	2, 2, 1	2, 2, 1	I_2, I_2, I_1	2 : 2
A2	1	1	1	19	-16	1	2	-	4, 1, 2	4, 1, 2	2, 1, 2	I_4, I_1, I_2	2 : 1
B1	0	1	1	79	-214	1	1	-	7, 4, 1	7, 4, 1	7, 2, 1	I_7, I_4, I_1	

616 $N = 616 = 2^3 \cdot 7 \cdot 11$ (5 isogeny classes) **616**

A1	0	0	0	85	86	1	2	-	10, 3, 2	0, 3, 2	2, 1, 2	III^*, I_3, I_2	2 : 2
A2	0	0	0	-355	702	1	2	+	11, 6, 1	0, 6, 1	1, 2, 1	II^*, I_6, I_1	2 : 1
B1	0	-1	0	3828	95348	0	2	-	8, 5, 6	0, 5, 6	2, 5, 2	I_1^*, I_5, I_6	2 : 2
B2	0	-1	0	-22792	936540	0	2	+	10, 10, 3	0, 10, 3	2, 10, 1	III^*, I_{10}, I_3	2 : 1
C1	0	1	0	-12	-32	0	2	-	8, 1, 2	0, 1, 2	2, 1, 2	I_1^*, I_1, I_2	2 : 2
C2	0	1	0	-232	-1440	0	2	+	10, 2, 1	0, 2, 1	2, 2, 1	III^*, I_2, I_1	2 : 1
D1	0	-1	0	-1	197	1	1	-	8, 2, 3	0, 2, 3	4, 2, 3	I_1^*, I_2, I_3	
E1	0	0	0	-26	-51	1	2	+	4, 1, 1	0, 1, 1	2, 1, 1	III, I_1, I_1	2 : 2
E2	0	0	0	-31	-30	1	4	+	8, 2, 2	0, 2, 2	4, 2, 2	I_1^*, I_2, I_2	2 : 1, 3, 4
E3	0	0	0	-251	1510	1	4	+	10, 4, 1	0, 4, 1	2, 4, 1	III^*, I_4, I_1	2 : 2
E4	0	0	0	109	-226	1	2	-	10, 1, 4	0, 1, 4	2, 1, 2	III^*, I_1, I_4	2 : 2

618 $N = 618 = 2 \cdot 3 \cdot 103$ (7 isogeny classes) **618**

A1	1	1	0	2	4	1	1	-	4, 1, 1	4, 1, 1	2, 1, 1	I_4, I_1, I_1	
B1	1	1	0	-2819	-58803	1	1	-	19, 1, 1	19, 1, 1	1, 1, 1	I_{19}, I_1, I_1	
C1	1	0	1	-21	34	1	3	-	1, 3, 1	1, 3, 1	1, 3, 1	I_1, I_3, I_1	3 : 2
C2	1	0	1	54	196	1	1	-	3, 1, 3	3, 1, 3	1, 1, 3	I_3, I_1, I_3	3 : 1
D1	1	0	1	325	-7018	1	3	-	4, 15, 1	4, 15, 1	2, 15, 1	I_4, I_{15}, I_1	3 : 2
D2	1	0	1	-20330	-1118500	1	1	-	12, 5, 3	12, 5, 3	2, 5, 3	I_{12}, I_5, I_3	3 : 1
E1	1	1	1	1	5	1	1	-	5, 1, 1	5, 1, 1	5, 1, 1	I_5, I_1, I_1	
F1	1	0	0	-185	1401	1	1	-	11, 7, 1	11, 7, 1	11, 7, 1	I_{11}, I_7, I_1	

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
620	$N = 620 = 2^2 \cdot 5 \cdot 31$ (3 isogeny classes)											620	
A1	0	1	0	-101	359	1	3	-	8, 1, 1	0, 1, 1	3, 1, 1	IV*, I ₁ , I ₁	3 : 2
A2	0	1	0	59	1495	1	1	-	8, 3, 3	0, 3, 3	1, 1, 3	IV*, I ₃ , I ₃	3 : 1
B1	0	0	0	-1052	13129	1	2	+	4, 5, 2	0, 5, 2	3, 5, 2	IV, I ₅ , I ₂	2 : 2
B2	0	0	0	-1207	9006	1	2	+	8, 10, 1	0, 10, 1	3, 10, 1	IV*, I ₁₀ , I ₁	2 : 1
C1	0	0	0	8	4	1	1	-	8, 1, 1	0, 1, 1	3, 1, 1	IV*, I ₁ , I ₁	
621	$N = 621 = 3^3 \cdot 23$ (2 isogeny classes)											621	
A1	1	-1	0	-123	548	0	1	+	11, 1	0, 1	1, 1	II*, I ₁	
B1	1	-1	1	-14	-16	1	1	+	5, 1	0, 1	1, 1	IV, I ₁	
622	$N = 622 = 2 \cdot 311$ (1 isogeny class)											622	
A1	1	-1	1	8	-5	1	1	-	7, 1	7, 1	7, 1	I ₇ , I ₁	
623	$N = 623 = 7 \cdot 89$ (1 isogeny class)											623	
A1	1	1	0	28	157	1	1	-	6, 1	6, 1	6, 1	I ₆ , I ₁	
624	$N = 624 = 2^4 \cdot 3 \cdot 13$ (10 isogeny classes)											624	
A1	0	-1	0	-3	6	1	2	-	4, 1, 2	0, 1, 2	1, 1, 2	II, I ₁ , I ₂	2 : 2
A2	0	-1	0	-68	240	1	2	+	8, 2, 1	0, 2, 1	2, 2, 1	I ₀ *, I ₂ , I ₁	2 : 1
B1	0	-1	0	5	-14	1	2	-	4, 3, 2	0, 3, 2	1, 1, 2	II, I ₃ , I ₂	2 : 2
B2	0	-1	0	-60	-144	1	2	+	8, 6, 1	0, 6, 1	2, 2, 1	I ₀ *, I ₆ , I ₁	2 : 1
C1	0	-1	0	-7	-2	0	2	+	4, 4, 1	0, 4, 1	1, 2, 1	II, I ₄ , I ₁	2 : 2
C2	0	-1	0	-52	160	0	4	+	8, 2, 2	0, 2, 2	2, 2, 2	I ₀ *, I ₂ , I ₂	2 : 1, 3, 4
C3	0	-1	0	-832	9520	0	2	+	10, 1, 1	0, 1, 1	2, 1, 1	I ₂ *, I ₁ , I ₁	2 : 2
C4	0	-1	0	8	448	0	4	-	10, 1, 4	0, 1, 4	4, 1, 4	I ₂ *, I ₁ , I ₄	2 : 2
D1	0	1	0	-3	0	0	2	+	4, 2, 1	0, 2, 1	1, 2, 1	II, I ₂ , I ₁	2 : 2
D2	0	1	0	12	12	0	2	-	8, 1, 2	0, 1, 2	2, 1, 2	I ₀ *, I ₁ , I ₂	2 : 1
E1	0	1	0	-651	-6228	0	2	+	4, 10, 3	0, 10, 3	1, 10, 1	II, I ₁₀ , I ₃	2 : 2
E2	0	1	0	564	-25668	0	2	-	8, 5, 6	0, 5, 6	2, 5, 2	I ₀ *, I ₅ , I ₆	2 : 1
F1	0	1	0	-39	-108	1	2	+	4, 2, 1	0, 2, 1	1, 2, 1	II, I ₂ , I ₁	2 : 2
F2	0	1	0	-44	-84	1	4	+	8, 4, 2	0, 4, 2	2, 4, 2	I ₀ *, I ₄ , I ₂	2 : 1, 3, 4
F3	0	1	0	-304	1892	1	4	+	10, 8, 1	0, 8, 1	4, 8, 1	I ₂ *, I ₈ , I ₁	2 : 2
F4	0	1	0	136	-444	1	4	-	10, 2, 4	0, 2, 4	2, 2, 4	I ₂ *, I ₂ , I ₄	2 : 2
G1	0	-1	0	-13	4	1	2	+	4, 6, 1	0, 6, 1	1, 2, 1	II, I ₆ , I ₁	2 : 2; 3 : 3
G2	0	-1	0	-148	-644	1	2	+	8, 3, 2	0, 3, 2	1, 1, 2	I ₀ *, I ₃ , I ₂	2 : 1; 3 : 4
G3	0	-1	0	-733	7888	1	2	+	4, 2, 3	0, 2, 3	1, 2, 3	II, I ₂ , I ₃	2 : 4; 3 : 1
G4	0	-1	0	-748	7564	1	2	+	8, 1, 6	0, 1, 6	1, 1, 6	I ₀ *, I ₁ , I ₆	2 : 3; 3 : 2
H1	0	1	0	8	20	0	2	-	12, 1, 1	0, 1, 1	4, 1, 1	I ₄ *, I ₁ , I ₁	2 : 2
H2	0	1	0	-72	180	0	4	+	12, 2, 2	0, 2, 2	4, 2, 2	I ₄ *, I ₂ , I ₂	2 : 1, 3, 4
H3	0	1	0	-312	-2028	0	2	+	12, 1, 4	0, 1, 4	2, 1, 4	I ₄ *, I ₁ , I ₄	2 : 2
H4	0	1	0	-1112	13908	0	4	+	12, 4, 1	0, 4, 1	4, 4, 1	I ₄ *, I ₄ , I ₁	2 : 2
I1	0	1	0	-312	-44460	0	2	-	28, 5, 1	16, 5, 1	4, 5, 1	I ₂₀ *, I ₅ , I ₁	2 : 2
I2	0	1	0	-20792	-1150380	0	4	+	20, 10, 2	8, 10, 2	4, 10, 2	I ₁₂ *, I ₁₀ , I ₂	2 : 1, 3, 4
I3	0	1	0	-331832	-73684908	0	2	+	16, 5, 4	4, 5, 4	2, 5, 4	I ₄ *, I ₁ , I ₄	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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624 $N = 624 = 2^4 \cdot 3 \cdot 13$ (continued) **624**

J1	0	1	0	-5	-6	0	2	+	4, 2, 1	0, 2, 1	1, 2, 1	II, I ₂ , I ₁	2 : 2
J2	0	1	0	-20	24	0	2	+	8, 1, 2	0, 1, 2	1, 1, 2	I ₀ [*] , I ₁ , I ₂	2 : 1

626 $N = 626 = 2 \cdot 313$ (2 isogeny classes) **626**

A1	1	-1	0	-7	9	1	2	+	2, 1	2, 1	2, 1	I ₂ , I ₁	2 : 2
A2	1	-1	0	-17	-13	1	2	+	1, 2	1, 2	1, 2	I ₁ , I ₂	2 : 1
B1	1	0	1	-2210	39796	0	1	-	19, 1	19, 1	1, 1	I ₁₉ , I ₁	

627 $N = 627 = 3 \cdot 11 \cdot 19$ (2 isogeny classes) **627**

A1	0	1	1	-1	-2	0	1	-	1, 1, 1	1, 1, 1	1, 1, 1	I ₁ , I ₁ , I ₁	
B1	0	1	1	-363	-2995	0	3	-	9, 3, 1	9, 3, 1	9, 3, 1	I ₉ , I ₃ , I ₁	3 : 2
B2	0	1	1	-30063	-2016358	0	1	-	3, 1, 3	3, 1, 3	3, 1, 3	I ₃ , I ₁ , I ₃	3 : 1

628 $N = 628 = 2^2 \cdot 157$ (1 isogeny class) **628**

A1	0	-1	0	4	8	0	1	-	8, 1	0, 1	1, 1	IV [*] , I ₁	
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629 $N = 629 = 17 \cdot 37$ (4 isogeny classes) **629**

A1	1	-1	0	11	-18	1	1	-	3, 1	3, 1	1, 1	I ₃ , I ₁	
B1	0	0	1	-211	1165	0	1	+	2, 3	2, 3	2, 1	I ₂ , I ₃	
C1	0	0	1	-40	48	1	1	+	4, 1	4, 1	4, 1	I ₄ , I ₁	
D1	1	-1	1	-171	1904	1	1	-	1, 5	1, 5	1, 5	I ₁ , I ₅	

630 $N = 630 = 2 \cdot 3^2 \cdot 5 \cdot 7$ (10 isogeny classes) **630**

A1	1	-1	0	-105	441	0	6	+	2, 3, 1, 3	2, 0, 1, 3	2, 2, 1, 3	I ₂ , III, I ₁ , I ₃	2 : 2; 3 : 3
A2	1	-1	0	-75	675	0	6	-	1, 3, 2, 6	1, 0, 2, 6	1, 2, 2, 6	I ₁ , III, I ₂ , I ₆	2 : 1; 3 : 4
A3	1	-1	0	-420	-2800	0	2	+	6, 9, 3, 1	6, 0, 3, 1	2, 2, 1, 1	I ₆ , III [*] , I ₃ , I ₁	2 : 4; 3 : 1
A4	1	-1	0	660	-15544	0	2	-	3, 9, 6, 2	3, 0, 6, 2	1, 2, 2, 2	I ₃ , III [*] , I ₆ , I ₂	2 : 3; 3 : 2
B1	1	-1	0	-5124	142160	0	2	+	14, 3, 1, 5	14, 0, 1, 5	2, 2, 1, 1	I ₁₄ , III, I ₁ , I ₅	2 : 2
B2	1	-1	0	-3204	248528	0	2	-	7, 3, 2, 10	7, 0, 2, 10	1, 2, 2, 2	I ₇ , III, I ₂ , I ₁₀	2 : 1
C1	1	-1	0	1890	-24300	0	2	-	16, 10, 2, 1	16, 4, 2, 1	2, 2, 2, 1	I ₁₆ , I ₄ [*] , I ₂ , I ₁	2 : 2
C2	1	-1	0	-9630	-210924	0	4	+	8, 14, 4, 2	8, 8, 4, 2	2, 4, 2, 2	I ₈ , I ₈ [*] , I ₄ , I ₂	2 : 1, 3, 4
C3	1	-1	0	-135630	-19186524	0	2	+	4, 22, 2, 1	4, 16, 2, 1	2, 4, 2, 1	I ₄ , I ₁₆ [*] , I ₂ , I ₁	2 : 2
C4	1	-1	0	-67950	6682500	0	4	+	4, 10, 8, 4	4, 4, 8, 4	2, 4, 2, 2	I ₄ , I ₄ [*] , I ₈ , I ₄	2 : 2, 5, 6
C5	1	-1	0	-1080450	432540000	0	4	+	2, 8, 4, 8	2, 2, 4, 8	2, 4, 2, 2	I ₂ , I ₂ [*] , I ₄ , I ₈	2 : 4, 7, 8
C6	1	-1	0	11430	21304296	0	2	-	2, 8, 16, 2	2, 2, 16, 2	2, 2, 2, 2	I ₂ , I ₂ [*] , I ₁₆ , I ₂	2 : 4
C7	1	-1	0	-17287200	27669604050	0	2	+	1, 7, 2, 4	1, 1, 2, 4	1, 2, 2, 2	I ₁ , I ₁ [*] , I ₂ , I ₄	2 : 5
C8	1	-1	0	-1073700	438205950	0	2	-	1, 7, 2, 16	1, 1, 2, 16	1, 4, 2, 2	I ₁ , I ₁ [*] , I ₂ , I ₁₆	2 : 5
D1	1	-1	0	90	436	1	2	-	8, 7, 1, 2	8, 1, 1, 2	2, 2, 1, 2	I ₈ , I ₁ [*] , I ₁ , I ₂	2 : 2
D2	1	-1	0	-630	4900	1	4	+	4, 8, 2, 4	4, 2, 2, 4	2, 4, 2, 4	I ₄ , I ₂ [*] , I ₂ , I ₄	2 : 1, 3, 4
D3	1	-1	0	-3330	-69080	1	2	+	2, 7, 1, 8	2, 1, 1, 8	2, 4, 1, 8	I ₂ , I ₁ [*] , I ₁ , I ₈	2 : 2
D4	1	-1	0	-9450	355936	1	4	+	2, 10, 4, 2	2, 4, 4, 2	2, 4, 2, 2	I ₂ , I ₄ [*] , I ₄ , I ₂	2 : 2, 5, 6
D5	1	-1	0	-151200	22667386	1	2	+	1, 8, 2, 1	1, 2, 2, 1	1, 2, 2, 1	I ₁ , I ₁ [*] , I ₂ , I ₁	2 : 4

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
630 630													
$N = 630 = 2 \cdot 3^2 \cdot 5 \cdot 7$ (continued)													
E1	1	-1	0	21	53	1	2	-	4, 6, 2, 1	4, 0, 2, 1	2, 2, 2, 1	I_4, I_0^*, I_2, I_1	2 : 2
E2	1	-1	0	-159	665	1	4	+	2, 6, 4, 2	2, 0, 4, 2	2, 4, 4, 2	I_2, I_0^*, I_4, I_2	2 : 1, 3, 4
E3	1	-1	0	-789	-7777	1	2	+	1, 6, 8, 1	1, 0, 8, 1	1, 2, 8, 1	I_1, I_0^*, I_8, I_1	2 : 2
E4	1	-1	0	-2409	46115	1	2	+	1, 6, 2, 4	1, 0, 2, 4	1, 2, 2, 2	I_1, I_0^*, I_2, I_4	2 : 2
F1	1	-1	0	-369	1053	0	2	+	12, 9, 1, 1	12, 3, 1, 1	2, 2, 1, 1	I_{12}, I_3^*, I_1, I_1	2 : 2; 3 : 3
F2	1	-1	0	-3249	-69795	0	4	+	6, 12, 2, 2	6, 6, 2, 2	2, 4, 2, 2	I_6, I_6^*, I_2, I_2	2 : 1, 4, 5; 3 : 6
F3	1	-1	0	-24129	1448685	0	6	+	4, 7, 3, 3	4, 1, 3, 3	2, 2, 3, 3	I_4, I_1^*, I_3, I_3	2 : 6; 3 : 1
F4	1	-1	0	-51849	-4531275	0	2	+	3, 9, 1, 4	3, 3, 1, 4	1, 4, 1, 4	I_3, I_3^*, I_1, I_4	2 : 2; 3 : 7
F5	1	-1	0	-729	-177147	0	2	-	3, 18, 4, 1	3, 12, 4, 1	1, 4, 4, 1	I_3, I_{12}^*, I_4, I_1	2 : 2; 3 : 8
F6	1	-1	0	-24309	1426113	0	12	+	2, 8, 6, 6	2, 2, 6, 6	2, 4, 6, 6	I_2, I_2^*, I_6, I_6	2 : 3, 7, 8; 3 : 2
F7	1	-1	0	-58059	-3373137	0	6	+	1, 7, 3, 12	1, 1, 3, 12	1, 4, 3, 12	I_1, I_1^*, I_3, I_{12}	2 : 6; 3 : 4
F8	1	-1	0	6561	4778595	0	6	-	1, 10, 12, 3	1, 4, 12, 3	1, 4, 12, 3	I_1, I_4^*, I_{12}, I_3	2 : 6; 3 : 5
G1	1	-1	1	-46118	-3792203	0	2	+	14, 9, 1, 5	14, 0, 1, 5	14, 2, 1, 1	I_{14}, III^*, I_1, I_5	2 : 2
G2	1	-1	1	-28838	-6681419	0	2	-	7, 9, 2, 10	7, 0, 2, 10	7, 2, 2, 2	I_7, III^*, I_2, I_{10}	2 : 1
H1	1	-1	1	-47	119	0	6	+	6, 3, 3, 1	6, 0, 3, 1	6, 2, 3, 1	I_6, III, I_3, I_1	2 : 2; 3 : 3
H2	1	-1	1	73	551	0	6	-	3, 3, 6, 2	3, 0, 6, 2	3, 2, 6, 2	I_3, III, I_6, I_2	2 : 1; 3 : 4
H3	1	-1	1	-947	-10961	0	2	+	2, 9, 1, 3	2, 0, 1, 3	2, 2, 1, 3	I_2, III^*, I_1, I_3	2 : 4; 3 : 1
H4	1	-1	1	-677	-17549	0	2	-	1, 9, 2, 6	1, 0, 2, 6	1, 2, 2, 6	I_1, III^*, I_2, I_6	2 : 3; 3 : 2
I1	1	-1	1	-4478	-114163	0	2	+	8, 9, 3, 1	8, 3, 3, 1	8, 2, 1, 1	I_8, I_3^*, I_3, I_1	2 : 2; 3 : 3
I2	1	-1	1	-5198	-74419	0	4	+	4, 12, 6, 2	4, 6, 6, 2	4, 4, 2, 2	I_4, I_6^*, I_6, I_2	2 : 1, 4, 5; 3 : 6
I3	1	-1	1	-13253	449597	0	6	+	24, 7, 1, 3	24, 1, 1, 3	24, 2, 1, 3	I_{24}, I_1^*, I_1, I_3	2 : 6; 3 : 1
I4	1	-1	1	-39218	2946557	0	2	+	2, 9, 12, 1	2, 3, 12, 1	2, 4, 2, 1	I_2, I_3^*, I_{12}, I_1	2 : 2; 3 : 7
I5	1	-1	1	17302	-560419	0	2	-	2, 18, 3, 4	2, 12, 3, 4	2, 4, 1, 4	I_2, I_{12}^*, I_3, I_4	2 : 2; 3 : 8
I6	1	-1	1	-197573	33848381	0	12	+	12, 8, 2, 6	12, 2, 2, 6	12, 4, 2, 6	I_{12}, I_2^*, I_2, I_6	2 : 3, 7, 8; 3 : 2
I7	1	-1	1	-3161093	2164026557	0	6	+	6, 7, 4, 3	6, 1, 4, 3	6, 4, 2, 3	I_6, I_1^*, I_4, I_3	2 : 6; 3 : 4
I8	1	-1	1	-183173	38980541	0	6	-	6, 10, 1, 12	6, 4, 1, 12	6, 4, 1, 12	I_6, I_4^*, I_1, I_{12}	2 : 6; 3 : 5
J1	1	-1	1	-32	51	0	4	+	4, 7, 1, 1	4, 1, 1, 1	4, 4, 1, 1	I_4, I_1^*, I_1, I_1	2 : 2
J2	1	-1	1	-212	-1101	0	4	+	2, 8, 2, 2	2, 2, 2, 2	2, 4, 2, 2	I_2, I_2^*, I_2, I_2	2 : 1, 3, 4
J3	1	-1	1	-3362	-74181	0	2	+	1, 10, 1, 1	1, 4, 1, 1	1, 4, 1, 1	I_1, I_4^*, I_1, I_1	2 : 2
J4	1	-1	1	58	-3909	0	2	-	1, 7, 4, 4	1, 1, 4, 4	1, 2, 4, 2	I_1, I_1^*, I_4, I_4	2 : 2

632 **632** $N = 632 = 2^3 \cdot 79$ (1 isogeny class)

A1	0	1	0	-16	16	1	1	+	10, 1	0, 1	2, 1	III^*, I_1	
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633 **633** $N = 633 = 3 \cdot 211$ (1 isogeny class)

A1	1	1	1	-17	-70	1	1	-	8, 1	8, 1	2, 1	I_8, I_1	
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635 **635** $N = 635 = 5 \cdot 127$ (2 isogeny classes)

A1	0	1	1	5	6	1	3	-	3, 1	3, 1	3, 1	I_3, I_1	3 : 2
A2	0	1	1	-45	-209	1	1	-	1, 3	1, 3	1, 3	I_1, I_3	3 : 1
B1	0	-1	1	-10	16	1	1	-	1, 1	1, 1	1, 1	I_1, I_1	

637 **637** $N = 637 = 7^2 \cdot 13$ (4 isogeny classes)

A1	1	-1	0	-107	454	1	1	-	4, 1	0, 1	1, 1	IV, I_1	7 : 2
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	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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637 $N = 637 = 7^2 \cdot 13$ (continued) **637**

B1	0	-1	1	-359	-2507	0	1	-	7, 1	1, 1	4, 1	I_1^*, I_1	3 : 2
B2	0	-1	1	621	-13238	0	1	-	9, 3	3, 3	4, 1	I_3^*, I_3	3 : 1, 3
B3	0	-1	1	-5749	415463	0	1	-	15, 1	9, 1	4, 1	I_9^*, I_1	3 : 2
C1	1	-1	0	-5252	-145223	1	1	-	10, 1	0, 1	1, 1	II^*, I_1	7 : 2
C2	1	-1	0	30763	6051758	1	1	-	10, 7	0, 7	1, 7	II^*, I_7	7 : 1
D1	0	0	1	49	-86	1	1	-	7, 1	1, 1	2, 1	I_1^*, I_1	

639 $N = 639 = 3^2 \cdot 71$ (1 isogeny class) **639**

A1	1	-1	1	4	-34	1	2	-	8, 1	2, 1	4, 1	I_2^*, I_1	2 : 2
A2	1	-1	1	-131	-520	1	2	+	7, 2	1, 2	4, 2	I_1^*, I_2	2 : 1

640 $N = 640 = 2^7 \cdot 5$ (8 isogeny classes) **640**

A1	0	0	0	-13	-18	1	2	+	7, 1	0, 1	1, 1	II, I_1	2 : 2
A2	0	0	0	-8	-32	1	2	-	14, 2	0, 2	2, 2	III^*, I_2	2 : 1
B1	0	0	0	-13	18	1	2	+	7, 1	0, 1	1, 1	II, I_1	2 : 2
B2	0	0	0	-8	32	1	2	-	14, 2	0, 2	2, 2	III^*, I_2	2 : 1
C1	0	0	0	-2	-4	0	2	-	8, 2	0, 2	2, 2	III, I_2	2 : 2
C2	0	0	0	-52	-144	0	2	+	13, 1	0, 1	4, 1	I_2^*, I_1	2 : 1
D1	0	-1	0	-15	-25	0	2	-	8, 4	0, 4	2, 4	III, I_4	2 : 2
D2	0	-1	0	-265	-1575	0	2	+	13, 2	0, 2	4, 2	I_2^*, I_2	2 : 1
E1	0	-1	0	-66	230	0	2	+	7, 2	0, 2	1, 2	II, I_2	2 : 2
E2	0	-1	0	-61	261	0	2	-	14, 4	0, 4	2, 2	III^*, I_4	2 : 1
F1	0	1	0	-66	-230	0	2	+	7, 2	0, 2	1, 2	II, I_2	2 : 2
F2	0	1	0	-61	-261	0	2	-	14, 4	0, 4	2, 2	III^*, I_4	2 : 1
G1	0	0	0	-2	4	1	2	-	8, 2	0, 2	2, 2	III, I_2	2 : 2
G2	0	0	0	-52	144	1	2	+	13, 1	0, 1	2, 1	I_2^*, I_1	2 : 1
H1	0	1	0	-15	25	1	2	-	8, 4	0, 4	2, 4	III, I_4	2 : 2
H2	0	1	0	-265	1575	1	2	+	13, 2	0, 2	2, 2	I_2^*, I_2	2 : 1

642 $N = 642 = 2 \cdot 3 \cdot 107$ (3 isogeny classes) **642**

A1	1	1	0	-49	85	0	2	+	10, 3, 1	10, 3, 1	2, 1, 1	I_{10}, I_3, I_1	2 : 2
A2	1	1	0	111	693	0	2	-	5, 6, 2	5, 6, 2	1, 2, 2	I_5, I_6, I_2	2 : 1
B1	1	0	1	140	-790	0	3	-	3, 12, 1	3, 12, 1	1, 12, 1	I_3, I_{12}, I_1	3 : 2
B2	1	0	1	-4315	-109978	0	1	-	9, 4, 3	9, 4, 3	1, 4, 1	I_9, I_4, I_3	3 : 1
C1	1	1	1	79	335	1	1	-	13, 4, 1	13, 4, 1	13, 2, 1	I_{13}, I_4, I_1	

643 $N = 643 = 643$ (1 isogeny class) **643**

A1	1	0	0	-4	3	2	1	-	1	1	1	I_1	
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644 $N = 644 = 2^2 \cdot 7 \cdot 23$ (2 isogeny classes) **644**

A1	0	1	0	6	-43	1	1	-	4, 4, 1	0, 4, 1	1, 2, 1	IV, I_4, I_1	
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	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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645 $N = 645 = 3 \cdot 5 \cdot 43$ (6 isogeny classes)**645**

A1	1	1	0	2	7	0	2	−	4, 1, 1	4, 1, 1	2, 1, 1	I_4, I_1, I_1	$2 : 2$
A2	1	1	0	−43	88	0	4	+	2, 2, 2	2, 2, 2	2, 2, 2	I_2, I_2, I_2	$2 : 1, 3, 4$
A3	1	1	0	−118	−407	0	2	+	1, 1, 4	1, 1, 4	1, 1, 4	I_1, I_1, I_4	$2 : 2$
A4	1	1	0	−688	6667	0	2	+	1, 4, 1	1, 4, 1	1, 2, 1	I_1, I_4, I_1	$2 : 2$
B1	1	1	0	−22	31	0	2	+	3, 2, 1	3, 2, 1	1, 2, 1	I_3, I_2, I_1	$2 : 2$
B2	1	1	0	3	126	0	2	−	6, 1, 2	6, 1, 2	2, 1, 2	I_6, I_1, I_2	$2 : 1$
C1	0	−1	1	−16780	855303	0	1	−	14, 2, 3	14, 2, 3	2, 2, 1	I_{14}, I_2, I_3	
D1	0	−1	1	−18000	−923542	0	1	−	6, 2, 1	6, 2, 1	2, 2, 1	I_6, I_2, I_1	
E1	0	1	1	1815	141239	1	1	−	12, 8, 1	12, 8, 1	12, 8, 1	I_{12}, I_8, I_1	
F1	0	1	1	10	44	1	1	−	6, 2, 1	6, 2, 1	6, 2, 1	I_6, I_2, I_1	

646 $N = 646 = 2 \cdot 17 \cdot 19$ (5 isogeny classes)**646**

A1	1	−1	0	−125	−507	0	2	+	6, 1, 2	6, 1, 2	2, 1, 2	I_6, I_1, I_2	$2 : 2$
A2	1	−1	0	−85	−867	0	2	−	3, 2, 4	3, 2, 4	1, 2, 4	I_3, I_2, I_4	$2 : 1$
B1	1	1	1	−77	−77	0	2	+	4, 3, 2	4, 3, 2	4, 1, 2	I_4, I_3, I_2	$2 : 2$
B2	1	1	1	303	−229	0	2	−	2, 6, 1	2, 6, 1	2, 2, 1	I_2, I_6, I_1	$2 : 1$
C1	1	0	0	−241	1413	0	2	+	2, 1, 4	2, 1, 4	2, 1, 2	I_2, I_1, I_4	$2 : 2$
C2	1	0	0	−3851	91663	0	2	+	1, 2, 2	1, 2, 2	1, 2, 2	I_1, I_2, I_2	$2 : 1$
D1	1	−1	1	−406	3237	1	2	+	12, 1, 2	12, 1, 2	12, 1, 2	I_{12}, I_1, I_2	$2 : 2$
D2	1	−1	1	−6486	202661	1	2	+	6, 2, 1	6, 2, 1	6, 2, 1	I_6, I_2, I_1	$2 : 1$
E1	1	0	0	−153	505	0	6	+	6, 3, 2	6, 3, 2	6, 3, 2	I_6, I_3, I_2	$2 : 2; 3 : 3$
E2	1	0	0	−913	−10287	0	6	+	3, 6, 1	3, 6, 1	3, 6, 1	I_3, I_6, I_1	$2 : 1; 3 : 4$
E3	1	0	0	−4573	−119379	0	2	+	2, 1, 6	2, 1, 6	2, 1, 6	I_2, I_1, I_6	$2 : 4; 3 : 1$
E4	1	0	0	−73163	−7623125	0	2	+	1, 2, 3	1, 2, 3	1, 2, 3	I_1, I_2, I_3	$2 : 3; 3 : 2$

648 $N = 648 = 2^3 \cdot 3^4$ (4 isogeny classes)**648**

A1	0	0	0	−3	14	1	1	−	10, 4	0, 0	2, 1	III^*, II	
B1	0	0	0	−3	−1	1	1	+	4, 4	0, 0	2, 1	III, II	
C1	0	0	0	−27	−378	0	1	−	10, 10	0, 0	2, 1	III^*, IV^*	
D1	0	0	0	−27	27	1	1	+	4, 10	0, 0	2, 3	III, IV^*	

649 $N = 649 = 11 \cdot 59$ (1 isogeny class)**649**

A1	1	0	0	−1	4	1	1	−	2, 1	2, 1	2, 1	I_2, I_1	
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650 $N = 650 = 2 \cdot 5^2 \cdot 13$ (13 isogeny classes)**650**

A1	1	−1	0	−167	−259	1	2	+	8, 7, 1	8, 1, 1	2, 2, 1	I_8, I_1^*, I_1	$2 : 2$
A2	1	−1	0	−2167	−38259	1	4	+	4, 8, 2	4, 2, 2	2, 4, 2	I_4, I_2^*, I_2	$2 : 1, 3, 4$
A3	1	−1	0	−34667	−2475759	1	2	+	2, 7, 1	2, 1, 1	2, 2, 1	I_2, I_1^*, I_1	$2 : 2$
A4	1	−1	0	−1667	−56759	1	2	−	2, 10, 4	2, 4, 4	2, 4, 2	I_2, I_4^*, I_4	$2 : 2$
B1	1	1	0	−130	−780	1	1	−	18, 2, 1	18, 0, 1	2, 1, 1	I_{18}, II, I_1	$3 : 2$
B2	1	1	0	−11330	−468940	1	1	−	6, 2, 3	6, 0, 3	2, 1, 1	I_6, II, I_3	$3 : 1$
C1	1	−1	0	−22	46	1	1	−	1, 2, 2	1, 0, 2	1, 1, 2	I_1, II, I_2	
D1	1	0	1	299	22048	0	1	−	7, 10, 2	7, 0, 2	1, 1, 2	I_7, II^*, I_2	
E1	1	0	1	−21026	−1175052	0	2	+	8, 11, 1	8, 5, 1	2, 4, 1	I_8, I_1^*, I_1	$2 : 2$

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
650 650													
$N = 650 = 2 \cdot 5^2 \cdot 13$ (continued)													
F1	1	-1	0	-67	341	0	1	-	7, 6, 1	7, 0, 1	1, 1, 1	I_7, I_0^*, I_1	7 : 2
F2	1	-1	0	-5317	-162409	0	1	-	1, 6, 7	1, 0, 7	1, 1, 7	I_1, I_0^*, I_7	7 : 1
G1	1	0	1	-26	48	1	3	-	2, 4, 1	2, 0, 1	2, 3, 1	I_2, IV, I_1	3 : 2
G2	1	0	1	99	248	1	1	-	6, 4, 3	6, 0, 3	2, 1, 3	I_6, IV, I_3	3 : 1
H1	1	1	1	12	31	0	1	-	1, 6, 1	1, 0, 1	1, 1, 1	I_1, I_0^*, I_1	3 : 2
H2	1	1	1	-113	-969	0	1	-	3, 6, 3	3, 0, 3	3, 1, 1	I_3, I_0^*, I_3	3 : 1, 3
H3	1	1	1	-11488	-478719	0	1	-	9, 6, 1	9, 0, 1	9, 1, 1	I_9, I_0^*, I_1	3 : 2
I1	1	1	1	-638	6031	0	1	-	2, 10, 1	2, 0, 1	2, 1, 1	I_2, II^*, I_1	3 : 2
I2	1	1	1	2487	31031	0	1	-	6, 10, 3	6, 0, 3	6, 1, 1	I_6, II^*, I_3	3 : 1
J1	1	1	1	-813	8531	0	2	+	4, 9, 1	4, 3, 1	4, 2, 1	I_4, I_3^*, I_1	2 : 2; 3 : 3
J2	1	1	1	-313	19531	0	2	-	2, 12, 2	2, 6, 2	2, 4, 2	I_2, I_6^*, I_2	2 : 1; 3 : 4
J3	1	1	1	-5188	-140219	0	2	+	12, 7, 3	12, 1, 3	12, 2, 1	I_{12}, I_1^*, I_3	2 : 4; 3 : 1
J4	1	1	1	2812	-524219	0	2	-	6, 8, 6	6, 2, 6	6, 4, 2	I_6, I_2^*, I_6	2 : 3; 3 : 2
K1	1	1	1	12	181	1	1	-	7, 4, 2	7, 0, 2	7, 3, 2	I_7, IV, I_2	
L1	1	0	0	-3263	-90983	0	3	-	18, 8, 1	18, 0, 1	18, 3, 1	I_{18}, IV^*, I_1	3 : 2
L2	1	0	0	-283263	-58050983	0	1	-	6, 8, 3	6, 0, 3	6, 1, 3	I_6, IV^*, I_3	3 : 1
M1	1	-1	1	-555	5197	0	1	-	1, 8, 2	1, 0, 2	1, 1, 2	I_1, IV^*, I_2	

651 651													
$N = 651 = 3 \cdot 7 \cdot 31$ (5 isogeny classes)													
A1	1	1	0	-5596	-164045	0	2	-	2, 10, 1	2, 10, 1	2, 10, 1	I_2, I_{10}, I_1	2 : 2
A2	1	1	0	-89631	-10365894	0	2	+	4, 5, 2	4, 5, 2	2, 5, 2	I_4, I_5, I_2	2 : 1
B1	1	1	0	-3	0	0	2	+	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	2 : 2
B2	1	1	0	12	15	0	2	-	1, 2, 2	1, 2, 2	1, 2, 2	I_1, I_2, I_2	2 : 1
C1	1	0	1	3	-5	1	2	-	2, 2, 1	2, 2, 1	2, 2, 1	I_2, I_2, I_1	2 : 2
C2	1	0	1	-32	-61	1	2	+	4, 1, 2	4, 1, 2	4, 1, 2	I_4, I_1, I_2	2 : 1
D1	1	0	0	36	-81	1	4	-	4, 4, 1	4, 4, 1	4, 4, 1	I_4, I_4, I_1	2 : 2
D2	1	0	0	-209	-816	1	4	+	8, 2, 2	8, 2, 2	8, 2, 2	I_8, I_2, I_2	2 : 1, 3, 4
D3	1	0	0	-3044	-64887	1	2	+	4, 1, 4	4, 1, 4	4, 1, 2	I_4, I_1, I_4	2 : 2
D4	1	0	0	-1294	17195	1	2	+	16, 1, 1	16, 1, 1	16, 1, 1	I_{16}, I_1, I_1	2 : 2
E1	0	1	1	23	-83	0	3	-	9, 1, 1	9, 1, 1	9, 1, 1	I_9, I_1, I_1	3 : 2
E2	0	1	1	-1057	-13610	0	3	-	3, 3, 3	3, 3, 3	3, 3, 3	I_3, I_3, I_3	3 : 1, 3
E3	0	1	1	-85687	-9682913	0	1	-	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	3 : 2

654 654													
$N = 654 = 2 \cdot 3 \cdot 109$ (2 isogeny classes)													
A1	1	0	1	-174	880	1	1	-	4, 8, 1	4, 8, 1	2, 8, 1	I_4, I_8, I_1	
B1	1	1	1	-56	1145	1	1	-	16, 4, 1	16, 4, 1	16, 2, 1	I_{16}, I_4, I_1	

655 655													
$N = 655 = 5 \cdot 131$ (1 isogeny class)													
A1	0	0	1	-13	18	2	1	-	2, 1	2, 1	2, 1	I_2, I_1	

656 656													
$N = 656 = 2^4 \cdot 41$ (3 isogeny classes)													
A1	0	0	0	-11	10	1	2	+	10, 1	0, 1	4, 1	I_2^*, I_1	2 : 2
A2	0	0	0	29	66	1	2	-	11, 2	0, 2	2, 2	I_3^*, I_2	2 : 1
B1	0	1	0	-12	-20	0	2	+	8, 1	0, 1	2, 1	I^*, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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656 **656**
 $N = 656 = 2^4 \cdot 41$ (continued)

C1	0	-1	0	-24	-16	0	2	+	14, 1	2, 1	4, 1	I_{6, I_1}^*	2 : 2
C2	0	-1	0	-184	1008	0	2	+	13, 2	1, 2	2, 2	I_{5, I_2}^*	2 : 1

657 **657**
 $N = 657 = 3^2 \cdot 73$ (4 isogeny classes)

A1	1	-1	1	-743	7494	0	2	+	16, 1	10, 1	4, 1	I_{10, I_1}^*	2 : 2
A2	1	-1	1	-11678	488634	0	2	+	11, 2	5, 2	2, 2	I_{5, I_2}^*	2 : 1
B1	0	0	1	-57	-167	0	1	-	7, 1	1, 1	4, 1	I_{1, I_1}^*	
C1	0	0	1	24	-36	1	1	-	9, 1	3, 1	2, 1	I_{3, I_1}^*	3 : 2
C2	0	0	1	-246	2043	1	3	-	7, 3	1, 3	2, 3	I_{1, I_3}^*	3 : 1
D1	1	-1	1	-11	10	1	2	+	6, 1	0, 1	4, 1	I_{0, I_1}^*	2 : 2
D2	1	-1	1	34	46	1	2	-	6, 2	0, 2	2, 2	I_{0, I_2}^*	2 : 1

658 **658**
 $N = 658 = 2 \cdot 7 \cdot 47$ (6 isogeny classes)

A1	1	1	0	-117008	18214144	0	1	-	30, 7, 1	30, 7, 1	2, 1, 1	I_{30, I_7, I_1}	
B1	1	1	0	-9	5	0	2	+	6, 1, 1	6, 1, 1	2, 1, 1	I_{6, I_1, I_1}	2 : 2
B2	1	1	0	-49	-147	0	2	+	3, 2, 2	3, 2, 2	1, 2, 2	I_{3, I_2, I_2}	2 : 1
C1	1	0	1	3	12	0	3	-	2, 3, 1	2, 3, 1	2, 3, 1	I_{2, I_3, I_1}	3 : 2
C2	1	0	1	-32	-338	0	1	-	6, 1, 3	6, 1, 3	2, 1, 1	I_{6, I_1, I_3}	3 : 1
D1	1	1	1	24	-23	1	1	-	12, 1, 1	12, 1, 1	12, 1, 1	I_{12, I_1, I_1}	
E1	1	-1	1	1668	19775	1	2	-	22, 4, 1	22, 4, 1	22, 4, 1	I_{22, I_4, I_1}	2 : 2
E2	1	-1	1	-8572	183615	1	2	+	11, 8, 2	11, 8, 2	11, 8, 2	I_{11, I_8, I_2}	2 : 1
F1	1	-1	1	-18	33	1	1	-	4, 1, 1	4, 1, 1	4, 1, 1	I_{4, I_1, I_1}	

659 **659**
 $N = 659 = 659$ (2 isogeny classes)

A1	1	1	0	-79	-306	1	1	+	1	1	1	I_1	
B1	0	1	1	-372	2641	0	1	-	1	1	1	I_1	

660 **660**
 $N = 660 = 2^2 \cdot 3 \cdot 5 \cdot 11$ (4 isogeny classes)

A1	0	-1	0	-21	-54	0	2	-	4, 2, 4, 1	0, 2, 4, 1	1, 2, 2, 1	IV, I_2, I_4, I_1	2 : 2
A2	0	-1	0	-396	-2904	0	2	+	8, 1, 2, 2	0, 1, 2, 2	1, 1, 2, 2	IV^*, I_1, I_2, I_2	2 : 1
B1	0	-1	0	-1	10	1	2	-	4, 2, 2, 1	0, 2, 2, 1	3, 2, 2, 1	IV, I_2, I_2, I_1	2 : 2
B2	0	-1	0	-76	280	1	2	+	8, 1, 1, 2	0, 1, 1, 2	3, 1, 1, 2	IV^*, I_1, I_1, I_2	2 : 1
C1	0	1	0	-41	120	1	6	-	4, 6, 2, 1	0, 6, 2, 1	3, 6, 2, 1	IV, I_6, I_2, I_1	2 : 2; 3 : 3
C2	0	1	0	-716	7140	1	6	+	8, 3, 1, 2	0, 3, 1, 2	3, 3, 1, 2	IV^*, I_3, I_1, I_2	2 : 1; 3 : 4
C3	0	1	0	319	-1356	1	2	-	4, 2, 6, 3	0, 2, 6, 3	1, 2, 2, 1	IV, I_2, I_6, I_3	2 : 4; 3 : 1
C4	0	1	0	-1556	-13356	1	2	+	8, 1, 3, 6	0, 1, 3, 6	1, 1, 1, 2	IV^*, I_1, I_3, I_6	2 : 3; 3 : 2
D1	0	1	0	219	-4500	0	6	-	4, 6, 4, 3	0, 6, 4, 3	3, 6, 2, 3	IV, I_6, I_4, I_3	2 : 2; 3 : 3
D2	0	1	0	-3156	-63900	0	6	+	8, 3, 2, 6	0, 3, 2, 6	3, 3, 2, 6	IV^*, I_3, I_2, I_6	2 : 1; 3 : 4
D3	0	1	0	-15621	-757296	0	2	-	4, 2, 12, 1	0, 2, 12, 1	1, 2, 2, 1	IV, I_2, I_{12}, I_1	2 : 4; 3 : 1
D4	0	1	0	-249996	-48194796	0	2	+	8, 1, 6, 2	0, 1, 6, 2	1, 1, 2, 2	IV^*, I_1, I_6, I_2	2 : 3; 3 : 2

662 **662**
 $N = 662 = 2 \cdot 331$ (1 isogeny class)

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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663 $N = 663 = 3 \cdot 13 \cdot 17$ (3 isogeny classes) **663**

A1	1	1	0	-262	-1745	0	2	+	6, 2, 1	6, 2, 1	2, 2, 1	I_6, I_2, I_1	2 : 2
A2	1	1	0	-327	-900	0	2	+	12, 1, 2	12, 1, 2	2, 1, 2	I_{12}, I_1, I_2	2 : 1
B1	1	1	1	-539	4592	1	4	+	2, 2, 1	2, 2, 1	2, 2, 1	I_2, I_2, I_1	2 : 2
B2	1	1	1	-544	4496	1	8	+	4, 4, 2	4, 4, 2	2, 4, 2	I_4, I_4, I_2	2 : 1, 3, 4
B3	1	1	1	-1389	-14094	1	4	+	8, 2, 4	8, 2, 4	2, 2, 4	I_8, I_2, I_4	2 : 2, 5, 6
B4	1	1	1	221	17042	1	4	-	2, 8, 1	2, 8, 1	2, 8, 1	I_2, I_8, I_1	2 : 2
B5	1	1	1	-20174	-1111138	1	2	+	16, 1, 2	16, 1, 2	2, 1, 2	I_{16}, I_1, I_2	2 : 3
B6	1	1	1	3876	-89910	1	2	-	4, 1, 8	4, 1, 8	2, 1, 8	I_4, I_1, I_8	2 : 3
C1	1	0	0	-33	-72	1	2	+	4, 2, 1	4, 2, 1	4, 2, 1	I_4, I_2, I_1	2 : 2
C2	1	0	0	-98	279	1	2	+	8, 1, 2	8, 1, 2	8, 1, 2	I_8, I_1, I_2	2 : 1

664 $N = 664 = 2^3 \cdot 83$ (3 isogeny classes) **664**

A1	0	0	0	-7	10	2	1	-	8, 1	0, 1	4, 1	I_1^*, I_1	
B1	0	1	0	1	2	1	1	-	4, 1	0, 1	2, 1	III, I_1	
C1	0	-1	0	-3	4	1	1	-	4, 1	0, 1	2, 1	III, I_1	

665 $N = 665 = 5 \cdot 7 \cdot 19$ (5 isogeny classes) **665**

A1	1	1	1	64	258	1	1	-	3, 5, 1	3, 5, 1	1, 5, 1	I_3, I_5, I_1	
B1	1	-1	0	-14	-17	1	2	+	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	2 : 2
B2	1	-1	0	-19	0	1	4	+	2, 2, 2	2, 2, 2	2, 2, 2	I_2, I_2, I_2	2 : 1, 3, 4
B3	1	-1	0	-194	1085	1	4	+	1, 1, 4	1, 1, 4	1, 1, 4	I_1, I_1, I_4	2 : 2
B4	1	-1	0	76	-57	1	2	-	4, 4, 1	4, 4, 1	4, 2, 1	I_4, I_4, I_1	2 : 2
C1	1	1	0	-2	1	1	1	-	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	
D1	0	-1	1	-210	6798	1	5	-	5, 5, 2	5, 5, 2	5, 5, 2	I_5, I_5, I_2	5 : 2
D2	0	-1	1	-16660	-1081562	1	1	-	1, 1, 10	1, 1, 10	1, 1, 2	I_1, I_1, I_{10}	5 : 1
E1	0	0	1	-97	-368	0	1	-	1, 1, 2	1, 1, 2	1, 1, 2	I_1, I_1, I_2	

666 $N = 666 = 2 \cdot 3^2 \cdot 37$ (7 isogeny classes) **666**

A1	1	-1	0	-231	-1315	0	1	-	5, 9, 1	5, 0, 1	1, 2, 1	I_5, III^*, I_1	
B1	1	-1	0	153	-4685	0	1	-	1, 17, 1	1, 11, 1	1, 2, 1	I_1, I_{11}^*, I_1	
C1	1	-1	0	18	108	1	1	-	3, 9, 1	3, 3, 1	1, 4, 1	I_3, I_3^*, I_1	3 : 2
C2	1	-1	0	-1332	19062	1	3	-	1, 7, 3	1, 1, 3	1, 4, 3	I_1, I_1^*, I_3	3 : 1
D1	1	-1	1	-26	57	1	1	-	5, 3, 1	5, 0, 1	5, 2, 1	I_5, III, I_1	
E1	1	-1	1	13	1235	1	1	-	13, 7, 1	13, 1, 1	13, 4, 1	I_{13}, I_1^*, I_1	
F1	1	-1	1	139	141	0	4	-	8, 9, 1	8, 3, 1	8, 4, 1	I_8, I_3^*, I_1	2 : 2
F2	1	-1	1	-581	1581	0	4	+	4, 12, 2	4, 6, 2	4, 4, 2	I_4, I_6^*, I_2	2 : 1, 3, 4
F3	1	-1	1	-5441	-151995	0	2	+	2, 9, 4	2, 3, 4	2, 2, 4	I_2, I_3^*, I_4	2 : 2
F4	1	-1	1	-7241	238677	0	2	+	2, 18, 1	2, 12, 1	2, 4, 1	I_2, I_{12}^*, I_1	2 : 2
G1	1	-1	1	-1640858	-808607271	0	1	-	23, 15, 1	23, 9, 1	23, 2, 1	I_{23}, I_9^*, I_1	

669 $N = 669 = 3 \cdot 223$ (1 isogeny class) **669**

A1	1	1	0	-1	-2	1	1	-	1, 1	1, 1	1, 1	I_1, I_1	
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670 $N = 670 = 2 \cdot 5 \cdot 67$ (4 isogeny classes) **670**

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
670 670													
$N = 670 = 2 \cdot 5 \cdot 67$ (continued)													
B1	1	0	1	2	6	1	3	–	1, 3, 1	1, 3, 1	1, 3, 1	I_1, I_3, I_1	3 : 2
B2	1	0	1	–23	–174	1	1	–	3, 1, 3	3, 1, 3	1, 1, 3	I_3, I_1, I_3	3 : 1
C1	1	–1	1	–13	21	1	1	–	5, 1, 1	5, 1, 1	5, 1, 1	I_5, I_1, I_1	
D1	1	0	0	44	–624	1	1	–	19, 1, 1	19, 1, 1	19, 1, 1	I_{19}, I_1, I_1	
672 672													
$N = 672 = 2^5 \cdot 3 \cdot 7$ (8 isogeny classes)													
A1	0	–1	0	2	4	1	2	–	6, 1, 2	0, 1, 2	2, 1, 2	III, I_1, I_2	2 : 2
A2	0	–1	0	–33	81	1	2	+	12, 2, 1	0, 2, 1	4, 2, 1	I_3^*, I_2, I_1	2 : 1
B1	0	1	0	210	1764	1	2	–	6, 5, 6	0, 5, 6	2, 5, 6	III, I_5, I_6	2 : 2
B2	0	1	0	–1505	17199	1	2	+	12, 10, 3	0, 10, 3	4, 10, 3	I_3^*, I_{10}, I_3	2 : 1
C1	0	–1	0	–22	40	0	4	+	6, 4, 2	0, 4, 2	2, 2, 2	III, I_4, I_2	2 : 2, 3, 4
C2	0	–1	0	–112	–392	0	2	+	9, 2, 4	0, 2, 4	1, 2, 2	I_0^*, I_2, I_4	2 : 1
C3	0	–1	0	–337	2497	0	4	+	12, 2, 1	0, 2, 1	4, 2, 1	I_3^*, I_2, I_1	2 : 1
C4	0	–1	0	48	180	0	2	–	9, 8, 1	0, 8, 1	2, 2, 1	I_0^*, I_8, I_1	2 : 1
D1	0	–1	0	210	–1764	0	2	–	6, 5, 6	0, 5, 6	2, 1, 2	III, I_5, I_6	2 : 2
D2	0	–1	0	–1505	–17199	0	2	+	12, 10, 3	0, 10, 3	2, 2, 1	I_3^*, I_{10}, I_3	2 : 1
E1	0	–1	0	–14	24	1	4	+	6, 2, 2	0, 2, 2	2, 2, 2	III, I_2, I_2	2 : 2, 3, 4
E2	0	–1	0	–49	–95	1	2	+	12, 4, 1	0, 4, 1	4, 2, 1	I_3^*, I_4, I_1	2 : 1
E3	0	–1	0	–224	1368	1	2	+	9, 1, 1	0, 1, 1	1, 1, 1	I_0^*, I_1, I_1	2 : 1
E4	0	–1	0	16	84	1	4	–	9, 1, 4	0, 1, 4	2, 1, 4	I_0^*, I_1, I_4	2 : 1
F1	0	1	0	–14	–24	1	4	+	6, 2, 2	0, 2, 2	2, 2, 2	III, I_2, I_2	2 : 2, 3, 4
F2	0	1	0	–224	–1368	1	2	+	9, 1, 1	0, 1, 1	2, 1, 1	I_0^*, I_1, I_1	2 : 1
F3	0	1	0	–49	95	1	4	+	12, 4, 1	0, 4, 1	4, 4, 1	I_3^*, I_4, I_1	2 : 1
F4	0	1	0	16	–84	1	2	–	9, 1, 4	0, 1, 4	1, 1, 2	I_0^*, I_1, I_4	2 : 1
G1	0	1	0	2	–4	0	2	–	6, 1, 2	0, 1, 2	2, 1, 2	III, I_1, I_2	2 : 2
G2	0	1	0	–33	–81	0	2	+	12, 2, 1	0, 2, 1	2, 2, 1	I_3^*, I_2, I_1	2 : 1
H1	0	1	0	–22	–40	0	4	+	6, 4, 2	0, 4, 2	2, 4, 2	III, I_4, I_2	2 : 2, 3, 4
H2	0	1	0	–337	–2497	0	2	+	12, 2, 1	0, 2, 1	4, 2, 1	I_3^*, I_2, I_1	2 : 1
H3	0	1	0	–112	392	0	4	+	9, 2, 4	0, 2, 4	2, 2, 4	I_0^*, I_2, I_4	2 : 1
H4	0	1	0	48	–180	0	2	–	9, 8, 1	0, 8, 1	1, 8, 1	I_0^*, I_8, I_1	2 : 1
674 674													
$N = 674 = 2 \cdot 337$ (3 isogeny classes)													
A1	1	0	1	3	0	1	1	–	3, 1	3, 1	1, 1	I_3, I_1	
B1	1	–1	1	–6	5	1	2	+	4, 1	4, 1	4, 1	I_4, I_1	2 : 2
B2	1	–1	1	14	21	1	2	–	2, 2	2, 2	2, 2	I_2, I_2	2 : 1
C1	1	–1	1	2064	18771	1	1	–	31, 1	31, 1	31, 1	I_{31}, I_1	
675 675													
$N = 675 = 3^3 \cdot 5^2$ (9 isogeny classes)													
A1	0	0	1	0	31	1	1	–	3, 6	0, 0	1, 2	II, I_0^*	3 : 2, 3
A2	0	0	1	0	–844	1	1	–	9, 6	0, 0	1, 2	IV^*, I_0^*	3 : 1, 4
A3	0	0	1	–750	7906	1	1	–	5, 6	0, 0	3, 2	IV, I_0^*	3 : 1
A4	0	0	1	–6750	–213469	1	1	–	11, 6	0, 0	1, 2	II^*, I_0^*	3 : 2
B1	1	–1	1	–5	2	1	1	+	5, 2	0, 0	3, 1	IV, II	
C1	0	0	1	0	6	0	3	–	3, 4	0, 0	1, 3	II, IV	3 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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675

$N = 675 = 3^3 \cdot 5^2$ (continued)

675

D1	1	-1	1	-1055	-3428	0	1	+	11, 8	0, 0	1, 1	II*, IV*	
E1	0	0	1	0	781	0	1	-	3, 10	0, 0	1, 1	II, II*	3 : 2
E2	0	0	1	0	-21094	0	1	-	9, 10	0, 0	3, 1	IV*, II*	3 : 1
F1	1	-1	0	-42	-19	0	1	+	11, 2	0, 0	1, 1	II*, II	
G1	0	0	1	-75	531	0	1	-	5, 8	0, 2	1, 2	IV, I ₂ *	
H1	0	0	1	-675	-14344	0	1	-	11, 8	0, 2	1, 2	II*, I ₂ *	
I1	1	-1	0	-117	166	1	1	+	5, 8	0, 0	1, 3	IV, IV*	

676

$N = 676 = 2^2 \cdot 13^2$ (5 isogeny classes)

676

A1	0	0	0	-676	-6591	0	2	+	4, 7	0, 1	3, 2	IV, I ₁ *	2 : 2
A2	0	0	0	169	-21970	0	2	-	8, 8	0, 2	3, 4	IV*, I ₂ *	2 : 1
B1	0	1	0	-4	-12	0	1	-	8, 2	0, 0	1, 1	IV*, II	3 : 2
B2	0	1	0	-524	-4796	0	1	-	8, 2	0, 0	3, 1	IV*, II	3 : 1
C1	0	1	0	-732	-23516	0	3	-	8, 8	0, 0	3, 3	IV*, IV*	3 : 2
C2	0	1	0	-88612	-10182444	0	1	-	8, 8	0, 0	1, 3	IV*, IV*	3 : 1
D1	0	0	0	-169	845	0	1	+	4, 4	0, 0	1, 1	IV, IV	
E1	0	0	0	-28561	1856465	0	1	+	4, 10	0, 0	3, 1	IV, II*	

677

$N = 677 = 677$ (1 isogeny class)

677

A1	1	1	1	2	0	1	1	-	1	1	1	I ₁	
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678

$N = 678 = 2 \cdot 3 \cdot 113$ (6 isogeny classes)

678

A1	1	1	0	-12	12	1	1	-	2, 1, 1	2, 1, 1	2, 1, 1	I ₂ , I ₁ , I ₁	
B1	1	0	1	6	-20	1	1	-	6, 3, 1	6, 3, 1	2, 3, 1	I ₆ , I ₃ , I ₁	
C1	1	1	1	-148	-427	1	2	+	14, 4, 1	14, 4, 1	14, 2, 1	I ₁₄ , I ₄ , I ₁	2 : 2
C2	1	1	1	492	-2475	1	2	-	7, 8, 2	7, 8, 2	7, 2, 2	I ₇ , I ₈ , I ₂	2 : 1
D1	1	0	0	-1661	26097	0	7	-	14, 7, 1	14, 7, 1	14, 7, 1	I ₁₄ , I ₇ , I ₁	7 : 2
D2	1	0	0	-7121	-2567403	0	1	-	2, 1, 7	2, 1, 7	2, 1, 7	I ₂ , I ₁ , I ₇	7 : 1
E1	1	0	0	-192	1008	0	4	+	4, 4, 1	4, 4, 1	4, 4, 1	I ₄ , I ₄ , I ₁	2 : 2
E2	1	0	0	-212	780	0	4	+	2, 8, 2	2, 8, 2	2, 8, 2	I ₂ , I ₈ , I ₂	2 : 1, 3, 4
E3	1	0	0	-1342	-18430	0	2	+	1, 16, 1	1, 16, 1	1, 16, 1	I ₁ , I ₁₆ , I ₁	2 : 2
E4	1	0	0	598	5478	0	2	-	1, 4, 4	1, 4, 4	1, 4, 4	I ₁ , I ₄ , I ₄	2 : 2
F1	1	0	0	-190	-1024	0	2	+	2, 4, 1	2, 4, 1	2, 4, 1	I ₂ , I ₄ , I ₁	2 : 2
F2	1	0	0	-180	-1134	0	2	-	1, 8, 2	1, 8, 2	1, 8, 2	I ₁ , I ₈ , I ₂	2 : 1

680

$N = 680 = 2^3 \cdot 5 \cdot 17$ (3 isogeny classes)

680

A1	0	0	0	-143	658	1	4	+	8, 2, 1	0, 2, 1	4, 2, 1	I ₁ *, I ₂ , I ₁	2 : 2
A2	0	0	0	-163	462	1	4	+	10, 4, 2	0, 4, 2	2, 2, 2	III*, I ₄ , I ₂	2 : 1, 3, 4
A3	0	0	0	-1163	-14938	1	2	+	11, 2, 4	0, 2, 4	1, 2, 4	II*, I ₂ , I ₄	2 : 2
A4	0	0	0	517	3318	1	2	-	11, 8, 1	0, 8, 1	1, 2, 1	II*, I ₈ , I ₁	2 : 2
B1	0	-1	0	0	-20	0	1	-	11, 1, 1	0, 1, 1	1, 1, 1	II*, I ₁ , I ₁	
C1	0	-1	0	-3540	-79900	0	2	+	8, 4, 1	0, 4, 1	4, 4, 1	I*, I ₄ , I ₁	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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681 $N = 681 = 3 \cdot 227$ (5 isogeny classes)**681**

A1	0	-1	1	-13	24	1	1	-	4, 1	4, 1	2, 1	I_4, I_1	
B1	1	1	0	-1154	-15345	0	4	+	10, 2	10, 2	2, 2	I_{10}, I_2	2 : 2, 3, 4
B2	1	1	0	-1149	-15480	0	2	+	5, 1	5, 1	1, 1	I_5, I_1	2 : 1
B3	1	1	0	-2369	20862	0	4	+	5, 4	5, 4	1, 4	I_5, I_4	2 : 1
B4	1	1	0	-19	-42812	0	2	-	20, 1	20, 1	2, 1	I_{20}, I_1	2 : 1
C1	0	-1	1	0	2	2	1	-	2, 1	2, 1	2, 1	I_2, I_1	
D1	0	1	1	-431	-3592	0	1	-	4, 1	4, 1	4, 1	I_4, I_1	
E1	0	1	1	-179	881	1	1	-	10, 1	10, 1	10, 1	I_{10}, I_1	

682 $N = 682 = 2 \cdot 11 \cdot 31$ (2 isogeny classes)**682**

A1	1	0	0	-33	73	1	3	-	9, 1, 1	9, 1, 1	9, 1, 1	I_9, I_1, I_1	3 : 2
A2	1	0	0	167	225	1	3	-	3, 3, 3	3, 3, 3	3, 1, 3	I_3, I_3, I_3	3 : 1, 3
A3	1	0	0	-2003	-39269	1	1	-	1, 9, 1	1, 9, 1	1, 1, 1	I_1, I_9, I_1	3 : 2
B1	1	-1	1	359	-6663	1	1	-	19, 3, 1	19, 3, 1	19, 3, 1	I_{19}, I_3, I_1	

684 $N = 684 = 2^2 \cdot 3^2 \cdot 19$ (3 isogeny classes)**684**

A1	0	0	0	-192	1028	1	1	-	8, 6, 1	0, 0, 1	3, 2, 1	IV^*, I_0^*, I_1	
B1	0	0	0	24	-511	1	2	-	4, 9, 2	0, 3, 2	3, 4, 2	IV, I_3^*, I_2	2 : 2
B2	0	0	0	-831	-8890	1	2	+	8, 12, 1	0, 6, 1	3, 4, 1	IV^*, I_6^*, I_1	2 : 1
C1	0	0	0	24	-268	0	1	-	8, 8, 1	0, 2, 1	1, 2, 1	IV^*, I_2^*, I_1	

685 $N = 685 = 5 \cdot 137$ (1 isogeny class)**685**

A1	1	-1	0	-5	6	1	1	-	1, 1	1, 1	1, 1	I_1, I_1	
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688 $N = 688 = 2^4 \cdot 43$ (3 isogeny classes)**688**

A1	0	0	0	4	-4	1	1	-	8, 1	0, 1	1, 1	I_0^*, I_1	
B1	0	-1	0	-13	-15	0	1	-	8, 1	0, 1	2, 1	I_0^*, I_1	3 : 2
B2	0	-1	0	67	-79	0	1	-	8, 3	0, 3	2, 1	I_0^*, I_3	3 : 1
C1	0	-1	0	-5	-19	1	1	-	12, 1	0, 1	1, 1	II^*, I_1	

689 $N = 689 = 13 \cdot 53$ (1 isogeny class)**689**

A1	1	0	0	-14	19	1	2	+	1, 1	1, 1	1, 1	I_1, I_1	2 : 2
A2	1	0	0	-9	34	1	2	-	2, 2	2, 2	2, 2	I_2, I_2	2 : 1

690 $N = 690 = 2 \cdot 3 \cdot 5 \cdot 23$ (11 isogeny classes)**690**

A1	1	1	0	172	-1968	1	2	-	14, 2, 4, 1	14, 2, 4, 1	2, 2, 2, 1	I_{14}, I_2, I_4, I_1	2 : 2
A2	1	1	0	-1748	-25392	1	2	+	7, 1, 8, 2	7, 1, 8, 2	1, 1, 2, 2	I_7, I_1, I_8, I_2	2 : 1
B1	1	1	0	167	-347	0	2	-	6, 7, 1, 2	6, 7, 1, 2	2, 1, 1, 2	I_6, I_7, I_1, I_2	2 : 2
B2	1	1	0	-753	-3843	0	2	+	3, 14, 2, 1	3, 14, 2, 1	1, 2, 2, 1	I_3, I_{14}, I_2, I_1	2 : 1
C1	1	1	0	-22777	-90852059	0	2	-	10, 18, 8, 1	10, 18, 8, 1	2, 2, 8, 1	I_{10}, I_{18}, I_8, I_1	2 : 2
C2	1	1	0	-3172057	-2148591611	0	2	+	5, 9, 16, 2	5, 9, 16, 2	1, 1, 16, 2	I_5, I_9, I_{16}, I_2	2 : 1
D1	1	1	0	-12	-36	0	2	-	2, 3, 1, 2	2, 3, 1, 2	2, 1, 1, 2	I_2, I_3, I_1, I_2	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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690

$N = 690 = 2 \cdot 3 \cdot 5 \cdot 23$ (continued)

690

E1	1	0	1	-604	-5734	1	2	+ 12, 5, 1, 1	12, 5, 1, 1	2, 5, 1, 1		I_{12}, I_5, I_1, I_1	2 : 2
E2	1	0	1	-924	922	1	4	+ 6, 10, 2, 2	6, 10, 2, 2	2, 10, 2, 2		I_6, I_{10}, I_2, I_2	2 : 1, 3, 4
E3	1	0	1	-10644	420826	1	2	+ 3, 5, 4, 4	3, 5, 4, 4	1, 5, 2, 4		I_3, I_5, I_4, I_4	2 : 2
E4	1	0	1	3676	8282	1	2	- 3, 20, 1, 1	3, 20, 1, 1	1, 20, 1, 1		I_3, I_{20}, I_1, I_1	2 : 2
F1	1	0	1	-13	8	0	2	+ 8, 1, 1, 1	8, 1, 1, 1	2, 1, 1, 1		I_8, I_1, I_1, I_1	2 : 2
F2	1	0	1	-93	-344	0	4	+ 4, 2, 2, 2	4, 2, 2, 2	2, 2, 2, 2		I_4, I_2, I_2, I_2	2 : 1, 3, 4
F3	1	0	1	-1473	-21872	0	2	+ 2, 1, 4, 1	2, 1, 4, 1	2, 1, 4, 1		I_2, I_1, I_4, I_1	2 : 2
F4	1	0	1	7	-1024	0	4	- 2, 4, 1, 4	2, 4, 1, 4	2, 4, 1, 4		I_2, I_4, I_1, I_4	2 : 2
G1	1	1	1	-4491	-207687	0	4	- 28, 4, 2, 1	28, 4, 2, 1	28, 2, 2, 1		I_{28}, I_4, I_2, I_1	2 : 2
G2	1	1	1	-86411	-9808711	0	4	+ 14, 8, 4, 2	14, 8, 4, 2	14, 2, 2, 2		I_{14}, I_8, I_4, I_2	2 : 1, 3, 4
G3	1	1	1	-1382411	-626186311	0	2	+ 7, 4, 2, 4	7, 4, 2, 4	7, 2, 2, 2		I_7, I_4, I_2, I_4	2 : 2
G4	1	1	1	-101131	-6258247	0	2	+ 7, 16, 8, 1	7, 16, 8, 1	7, 2, 2, 1		I_7, I_{16}, I_8, I_1	2 : 2
H1	1	1	1	4	29	1	2	- 6, 2, 2, 1	6, 2, 2, 1	6, 2, 2, 1		I_6, I_2, I_2, I_1	2 : 2
H2	1	1	1	-116	413	1	2	+ 3, 1, 4, 2	3, 1, 4, 2	3, 1, 2, 2		I_3, I_1, I_4, I_2	2 : 1
I1	1	0	0	134	-604	0	2	- 6, 1, 5, 2	6, 1, 5, 2	6, 1, 1, 2		I_6, I_1, I_5, I_2	2 : 2
I2	1	0	0	-786	-5940	0	2	+ 3, 2, 10, 1	3, 2, 10, 1	3, 2, 2, 1		I_3, I_2, I_{10}, I_1	2 : 1
J1	1	0	0	-245	-1503	0	2	- 10, 1, 1, 2	10, 1, 1, 2	10, 1, 1, 2		I_{10}, I_1, I_1, I_2	2 : 2
J2	1	0	0	-3925	-94975	0	2	+ 5, 2, 2, 1	5, 2, 2, 1	5, 2, 2, 1		I_5, I_2, I_2, I_1	2 : 1
K1	1	0	0	-420	3600	0	8	- 8, 8, 2, 1	8, 8, 2, 1	8, 8, 2, 1		I_8, I_8, I_2, I_1	2 : 2
K2	1	0	0	-6900	220032	0	8	+ 4, 4, 4, 2	4, 4, 4, 2	4, 4, 4, 2		I_4, I_4, I_4, I_2	2 : 1, 3, 4
K3	1	0	0	-7080	207900	0	4	+ 2, 2, 8, 4	2, 2, 8, 4	2, 2, 8, 2		I_2, I_2, I_8, I_4	2 : 2, 5, 6
K4	1	0	0	-110400	14109732	0	4	+ 2, 2, 2, 1	2, 2, 2, 1	2, 2, 2, 1		I_2, I_2, I_2, I_1	2 : 2
K5	1	0	0	-25830	-1370850	0	2	+ 1, 1, 4, 8	1, 1, 4, 8	1, 1, 4, 2		I_1, I_1, I_4, I_8	2 : 3
K6	1	0	0	8790	1010922	0	2	- 1, 1, 16, 2	1, 1, 16, 2	1, 1, 16, 2		I_1, I_1, I_{16}, I_2	2 : 3

692

$N = 692 = 2^2 \cdot 173$ (1 isogeny class)

692

A1	0	1	0	-52	180	0	2	- 8, 2	0, 2	1, 2		IV^*, I_2	2 : 2
A2	0	1	0	-57	148	0	2	+ 4, 1	0, 1	1, 1		IV, I_1	2 : 1

693

$N = 693 = 3^2 \cdot 7 \cdot 11$ (4 isogeny classes)

693

A1	1	-1	1	31	-264	0	2	- 6, 3, 2	0, 3, 2	2, 1, 2		I_0^*, I_3, I_2	2 : 2
A2	1	-1	1	-464	-3432	0	2	+ 6, 6, 1	0, 6, 1	2, 2, 1		I_0^*, I_6, I_1	2 : 1
B1	0	0	1	18	-7	1	1	- 6, 2, 1	0, 2, 1	1, 2, 1		I_0^*, I_2, I_1	
C1	0	0	1	-804	-8775	0	1	- 6, 2, 1	0, 2, 1	1, 2, 1		I_0^*, I_2, I_1	3 : 2
C2	0	0	1	-444	-16650	0	3	- 6, 6, 3	0, 6, 3	1, 6, 3		I_0^*, I_6, I_3	3 : 1, 3
C3	0	0	1	3966	430965	0	3	- 6, 2, 9	0, 2, 9	1, 2, 9		I_0^*, I_2, I_9	3 : 2
D1	1	-1	0	-306	-1985	0	2	+ 7, 2, 1	1, 2, 1	4, 2, 1		I_1^*, I_2, I_1	2 : 2
D2	1	-1	0	-351	-1328	0	4	+ 8, 4, 2	2, 4, 2	4, 4, 2		I_2^*, I_4, I_2	2 : 1, 3, 4
D3	1	-1	0	-2556	49387	0	4	+ 10, 2, 4	4, 2, 4	4, 2, 4		I_4^*, I_2, I_4	2 : 2, 5, 6
D4	1	-1	0	1134	-10535	0	2	- 7, 8, 1	1, 8, 1	2, 8, 1		I_1^*, I_8, I_1	2 : 2
D5	1	-1	0	-40671	3167194	0	2	+ 14, 1, 2	8, 1, 2	4, 1, 2		I_8^*, I_1, I_2	2 : 3
D6	1	-1	0	279	150880	0	2	- 8, 1, 8	2, 1, 8	2, 1, 8		I_2^*, I_1, I_8	2 : 3

696

$N = 696 = 2^3 \cdot 3 \cdot 29$ (7 isogeny classes)

696

A1	0	-1	0	-88	349	1	1	- 4, 3, 1	0, 3, 1	2, 1, 1		III, I_3, I_1	
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	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
696	$N = 696 = 2^3 \cdot 3 \cdot 29$ (continued)												696
C1	0	1	0	12	9	1	1	-	4, 5, 1	0, 5, 1	2, 5, 1	III, I ₅ , I ₁	
D1	0	-1	0	-5920	177388	0	1	-	11, 5, 3	0, 5, 3	1, 1, 1	II*, I ₅ , I ₃	
E1	0	-1	0	-36	-87	0	1	-	4, 1, 3	0, 1, 3	2, 1, 1	III, I ₁ , I ₃	
F1	0	-1	0	56	-1415	1	1	-	4, 7, 3	0, 7, 3	2, 1, 3	III, I ₇ , I ₃	
G1	0	1	0	-4	5	1	1	-	4, 3, 1	0, 3, 1	2, 3, 1	III, I ₃ , I ₁	
699	$N = 699 = 3 \cdot 233$ (1 isogeny class)												699
A1	0	1	1	-10	-17	0	1	-	3, 1	3, 1	3, 1	I ₃ , I ₁	
700	$N = 700 = 2^2 \cdot 5^2 \cdot 7$ (10 isogeny classes)												700
A1	0	-1	0	-133	-2863	0	1	-	8, 9, 1	0, 3, 1	1, 2, 1	IV*, I ₃ *, I ₁	3 : 2
A2	0	-1	0	-20133	-1092863	0	1	-	8, 7, 3	0, 1, 3	3, 2, 1	IV*, I ₁ *, I ₃	3 : 1
B1	0	-1	0	2	-3	0	1	-	4, 2, 1	0, 0, 1	1, 1, 1	IV, II, I ₁	3 : 2
B2	0	-1	0	-98	-343	0	1	-	4, 2, 3	0, 0, 3	3, 1, 1	IV, II, I ₃	3 : 1
C1	0	0	0	-5	5	1	1	-	4, 2, 1	0, 0, 1	3, 1, 1	IV, II, I ₁	
D1	0	0	0	800	26500	1	1	-	8, 7, 5	0, 1, 5	3, 4, 5	IV*, I ₁ *, I ₅	
E1	0	0	0	-2000	-34375	1	2	+	4, 9, 2	0, 0, 2	3, 2, 2	IV, III*, I ₂	2 : 2
E2	0	0	0	-1375	-56250	1	2	-	8, 9, 4	0, 0, 4	3, 2, 2	IV*, III*, I ₄	2 : 1
F1	0	0	0	-125	625	1	1	-	4, 8, 1	0, 0, 1	1, 3, 1	IV, IV*, I ₁	
G1	0	0	0	-40	100	1	1	-	8, 3, 1	0, 0, 1	3, 2, 1	IV*, III, I ₁	
H1	0	0	0	-80	-275	0	2	+	4, 3, 2	0, 0, 2	1, 2, 2	IV, III, I ₂	2 : 2
H2	0	0	0	-55	-450	0	2	-	8, 3, 4	0, 0, 4	1, 2, 4	IV*, III, I ₄	2 : 1
I1	0	1	0	42	-287	0	3	-	4, 8, 1	0, 0, 1	3, 3, 1	IV, IV*, I ₁	3 : 2
I2	0	1	0	-2458	-47787	0	1	-	4, 8, 3	0, 0, 3	1, 1, 3	IV, IV*, I ₃	3 : 1
J1	0	0	0	-1000	12500	0	1	-	8, 9, 1	0, 0, 1	1, 2, 1	IV*, III*, I ₁	
701	$N = 701 = 701$ (1 isogeny class)												701
A1	0	-1	1	-2	1	0	1	+	1	1	1	I ₁	
702	$N = 702 = 2 \cdot 3^3 \cdot 13$ (16 isogeny classes)												702
A1	1	-1	0	-9	-19	1	1	-	5, 3, 2	5, 0, 2	1, 1, 2	I ₅ , II, I ₂	
B1	1	-1	0	-3	-1	1	1	+	1, 3, 1	1, 0, 1	1, 1, 1	I ₁ , II, I ₁	
C1	1	-1	0	39	35	0	1	-	1, 11, 1	1, 0, 1	1, 1, 1	I ₁ , II*, I ₁	
D1	1	-1	0	-366	-2476	0	1	+	7, 11, 1	7, 0, 1	1, 1, 1	I ₇ , II*, I ₁	
E1	1	-1	0	-5826	173076	0	3	-	9, 3, 6	9, 0, 6	1, 1, 6	I ₉ , II, I ₆	3 : 2, 3
E2	1	-1	0	11919	881693	0	1	-	27, 9, 2	27, 0, 2	1, 1, 2	I ₂₇ , IV*, I ₂	3 : 1
E3	1	-1	0	-472266	125037036	0	3	-	3, 5, 2	3, 0, 2	1, 3, 2	I ₃ , IV, I ₂	3 : 1
F1	1	-1	0	-648	9536	0	1	-	11, 3, 5	11, 0, 5	1, 1, 1	I ₁₁ , II, I ₅	
G1	1	-1	0	-165	533	0	1	+	19, 3, 1	19, 0, 1	1, 1, 1	I ₁₉ , II, I ₁	
H1	1	-1	0	-132	618	1	3	+	1, 3, 1	1, 0, 1	1, 1, 1	I ₁ , II, I ₁	3 : 2
H2	1	-1	0	-177	197	1	3	+	3, 9, 3	3, 0, 3	1, 3, 3	I ₃ , IV*, I ₃	3 : 1, 3
H3	1	-1	0	-8952	-323776	1	1	+	9, 11, 1	9, 0, 1	1, 1, 1	I ₉ , II*, I ₁	3 : 2
I1	1	-1	1	-29	55	0	1	+	1, 9, 1	1, 0, 1	1, 1, 1	I ₁ , IV*, I ₁	

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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702

$N = 702 = 2 \cdot 3^3 \cdot 13$ (continued)

702

K1	1	-1	1	-41	105	1	1	+	7, 5, 1	7, 0, 1	7, 3, 1	I_7, IV, I_1	
L1	1	-1	1	-83	595	1	1	-	5, 9, 2	5, 0, 2	5, 3, 2	I_5, IV^*, I_2	
M1	1	-1	1	-1487	-12905	1	1	+	19, 9, 1	19, 0, 1	19, 3, 1	I_{19}, IV^*, I_1	
N1	1	-1	1	-20	-1	0	3	+	3, 3, 3	3, 0, 3	3, 1, 3	I_3, II, I_3	3 : 2, 3
N2	1	-1	1	-1190	-15497	0	1	+	1, 9, 1	1, 0, 1	1, 3, 1	I_1, IV^*, I_1	3 : 1
N3	1	-1	1	-995	12323	0	3	+	9, 5, 1	9, 0, 1	9, 1, 1	I_9, IV, I_1	3 : 1
O1	1	-1	1	4	-3	0	1	-	1, 5, 1	1, 0, 1	1, 1, 1	I_1, IV, I_1	
P1	1	-1	1	1324	-33097	0	3	-	27, 3, 2	27, 0, 2	27, 1, 2	I_{27}, II, I_2	3 : 2
P2	1	-1	1	-52436	-4620617	0	3	-	9, 9, 6	9, 0, 6	9, 3, 6	I_9, IV^*, I_6	3 : 1, 3
P3	1	-1	1	-4250396	-3371749577	0	1	-	3, 11, 2	3, 0, 2	3, 1, 2	I_3, II^*, I_2	3 : 2

703

$N = 703 = 19 \cdot 37$ (2 isogeny classes)

703

A1	0	0	1	-736	1057	0	1	+	2, 5	2, 5	2, 1	I_2, I_5	
B1	0	0	1	1	-8	1	1	-	1, 2	1, 2	1, 2	I_1, I_2	

704

$N = 704 = 2^6 \cdot 11$ (12 isogeny classes)

704

A1	0	1	0	-1	1	1	1	-	6, 1	0, 1	1, 1	II, I_1	5 : 2
A2	0	1	0	-41	-199	1	1	-	6, 5	0, 5	1, 1	II, I_5	5 : 1, 3
A3	0	1	0	-31281	-2139919	1	1	-	6, 1	0, 1	1, 1	II, I_1	5 : 2
B1	0	-1	0	1	1	1	1	-	6, 1	0, 1	1, 1	II, I_1	
C1	0	1	0	1	-1	0	1	-	6, 1	0, 1	1, 1	II, I_1	
D1	0	-1	0	11	-19	0	1	-	14, 1	0, 1	1, 1	II^*, I_1	3 : 2
D2	0	-1	0	-309	-2003	0	1	-	14, 3	0, 3	1, 3	II^*, I_3	3 : 1
E1	0	0	0	-16	32	0	1	-	14, 1	0, 1	1, 1	II^*, I_1	
F1	0	1	0	11	19	0	1	-	14, 1	0, 1	1, 1	II^*, I_1	3 : 2
F2	0	1	0	-309	2003	0	1	-	14, 3	0, 3	1, 1	II^*, I_3	3 : 1
G1	0	-1	0	-11	-11	0	1	-	6, 1	0, 1	1, 1	II, I_1	
H1	0	0	0	2	14	0	1	-	6, 3	0, 3	1, 1	II, I_3	
I1	0	0	0	-16	-32	0	1	-	14, 1	0, 1	1, 1	II^*, I_1	
J1	0	1	0	-11	11	1	1	-	6, 1	0, 1	1, 1	II, I_1	
K1	0	-1	0	-1	-1	1	1	-	6, 1	0, 1	1, 1	II, I_1	5 : 2
K2	0	-1	0	-41	199	1	1	-	6, 5	0, 5	1, 5	II, I_5	5 : 1, 3
K3	0	-1	0	-31281	2139919	1	1	-	6, 1	0, 1	1, 1	II, I_1	5 : 2
L1	0	0	0	2	-14	1	1	-	6, 3	0, 3	1, 3	II, I_3	

705

$N = 705 = 3 \cdot 5 \cdot 47$ (6 isogeny classes)

705

A1	0	-1	1	-5781	175862	1	1	-	14, 5, 1	14, 5, 1	2, 1, 1	I_{14}, I_5, I_1	
B1	1	1	1	-120	42282	1	1	-	3, 3, 5	3, 3, 5	1, 3, 5	I_3, I_3, I_5	
C1	0	1	1	9	20	0	3	-	6, 1, 1	6, 1, 1	6, 1, 1	I_6, I_1, I_1	3 : 2
C2	0	1	1	-81	-619	0	1	-	2, 3, 3	2, 3, 3	2, 1, 1	I_2, I_3, I_3	3 : 1
D1	1	0	1	6	1	1	1	-	1, 3, 1	1, 3, 1	1, 1, 1	I_1, I_3, I_1	

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
705 705 $N = 705 = 3 \cdot 5 \cdot 47$ (continued)													
F1	1	0	1	-368	2681	0	2	+	1, 3, 1	1, 3, 1	1, 3, 1	I_1, I_3, I_1	2 : 2
F2	1	0	1	-373	2603	0	4	+	2, 6, 2	2, 6, 2	2, 6, 2	I_2, I_6, I_2	2 : 1, 3, 4
F3	1	0	1	-1078	-10369	0	2	+	1, 12, 1	1, 12, 1	1, 12, 1	I_1, I_{12}, I_1	2 : 2
F4	1	0	1	252	10603	0	4	-	4, 3, 4	4, 3, 4	4, 3, 4	I_4, I_3, I_4	2 : 2
706 706 $N = 706 = 2 \cdot 353$ (4 isogeny classes)													
A1	1	1	0	1	-1	1	1	-	1, 1	1, 1	1, 1	I_1, I_1	
B1	1	-1	1	-118	2693	1	1	-	23, 1	23, 1	23, 1	I_{23}, I_1	
C1	1	-1	1	-7	-5	1	2	+	2, 1	2, 1	2, 1	I_2, I_1	2 : 2
C2	1	-1	1	3	-25	1	2	-	1, 2	1, 2	1, 2	I_1, I_2	2 : 1
D1	1	0	0	-18	4	1	2	+	10, 1	10, 1	10, 1	I_{10}, I_1	2 : 2
D2	1	0	0	-178	-924	1	2	+	5, 2	5, 2	5, 2	I_5, I_2	2 : 1
707 707 $N = 707 = 7 \cdot 101$ (1 isogeny class)													
A1	0	1	1	-12	12	2	1	+	2, 1	2, 1	2, 1	I_2, I_1	
708 708 $N = 708 = 2^2 \cdot 3 \cdot 59$ (1 isogeny class)													
A1	0	-1	0	11	34	0	2	-	4, 6, 1	0, 6, 1	1, 2, 1	IV, I_6, I_1	2 : 2
A2	0	-1	0	-124	520	0	2	+	8, 3, 2	0, 3, 2	1, 1, 2	IV^*, I_3, I_2	2 : 1
709 709 $N = 709 = 709$ (1 isogeny class)													
A1	0	-1	1	-2	0	2	1	+	1	1	1	I_1	
710 710 $N = 710 = 2 \cdot 5 \cdot 71$ (4 isogeny classes)													
A1	1	1	0	-27	-59	1	1	+	3, 4, 1	3, 4, 1	1, 4, 1	I_3, I_4, I_1	
B1	1	1	1	-416	3009	1	1	+	17, 2, 1	17, 2, 1	17, 2, 1	I_{17}, I_2, I_1	
C1	1	1	1	-70	195	1	1	+	7, 2, 1	7, 2, 1	7, 2, 1	I_7, I_2, I_1	
D1	1	1	1	-1105	11727	0	5	+	5, 10, 1	5, 10, 1	5, 10, 1	I_5, I_{10}, I_1	5 : 2
D2	1	1	1	-181355	-29801973	0	1	+	1, 2, 5	1, 2, 5	1, 2, 5	I_1, I_2, I_5	5 : 1
711 711 $N = 711 = 3^2 \cdot 79$ (3 isogeny classes)													
A1	1	-1	0	3	-2	1	1	-	3, 1	0, 1	2, 1	III, I_1	
B1	1	-1	1	25	28	1	1	-	9, 1	0, 1	2, 1	III^*, I_1	
C1	1	-1	0	-18	-23	0	1	+	6, 1	0, 1	1, 1	I_0^*, I_1	
712 712 $N = 712 = 2^3 \cdot 89$ (1 isogeny class)													
A1	0	1	0	-32	-80	0	2	+	10, 1	0, 1	2, 1	III^*, I_1	2 : 2
A2	0	1	0	-72	112	0	2	+	11, 2	0, 2	1, 2	II^*, I_2	2 : 1
713 713 $N = 713 = 23 \cdot 31$ (1 isogeny class)													
A1	1	0	1	-1	1	1	1	-	1, 1	1, 1	1, 1	I_1, I_1	
714 714 $N = 714 = 2 \cdot 3 \cdot 7 \cdot 17$ (9 isogeny classes)													
A1	1	1	0	-3334	81940	1	2	-	14, 8, 3, 1	14, 8, 3, 1	2, 2, 1, 1	I_{14}, I_8, I_3, I_1	2 : 2
A2	1	1	0	-55174	4965268	1	2	+	7, 4, 6, 2	7, 4, 6, 2	1, 2, 2, 2	I_7, I_4, I_6, I_2	2 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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714

$N = 714 = 2 \cdot 3 \cdot 7 \cdot 17$ (continued)

714

C1	1	1	0	-14597	-686643	0	1	-	17, 3, 5, 1	17, 3, 5, 1	1, 1, 5, 1	I_{17}, I_3, I_5, I_1	
D1	1	1	0	-21	45	1	2	-	6, 4, 1, 1	6, 4, 1, 1	2, 2, 1, 1	I_6, I_4, I_1, I_1	2 : 2
D2	1	1	0	-381	2709	1	2	+	3, 2, 2, 2	3, 2, 2, 2	1, 2, 2, 2	I_3, I_2, I_2, I_2	2 : 1
E1	1	1	1	-2204	-41731	0	1	-	7, 3, 1, 5	7, 3, 1, 5	7, 1, 1, 1	I_7, I_3, I_1, I_5	
F1	1	1	1	1	101	1	4	-	12, 2, 1, 1	12, 2, 1, 1	12, 2, 1, 1	I_{12}, I_2, I_1, I_1	2 : 2
F2	1	1	1	-319	2021	1	4	+	6, 4, 2, 2	6, 4, 2, 2	6, 2, 2, 2	I_6, I_4, I_2, I_2	2 : 1, 3, 4
F3	1	1	1	-679	-3883	1	2	+	3, 2, 4, 4	3, 2, 4, 4	3, 2, 2, 4	I_3, I_2, I_4, I_4	2 : 2
F4	1	1	1	-5079	137205	1	2	+	3, 8, 1, 1	3, 8, 1, 1	3, 2, 1, 1	I_3, I_8, I_1, I_1	2 : 2
G1	1	1	1	-70244	7127525	0	8	+	24, 4, 4, 1	24, 4, 4, 1	24, 2, 4, 1	I_{24}, I_4, I_4, I_1	2 : 2
G2	1	1	1	-90724	2605541	0	8	+	12, 8, 8, 2	12, 8, 8, 2	12, 2, 8, 2	I_{12}, I_8, I_8, I_2	2 : 1, 3, 4
G3	1	1	1	-859044	-304722459	0	4	+	6, 16, 4, 4	6, 16, 4, 4	6, 2, 4, 4	I_6, I_{16}, I_4, I_4	2 : 2, 5, 6
G4	1	1	1	349916	20936165	0	4	-	6, 4, 16, 1	6, 4, 16, 1	6, 2, 16, 1	I_6, I_4, I_{16}, I_1	2 : 2
G5	1	1	1	-13718604	-19563199515	0	2	+	3, 8, 2, 8	3, 8, 2, 8	3, 2, 2, 8	I_3, I_8, I_2, I_8	2 : 3
G6	1	1	1	-292604	-699871003	0	2	-	3, 32, 2, 2	3, 32, 2, 2	3, 2, 2, 2	I_3, I_{32}, I_2, I_2	2 : 3
H1	1	1	1	1	-1	0	1	-	1, 1, 1, 1	1, 1, 1, 1	1, 1, 1, 1	I_1, I_1, I_1, I_1	
I1	1	0	0	108	11664	0	9	-	9, 9, 3, 1	9, 9, 3, 1	9, 9, 3, 1	I_9, I_9, I_3, I_1	3 : 2
I2	1	0	0	-972	-315144	0	3	-	3, 3, 9, 3	3, 3, 9, 3	3, 3, 9, 1	I_3, I_3, I_9, I_3	3 : 1, 3
I3	1	0	0	-381702	-90803346	0	1	-	1, 1, 3, 9	1, 1, 3, 9	1, 1, 3, 1	I_1, I_1, I_3, I_9	3 : 2

715

$N = 715 = 5 \cdot 11 \cdot 13$ (2 isogeny classes)

715

A1	0	1	1	-5	6	1	3	-	3, 1, 1	3, 1, 1	3, 1, 1	I_3, I_1, I_1	3 : 2
A2	0	1	1	45	-129	1	1	-	1, 3, 3	1, 3, 3	1, 1, 3	I_1, I_3, I_3	3 : 1
B1	0	0	1	43	-2088	1	1	-	7, 1, 3	7, 1, 3	7, 1, 3	I_7, I_1, I_3	

718

$N = 718 = 2 \cdot 359$ (3 isogeny classes)

718

A1	1	-1	0	-17	-163	0	1	-	15, 1	15, 1	1, 1	I_{15}, I_1	
B1	1	0	1	-5	0	2	1	+	4, 1	4, 1	2, 1	I_4, I_1	
C1	1	-1	1	-514	4609	1	1	+	12, 1	12, 1	12, 1	I_{12}, I_1	

720

$N = 720 = 2^4 \cdot 3^2 \cdot 5$ (10 isogeny classes)

720

A1	0	0	0	-3	18	1	2	-	10, 3, 1	0, 0, 1	4, 2, 1	I_2^*, III, I_1	2 : 2
A2	0	0	0	-123	522	1	2	+	11, 3, 2	0, 0, 2	4, 2, 2	I_3^*, III, I_2	2 : 1
B1	0	0	0	-27	-486	0	2	-	10, 9, 1	0, 0, 1	4, 2, 1	I_2^*, III^*, I_1	2 : 2
B2	0	0	0	-1107	-14094	0	2	+	11, 9, 2	0, 0, 2	2, 2, 2	I_3^*, III^*, I_2	2 : 1
C1	0	0	0	-138	623	0	2	+	4, 8, 1	0, 2, 1	1, 2, 1	II, I_2^*, I_1	2 : 2
C2	0	0	0	-183	182	0	4	+	8, 10, 2	0, 4, 2	2, 4, 2	I_0^*, I_4^*, I_2	2 : 1, 3, 4
C3	0	0	0	-1803	-29302	0	4	+	10, 8, 4	0, 2, 4	4, 4, 2	I_2^*, I_2^*, I_4	2 : 2, 5, 6
C4	0	0	0	717	1442	0	2	-	10, 14, 1	0, 8, 1	2, 4, 1	I_2^*, I_8^*, I_1	2 : 2
C5	0	0	0	-28803	-1881502	0	2	+	11, 7, 2	0, 1, 2	2, 4, 2	I_3^*, I_1^*, I_2	2 : 3
C6	0	0	0	-723	-64078	0	2	-	11, 7, 8	0, 1, 8	4, 2, 2	I_3^*, I_1^*, I_8	2 : 3
D1	0	0	0	-18	27	0	2	+	4, 6, 1	0, 0, 1	1, 2, 1	II, I_0^*, I_1	2 : 2
D2	0	0	0	-63	-162	0	4	+	8, 6, 2	0, 0, 2	2, 4, 2	I_0^*, I_0^*, I_2	2 : 1, 3, 4
D3	0	0	0	-963	-11502	0	2	+	10, 6, 1	0, 0, 1	4, 2, 1	I_0^*, I_0^*, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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720 $N = 720 = 2^4 \cdot 3^2 \cdot 5$ (continued)**720**

E1	0	0	0	33	-34	1	2	-	8, 7, 1	0, 1, 1	2, 2, 1	I_0^*, I_1^*, I_1	2 : 2
E2	0	0	0	-147	-286	1	4	+	10, 8, 2	0, 2, 2	4, 4, 2	I_2^*, I_2^*, I_2	2 : 1, 3, 4
E3	0	0	0	-1947	-33046	1	2	+	11, 10, 1	0, 4, 1	2, 4, 1	I_3^*, I_4^*, I_1	2 : 2
E4	0	0	0	-1227	16346	1	4	+	11, 7, 4	0, 1, 4	4, 4, 4	I_3^*, I_1^*, I_4	2 : 2
F1	0	0	0	-123	-598	0	2	-	18, 3, 1	6, 0, 1	4, 2, 1	I_{10}^*, III, I_1	2 : 2; 3 : 3
F2	0	0	0	-2043	-35542	0	2	+	15, 3, 2	3, 0, 2	2, 2, 2	I_7^*, III, I_2	2 : 1; 3 : 4
F3	0	0	0	837	2538	0	2	-	14, 9, 3	2, 0, 3	4, 2, 1	I_6^*, III^*, I_3	2 : 4; 3 : 1
F4	0	0	0	-3483	20682	0	2	+	13, 9, 6	1, 0, 6	2, 2, 2	I_5^*, III^*, I_6	2 : 3; 3 : 2
G1	0	0	0	93	-94	1	2	-	14, 3, 3	2, 0, 3	4, 2, 3	I_6^*, III, I_3	2 : 2; 3 : 3
G2	0	0	0	-387	-766	1	2	+	13, 3, 6	1, 0, 6	4, 2, 6	I_5^*, III, I_6	2 : 1; 3 : 4
G3	0	0	0	-1107	16146	1	2	-	18, 9, 1	6, 0, 1	4, 2, 1	I_{10}^*, III^*, I_1	2 : 4; 3 : 1
G4	0	0	0	-18387	959634	1	2	+	15, 9, 2	3, 0, 2	4, 2, 2	I_7^*, III^*, I_2	2 : 3; 3 : 2
H1	0	0	0	-3	322	1	2	-	12, 7, 1	0, 1, 1	4, 4, 1	I_4^*, I_1^*, I_1	2 : 2
H2	0	0	0	-723	7378	1	4	+	12, 8, 2	0, 2, 2	4, 4, 2	I_4^*, I_2^*, I_2	2 : 1, 3, 4
H3	0	0	0	-1443	-9758	1	4	+	12, 10, 4	0, 4, 4	4, 4, 2	I_4^*, I_4^*, I_4	2 : 2, 5, 6
H4	0	0	0	-11523	476098	1	2	+	12, 7, 1	0, 1, 1	2, 2, 1	I_4^*, I_1^*, I_1	2 : 2
H5	0	0	0	-19443	-1042958	1	4	+	12, 14, 2	0, 8, 2	4, 4, 2	I_4^*, I_8^*, I_2	2 : 3, 7, 8
H6	0	0	0	5037	-73262	1	2	-	12, 8, 8	0, 2, 8	2, 2, 2	I_4^*, I_2^*, I_8	2 : 3
H7	0	0	0	-311043	-66769598	1	2	+	12, 10, 1	0, 4, 1	4, 2, 1	I_4^*, I_4^*, I_1	2 : 5
H8	0	0	0	-15843	-1441118	1	2	-	12, 22, 1	0, 16, 1	2, 4, 1	I_4^*, I_{16}^*, I_1	2 : 5
I1	0	0	0	-12	11	0	2	+	4, 6, 1	0, 0, 1	1, 2, 1	II, I_0^*, I_1	2 : 2; 3 : 3
I2	0	0	0	33	74	0	2	-	8, 6, 2	0, 0, 2	1, 2, 2	I_0^*, I_0^*, I_2	2 : 1; 3 : 4
I3	0	0	0	-372	-2761	0	2	+	4, 6, 3	0, 0, 3	1, 2, 3	II, I_0^*, I_3	2 : 4; 3 : 1
I4	0	0	0	-327	-3454	0	2	-	8, 6, 6	0, 0, 6	1, 2, 6	I_0^*, I_0^*, I_6	2 : 3; 3 : 2
J1	0	0	0	213	3674	0	2	-	16, 9, 1	4, 3, 1	4, 2, 1	I_8^*, I_3^*, I_1	2 : 2; 3 : 3
J2	0	0	0	-2667	48026	0	4	+	14, 12, 2	2, 6, 2	4, 4, 2	I_6^*, I_6^*, I_2	2 : 1, 4, 5; 3 : 6
J3	0	0	0	-1947	-108214	0	2	-	24, 7, 3	12, 1, 3	4, 2, 3	I_{16}^*, I_1^*, I_3	2 : 6; 3 : 1
J4	0	0	0	-9867	-324934	0	2	+	13, 18, 1	1, 12, 1	4, 4, 1	I_5^*, I_{12}^*, I_1	2 : 2; 3 : 7
J5	0	0	0	-41547	3259514	0	4	+	13, 9, 4	1, 3, 4	2, 4, 4	I_5^*, I_3^*, I_4	2 : 2; 3 : 8
J6	0	0	0	-48027	-4043446	0	4	+	18, 8, 6	6, 2, 6	4, 4, 6	I_{10}^*, I_2^*, I_6	2 : 3, 7, 8; 3 : 2
J7	0	0	0	-768027	-259067446	0	2	+	15, 10, 3	3, 4, 3	4, 4, 3	I_7^*, I_4^*, I_3	2 : 6; 3 : 4
J8	0	0	0	-65307	-874294	0	4	+	15, 7, 12	3, 1, 12	2, 4, 12	I_7^*, I_1^*, I_{12}	2 : 6; 3 : 5

722 $N = 722 = 2 \cdot 19^2$ (6 isogeny classes)**722**

A1	1	0	1	714	-16080	1	3	-	3, 8	3, 0	1, 3	I_3, IV^*	3 : 2
A2	1	0	1	-33581	-2375576	1	1	-	9, 8	9, 0	1, 3	I_9, IV^*	3 : 1
B1	1	-1	0	-1	-11	1	1	-	3, 3	3, 0	1, 2	I_3, III	
C1	1	0	1	-8	-8138	0	1	-	5, 7	5, 1	1, 2	I_5, I_1^*	5 : 2
C2	1	0	1	-25278	1710222	0	1	-	1, 11	1, 5	1, 2	I_1, I_5^*	5 : 1
D1	1	-1	1	-429	77485	0	1	-	3, 9	3, 0	3, 2	I_3, III^*	
E1	1	1	1	-5603	-163815	1	1	-	3, 7	3, 1	3, 4	I_3, I_1^*	3 : 2
E2	1	1	1	3422	-612177	1	1	-	9, 9	9, 3	9, 4	I_9, I_3^*	3 : 1, 3
E3	1	1	1	-30873	16782247	1	1	-	27, 7	27, 1	27, 4	I_{27}, I_1^*	3 : 2
F1	1	1	1	2	3	1	1	-	3, 2	3, 0	3, 1	I_3, II	3 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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723 **723**
 $N = 723 = 3 \cdot 241$ (2 isogeny classes)

A1	1	1	1	-4	-4	1	2	+	2, 1	2, 1	2, 1	I_2, I_1	2 : 2
A2	1	1	1	11	-10	1	2	-	1, 2	1, 2	1, 2	I_1, I_2	2 : 1
B1	0	1	1	-3	-4	1	1	-	1, 1	1, 1	1, 1	I_1, I_1	

725 **725**
 $N = 725 = 5^2 \cdot 29$ (1 isogeny class)

A1	1	-1	0	-67	216	1	2	+	7, 1	1, 1	2, 1	I_1^*, I_1	2 : 2
A2	1	-1	0	58	841	1	2	-	8, 2	2, 2	4, 2	I_2^*, I_2	2 : 1

726 **726**
 $N = 726 = 2 \cdot 3 \cdot 11^2$ (9 isogeny classes)

A1	1	1	0	-35	-51	1	2	+	6, 3, 3	6, 3, 0	2, 1, 2	I_6, I_3, III	2 : 2
A2	1	1	0	-475	-4187	1	2	+	3, 6, 3	3, 6, 0	1, 2, 2	I_3, I_6, III	2 : 1
B1	1	1	0	21657	-1855179	0	1	-	10, 4, 10	10, 4, 0	2, 2, 1	I_{10}, I_4, II^*	
C1	1	1	0	-244	-128	0	2	+	4, 1, 7	4, 1, 1	2, 1, 2	I_4, I_1, I_1^*	2 : 2
C2	1	1	0	-2664	51660	0	4	+	2, 2, 8	2, 2, 2	2, 2, 4	I_2, I_2, I_2^*	2 : 1, 3, 4
C3	1	1	0	-42594	3365850	0	2	+	1, 1, 7	1, 1, 1	1, 1, 4	I_1, I_1, I_1^*	2 : 2
C4	1	1	0	-1454	100302	0	2	-	1, 4, 10	1, 4, 4	1, 2, 4	I_1, I_4, I_4^*	2 : 2
D1	1	0	1	-14	20	1	1	-	2, 4, 2	2, 4, 0	2, 4, 1	I_2, I_4, II	
E1	1	0	1	-5448	-113258	1	2	+	10, 5, 7	10, 5, 1	2, 5, 4	I_{10}, I_5, I_1^*	2 : 2; 5 : 3
E2	1	0	1	13912	-732778	1	2	-	5, 10, 8	5, 10, 2	1, 10, 4	I_5, I_{10}, I_2^*	2 : 1; 5 : 4
E3	1	0	1	-1217868	517205302	1	2	+	2, 1, 11	2, 1, 5	2, 1, 4	I_2, I_1, I_5^*	2 : 4; 5 : 1
E4	1	0	1	-1216658	518284622	1	2	-	1, 2, 16	1, 2, 10	1, 2, 4	I_1, I_2, I_{10}^*	2 : 3; 5 : 2
F1	1	1	1	-4298	46487	0	2	+	6, 3, 9	6, 3, 0	6, 1, 2	I_6, I_3, III^*	2 : 2
F2	1	1	1	-57538	5285303	0	2	+	3, 6, 9	3, 6, 0	3, 2, 2	I_3, I_6, III^*	2 : 1
G1	1	1	1	179	1475	1	1	-	10, 4, 4	10, 4, 0	10, 2, 3	I_{10}, I_4, IV	
H1	1	0	0	-668	-6324	0	2	+	2, 3, 7	2, 3, 1	2, 3, 2	I_2, I_3, I_1^*	2 : 2; 3 : 3
H2	1	0	0	542	-26410	0	2	-	1, 6, 8	1, 6, 2	1, 6, 4	I_1, I_6, I_2^*	2 : 1; 3 : 4
H3	1	0	0	-9743	367929	0	2	+	6, 1, 9	6, 1, 3	6, 1, 2	I_6, I_1, I_3^*	2 : 4; 3 : 1
H4	1	0	0	-4903	734801	0	2	-	3, 2, 12	3, 2, 6	3, 2, 4	I_3, I_2, I_6^*	2 : 3; 3 : 2
I1	1	0	0	-1636	-28588	0	1	-	2, 4, 8	2, 4, 0	2, 4, 1	I_2, I_4, IV^*	

728 **728**
 $N = 728 = 2^3 \cdot 7 \cdot 13$ (4 isogeny classes)

A1	0	-1	0	-8	-20	0	1	-	11, 1, 1	0, 1, 1	1, 1, 1	II^*, I_1, I_1	
B1	0	-1	0	1071	8501	0	1	-	8, 1, 7	0, 1, 7	4, 1, 1	I_1^*, I_1, I_7	
C1	0	0	0	-68	-236	1	1	-	8, 1, 3	0, 1, 3	2, 1, 3	I_1^*, I_1, I_3	
D1	0	1	0	-1	51	1	1	-	8, 3, 1	0, 3, 1	2, 3, 1	I_1^*, I_3, I_1	

730 **730**
 $N = 730 = 2 \cdot 5 \cdot 73$ (11 isogeny classes)

A1	1	-1	0	-865	-9219	0	2	+	16, 4, 1	16, 4, 1	2, 2, 1	I_{16}, I_4, I_1	2 : 2
A2	1	-1	0	415	-35075	0	2	-	8, 8, 2	8, 8, 2	2, 2, 2	I_8, I_8, I_2	2 : 1
B1	1	0	1	96	-658	0	3	-	7, 1, 3	7, 1, 3	1, 1, 3	I_7, I_1, I_3	3 : 2
B2	1	0	1	-3919	-94974	0	1	-	21, 3, 1	21, 3, 1	1, 1, 1	I_{21}, I_3, I_1	3 : 1
C1	1	-1	0	-2440	47006	0	1	+	1, 7, 1	1, 7, 1	1, 1, 1	I_1, I_7, I_1	
D1	1	1	0	-1897	29189	0	1	+	27, 1, 1	27, 1, 1	1, 1, 1	I_{27}, I_1, I_1	

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
730	$N = 730 = 2 \cdot 5 \cdot 73$ (continued)											730	
F1	1	-1	0	-949	11493	1	2	+	4, 4, 1	4, 4, 1	2, 4, 1	I_4, I_4, I_1	2 : 2
F2	1	-1	0	-929	11985	1	2	-	2, 8, 2	2, 8, 2	2, 8, 2	I_2, I_8, I_2	2 : 1
G1	1	-1	0	-4	-2	1	1	+	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	
H1	1	0	0	19	-5	0	1	-	1, 5, 1	1, 5, 1	1, 1, 1	I_1, I_5, I_1	
I1	1	1	1	-26	39	1	1	+	7, 1, 1	7, 1, 1	7, 1, 1	I_7, I_1, I_1	
J1	1	1	1	-405	-1925	1	1	+	9, 7, 1	9, 7, 1	9, 7, 1	I_9, I_7, I_1	
K1	1	0	0	-15	17	0	3	+	3, 3, 1	3, 3, 1	3, 3, 1	I_3, I_3, I_1	3 : 2
K2	1	0	0	-365	-2713	0	1	+	1, 1, 3	1, 1, 3	1, 1, 3	I_1, I_1, I_3	3 : 1
731	$N = 731 = 17 \cdot 43$ (1 isogeny class)											731	
A1	1	0	1	-539	4765	1	1	-	3, 1	3, 1	1, 1	I_3, I_1	
732	$N = 732 = 2^2 \cdot 3 \cdot 61$ (3 isogeny classes)											732	
A1	0	-1	0	-17	30	0	2	+	4, 4, 1	0, 4, 1	1, 2, 1	IV, I_4, I_1	2 : 2
A2	0	-1	0	28	120	0	2	-	8, 2, 2	0, 2, 2	1, 2, 2	IV^*, I_2, I_2	2 : 1
B1	0	-1	0	-100	424	1	1	-	8, 4, 1	0, 4, 1	3, 2, 1	IV^*, I_4, I_1	
C1	0	1	0	-29	36	1	2	+	4, 6, 1	0, 6, 1	3, 6, 1	IV, I_6, I_1	2 : 2
C2	0	1	0	-164	-828	1	2	+	8, 3, 2	0, 3, 2	3, 3, 2	IV^*, I_3, I_2	2 : 1
733	$N = 733 = 733$ (1 isogeny class)											733	
A1	1	1	0	-75	-284	0	1	+	1	1	1	I_1	
734	$N = 734 = 2 \cdot 367$ (1 isogeny class)											734	
A1	1	1	1	-3	-31	0	2	-	10, 1	10, 1	10, 1	I_{10}, I_1	2 : 2
A2	1	1	1	-163	-863	0	2	+	5, 2	5, 2	5, 2	I_5, I_2	2 : 1
735	$N = 735 = 3 \cdot 5 \cdot 7^2$ (6 isogeny classes)											735	
A1	1	1	0	-123	-552	0	2	+	1, 1, 7	1, 1, 1	1, 1, 4	I_1, I_1, I_1^*	2 : 2
A2	1	1	0	-368	1947	0	4	+	2, 2, 8	2, 2, 2	2, 2, 4	I_2, I_2, I_2^*	2 : 1, 3, 4
A3	1	1	0	-5513	155268	0	2	+	1, 4, 7	1, 4, 1	1, 2, 2	I_1, I_4, I_1^*	2 : 2
A4	1	1	0	857	13462	0	2	-	4, 1, 10	4, 1, 4	2, 1, 4	I_4, I_1, I_4^*	2 : 2
B1	0	-1	1	-15206	-1184338	0	1	-	7, 4, 10	7, 4, 0	1, 2, 1	I_7, I_4, II^*	
C1	0	-1	1	5	6	1	1	-	3, 2, 2	3, 2, 0	1, 2, 1	I_3, I_2, II	3 : 2
C2	0	-1	1	-205	1203	1	1	-	1, 6, 2	1, 6, 0	1, 6, 1	I_1, I_6, II	3 : 1
D1	0	1	1	229	-2614	0	3	-	3, 2, 8	3, 2, 0	3, 2, 3	I_3, I_2, IV^*	3 : 2
D2	0	1	1	-10061	-392605	0	1	-	1, 6, 8	1, 6, 0	1, 2, 3	I_1, I_6, IV^*	3 : 1
E1	1	0	0	-1	-64	1	2	-	1, 1, 6	1, 1, 0	1, 1, 2	I_1, I_1, I_0^*	2 : 2
E2	1	0	0	-246	-1485	1	4	+	2, 2, 6	2, 2, 0	2, 2, 4	I_2, I_2, I_0^*	2 : 1, 3, 4
E3	1	0	0	-3921	-94830	1	2	+	1, 1, 6	1, 1, 0	1, 1, 4	I_1, I_1, I_0^*	2 : 2
E4	1	0	0	-491	1896	1	4	+	4, 4, 6	4, 4, 0	4, 2, 4	I_4, I_4, I_0^*	2 : 2, 5, 6
E5	1	0	0	-6616	206471	1	4	+	8, 2, 6	8, 2, 0	8, 2, 4	I_8, I_2, I_0^*	2 : 4, 7, 8
E6	1	0	0	1714	14685	1	2	-	2, 8, 6	2, 8, 0	2, 2, 2	I_2, I_8, I_0^*	2 : 4
E7	1	0	0	-105841	13244636	1	2	+	4, 1, 6	4, 1, 0	4, 1, 2	I_4, I_1, I_0^*	2 : 5
E8	1	0	0	-5391	285606	1	2	-	16, 1, 6	16, 1, 0	16, 1, 4	I_{16}, I_1, I_0^*	2 : 5

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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737 $N = 737 = 11 \cdot 67$ (1 isogeny class) **737**

A1	0	-1	1	406	-686	1	1	-	4, 3	4, 3	4, 3	I_4, I_3	
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738 $N = 738 = 2 \cdot 3^2 \cdot 41$ (10 isogeny classes) **738**

A1	1	-1	0	66	116	1	1	-	5, 9, 1	5, 0, 1	1, 2, 1	I_5, III^*, I_1	
B1	1	-1	0	-1575	751869	0	1	-	25, 11, 1	25, 5, 1	1, 2, 1	I_{25}, I_5^*, I_1	5 : 2
B2	1	-1	0	-5215815	4586220189	0	1	-	5, 7, 5	5, 1, 5	1, 2, 1	I_5, I_1^*, I_5	5 : 1
C1	1	-1	0	-81	-243	0	2	+	4, 8, 1	4, 2, 1	2, 2, 1	I_4, I_2^*, I_1	2 : 2
C2	1	-1	0	-261	1377	0	4	+	2, 10, 2	2, 4, 2	2, 4, 2	I_2, I_4^*, I_2	2 : 1, 3, 4
C3	1	-1	0	-3951	96579	0	2	+	1, 14, 1	1, 8, 1	1, 4, 1	I_1, I_8^*, I_1	2 : 2
C4	1	-1	0	549	7695	0	2	-	1, 8, 4	1, 2, 4	1, 4, 2	I_1, I_2^*, I_4	2 : 2
D1	1	-1	0	-2430	46732	1	1	-	3, 13, 1	3, 7, 1	1, 4, 1	I_3, I_7^*, I_1	
E1	1	-1	1	7	-7	1	1	-	5, 3, 1	5, 0, 1	5, 2, 1	I_5, III, I_1	
F1	1	-1	1	-374	2949	1	1	-	11, 7, 1	11, 1, 1	11, 4, 1	I_{11}, I_1^*, I_1	
G1	1	-1	1	-599	-5457	0	2	+	6, 10, 1	6, 4, 1	6, 2, 1	I_6, I_4^*, I_1	2 : 2
G2	1	-1	1	-239	-12225	0	2	-	3, 14, 2	3, 8, 2	3, 4, 2	I_3, I_8^*, I_2	2 : 1
H1	1	-1	1	-4085069	3178971893	0	2	+	14, 18, 1	14, 12, 1	14, 2, 1	I_{14}, I_{12}^*, I_1	2 : 2
H2	1	-1	1	-4079309	3188379125	0	2	-	7, 30, 2	7, 24, 2	7, 4, 2	I_7, I_{24}^*, I_2	2 : 1
I1	1	-1	1	-14	-7	0	2	+	2, 6, 1	2, 0, 1	2, 2, 1	I_2, I_0^*, I_1	2 : 2
I2	1	-1	1	-104	425	0	2	+	1, 6, 2	1, 0, 2	1, 2, 2	I_1, I_0^*, I_2	2 : 1
J1	1	-1	1	-14	-61	0	1	-	1, 9, 1	1, 3, 1	1, 2, 1	I_1, I_3^*, I_1	3 : 2
J2	1	-1	1	121	1559	0	3	-	3, 7, 3	3, 1, 3	3, 2, 3	I_3, I_1^*, I_3	3 : 1

739 $N = 739 = 739$ (1 isogeny class) **739**

A1	0	0	1	1	1	0	1	-	1	1	1	I_1	
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740 $N = 740 = 2^2 \cdot 5 \cdot 37$ (3 isogeny classes) **740**

A1	0	0	0	-219448	39364772	0	1	+	8, 8, 5	0, 8, 5	3, 2, 1	IV^*, I_8, I_5	
B1	0	1	0	-181	-425	1	3	+	8, 2, 3	0, 2, 3	3, 2, 3	IV^*, I_2, I_3	3 : 2
B2	0	1	0	-12021	-511321	1	1	+	8, 6, 1	0, 6, 1	1, 2, 1	IV^*, I_6, I_1	3 : 1
C1	0	-1	0	-45	25	1	1	+	8, 4, 1	0, 4, 1	3, 4, 1	IV^*, I_4, I_1	

741 $N = 741 = 3 \cdot 13 \cdot 19$ (5 isogeny classes) **741**

A1	1	1	0	-2	-3	0	1	-	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	
B1	1	1	0	5571	-41634	0	1	-	7, 3, 5	7, 3, 5	1, 3, 1	I_7, I_3, I_5	
C1	1	0	1	-5227	-155497	0	1	-	11, 5, 1	11, 5, 1	11, 1, 1	I_{11}, I_5, I_1	
D1	0	1	1	101470	57781877	0	1	-	10, 4, 7	10, 4, 7	10, 2, 1	I_{10}, I_4, I_7	
E1	0	1	1	-5	23	1	1	-	4, 2, 1	4, 2, 1	4, 2, 1	I_4, I_2, I_1	

742 $N = 742 = 2 \cdot 7 \cdot 53$ (7 isogeny classes) **742**

A1	1	-1	0	-5	7	1	1	-	1, 2, 1	1, 2, 1	1, 2, 1	I_1, I_2, I_1	
B1	1	1	0	-63	245	0	2	-	4, 3, 2	4, 3, 2	2, 1, 2	I_4, I_3, I_2	2 : 2
B2	1	1	0	-1123	14025	0	2	+	2, 6, 1	2, 6, 1	2, 2, 1	I_2, I_6, I_1	2 : 1
C1	1	-1	0	727	11853	0	1	-	25, 2, 1	25, 2, 1	1, 2, 1	I_{25}, I_2, I_1	

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
742	$N = 742 = 2 \cdot 7 \cdot 53$ (continued)											742	
E1	1	1	0	-29612	2027600	1	2	-	10, 5, 4	10, 5, 4	2, 5, 4	I_{10}, I_5, I_4	2 : 2
E2	1	1	0	-479052	127421360	1	2	+	5, 10, 2	5, 10, 2	1, 10, 2	I_5, I_{10}, I_2	2 : 1
F1	1	-1	1	-81	11797	0	1	-	2, 10, 1	2, 10, 1	2, 2, 1	I_2, I_{10}, I_1	
G1	1	1	1	-14	75	1	1	-	10, 2, 1	10, 2, 1	10, 2, 1	I_{10}, I_2, I_1	
744	$N = 744 = 2^3 \cdot 3 \cdot 31$ (7 isogeny classes)											744	
A1	0	-1	0	4	-3	1	1	-	4, 2, 1	0, 2, 1	2, 2, 1	III, I_2, I_1	
B1	0	1	0	-279	-1890	0	2	+	4, 3, 1	0, 3, 1	2, 3, 1	III, I_3, I_1	2 : 2
B2	0	1	0	-284	-1824	0	4	+	8, 6, 2	0, 6, 2	2, 6, 2	I_1^*, I_6, I_2	2 : 1, 3, 4
B3	0	1	0	-904	8096	0	4	+	10, 12, 1	0, 12, 1	2, 12, 1	III^*, I_{12}, I_1	2 : 2
B4	0	1	0	256	-7440	0	2	-	10, 3, 4	0, 3, 4	2, 3, 2	III^*, I_3, I_4	2 : 2
C1	0	1	0	8	89	1	1	-	4, 8, 1	0, 8, 1	2, 8, 1	III, I_8, I_1	
D1	0	-1	0	936	-25839	0	1	-	4, 6, 5	0, 6, 5	2, 2, 1	III, I_6, I_5	
E1	0	-1	0	-32	-84	0	1	-	11, 3, 1	0, 3, 1	1, 1, 1	II^*, I_3, I_1	
F1	0	-1	0	-140	753	1	1	-	4, 4, 3	0, 4, 3	2, 2, 3	III, I_4, I_3	
G1	0	1	0	-96	333	1	1	-	4, 6, 1	0, 6, 1	2, 6, 1	III, I_6, I_1	
747	$N = 747 = 3^2 \cdot 83$ (5 isogeny classes)											747	
A1	1	-1	1	-56	-134	1	2	+	9, 1	0, 1	2, 1	III^*, I_1	2 : 2
A2	1	-1	1	-191	892	1	2	+	9, 2	0, 2	2, 2	III^*, I_2	2 : 1
B1	1	-1	0	-6	7	0	2	+	3, 1	0, 1	2, 1	III, I_1	2 : 2
B2	1	-1	0	-21	-26	0	2	+	3, 2	0, 2	2, 2	III, I_2	2 : 1
C1	1	-1	0	-495	-4118	1	1	-	9, 1	3, 1	2, 1	I_3^*, I_1	
D1	1	-1	0	9	4	1	1	-	6, 1	0, 1	1, 1	I_0^*, I_1	
E1	1	-1	1	13	-12	1	1	-	7, 1	1, 1	2, 1	I_1^*, I_1	
748	$N = 748 = 2^2 \cdot 11 \cdot 17$ (1 isogeny class)											748	
A1	0	0	0	-496	-4252	0	1	-	8, 1, 2	0, 1, 2	3, 1, 2	IV^*, I_1, I_2	
749	$N = 749 = 7 \cdot 107$ (1 isogeny class)											749	
A1	1	0	0	-4	-5	1	1	-	2, 1	2, 1	2, 1	I_2, I_1	
752	$N = 752 = 2^4 \cdot 47$ (1 isogeny class)											752	
A1	0	0	0	5	42	1	2	-	14, 1	2, 1	4, 1	I_6^*, I_1	2 : 2
A2	0	0	0	-155	714	1	2	+	13, 2	1, 2	4, 2	I_5^*, I_2	2 : 1
753	$N = 753 = 3 \cdot 251$ (3 isogeny classes)											753	
A1	0	-1	1	-4	-3	0	1	-	2, 1	2, 1	2, 1	I_2, I_1	
B1	0	1	1	-9	20	0	3	-	6, 1	6, 1	6, 1	I_6, I_1	3 : 2
B2	0	1	1	81	-475	0	1	-	2, 3	2, 3	2, 1	I_2, I_3	3 : 1
C1	0	1	1	5	7	1	1	-	4, 1	4, 1	4, 1	I_4, I_1	
754	$N = 754 = 2 \cdot 13 \cdot 29$ (4 isogeny classes)											754	
A1	1	0	1	-377	2782	0	3	-	1 3 2	1 3 2	1 3 2	I_1, I_2, I_2	3 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
754 754													
$N = 754 = 2 \cdot 13 \cdot 29$ (continued)													
B1	1	0	1	-10758	428760	1	1	-	13, 1, 4	13, 1, 4	1, 1, 4	I_{13}, I_1, I_4	
C1	1	0	1	-7	-6	1	2	+	4, 1, 1	4, 1, 1	2, 1, 1	I_4, I_1, I_1	2 : 2
C2	1	0	1	13	-30	1	2	-	2, 2, 2	2, 2, 2	2, 2, 2	I_2, I_2, I_2	2 : 1
D1	1	0	0	43	-31	1	1	-	9, 1, 2	9, 1, 2	9, 1, 2	I_9, I_1, I_2	
755 755													
$N = 755 = 5 \cdot 151$ (6 isogeny classes)													
A1	0	0	1	2	-1	1	1	-	1, 1	1, 1	1, 1	I_1, I_1	
B1	1	0	1	1	1	1	1	-	1, 1	1, 1	1, 1	I_1, I_1	
C1	1	0	1	1	-3	0	2	-	2, 1	2, 1	2, 1	I_2, I_1	2 : 2
C2	1	0	1	-24	-43	0	2	+	1, 2	1, 2	1, 2	I_1, I_2	2 : 1
D1	0	1	1	0	1	0	1	-	1, 1	1, 1	1, 1	I_1, I_1	
E1	0	0	1	-7	7	0	1	-	1, 1	1, 1	1, 1	I_1, I_1	
F1	0	0	1	-56917	-5226543	0	1	-	13, 1	13, 1	13, 1	I_{13}, I_1	
756 756													
$N = 756 = 2^2 \cdot 3^3 \cdot 7$ (6 isogeny classes)													
A1	0	0	0	-432	3348	0	1	+	8, 11, 1	0, 0, 1	1, 1, 1	IV^*, II^*, I_1	
B1	0	0	0	-24	-44	1	1	+	8, 3, 1	0, 0, 1	1, 1, 1	IV^*, II, I_1	3 : 2
B2	0	0	0	-264	1636	1	3	+	8, 5, 3	0, 0, 3	3, 3, 3	IV^*, IV, I_3	3 : 1
C1	0	0	0	-48	-124	1	1	+	8, 5, 1	0, 0, 1	3, 1, 1	IV^*, IV, I_1	
D1	0	0	0	-216	1188	0	3	+	8, 9, 1	0, 0, 1	3, 3, 1	IV^*, IV^*, I_1	3 : 2
D2	0	0	0	-2376	-44172	0	1	+	8, 11, 3	0, 0, 3	1, 1, 3	IV^*, II^*, I_3	3 : 1
E1	0	0	0	9	-2	0	1	-	8, 3, 1	0, 0, 1	1, 1, 1	IV^*, II, I_1	3 : 2
E2	0	0	0	-111	502	0	3	-	8, 5, 3	0, 0, 3	3, 1, 3	IV^*, IV, I_3	3 : 1
F1	0	0	0	81	54	0	3	-	8, 9, 1	0, 0, 1	3, 3, 1	IV^*, IV^*, I_1	3 : 2
F2	0	0	0	-999	-13554	0	1	-	8, 11, 3	0, 0, 3	1, 1, 3	IV^*, II^*, I_3	3 : 1
758 758													
$N = 758 = 2 \cdot 379$ (2 isogeny classes)													
A1	1	0	1	11	0	1	1	-	8, 1	8, 1	2, 1	I_8, I_1	
B1	1	1	1	-44	-131	0	1	-	4, 1	4, 1	4, 1	I_4, I_1	
759 759													
$N = 759 = 3 \cdot 11 \cdot 23$ (2 isogeny classes)													
A1	1	1	1	-23	-628	1	2	-	10, 2, 1	10, 2, 1	2, 2, 1	I_{10}, I_2, I_1	2 : 2
A2	1	1	1	-1238	-17152	1	2	+	5, 4, 2	5, 4, 2	1, 4, 2	I_5, I_4, I_2	2 : 1
B1	1	0	0	31	-192	1	4	-	8, 2, 1	8, 2, 1	8, 2, 1	I_8, I_2, I_1	2 : 2
B2	1	0	0	-374	-2541	1	8	+	4, 4, 2	4, 4, 2	4, 4, 2	I_4, I_4, I_2	2 : 1, 3, 4
B3	1	0	0	-5819	-171336	1	4	+	2, 2, 4	2, 2, 4	2, 2, 2	I_2, I_2, I_4	2 : 2, 5, 6
B4	1	0	0	-1409	17538	1	4	+	2, 8, 1	2, 8, 1	2, 8, 1	I_2, I_8, I_1	2 : 2
B5	1	0	0	-93104	-10942305	1	2	+	1, 1, 2	1, 1, 2	1, 1, 2	I_1, I_1, I_2	2 : 3
B6	1	0	0	-5654	-181467	1	2	-	1, 1, 8	1, 1, 8	1, 1, 2	I_1, I_1, I_8	2 : 3
760 760													
$N = 760 = 2^3 \cdot 5 \cdot 19$ (5 isogeny classes)													
A1	0	-1	0	5	0	0	2	-	4, 2, 1	0, 2, 1	2, 2, 1	III, I_2, I_1	2 : 2
A2	0	-1	0	-20	20	0	2	+	8, 1, 2	0, 1, 2	2, 1, 2	I_1^*, I_1, I_2	2 : 1
B1	0	1	0	-26035	-1626942	0	2	-	4, 14, 1	0, 14, 1	2, 14, 1	III, I_1, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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760 $N = 760 = 2^3 \cdot 5 \cdot 19$ (continued)**760**

C1	0	0	0	-67	926	0	1	-	11, 2, 3	0, 2, 3	1, 2, 1	$\text{II}^*, \text{I}_2, \text{I}_3$	
D1	0	1	0	-35	58	1	2	+	4, 3, 2	0, 3, 2	2, 3, 2	$\text{III}, \text{I}_3, \text{I}_2$	2 : 2
D2	0	1	0	60	400	1	2	-	8, 6, 1	0, 6, 1	2, 6, 1	$\text{I}_1^*, \text{I}_6, \text{I}_1$	2 : 1
E1	0	0	0	-2	21	1	4	-	4, 4, 1	0, 4, 1	2, 4, 1	$\text{III}, \text{I}_4, \text{I}_1$	2 : 2
E2	0	0	0	-127	546	1	4	+	8, 2, 2	0, 2, 2	4, 2, 2	$\text{I}_1^*, \text{I}_2, \text{I}_2$	2 : 1, 3, 4
E3	0	0	0	-227	-434	1	2	+	10, 1, 4	0, 1, 4	2, 1, 2	$\text{III}^*, \text{I}_1, \text{I}_4$	2 : 2
E4	0	0	0	-2027	35126	1	2	+	10, 1, 1	0, 1, 1	2, 1, 1	$\text{III}^*, \text{I}_1, \text{I}_1$	2 : 2

762 $N = 762 = 2 \cdot 3 \cdot 127$ (7 isogeny classes)**762**

A1	1	0	1	-6	-8	0	1	-	5, 1, 1	5, 1, 1	1, 1, 1	$\text{I}_5, \text{I}_1, \text{I}_1$	
B1	1	0	1	-17677	-9208	0	1	+	35, 4, 1	35, 4, 1	1, 4, 1	$\text{I}_{35}, \text{I}_4, \text{I}_1$	
C1	1	0	1	-10	-10	1	1	+	1, 4, 1	1, 4, 1	1, 4, 1	$\text{I}_1, \text{I}_4, \text{I}_1$	
D1	1	1	1	-21	27	1	1	+	5, 2, 1	5, 2, 1	5, 2, 1	$\text{I}_5, \text{I}_2, \text{I}_1$	
E1	1	0	0	-267	1521	1	1	+	11, 6, 1	11, 6, 1	11, 6, 1	$\text{I}_{11}, \text{I}_6, \text{I}_1$	
F1	1	0	0	-8	-216	0	3	-	3, 9, 1	3, 9, 1	3, 9, 1	$\text{I}_3, \text{I}_9, \text{I}_1$	3 : 2
F2	1	0	0	-2978	-62802	0	1	-	1, 3, 3	1, 3, 3	1, 3, 3	$\text{I}_1, \text{I}_3, \text{I}_3$	3 : 1
G1	1	0	0	-101946	12401892	0	7	+	21, 14, 1	21, 14, 1	21, 14, 1	$\text{I}_{21}, \text{I}_{14}, \text{I}_1$	7 : 2
G2	1	0	0	-22361106	-40701264948	0	1	+	3, 2, 7	3, 2, 7	3, 2, 7	$\text{I}_3, \text{I}_2, \text{I}_7$	7 : 1

763 $N = 763 = 7 \cdot 109$ (1 isogeny class)**763**

A1	0	0	1	-5	10	1	1	-	3, 1	3, 1	3, 1	I_3, I_1	
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765 $N = 765 = 3^2 \cdot 5 \cdot 17$ (3 isogeny classes)**765**

A1	1	-1	0	-150	791	0	2	-	9, 1, 2	0, 1, 2	2, 1, 2	$\text{III}^*, \text{I}_1, \text{I}_2$	2 : 2
A2	1	-1	0	-2445	47150	0	2	+	9, 2, 1	0, 2, 1	2, 2, 1	$\text{III}^*, \text{I}_2, \text{I}_1$	2 : 1
B1	1	-1	1	-17	-24	0	2	-	3, 1, 2	0, 1, 2	2, 1, 2	$\text{III}, \text{I}_1, \text{I}_2$	2 : 2
B2	1	-1	1	-272	-1656	0	2	+	3, 2, 1	0, 2, 1	2, 2, 1	$\text{III}, \text{I}_2, \text{I}_1$	2 : 1
C1	1	-1	1	-77	276	1	2	+	6, 2, 1	0, 2, 1	2, 2, 1	$\text{I}_0^*, \text{I}_2, \text{I}_1$	2 : 2
C2	1	-1	1	-32	564	1	2	-	6, 4, 2	0, 4, 2	2, 4, 2	$\text{I}_0^*, \text{I}_4, \text{I}_2$	2 : 1

766 $N = 766 = 2 \cdot 383$ (1 isogeny class)**766**

A1	1	1	0	11	45	0	1	-	11, 1	11, 1	1, 1	$\text{I}_{11}, \text{I}_1$	
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768 $N = 768 = 2^8 \cdot 3$ (8 isogeny classes)**768**

A1	0	-1	0	-23	51	1	2	+	9, 2	0, 2	2, 2	III, I_2	2 : 2
A2	0	-1	0	-13	85	1	2	-	15, 4	0, 4	2, 2	III^*, I_4	2 : 1
B1	0	-1	0	1	3	1	2	-	9, 2	0, 2	2, 2	III, I_2	2 : 2; 5 : 3
B2	0	-1	0	-29	69	1	2	+	15, 1	0, 1	2, 1	III^*, I_1	2 : 1; 5 : 4
B3	0	-1	0	-159	-765	1	2	-	9, 10	0, 10	2, 2	$\text{III}, \text{I}_{10}$	2 : 4; 5 : 1
B4	0	-1	0	-2589	-49851	1	2	+	15, 5	0, 5	2, 1	III^*, I_5	2 : 3; 5 : 2
C1	0	1	0	-23	-51	0	2	+	9, 2	0, 2	2, 2	III, I_2	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
768 768													
$N = 768 = 2^8 \cdot 3$ (continued)													
D1	0	1	0	-7	5	0	2	+	9, 1	0, 1	2, 1	III, I ₁	2 : 2; 5 : 3
D2	0	1	0	3	27	0	2	-	15, 2	0, 2	2, 2	III*, I ₂	2 : 1; 5 : 4
D3	0	1	0	-647	-6555	0	2	+	9, 5	0, 5	2, 5	III, I ₅	2 : 4; 5 : 1
D4	0	1	0	-637	-6757	0	2	-	15, 10	0, 10	2, 10	III*, I ₁₀	2 : 3; 5 : 2
E1	0	-1	0	-3	-9	0	2	-	9, 4	0, 4	2, 2	III, I ₄	2 : 2
E2	0	-1	0	-93	-315	0	2	+	15, 2	0, 2	2, 2	III*, I ₂	2 : 1
F1	0	-1	0	-7	-5	0	2	+	9, 1	0, 1	2, 1	III, I ₁	2 : 2; 5 : 3
F2	0	-1	0	3	-27	0	2	-	15, 2	0, 2	2, 2	III*, I ₂	2 : 1; 5 : 4
F3	0	-1	0	-647	6555	0	2	+	9, 5	0, 5	2, 1	III, I ₅	2 : 4; 5 : 1
F4	0	-1	0	-637	6757	0	2	-	15, 10	0, 10	2, 2	III*, I ₁₀	2 : 3; 5 : 2
G1	0	1	0	-3	9	1	2	-	9, 4	0, 4	2, 4	III, I ₄	2 : 2
G2	0	1	0	-93	315	1	2	+	15, 2	0, 2	2, 2	III*, I ₂	2 : 1
H1	0	1	0	1	-3	1	2	-	9, 2	0, 2	2, 2	III, I ₂	2 : 2; 5 : 3
H2	0	1	0	-29	-69	1	2	+	15, 1	0, 1	2, 1	III*, I ₁	2 : 1; 5 : 4
H3	0	1	0	-159	765	1	2	-	9, 10	0, 10	2, 10	III, I ₁₀	2 : 4; 5 : 1
H4	0	1	0	-2589	49851	1	2	+	15, 5	0, 5	2, 5	III*, I ₅	2 : 3; 5 : 2

770 770													
$N = 770 = 2 \cdot 5 \cdot 7 \cdot 11$ (7 isogeny classes)													
A1	1	1	0	-3	-7	0	2	-	2, 1, 2, 1	2, 1, 2, 1	2, 1, 2, 1	I ₂ , I ₁ , I ₂ , I ₁	2 : 2
A2	1	1	0	-73	-273	0	2	+	1, 2, 1, 2	1, 2, 1, 2	1, 2, 1, 2	I ₁ , I ₂ , I ₁ , I ₂	2 : 1
B1	1	0	1	-914	10596	0	6	-	6, 1, 6, 1	6, 1, 6, 1	2, 1, 6, 1	I ₆ , I ₁ , I ₆ , I ₁	2 : 2; 3 : 3
B2	1	0	1	-14634	680132	0	6	+	3, 2, 3, 2	3, 2, 3, 2	1, 2, 3, 2	I ₃ , I ₂ , I ₃ , I ₂	2 : 1; 3 : 4
B3	1	0	1	2271	56852	0	2	-	18, 3, 2, 3	18, 3, 2, 3	2, 1, 2, 1	I ₁₈ , I ₃ , I ₂ , I ₃	2 : 4; 3 : 1
B4	1	0	1	-15649	580116	0	2	+	9, 6, 1, 6	9, 6, 1, 6	1, 2, 1, 2	I ₉ , I ₆ , I ₁ , I ₆	2 : 3; 3 : 2
C1	1	-1	0	-12089	-612755	0	2	-	8, 5, 8, 1	8, 5, 8, 1	2, 5, 2, 1	I ₈ , I ₅ , I ₈ , I ₁	2 : 2
C2	1	-1	0	-204169	-35456067	0	4	+	4, 10, 4, 2	4, 10, 4, 2	2, 10, 2, 2	I ₄ , I ₁₀ , I ₄ , I ₂	2 : 1, 3, 4
C3	1	-1	0	-3266669	-2271693567	0	2	+	2, 5, 2, 4	2, 5, 2, 4	2, 5, 2, 2	I ₂ , I ₅ , I ₂ , I ₄	2 : 2
C4	1	-1	0	-214949	-31495495	0	4	+	2, 20, 2, 1	2, 20, 2, 1	2, 20, 2, 1	I ₂ , I ₂₀ , I ₂ , I ₁	2 : 2
D1	1	0	1	32	558	1	2	-	8, 4, 1, 2	8, 4, 1, 2	2, 4, 1, 2	I ₈ , I ₄ , I ₁ , I ₂	2 : 2
D2	1	0	1	-848	9006	1	2	+	4, 8, 2, 1	4, 8, 2, 1	2, 8, 2, 1	I ₄ , I ₈ , I ₂ , I ₁	2 : 1
E1	1	-1	0	-29	-635	1	2	-	16, 1, 2, 1	16, 1, 2, 1	2, 1, 2, 1	I ₁₆ , I ₁ , I ₂ , I ₁	2 : 2
E2	1	-1	0	-1309	-17787	1	4	+	8, 2, 4, 2	8, 2, 4, 2	2, 2, 4, 2	I ₈ , I ₂ , I ₄ , I ₂	2 : 1, 3, 4
E3	1	-1	0	-20909	-1158507	1	2	+	4, 1, 2, 4	4, 1, 2, 4	2, 1, 2, 2	I ₄ , I ₁ , I ₂ , I ₄	2 : 2
E4	1	-1	0	-2189	9845	1	4	+	4, 4, 8, 1	4, 4, 8, 1	2, 4, 8, 1	I ₄ , I ₄ , I ₈ , I ₁	2 : 2
F1	1	0	0	-56	3136	1	6	-	12, 2, 3, 2	12, 2, 3, 2	12, 2, 3, 2	I ₁₂ , I ₂ , I ₃ , I ₂	2 : 2; 3 : 3
F2	1	0	0	-3576	81280	1	6	+	6, 4, 6, 1	6, 4, 6, 1	6, 2, 6, 1	I ₆ , I ₄ , I ₆ , I ₁	2 : 1; 3 : 4
F3	1	0	0	504	-84560	1	2	-	4, 6, 1, 6	4, 6, 1, 6	4, 2, 1, 2	I ₄ , I ₆ , I ₁ , I ₆	2 : 4; 3 : 1
F4	1	0	0	-26116	-1580604	1	2	+	2, 12, 2, 3	2, 12, 2, 3	2, 2, 2, 1	I ₂ , I ₁₂ , I ₂ , I ₃	2 : 3; 3 : 2
G1	1	0	0	10	100	0	6	-	6, 3, 2, 1	6, 3, 2, 1	6, 3, 2, 1	I ₆ , I ₃ , I ₂ , I ₁	2 : 2; 3 : 3
G2	1	0	0	-270	1612	0	6	+	3, 6, 1, 2	3, 6, 1, 2	3, 6, 1, 2	I ₃ , I ₆ , I ₁ , I ₂	2 : 1; 3 : 4
G3	1	0	0	-90	-2720	0	2	-	2, 1, 6, 3	2, 1, 6, 3	2, 1, 6, 1	I ₂ , I ₁ , I ₆ , I ₃	2 : 4; 3 : 1
G4	1	0	0	-3520	-80238	0	2	+	1, 2, 3, 6	1, 2, 3, 6	1, 2, 3, 2	I ₁ , I ₂ , I ₃ , I ₆	2 : 3; 3 : 2

774 774													
$N = 774 = 2 \cdot 3^2 \cdot 43$ (9 isogeny classes)													
A1	1	-1	0	57	-243	0	3	-	4, 3, 3	4, 0, 3	2, 2, 3	I, III, I ₂	3 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
774 774													
$N = 774 = 2 \cdot 3^2 \cdot 43$ (continued)													
B1	1	-1	0	-216	832	0	2	+	12, 7, 1	12, 1, 1	2, 2, 1	I_{12}, I_1^*, I_1	2 : 2
B2	1	-1	0	-3096	67072	0	4	+	6, 8, 2	6, 2, 2	2, 4, 2	I_6, I_2^*, I_2	2 : 1, 3, 4
B3	1	-1	0	-49536	4255960	0	2	+	3, 7, 1	3, 1, 1	1, 4, 1	I_3, I_1^*, I_1	2 : 2
B4	1	-1	0	-2736	82984	0	2	-	3, 10, 4	3, 4, 4	1, 4, 2	I_3, I_4^*, I_4	2 : 2
C1	1	-1	0	-397116	-96224252	0	1	-	2, 25, 1	2, 19, 1	2, 4, 1	I_2, I_{19}^*, I_1	
D1	1	-1	0	1431	-46899	1	1	-	14, 13, 1	14, 7, 1	2, 2, 1	I_{14}, I_7^*, I_1	7 : 2
D2	1	-1	0	-539109	152510121	1	1	-	2, 7, 7	2, 1, 7	2, 2, 7	I_2, I_1^*, I_7	7 : 1
E1	1	-1	0	-18	0	1	2	+	2, 7, 1	2, 1, 1	2, 4, 1	I_2, I_1^*, I_1	2 : 2
E2	1	-1	0	72	-54	1	2	-	1, 8, 2	1, 2, 2	1, 4, 2	I_1, I_2^*, I_2	2 : 1
F1	1	-1	1	-209	1217	1	3	-	12, 3, 1	12, 0, 1	12, 2, 1	I_{12}, III, I_1	3 : 2
F2	1	-1	1	511	6049	1	1	-	4, 9, 3	4, 0, 3	4, 2, 3	I_4, III^*, I_3	3 : 1
G1	1	-1	1	22	105	1	1	-	6, 7, 1	6, 1, 1	6, 4, 1	I_6, I_1^*, I_1	
H1	1	-1	1	-17249	-866127	0	2	+	14, 13, 1	14, 7, 1	14, 2, 1	I_{14}, I_7^*, I_1	2 : 2
H2	1	-1	1	-11489	-1458255	0	2	-	7, 20, 2	7, 14, 2	7, 4, 2	I_7, I_{14}^*, I_2	2 : 1
I1	1	-1	1	-131	-601	0	1	-	2, 11, 1	2, 5, 1	2, 2, 1	I_2, I_5^*, I_1	

775 775													
$N = 775 = 5^2 \cdot 31$ (3 isogeny classes)													
A1	0	1	1	-33	94	1	1	-	7, 1	1, 1	2, 1	I_1^*, I_1	
B1	1	0	1	-26	-177	0	2	-	8, 1	2, 1	4, 1	I_2^*, I_1	2 : 2
B2	1	0	1	-651	-6427	0	2	+	7, 2	1, 2	2, 2	I_1^*, I_2	2 : 1
C1	0	1	1	242	1269	0	1	-	11, 1	5, 1	4, 1	I_5^*, I_1	5 : 2
C2	0	1	1	-21008	-1181231	0	1	-	7, 5	1, 5	4, 5	I_1^*, I_5	5 : 1

776 776													
$N = 776 = 2^3 \cdot 97$ (1 isogeny class)													
A1	0	0	0	-31	66	1	2	+	8, 1	0, 1	4, 1	I_1^*, I_1	2 : 2
A2	0	0	0	-11	150	1	2	-	10, 2	0, 2	2, 2	III^*, I_2	2 : 1

777 777													
$N = 777 = 3 \cdot 7 \cdot 37$ (7 isogeny classes)													
A1	1	1	0	-16	19	0	2	+	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	2 : 2
A2	1	1	0	-21	0	0	4	+	2, 2, 2	2, 2, 2	2, 2, 2	I_2, I_2, I_2	2 : 1, 3, 4
A3	1	1	0	-206	-1221	0	2	+	4, 4, 1	4, 4, 1	2, 2, 1	I_4, I_4, I_1	2 : 2
A4	1	1	0	84	105	0	4	-	1, 1, 4	1, 1, 4	1, 1, 4	I_1, I_1, I_4	2 : 2
B1	0	-1	1	-2531950	1551713040	0	1	-	10, 13, 1	10, 13, 1	2, 1, 1	I_{10}, I_{13}, I_1	
C1	0	-1	1	-169	-792	0	1	-	4, 1, 1	4, 1, 1	2, 1, 1	I_4, I_1, I_1	
D1	1	1	1	-14	26	1	4	-	1, 4, 1	1, 4, 1	1, 4, 1	I_1, I_4, I_1	2 : 2
D2	1	1	1	-259	1496	1	4	+	2, 2, 2	2, 2, 2	2, 2, 2	I_2, I_2, I_2	2 : 1, 3, 4
D3	1	1	1	-294	1020	1	2	+	4, 1, 4	4, 1, 4	2, 1, 4	I_4, I_1, I_4	2 : 2
D4	1	1	1	-4144	100952	1	2	+	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	2 : 2
E1	1	0	1	-1312	-18391	1	2	+	5, 1, 1	5, 1, 1	5, 1, 1	I_5, I_1, I_1	2 : 2
E2	1	0	1	-1317	-18245	1	4	+	10, 2, 2	10, 2, 2	10, 2, 2	I_{10}, I_2, I_2	2 : 1, 3, 4
E3	1	0	1	-2612	23195	1	4	+	20, 1, 1	20, 1, 1	20, 1, 1	I_{20}, I_1, I_1	2 : 2
E4	1	0	1	-102	-50321	1	2	-	5, 4, 4	5, 4, 4	5, 2, 4	I_5, I_4, I_4	2 : 2
F1	0	1	1	0	2	1	1	-	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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780 $N = 780 = 2^2 \cdot 3 \cdot 5 \cdot 13$ (4 isogeny classes) **780**

A1	0	-1	0	-105	450	1	2	+	4, 4, 2, 1	0, 4, 2, 1	3, 2, 2, 1	IV, I ₄ , I ₂ , I ₁	2 : 2
A2	0	-1	0	-60	792	1	2	-	8, 2, 4, 2	0, 2, 4, 2	3, 2, 4, 2	IV*, I ₂ , I ₄ , I ₂	2 : 1
B1	0	-1	0	195	-195975	0	1	-	8, 13, 5, 1	0, 13, 5, 1	1, 1, 5, 1	IV*, I ₁₃ , I ₅ , I ₁	
C1	0	1	0	-81	0	1	2	+	4, 8, 2, 1	0, 8, 2, 1	3, 8, 2, 1	IV, I ₈ , I ₂ , I ₁	2 : 2
C2	0	1	0	324	324	1	2	-	8, 4, 4, 2	0, 4, 4, 2	3, 4, 2, 2	IV*, I ₄ , I ₄ , I ₂	2 : 1
D1	0	1	0	19	15	0	3	-	8, 3, 1, 1	0, 3, 1, 1	3, 3, 1, 1	IV*, I ₃ , I ₁ , I ₁	3 : 2
D2	0	1	0	-221	-1521	0	1	-	8, 1, 3, 3	0, 1, 3, 3	1, 1, 1, 3	IV*, I ₁ , I ₃ , I ₃	3 : 1

781 $N = 781 = 11 \cdot 71$ (2 isogeny classes) **781**

A1	0	0	1	-1378	347	0	1	+	9, 1	9, 1	1, 1	I ₉ , I ₁	
B1	0	0	1	-808	8840	1	1	+	3, 1	3, 1	3, 1	I ₃ , I ₁	

782 $N = 782 = 2 \cdot 17 \cdot 23$ (5 isogeny classes) **782**

A1	1	0	1	5	6	1	2	-	6, 1, 1	6, 1, 1	2, 1, 1	I ₆ , I ₁ , I ₁	2 : 2
A2	1	0	1	-35	54	1	2	+	3, 2, 2	3, 2, 2	1, 2, 2	I ₃ , I ₂ , I ₂	2 : 1
B1	1	0	0	-60	-184	0	1	-	3, 1, 1	3, 1, 1	3, 1, 1	I ₃ , I ₁ , I ₁	
C1	1	0	0	-99153	-12025559	0	2	+	14, 1, 4	14, 1, 4	14, 1, 2	I ₁₄ , I ₁ , I ₄	2 : 2
C2	1	0	0	-99793	-11862615	0	2	+	7, 2, 8	7, 2, 8	7, 2, 2	I ₇ , I ₂ , I ₈	2 : 1
D1	1	-1	1	0	1	0	1	-	1, 1, 1	1, 1, 1	1, 1, 1	I ₁ , I ₁ , I ₁	
E1	1	-1	1	-529	385	0	4	+	20, 1, 2	20, 1, 2	20, 1, 2	I ₂₀ , I ₁ , I ₂	2 : 2
E2	1	-1	1	-5649	-161407	0	4	+	10, 2, 4	10, 2, 4	10, 2, 4	I ₁₀ , I ₂ , I ₄	2 : 1, 3, 4
E3	1	-1	1	-90289	-10419775	0	2	+	5, 4, 2	5, 4, 2	5, 4, 2	I ₅ , I ₄ , I ₂	2 : 2
E4	1	-1	1	-2929	-319167	0	2	-	5, 1, 8	5, 1, 8	5, 1, 8	I ₅ , I ₁ , I ₈	2 : 2

784 $N = 784 = 2^4 \cdot 7^2$ (10 isogeny classes) **784**

A1	0	1	0	-16	-29	1	1	+	4, 4	0, 0	1, 1	II, IV	
B1	0	0	0	-343	2401	1	1	+	4, 8	0, 0	1, 3	II, IV*	
C1	0	0	0	49	686	0	2	-	8, 7	0, 1	2, 2	I ₀ , I ₁ *	2 : 2
C2	0	0	0	-931	10290	0	4	+	10, 8	0, 2	4, 4	I ₂ , I ₂ *	2 : 1, 3, 4
C3	0	0	0	-2891	-47334	0	2	+	11, 10	0, 4	2, 4	I ₃ , I ₄ *	2 : 2
C4	0	0	0	-14651	682570	0	4	+	11, 7	0, 1	4, 4	I ₃ , I ₁ *	2 : 2
D1	0	-1	0	-800	8359	0	1	+	4, 10	0, 0	1, 1	II, II*	
E1	0	-1	0	-16	-1392	0	2	-	10, 7	0, 1	4, 4	I ₂ , I ₁ *	2 : 2
E2	0	-1	0	-1976	-32752	0	2	+	11, 8	0, 2	4, 4	I ₃ , I ₂ *	2 : 1
F1	0	0	0	-7	-7	0	1	+	4, 2	0, 0	1, 1	II, II	
G1	0	-1	0	-114	127	0	1	+	4, 8	0, 0	1, 1	II, IV*	3 : 2
G2	0	-1	0	-6974	226507	0	1	+	4, 8	0, 0	1, 1	II, IV*	3 : 1
H1	0	0	0	-35	98	1	2	-	12, 3	0, 0	4, 2	I ₄ , III	2 : 2; 7 : 3
H2	0	0	0	-595	5586	1	2	+	12, 3	0, 0	2, 2	I ₄ , III	2 : 1; 7 : 4
H3	0	0	0	-1715	-33614	1	2	-	12, 9	0, 0	4, 2	I ₄ , III*	2 : 4; 7 : 1
H4	0	0	0	-29155	-1915998	1	2	+	12, 9	0, 0	2, 2	I ₄ , III*	2 : 3; 7 : 2
I1	0	1	0	-2	-1	1	1	+	4, 2	0, 0	1, 1	II, II	3 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
784 784													
$N = 784 = 2^4 \cdot 7^2$ (continued)													
J1	0	1	0	-408	6292	1	2	-	14, 7	2, 1	4, 4	I_6^*, I_1^*	2 : 2; 3 : 3
J2	0	1	0	-8248	285396	1	2	+	13, 8	1, 2	2, 4	I_5^*, I_2^*	2 : 1; 3 : 4
J3	0	1	0	3512	-133260	1	2	-	18, 9	6, 3	4, 4	I_{10}^*, I_3^*	2 : 4; 3 : 1, 5
J4	0	1	0	-27848	-1475468	1	2	+	15, 12	3, 6	2, 4	I_7^*, I_6^*	2 : 3; 3 : 2, 6
J5	0	1	0	-133688	-18913196	1	2	-	30, 7	18, 1	4, 4	I_{22}^*, I_1^*	2 : 6; 3 : 3
J6	0	1	0	-2140728	-1206278060	1	2	+	21, 8	9, 2	2, 4	I_{13}^*, I_2^*	2 : 5; 3 : 4
786 786													
$N = 786 = 2 \cdot 3 \cdot 131$ (13 isogeny classes)													
A1	1	1	0	-8	6	1	1	+	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	
B1	1	1	0	-281	1701	1	1	-	6, 3, 1	6, 3, 1	2, 1, 1	I_6, I_3, I_1	
C1	1	1	0	1217	6622405	1	1	-	9, 24, 1	9, 24, 1	1, 2, 1	I_9, I_{24}, I_1	
D1	1	1	0	-3418	-78356	0	1	+	3, 7, 1	3, 7, 1	1, 1, 1	I_3, I_7, I_1	
E1	1	1	0	-29	-3	0	2	+	12, 1, 1	12, 1, 1	2, 1, 1	I_{12}, I_1, I_1	2 : 2
E2	1	1	0	-349	2365	0	4	+	6, 2, 2	6, 2, 2	2, 2, 2	I_6, I_2, I_2	2 : 1, 3, 4
E3	1	1	0	-5589	158517	0	2	+	3, 4, 1	3, 4, 1	1, 2, 1	I_3, I_4, I_1	2 : 2
E4	1	1	0	-229	4165	0	2	-	3, 1, 4	3, 1, 4	1, 1, 4	I_3, I_1, I_4	2 : 2
F1	1	0	1	-40	92	0	3	+	1, 3, 1	1, 3, 1	1, 3, 1	I_1, I_3, I_1	3 : 2
F2	1	0	1	-145	-580	0	1	+	3, 1, 3	3, 1, 3	1, 1, 1	I_3, I_1, I_3	3 : 1
G1	1	0	1	-103	-406	1	1	+	11, 1, 1	11, 1, 1	1, 1, 1	I_{11}, I_1, I_1	
H1	1	0	1	-17	56	1	1	-	2, 7, 1	2, 7, 1	2, 7, 1	I_2, I_7, I_1	
I1	1	1	1	-71	-259	0	2	+	6, 3, 1	6, 3, 1	6, 1, 1	I_6, I_3, I_1	2 : 2
I2	1	1	1	-31	-499	0	2	-	3, 6, 2	3, 6, 2	3, 2, 2	I_3, I_6, I_2	2 : 1
J1	1	1	1	-861	9267	1	1	+	21, 1, 1	21, 1, 1	21, 1, 1	I_{21}, I_1, I_1	
K1	1	1	1	10	11	1	1	-	3, 4, 1	3, 4, 1	3, 2, 1	I_3, I_4, I_1	
L1	1	0	0	-42	36	1	1	+	7, 5, 1	7, 5, 1	7, 5, 1	I_7, I_5, I_1	
M1	1	0	0	-2135	35913	0	5	+	5, 15, 1	5, 15, 1	5, 15, 1	I_5, I_{15}, I_1	5 : 2
M2	1	0	0	-227045	-41659377	0	1	+	1, 3, 5	1, 3, 5	1, 3, 5	I_1, I_3, I_5	5 : 1
790 790													
$N = 790 = 2 \cdot 5 \cdot 79$ (1 isogeny class)													
A1	1	0	0	-25	57	1	2	-	8, 2, 1	8, 2, 1	8, 2, 1	I_8, I_2, I_1	2 : 2
A2	1	0	0	-425	3337	1	2	+	4, 1, 2	4, 1, 2	4, 1, 2	I_4, I_1, I_2	2 : 1
791 791													
$N = 791 = 7 \cdot 113$ (3 isogeny classes)													
A1	1	0	1	-31	117	0	2	-	3, 2	3, 2	1, 2	I_3, I_2	2 : 2
A2	1	0	1	-596	5541	0	2	+	6, 1	6, 1	2, 1	I_6, I_1	2 : 1
B1	1	0	1	-38	-93	0	2	-	1, 2	1, 2	1, 2	I_1, I_2	2 : 2
B2	1	0	1	-603	-5743	0	2	+	2, 1	2, 1	2, 1	I_2, I_1	2 : 1
C1	1	-1	1	-19	-14	1	4	+	4, 1	4, 1	4, 1	I_4, I_1	2 : 2
C2	1	-1	1	-264	-1582	1	4	+	2, 2	2, 2	2, 2	I_2, I_2	2 : 1, 3, 4
C3	1	-1	1	-4219	-104412	1	2	+	1, 1	1, 1	1, 1	I_1, I_1	2 : 2
C4	1	-1	1	-229	-2044	1	2	-	1, 4	1, 4	1, 4	I_1, I_4	2 : 2
792 792													
$N = 792 = 2^3 \cdot 3^2 \cdot 11$ (7 isogeny classes)													
A1	0	0	0	-135	-486	1	2	+	8, 9, 1	0, 0, 1	2, 2, 1	I^*, III^*, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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792

$N = 792 = 2^3 \cdot 3^2 \cdot 11$ (continued)

792

B1	0	0	0	-75	-74	0	2	+	10, 7, 1	0, 1, 1	2, 2, 1	III*, I ₁ *, I ₁	2 : 2
B2	0	0	0	285	-578	0	2	-	11, 8, 2	0, 2, 2	1, 4, 2	II*, I ₂ *, I ₂	2 : 1
C1	0	0	0	-15	18	1	2	+	8, 3, 1	0, 0, 1	4, 2, 1	I ₁ *, III, I ₁	2 : 2
C2	0	0	0	-75	-234	1	2	+	10, 3, 2	0, 0, 2	2, 2, 2	III*, III, I ₂	2 : 1
D1	0	0	0	-111	434	1	4	+	8, 7, 1	0, 1, 1	4, 4, 1	I ₁ *, I ₁ *, I ₁	2 : 2
D2	0	0	0	-291	-1330	1	4	+	10, 8, 2	0, 2, 2	2, 4, 2	III*, I ₂ *, I ₂	2 : 1, 3, 4
D3	0	0	0	-4251	-106666	1	2	+	11, 10, 1	0, 4, 1	1, 4, 1	II*, I ₄ *, I ₁	2 : 2
D4	0	0	0	789	-8890	1	2	-	11, 7, 4	0, 1, 4	1, 2, 2	II*, I ₁ *, I ₄	2 : 2
E1	0	0	0	6	-155	0	2	-	4, 10, 1	0, 4, 1	2, 4, 1	III, I ₄ *, I ₁	2 : 2
E2	0	0	0	-399	-2990	0	4	+	8, 8, 2	0, 2, 2	4, 4, 2	I ₁ *, I ₂ *, I ₂	2 : 1, 3, 4
E3	0	0	0	-6339	-194258	0	2	+	10, 7, 1	0, 1, 1	2, 2, 1	III*, I ₁ *, I ₁	2 : 2
E4	0	0	0	-939	6838	0	4	+	10, 7, 4	0, 1, 4	2, 4, 4	III*, I ₁ *, I ₄	2 : 2
F1	0	0	0	-36	-108	0	1	-	8, 6, 1	0, 0, 1	2, 1, 1	I ₁ *, I ₀ *, I ₁	
G1	0	0	0	-72147	7458910	0	2	+	10, 13, 1	0, 7, 1	2, 2, 1	III*, I ₇ *, I ₁	2 : 2
G2	0	0	0	-71787	7537030	0	2	-	11, 20, 2	0, 14, 2	1, 4, 2	II*, I ₁₄ *, I ₂	2 : 1

793

$N = 793 = 13 \cdot 61$ (1 isogeny class)

793

A1	1	-1	0	-16	-21	1	2	+	1, 1	1, 1	1, 1	I ₁ , I ₁	2 : 2
A2	1	-1	0	-11	-38	1	2	-	2, 2	2, 2	2, 2	I ₂ , I ₂	2 : 1

794

$N = 794 = 2 \cdot 397$ (4 isogeny classes)

794

A1	1	0	1	-3	2	2	1	-	2, 1	2, 1	2, 1	I ₂ , I ₁	
B1	1	0	0	-57	161	1	3	-	3, 1	3, 1	3, 1	I ₃ , I ₁	3 : 2
B2	1	0	0	13	539	1	1	-	1, 3	1, 3	1, 3	I ₁ , I ₃	3 : 1
C1	1	1	1	4	-3	1	1	-	5, 1	5, 1	5, 1	I ₅ , I ₁	
D1	1	0	0	47	-471	1	3	-	18, 1	18, 1	18, 1	I ₁₈ , I ₁	3 : 2
D2	1	0	0	-3473	-79127	1	3	-	6, 3	6, 3	6, 3	I ₆ , I ₃	3 : 1, 3
D3	1	0	0	-281373	-57471035	1	1	-	2, 1	2, 1	2, 1	I ₂ , I ₁	3 : 2

795

$N = 795 = 3 \cdot 5 \cdot 53$ (4 isogeny classes)

795

A1	1	1	0	-8	3	1	2	+	4, 1, 1	4, 1, 1	2, 1, 1	I ₄ , I ₁ , I ₁	2 : 2
A2	1	1	0	-53	-168	1	4	+	2, 2, 2	2, 2, 2	2, 2, 2	I ₂ , I ₂ , I ₂	2 : 1, 3, 4
A3	1	1	0	-848	-9867	1	2	+	1, 4, 1	1, 4, 1	1, 2, 1	I ₁ , I ₄ , I ₁	2 : 2
A4	1	1	0	22	-513	1	2	-	1, 1, 4	1, 1, 4	1, 1, 2	I ₁ , I ₁ , I ₄	2 : 2
B1	0	-1	1	-221	-1198	0	1	-	3, 5, 1	3, 5, 1	1, 1, 1	I ₃ , I ₅ , I ₁	
C1	0	1	1	-491	15251	0	3	-	15, 3, 1	15, 3, 1	15, 1, 1	I ₁₅ , I ₃ , I ₁	3 : 2
C2	0	1	1	4369	-387400	0	1	-	5, 9, 3	5, 9, 3	5, 1, 1	I ₅ , I ₉ , I ₃	3 : 1
D1	1	0	1	21	-23	0	2	-	3, 4, 1	3, 4, 1	3, 2, 1	I ₃ , I ₄ , I ₁	2 : 2
D2	1	0	1	-104	-223	0	4	+	6, 2, 2	6, 2, 2	6, 2, 2	I ₆ , I ₂ , I ₂	2 : 1, 3, 4
D3	1	0	1	-1429	-20893	0	2	+	12, 1, 1	12, 1, 1	12, 1, 1	I ₁₂ , I ₁ , I ₁	2 : 2
D4	1	0	1	-779	8147	0	2	+	3, 1, 4	3, 1, 4	3, 1, 2	I ₃ , I ₁ , I ₄	2 : 2

797

$N = 797 = 797$ (1 isogeny class)

797

	$a_1 a_2 a_3$	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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798 $N = 798 = 2 \cdot 3 \cdot 7 \cdot 19$ (9 isogeny classes)**798**

A1	1 1 0	-10	4	1	2	+	4, 1, 2, 1	4, 1, 2, 1	2, 1, 2, 1	I_4, I_1, I_2, I_1	2 : 2
A2	1 1 0	-150	648	1	2	+	2, 2, 1, 2	2, 2, 1, 2	2, 2, 1, 2	I_2, I_2, I_1, I_2	2 : 1
B1	1 0 1	-80	-226	0	2	+	12, 1, 2, 1	12, 1, 2, 1	2, 1, 2, 1	I_{12}, I_1, I_2, I_1	2 : 2
B2	1 0 1	-400	2846	0	4	+	6, 2, 4, 2	6, 2, 4, 2	2, 2, 2, 2	I_6, I_2, I_4, I_2	2 : 1, 3, 4
B3	1 0 1	-6280	191006	0	2	+	3, 1, 2, 4	3, 1, 2, 4	1, 1, 2, 2	I_3, I_1, I_2, I_4	2 : 2
B4	1 0 1	360	12574	0	2	-	3, 4, 8, 1	3, 4, 8, 1	1, 4, 2, 1	I_3, I_4, I_8, I_1	2 : 2
C1	1 0 1	-92	326	1	2	+	2, 5, 2, 1	2, 5, 2, 1	2, 5, 2, 1	I_2, I_5, I_2, I_1	2 : 2
C2	1 0 1	-22	830	1	2	-	1, 10, 1, 2	1, 10, 1, 2	1, 10, 1, 2	I_1, I_{10}, I_1, I_2	2 : 1
D1	1 0 1	-162	-476	1	2	+	4, 5, 4, 1	4, 5, 4, 1	2, 5, 4, 1	I_4, I_5, I_4, I_1	2 : 2
D2	1 0 1	-1142	14420	1	4	+	2, 10, 2, 2	2, 10, 2, 2	2, 10, 2, 2	I_2, I_{10}, I_2, I_2	2 : 1, 3, 4
D3	1 0 1	-18152	939764	1	2	+	1, 5, 1, 4	1, 5, 1, 4	1, 5, 1, 2	I_1, I_5, I_1, I_4	2 : 2
D4	1 0 1	188	46340	1	2	-	1, 20, 1, 1	1, 20, 1, 1	1, 20, 1, 1	I_1, I_{20}, I_1, I_1	2 : 2
E1	1 0 1	-7801	264524	0	6	+	4, 9, 2, 1	4, 9, 2, 1	2, 9, 2, 1	I_4, I_9, I_2, I_1	2 : 2; 3 : 3
E2	1 0 1	-7941	254500	0	6	+	2, 18, 1, 2	2, 18, 1, 2	2, 18, 1, 2	I_2, I_{18}, I_1, I_2	2 : 1; 3 : 4
E3	1 0 1	-11176	13046	0	6	+	12, 3, 6, 3	12, 3, 6, 3	2, 3, 6, 3	I_{12}, I_3, I_6, I_3	2 : 4; 3 : 1, 5
E4	1 0 1	-120936	-16143626	0	6	+	6, 6, 3, 6	6, 6, 3, 6	2, 6, 3, 6	I_6, I_6, I_3, I_6	2 : 3; 3 : 2, 6
E5	1 0 1	-611671	-184179718	0	2	+	36, 1, 2, 1	36, 1, 2, 1	2, 1, 2, 1	I_{36}, I_1, I_2, I_1	2 : 6; 3 : 3
E6	1 0 1	-9786711	-11785100294	0	2	+	18, 2, 1, 2	18, 2, 1, 2	2, 2, 1, 2	I_{18}, I_2, I_1, I_2	2 : 5; 3 : 4
F1	1 0 1	-39	-86	0	2	+	8, 1, 2, 1	8, 1, 2, 1	2, 1, 2, 1	I_8, I_1, I_2, I_1	2 : 2
F2	1 0 1	-599	-5686	0	2	+	4, 2, 1, 2	4, 2, 1, 2	2, 2, 1, 2	I_4, I_2, I_1, I_2	2 : 1
G1	1 1 1	-354	-2193	1	2	+	10, 1, 2, 3	10, 1, 2, 3	10, 1, 2, 3	I_{10}, I_1, I_2, I_3	2 : 2
G2	1 1 1	766	-12049	1	2	-	5, 2, 1, 6	5, 2, 1, 6	5, 2, 1, 6	I_5, I_2, I_1, I_6	2 : 1
H1	1 0 0	-1015	11561	1	2	+	12, 7, 2, 1	12, 7, 2, 1	12, 7, 2, 1	I_{12}, I_7, I_2, I_1	2 : 2
H2	1 0 0	-3255	-57879	1	2	+	6, 14, 1, 2	6, 14, 1, 2	6, 14, 1, 2	I_6, I_{14}, I_1, I_2	2 : 1
I1	1 0 0	3	-15	0	2	-	8, 1, 1, 1	8, 1, 1, 1	8, 1, 1, 1	I_8, I_1, I_1, I_1	2 : 2
I2	1 0 0	-77	-255	0	4	+	4, 2, 2, 2	4, 2, 2, 2	4, 2, 2, 2	I_4, I_2, I_2, I_2	2 : 1, 3, 4
I3	1 0 0	-1217	-16443	0	2	+	2, 1, 4, 1	2, 1, 4, 1	2, 1, 2, 1	I_2, I_1, I_4, I_1	2 : 2
I4	1 0 0	-217	893	0	4	+	2, 4, 1, 4	2, 4, 1, 4	2, 4, 1, 4	I_2, I_4, I_1, I_4	2 : 2

799 $N = 799 = 17 \cdot 47$ (2 isogeny classes)**799**

A1	1 1 1	-16	16	0	2	+	1, 2	1, 2	1, 2	I_1, I_2	2 : 2
A2	1 1 1	-251	1426	0	2	+	2, 1	2, 1	2, 1	I_2, I_1	2 : 1
B1	1 1 1	-118	418	1	2	+	3, 2	3, 2	3, 2	I_3, I_2	2 : 2
B2	1 1 1	-353	-2120	1	2	+	6, 1	6, 1	6, 1	I_6, I_1	2 : 1

800 $N = 800 = 2^5 \cdot 5^2$ (9 isogeny classes)**800**

A1	0 0 0	-25	0	1	4	+	6, 6	0, 0	2, 4	III, I_0^*	2 : 2, 3, 4
A2	0 0 0	-275	-1750	1	2	+	9, 6	0, 0	1, 2	I_0^*, I_0^*	2 : 1
A3	0 0 0	-275	1750	1	2	+	9, 6	0, 0	2, 2	I_0^*, I_0^*	2 : 1
A4	0 0 0	100	0	1	2	-	12, 6	0, 0	2, 4	I_3^*, I_0^*	2 : 1
B1	0 1 0	-8	8	1	1	-	9, 2	0, 0	2, 1	I_0^*, II	
C1	0 1 0	-158	-812	1	2	+	6, 7	0, 1	2, 2	III, I_1^*	2 : 2
C2	0 1 0	-33	-1937	1	2	-	12, 8	0, 2	4, 4	I_3^*, I_2^*	2 : 1
D1	0 0 0	-125	0	0	2	+	6, 9	0, 0	2, 2	III, III^*	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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800 **800**
 $N = 800 = 2^5 \cdot 5^2$ (continued)

E1	0	1	0	-208	-1412	0	1	-	9, 8	0, 0	1, 3	I_0^*, IV^*	
F1	0	-1	0	-8	-8	0	1	-	9, 2	0, 0	1, 1	I_0^*, II	
G1	0	-1	0	-158	812	0	2	+	6, 7	0, 1	2, 2	III, I_1^*	2 : 2
G2	0	-1	0	-33	1937	0	2	-	12, 8	0, 2	2, 4	I_3^*, I_2^*	2 : 1
H1	0	0	0	-5	0	1	2	+	6, 3	0, 0	2, 2	III, III	2 : 2
H2	0	0	0	20	0	1	2	-	12, 3	0, 0	2, 2	I_3^*, III	2 : 1
I1	0	-1	0	-208	1412	1	1	-	9, 8	0, 0	2, 3	I_0^*, IV^*	

801 **801**
 $N = 801 = 3^2 \cdot 89$ (4 isogeny classes)

A1	0	0	1	-3972	-169349	0	1	-	23, 1	17, 1	2, 1	I_{17}^*, I_1	
B1	1	-1	1	-14	-12	0	2	+	6, 1	0, 1	2, 1	I_0^*, I_1	2 : 2
B2	1	-1	1	31	-102	0	2	-	6, 2	0, 2	2, 2	I_0^*, I_2	2 : 1
C1	0	0	1	-30	-90	1	1	-	9, 1	3, 1	4, 1	I_3^*, I_1	3 : 2
C2	0	0	1	240	1233	1	3	-	7, 3	1, 3	4, 3	I_1^*, I_3	3 : 1
D1	1	-1	0	-9	-14	1	1	-	6, 1	0, 1	1, 1	I_0^*, I_1	

802 **802**
 $N = 802 = 2 \cdot 401$ (2 isogeny classes)

A1	1	-1	1	2	-1	0	1	-	1, 1	1, 1	1, 1	I_1, I_1	
B1	1	0	0	-9	-11	0	2	+	2, 1	2, 1	2, 1	I_2, I_1	2 : 2
B2	1	0	0	-19	15	0	2	+	1, 2	1, 2	1, 2	I_1, I_2	2 : 1

804 **804**
 $N = 804 = 2^2 \cdot 3 \cdot 67$ (4 isogeny classes)

A1	0	-1	0	59	-122	0	2	-	4, 5, 2	0, 5, 2	3, 1, 2	IV, I_5, I_2	2 : 2
A2	0	-1	0	-276	-792	0	2	+	8, 10, 1	0, 10, 1	3, 2, 1	IV^*, I_{10}, I_1	2 : 1
B1	0	-1	0	-1373	-19191	1	1	-	8, 10, 1	0, 10, 1	3, 2, 1	IV^*, I_{10}, I_1	
C1	0	-1	0	-12	24	1	1	-	8, 1, 1	0, 1, 1	3, 1, 1	IV^*, I_1, I_1	
D1	0	1	0	84	36	1	1	-	8, 7, 1	0, 7, 1	3, 7, 1	IV^*, I_7, I_1	

805 **805**
 $N = 805 = 5 \cdot 7 \cdot 23$ (4 isogeny classes)

A1	0	-1	1	23004	2393001	1	1	-	5, 11, 2	5, 11, 2	1, 1, 2	I_5, I_{11}, I_2	
B1	1	-1	1	-163	-758	0	2	+	2, 3, 1	2, 3, 1	2, 1, 1	I_2, I_3, I_1	2 : 2
B2	1	-1	1	-138	-1018	0	2	-	1, 6, 2	1, 6, 2	1, 2, 2	I_1, I_6, I_2	2 : 1
C1	1	-1	1	2	2356	0	4	-	2, 3, 4	2, 3, 4	2, 1, 4	I_2, I_3, I_4	2 : 2
C2	1	-1	1	-2643	52082	0	4	+	4, 6, 2	4, 6, 2	2, 2, 2	I_4, I_6, I_2	2 : 1, 3, 4
C3	1	-1	1	-5518	-79018	0	2	+	2, 12, 1	2, 12, 1	2, 2, 1	I_2, I_{12}, I_1	2 : 2
C4	1	-1	1	-42088	3333906	0	2	+	8, 3, 1	8, 3, 1	2, 1, 1	I_8, I_3, I_1	2 : 2
D1	0	0	1	-13	49	0	1	-	1, 3, 2	1, 3, 2	1, 1, 2	I_1, I_3, I_2	

806 **806**
 $N = 806 = 2 \cdot 13 \cdot 31$ (6 isogeny classes)

A1	1	0	1	-3	30	1	1	-	5, 1, 2	5, 1, 2	1, 1, 2	I_5, I_1, I_2	
B1	1	1	0	52	-176	1	1	-	11, 1, 2	11, 1, 2	1, 1, 2	I_{11}, I_1, I_2	
C1	1	0	0	-97	361	1	1	-	5, 1, 2	5, 1, 2	5, 1, 2	I_5, I_1, I_2	

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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806 $N = 806 = 2 \cdot 13 \cdot 31$ (continued)**806**

E1	1	0	0	2511	39401	0	3	−	27, 1, 2	27, 1, 2	27, 1, 2	I_{27}, I_1, I_2	3 : 2
E2	1	0	0	−25649	−2195479	0	3	−	9, 3, 6	9, 3, 6	9, 3, 6	I_9, I_3, I_6	3 : 1, 3
E3	1	0	0	−2293609	−1337178239	0	1	−	3, 9, 2	3, 9, 2	3, 9, 2	I_3, I_9, I_2	3 : 2
F1	1	1	1	−14105	638919	0	5	−	5, 5, 2	5, 5, 2	5, 5, 2	I_5, I_5, I_2	5 : 2
F2	1	1	1	66885	2264179	0	1	−	1, 1, 10	1, 1, 10	1, 1, 10	I_1, I_1, I_{10}	5 : 1

807 $N = 807 = 3 \cdot 269$ (1 isogeny class)**807**

A1	0	1	1	−49	115	0	3	+	6, 1	6, 1	6, 1	I_6, I_1	3 : 2
A2	0	1	1	−409	−3260	0	1	+	2, 3	2, 3	2, 1	I_2, I_3	3 : 1

808 $N = 808 = 2^3 \cdot 101$ (2 isogeny classes)**808**

A1	0	0	0	−11	−26	0	1	−	11, 1	0, 1	1, 1	II^*, I_1	
B1	0	−1	0	−129	−523	0	1	+	8, 1	0, 1	2, 1	I_1^*, I_1	

810 $N = 810 = 2 \cdot 3^4 \cdot 5$ (8 isogeny classes)**810**

A1	1	−1	0	−9	15	0	3	−	1, 4, 3	1, 0, 3	1, 1, 3	I_1, II, I_3	3 : 2
A2	1	−1	0	66	−100	0	1	−	3, 12, 1	3, 0, 1	1, 1, 1	I_3, II^*, I_1	3 : 1
B1	1	−1	0	36	120	0	3	−	3, 4, 6	3, 0, 6	1, 1, 6	I_3, II, I_6	3 : 2
B2	1	−1	0	−339	−4555	0	1	−	9, 12, 2	9, 0, 2	1, 1, 2	I_9, II^*, I_2	3 : 1
C1	1	−1	0	−114	−10252	0	3	−	5, 6, 9	5, 0, 9	1, 3, 9	I_5, IV, I_9	3 : 2
C2	1	−1	0	−39489	−3010627	0	1	−	15, 10, 3	15, 0, 3	1, 3, 3	I_{15}, IV^*, I_3	3 : 1
D1	1	−1	0	−24	80	1	3	−	4, 6, 3	4, 0, 3	2, 3, 3	I_4, IV, I_3	3 : 2
D2	1	−1	0	201	−1315	1	1	−	12, 10, 1	12, 0, 1	2, 1, 1	I_{12}, IV^*, I_1	3 : 1
E1	1	−1	1	7	1	0	3	−	3, 6, 1	3, 0, 1	3, 3, 1	I_3, IV, I_1	3 : 2
E2	1	−1	1	−83	−323	0	1	−	1, 10, 3	1, 0, 3	1, 3, 1	I_1, IV^*, I_3	3 : 1
F1	1	−1	1	22	41	0	3	−	12, 4, 1	12, 0, 1	12, 1, 1	I_{12}, II, I_1	3 : 2
F2	1	−1	1	−218	−1943	0	1	−	4, 12, 3	4, 0, 3	4, 1, 1	I_4, II^*, I_3	3 : 1
G1	1	−1	1	−4388	112967	0	3	−	15, 4, 3	15, 0, 3	15, 1, 1	I_{15}, II, I_3	3 : 2
G2	1	−1	1	−1028	277831	0	1	−	5, 12, 9	5, 0, 9	5, 1, 1	I_5, II^*, I_9	3 : 1
H1	1	−1	1	−38	181	1	3	−	9, 6, 2	9, 0, 2	9, 3, 2	I_9, IV, I_2	3 : 2
H2	1	−1	1	322	−3563	1	1	−	3, 10, 6	3, 0, 6	3, 1, 2	I_3, IV^*, I_6	3 : 1

811 $N = 811 = 811$ (1 isogeny class)**811**

A1	0	0	1	−2	−2	1	1	−	1	1	1	I_1	
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812 $N = 812 = 2^2 \cdot 7 \cdot 29$ (2 isogeny classes)**812**

A1	0	0	0	−40	−124	0	1	−	8, 3, 1	0, 3, 1	3, 1, 1	IV^*, I_3, I_1	
B1	0	−1	0	−36	232	1	1	−	8, 4, 1	0, 4, 1	3, 4, 1	IV^*, I_4, I_1	

813 $N = 813 = 3 \cdot 271$ (2 isogeny classes)**813**

A1	0	1	1	−2	−1	0	1	+	1, 1	1, 1	1, 1	I_1, I_1	
B1	0	1	1	−73	190	1	3	+	9, 1	9, 1	9, 1	I_9, I_1	3 : 2
B2	0	1	1	−1423	−21113	1	3	+	3, 3	3, 3	3, 3	I_3, I_3	3 : 1, 3

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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814 $N = 814 = 2 \cdot 11 \cdot 37$ (2 isogeny classes) **814**

A1	1	0	1	5	30	1	3	−	3, 3, 1	3, 3, 1	1, 3, 1	I_3, I_3, I_1	3 : 2
A2	1	0	1	−50	−828	1	1	−	9, 1, 3	9, 1, 3	1, 1, 3	I_9, I_1, I_3	3 : 1
B1	1	−1	1	−28	63	1	1	−	5, 1, 1	5, 1, 1	5, 1, 1	I_5, I_1, I_1	

815 $N = 815 = 5 \cdot 163$ (1 isogeny class) **815**

A1	0	1	1	15	−69	1	3	−	6, 1	6, 1	6, 1	I_6, I_1	3 : 2
A2	0	1	1	−985	−12244	1	1	−	2, 3	2, 3	2, 3	I_2, I_3	3 : 1

816 $N = 816 = 2^4 \cdot 3 \cdot 17$ (10 isogeny classes) **816**

A1	0	−1	0	−48	144	1	2	+	10, 2, 1	0, 2, 1	4, 2, 1	I_2^*, I_2, I_1	2 : 2
A2	0	−1	0	−8	336	1	2	−	11, 4, 2	0, 4, 2	2, 2, 2	I_3^*, I_4, I_2	2 : 1
B1	0	−1	0	−52	−128	0	2	+	8, 2, 1	0, 2, 1	2, 2, 1	I_0^*, I_2, I_1	2 : 2
B2	0	−1	0	−72	0	0	4	+	10, 4, 2	0, 4, 2	4, 2, 2	I_2^*, I_4, I_2	2 : 1, 3, 4
B3	0	−1	0	−752	8160	0	2	+	11, 8, 1	0, 8, 1	2, 2, 1	I_3^*, I_8, I_1	2 : 2
B4	0	−1	0	288	−288	0	4	−	11, 2, 4	0, 2, 4	4, 2, 4	I_3^*, I_2, I_4	2 : 2
C1	0	−1	0	−17	−51	0	1	−	8, 5, 1	0, 5, 1	1, 1, 1	I_0^*, I_5, I_1	
D1	0	1	0	511	1899	0	1	−	8, 3, 5	0, 3, 5	1, 3, 1	I_0^*, I_3, I_5	
E1	0	−1	0	−4088	−99216	0	2	+	18, 6, 1	6, 6, 1	4, 2, 1	I_{10}^*, I_6, I_1	2 : 2; 3 : 3
E2	0	−1	0	−3448	−131984	0	2	−	15, 12, 2	3, 12, 2	4, 2, 2	I_7^*, I_{12}, I_2	2 : 1; 3 : 4
E3	0	−1	0	−12008	386928	0	2	+	30, 2, 3	18, 2, 3	4, 2, 1	I_{22}^*, I_2, I_3	2 : 4; 3 : 1
E4	0	−1	0	28952	2418544	0	2	−	21, 4, 6	9, 4, 6	4, 2, 2	I_{13}^*, I_4, I_6	2 : 3; 3 : 2
F1	0	−1	0	11	61	0	1	−	12, 3, 1	0, 3, 1	1, 1, 1	II^*, I_3, I_1	3 : 2
F2	0	−1	0	−949	11581	0	1	−	12, 1, 3	0, 1, 3	1, 1, 1	II^*, I_1, I_3	3 : 1
G1	0	−1	0	−5	9	1	1	−	8, 1, 1	0, 1, 1	2, 1, 1	I_0^*, I_1, I_1	
H1	0	−1	0	−544	−4352	1	2	+	20, 4, 1	8, 4, 1	4, 2, 1	I_{12}^*, I_4, I_1	2 : 2
H2	0	−1	0	−1824	25344	1	4	+	16, 8, 2	4, 8, 2	4, 2, 2	I_8^*, I_8, I_2	2 : 1, 3, 4
H3	0	−1	0	−27744	1787904	1	8	+	14, 4, 4	2, 4, 4	4, 2, 4	I_6^*, I_4, I_4	2 : 2, 5, 6
H4	0	−1	0	3616	142848	1	2	−	14, 16, 1	2, 16, 1	2, 2, 1	I_6^*, I_{16}, I_1	2 : 2
H5	0	−1	0	−443904	113984640	1	4	+	13, 2, 2	1, 2, 2	4, 2, 2	I_5^*, I_2, I_2	2 : 3
H6	0	−1	0	−26304	1980288	1	4	−	13, 2, 8	1, 2, 8	2, 2, 8	I_5^*, I_2, I_8	2 : 3
I1	0	1	0	−1621	24623	1	1	−	8, 11, 1	0, 11, 1	2, 11, 1	I_0^*, I_{11}, I_1	
J1	0	1	0	−40	−76	1	2	+	14, 2, 1	2, 2, 1	4, 2, 1	I_6^*, I_2, I_1	2 : 2
J2	0	1	0	120	−396	1	2	−	13, 4, 2	1, 4, 2	4, 4, 2	I_5^*, I_4, I_2	2 : 1

817 $N = 817 = 19 \cdot 43$ (2 isogeny classes) **817**

A1	0	1	1	1	6	2	1	−	2, 1	2, 1	2, 1	I_2, I_1	
B1	0	1	1	−16649	821406	1	1	−	2, 5	2, 5	2, 5	I_2, I_5	

819 $N = 819 = 3^2 \cdot 7 \cdot 13$ (6 isogeny classes) **819**

A1	1	−1	0	−42	−73	1	2	+	9, 1, 1	0, 1, 1	2, 1, 1	III^*, I_1, I_1	2 : 2
A2	1	−1	0	93	−532	1	2	−	9, 2, 2	0, 2, 2	2, 2, 2	III^*, I_2, I_2	2 : 1
B1	1	−1	1	−5	4	1	2	+	3, 1, 1	0, 1, 1	2, 1, 1	III, I_1, I_1	2 : 2
B2	1	−1	1	10	16	1	2	−	3, 2, 2	0, 2, 2	2, 2, 2	III, I_2, I_2	2 : 1
C1	0	0	1	9	−7	0	1	−	6, 1, 1	0, 1, 1	2, 1, 1	I_0^*, I_1, I_1	

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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819 $N = 819 = 3^2 \cdot 7 \cdot 13$ (continued)**819**

E1	0	0	1	-66	-207	0	1	-	6, 1, 1	0, 1, 1	2, 1, 1	I_0^*, I_1, I_1	3 : 2
E2	0	0	1	114	-1026	0	3	-	6, 3, 3	0, 3, 3	2, 3, 3	I_0^*, I_3, I_3	3 : 1, 3
E3	0	0	1	-1056	32553	0	3	-	6, 9, 1	0, 9, 1	2, 9, 1	I_0^*, I_9, I_1	3 : 2
F1	0	0	1	-237	-1607	0	1	-	10, 3, 1	4, 3, 1	2, 3, 1	I_4^*, I_3, I_1	

822 $N = 822 = 2 \cdot 3 \cdot 137$ (6 isogeny classes)**822**

A1	1	1	0	-3	-9	1	1	-	1, 4, 1	1, 4, 1	1, 2, 1	I_1, I_4, I_1	
B1	1	0	1	-18716	-987046	0	2	+	10, 8, 1	10, 8, 1	2, 8, 1	I_{10}, I_8, I_1	2 : 2
B2	1	0	1	-18556	-1004710	0	2	-	5, 16, 2	5, 16, 2	1, 16, 2	I_5, I_{16}, I_2	2 : 1
C1	1	0	1	-1122	14548	0	3	-	5, 12, 1	5, 12, 1	1, 12, 1	I_5, I_{12}, I_1	3 : 2
C2	1	0	1	4143	72868	0	1	-	15, 4, 3	15, 4, 3	1, 4, 1	I_{15}, I_4, I_3	3 : 1
D1	1	0	1	31	20	1	1	-	6, 5, 1	6, 5, 1	2, 5, 1	I_6, I_5, I_1	
E1	1	0	0	-47	57	0	4	+	12, 2, 1	12, 2, 1	12, 2, 1	I_{12}, I_2, I_1	2 : 2
E2	1	0	0	-367	-2695	0	4	+	6, 4, 2	6, 4, 2	6, 4, 2	I_6, I_4, I_2	2 : 1, 3, 4
E3	1	0	0	-5847	-172575	0	2	+	3, 8, 1	3, 8, 1	3, 8, 1	I_3, I_8, I_1	2 : 2
E4	1	0	0	-7	-7663	0	2	-	3, 2, 4	3, 2, 4	3, 2, 4	I_3, I_2, I_4	2 : 2
F1	1	0	0	-4	-4	0	1	-	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	

825 $N = 825 = 3 \cdot 5^2 \cdot 11$ (3 isogeny classes)**825**

A1	0	-1	1	-23	53	1	1	-	3, 2, 2	3, 0, 2	1, 1, 2	I_3, II, I_2	3 : 2
A2	0	-1	1	127	38	1	1	-	1, 2, 6	1, 0, 6	1, 1, 2	I_1, II, I_6	3 : 1
B1	1	0	0	-163	-808	1	2	+	3, 6, 1	3, 0, 1	3, 2, 1	I_3, I_0^*, I_1	2 : 2
B2	1	0	0	-288	567	1	4	+	6, 6, 2	6, 0, 2	6, 4, 2	I_6, I_0^*, I_2	2 : 1, 3, 4
B3	1	0	0	-3663	84942	1	2	+	3, 6, 4	3, 0, 4	3, 2, 4	I_3, I_0^*, I_4	2 : 2
B4	1	0	0	1087	4692	1	2	-	12, 6, 1	12, 0, 1	12, 4, 1	I_{12}, I_0^*, I_1	2 : 2
C1	0	1	1	-583	5494	1	3	-	3, 8, 2	3, 0, 2	3, 3, 2	I_3, IV^*, I_2	3 : 2
C2	0	1	1	3167	11119	1	1	-	1, 8, 6	1, 0, 6	1, 1, 2	I_1, IV^*, I_6	3 : 1

826 $N = 826 = 2 \cdot 7 \cdot 59$ (2 isogeny classes)**826**

A1	1	1	0	21	-49	0	1	-	1, 5, 1	1, 5, 1	1, 1, 1	I_1, I_5, I_1	
B1	1	1	0	-136	-672	0	1	-	5, 3, 1	5, 3, 1	1, 3, 1	I_5, I_3, I_1	

827 $N = 827 = 827$ (1 isogeny class)**827**

A1	0	0	1	-10	12	1	1	-	1	1	1	I_1	
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828 $N = 828 = 2^2 \cdot 3^2 \cdot 23$ (4 isogeny classes)**828**

A1	0	0	0	-24	45	0	2	+	4, 3, 1	0, 0, 1	1, 2, 1	IV, III, I_1	2 : 2
A2	0	0	0	-39	-18	0	2	+	8, 3, 2	0, 0, 2	1, 2, 2	IV^*, III, I_2	2 : 1
B1	0	0	0	-216	-1215	1	2	+	4, 9, 1	0, 0, 1	3, 2, 1	IV, III^*, I_1	2 : 2
B2	0	0	0	-351	486	1	2	+	8, 9, 2	0, 0, 2	3, 2, 2	IV^*, III^*, I_2	2 : 1
C1	0	0	0	-9	-27	1	1	-	4, 6, 1	0, 0, 1	1, 1, 1	IV, I_0^*, I_1	
D1	0	0	0	15	-11	0	1	-	4, 6, 1	0, 0, 1	1, 1, 1	IV, I_0^*, I_1	3 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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829 $N = 829 = 829$ (1 isogeny class) **829**

A1	0	0	1	-4	-3	1	1	+	1	1	1	I_1	
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830 $N = 830 = 2 \cdot 5 \cdot 83$ (3 isogeny classes) **830**

A1	1	0	1	37	-62	0	3	-	2, 6, 1	2, 6, 1	2, 6, 1	I_2, I_6, I_1	3 : 2
A2	1	0	1	-838	-9512	0	1	-	6, 2, 3	6, 2, 3	2, 2, 1	I_6, I_2, I_3	3 : 1
B1	1	1	1	-11185	456015	1	1	-	16, 8, 1	16, 8, 1	16, 8, 1	I_{16}, I_8, I_1	
C1	1	-1	1	3	69	1	1	-	10, 2, 1	10, 2, 1	10, 2, 1	I_{10}, I_2, I_1	

831 $N = 831 = 3 \cdot 277$ (1 isogeny class) **831**

A1	1	0	0	-68	285	1	1	-	10, 1	10, 1	10, 1	I_{10}, I_1	
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832 $N = 832 = 2^6 \cdot 13$ (10 isogeny classes) **832**

A1	0	1	0	-1	31	1	1	-	15, 1	0, 1	4, 1	I_5^*, I_1	
B1	0	-1	0	-1	-31	1	1	-	15, 1	0, 1	4, 1	I_5^*, I_1	
C1	0	-1	0	31	97	1	1	-	19, 1	1, 1	4, 1	I_9^*, I_1	3 : 2
C2	0	-1	0	-289	-3679	1	1	-	21, 3	3, 3	4, 1	I_{11}^*, I_3	3 : 1, 3
C3	0	-1	0	-29409	-1931423	1	1	-	27, 1	9, 1	4, 1	I_{17}^*, I_1	3 : 2
D1	0	0	0	-16	-24	0	2	+	10, 1	0, 1	2, 1	I_0^*, I_1	2 : 2
D2	0	0	0	4	-80	0	2	-	14, 2	0, 2	2, 2	I_4^*, I_2	2 : 1
E1	0	-1	0	-65	-191	0	1	-	17, 1	0, 1	2, 1	I_7^*, I_1	
F1	0	0	0	-172	1328	0	1	-	25, 1	7, 1	2, 1	I_{15}^*, I_1	7 : 2
F2	0	0	0	-13612	-670672	0	1	-	19, 7	1, 7	2, 7	I_9^*, I_7	7 : 1
G1	0	1	0	31	-97	0	1	-	19, 1	1, 1	2, 1	I_9^*, I_1	3 : 2
G2	0	1	0	-289	3679	0	1	-	21, 3	3, 3	2, 1	I_{11}^*, I_3	3 : 1, 3
G3	0	1	0	-29409	1931423	0	1	-	27, 1	9, 1	2, 1	I_{17}^*, I_1	3 : 2
H1	0	0	0	-16	24	1	2	+	10, 1	0, 1	2, 1	I_0^*, I_1	2 : 2
H2	0	0	0	4	80	1	2	-	14, 2	0, 2	4, 2	I_4^*, I_2	2 : 1
I1	0	1	0	-65	191	1	1	-	17, 1	0, 1	4, 1	I_7^*, I_1	
J1	0	0	0	-172	-1328	1	1	-	25, 1	7, 1	4, 1	I_{15}^*, I_1	7 : 2
J2	0	0	0	-13612	670672	1	1	-	19, 7	1, 7	4, 7	I_9^*, I_7	7 : 1

833 $N = 833 = 7^2 \cdot 17$ (1 isogeny class) **833**

A1	1	-1	1	-34	-24	0	2	+	6, 1	0, 1	2, 1	I_0^*, I_1	2 : 2
A2	1	-1	1	-279	1838	0	4	+	6, 2	0, 2	4, 2	I_0^*, I_2	2 : 1, 3, 4
A3	1	-1	1	-4444	115126	0	2	+	6, 1	0, 1	2, 1	I_0^*, I_1	2 : 2
A4	1	-1	1	-34	4778	0	2	-	6, 4	0, 4	2, 2	I_0^*, I_4	2 : 2

834 $N = 834 = 2 \cdot 3 \cdot 139$ (7 isogeny classes) **834**

A1	1	0	1	-11795	-233746	0	2	+	28, 7, 1	28, 7, 1	2, 7, 1	I_{28}, I_7, I_1	2 : 2
A2	1	0	1	-93715	10874606	0	4	+	14, 14, 2	14, 14, 2	2, 14, 2	I_{14}, I_{14}, I_2	2 : 1, 3, 4
A3	1	0	1	-1493395	702316526	0	2	+	7, 7, 4	7, 7, 4	1, 7, 2	I_7, I_7, I_4	2 : 2
A4	1	0	1	-4755	30694894	0	2	-	7, 28, 1	7, 28, 1	1, 28, 1	I_7, I_{28}, I_1	2 : 2
B1	1	0	1	-60	-182	0	1	-	4, 1, 1	4, 1, 1	2, 1, 1	I_4, I_1, I_1	

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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834 $N = 834 = 2 \cdot 3 \cdot 139$ (continued)**834**

D1	1	1	1	-8	5	0	2	+	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	2 : 2
D2	1	1	1	2	29	0	2	-	1, 2, 2	1, 2, 2	1, 2, 2	I_1, I_2, I_2	2 : 1
E1	1	1	1	2	-1	1	1	-	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	
F1	1	1	1	-1027	12257	1	1	-	14, 4, 1	14, 4, 1	14, 2, 1	I_{14}, I_4, I_1	
G1	1	0	0	-70	356	1	5	-	10, 5, 1	10, 5, 1	10, 5, 1	I_{10}, I_5, I_1	5 : 2
G2	1	0	0	-1090	-40504	1	1	-	2, 1, 5	2, 1, 5	2, 1, 1	I_2, I_1, I_5	5 : 1

836 $N = 836 = 2^2 \cdot 11 \cdot 19$ (2 isogeny classes)**836**

A1	0	-1	0	-5	-47	1	1	-	8, 1, 2	0, 1, 2	1, 1, 2	IV^*, I_1, I_2	
B1	0	-1	0	3	-10	0	2	-	4, 2, 1	0, 2, 1	3, 2, 1	IV, I_2, I_1	2 : 2
B2	0	-1	0	-52	-120	0	2	+	8, 1, 2	0, 1, 2	3, 1, 2	IV^*, I_1, I_2	2 : 1

840 $N = 840 = 2^3 \cdot 3 \cdot 5 \cdot 7$ (10 isogeny classes)**840**

A1	0	-1	0	-316	-2060	1	2	+	8, 3, 1, 1	0, 3, 1, 1	2, 1, 1, 1	I_1^*, I_3, I_1, I_1	2 : 2
A2	0	-1	0	-336	-1764	1	4	+	10, 6, 2, 2	0, 6, 2, 2	2, 2, 2, 2	III^*, I_6, I_2, I_2	2 : 1, 3, 4
A3	0	-1	0	-1736	26796	1	2	+	11, 12, 1, 1	0, 12, 1, 1	1, 2, 1, 1	II^*, I_{12}, I_1, I_1	2 : 2
A4	0	-1	0	744	-11700	1	2	-	11, 3, 4, 4	0, 3, 4, 4	1, 1, 2, 2	II^*, I_3, I_4, I_4	2 : 2
B1	0	-1	0	9	-84	0	4	-	4, 1, 2, 4	0, 1, 2, 4	2, 1, 2, 4	III, I_1, I_2, I_4	2 : 2
B2	0	-1	0	-236	-1260	0	4	+	8, 2, 4, 2	0, 2, 4, 2	2, 2, 2, 2	I_1^*, I_2, I_4, I_2	2 : 1, 3, 4
B3	0	-1	0	-3736	-86660	0	2	+	10, 4, 2, 1	0, 4, 2, 1	2, 2, 2, 1	III^*, I_4, I_2, I_1	2 : 2
B4	0	-1	0	-656	4956	0	2	+	10, 1, 8, 1	0, 1, 8, 1	2, 1, 2, 1	III^*, I_1, I_8, I_1	2 : 2
C1	0	-1	0	-15	12	0	4	+	4, 1, 4, 1	0, 1, 4, 1	2, 1, 4, 1	III, I_1, I_4, I_1	2 : 2
C2	0	-1	0	-140	-588	0	4	+	8, 2, 2, 2	0, 2, 2, 2	2, 2, 2, 2	I_1^*, I_2, I_2, I_2	2 : 1, 3, 4
C3	0	-1	0	-2240	-40068	0	2	+	10, 1, 1, 1	0, 1, 1, 1	2, 1, 1, 1	III^*, I_1, I_1, I_1	2 : 2
C4	0	-1	0	-40	-1508	0	2	-	10, 4, 1, 4	0, 4, 1, 4	2, 2, 1, 2	III^*, I_4, I_1, I_4	2 : 2
D1	0	1	0	-27991	-1811530	0	2	+	4, 5, 8, 3	0, 5, 8, 3	2, 5, 2, 1	III, I_5, I_8, I_3	2 : 2
D2	0	1	0	-31116	-1385280	0	4	+	8, 10, 4, 6	0, 10, 4, 6	2, 10, 2, 2	I_1^*, I_{10}, I_4, I_6	2 : 1, 3, 4
D3	0	1	0	-202616	34012320	0	4	+	10, 20, 2, 3	0, 20, 2, 3	2, 20, 2, 1	III^*, I_{20}, I_2, I_3	2 : 2
D4	0	1	0	90384	-9452880	0	2	-	10, 5, 2, 12	0, 5, 2, 12	2, 5, 2, 2	III^*, I_5, I_2, I_{12}	2 : 2
E1	0	-1	0	9	0	1	2	-	4, 4, 1, 1	0, 4, 1, 1	2, 2, 1, 1	III, I_4, I_1, I_1	2 : 2
E2	0	-1	0	-36	36	1	4	+	8, 2, 2, 2	0, 2, 2, 2	4, 2, 2, 2	I_1^*, I_2, I_2, I_2	2 : 1, 3, 4
E3	0	-1	0	-336	-2244	1	2	+	10, 1, 1, 4	0, 1, 1, 4	2, 1, 1, 4	III^*, I_1, I_1, I_4	2 : 2
E4	0	-1	0	-456	3900	1	2	+	10, 1, 4, 1	0, 1, 4, 1	2, 1, 2, 1	III^*, I_1, I_4, I_1	2 : 2
F1	0	-1	0	-175	952	1	4	+	4, 1, 2, 1	0, 1, 2, 1	2, 1, 2, 1	III, I_1, I_2, I_1	2 : 2
F2	0	-1	0	-180	900	1	8	+	8, 2, 4, 2	0, 2, 4, 2	4, 2, 4, 2	I_1^*, I_2, I_4, I_2	2 : 1, 3, 4
F3	0	-1	0	-680	-5700	1	4	+	10, 4, 2, 4	0, 4, 2, 4	2, 2, 2, 2	III^*, I_4, I_2, I_4	2 : 2, 5, 6
F4	0	-1	0	240	4092	1	4	-	10, 1, 8, 1	0, 1, 8, 1	2, 1, 8, 1	III^*, I_1, I_8, I_1	2 : 2
F5	0	-1	0	-10480	-409460	1	2	+	11, 8, 1, 2	0, 8, 1, 2	1, 2, 1, 2	II^*, I_8, I_1, I_2	2 : 3
F6	0	-1	0	1120	-32340	1	2	-	11, 2, 1, 8	0, 2, 1, 8	1, 2, 1, 2	II^*, I_2, I_1, I_8	2 : 3
G1	0	-1	0	-735	7920	0	4	+	4, 2, 1, 2	0, 2, 1, 2	2, 2, 1, 2	III, I_2, I_1, I_2	2 : 2
G2	0	-1	0	-740	7812	0	8	+	8, 4, 2, 4	0, 4, 2, 4	4, 2, 2, 4	I_1^*, I_4, I_2, I_4	2 : 1, 3, 4
G3	0	-1	0	-1720	-16100	0	4	+	10, 8, 4, 2	0, 8, 4, 2	2, 2, 4, 2	III^*, I_8, I_4, I_2	2 : 2, 5, 6
G4	0	-1	0	160	24732	0	4	-	10, 2, 1, 8	0, 2, 1, 8	2, 2, 1, 8	III^*, I_2, I_1, I_8	2 : 2
G5	0	-1	0	-24400	-1458548	0	2	+	11, 4, 8, 1	0, 4, 8, 1	1, 2, 8, 1	II^*, I_1, I_8, I_1	2 : 3

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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840 **840**
 $N = 840 = 2^3 \cdot 3 \cdot 5 \cdot 7$ (continued)

H1	0	1	0	-71	-246	1	2	+	4, 3, 4, 1	0, 3, 4, 1	2, 3, 2, 1	III, I ₃ , I ₄ , I ₁	2 : 2
H2	0	1	0	-196	704	1	4	+	8, 6, 2, 2	0, 6, 2, 2	4, 6, 2, 2	I ₁ [*] , I ₆ , I ₂ , I ₂	2 : 1, 3, 4
H3	0	1	0	-2896	59024	1	2	+	10, 3, 1, 4	0, 3, 1, 4	2, 3, 1, 2	III [*] , I ₃ , I ₁ , I ₄	2 : 2
H4	0	1	0	504	5184	1	2	-	10, 12, 1, 1	0, 12, 1, 1	2, 12, 1, 1	III [*] , I ₁₂ , I ₁ , I ₁	2 : 2
I1	0	1	0	-36	-96	0	2	+	8, 1, 1, 1	0, 1, 1, 1	4, 1, 1, 1	I ₁ [*] , I ₁ , I ₁ , I ₁	2 : 2
I2	0	1	0	-56	0	0	4	+	10, 2, 2, 2	0, 2, 2, 2	2, 2, 2, 2	III [*] , I ₂ , I ₂ , I ₂	2 : 1, 3, 4
I3	0	1	0	-656	6240	0	2	+	11, 1, 1, 4	0, 1, 1, 4	1, 1, 1, 4	II [*] , I ₁ , I ₁ , I ₄	2 : 2
I4	0	1	0	224	224	0	2	-	11, 4, 4, 1	0, 4, 4, 1	1, 4, 2, 1	II [*] , I ₄ , I ₄ , I ₁	2 : 2
J1	0	1	0	-15	90	0	4	-	4, 8, 1, 1	0, 8, 1, 1	2, 8, 1, 1	III, I ₈ , I ₁ , I ₁	2 : 2
J2	0	1	0	-420	3168	0	8	+	8, 4, 2, 2	0, 4, 2, 2	4, 4, 2, 2	I ₁ [*] , I ₄ , I ₂ , I ₂	2 : 1, 3, 4
J3	0	1	0	-600	0	0	4	+	10, 2, 4, 4	0, 2, 4, 4	2, 2, 4, 2	III [*] , I ₂ , I ₄ , I ₄	2 : 2, 5, 6
J4	0	1	0	-6720	209808	0	4	+	10, 2, 1, 1	0, 2, 1, 1	2, 2, 1, 1	III [*] , I ₂ , I ₁ , I ₁	2 : 2
J5	0	1	0	-6480	-202272	0	2	+	11, 1, 8, 2	0, 1, 8, 2	1, 1, 8, 2	II [*] , I ₁ , I ₈ , I ₂	2 : 3
J6	0	1	0	2400	2400	0	2	-	11, 1, 2, 8	0, 1, 2, 8	1, 1, 2, 2	II [*] , I ₁ , I ₂ , I ₈	2 : 3

842 **842**
 $N = 842 = 2 \cdot 421$ (2 isogeny classes)

A1	1	0	1	-10	-12	1	1	+	3, 1	3, 1	1, 1	I ₃ , I ₁	
B1	1	0	0	-59	145	1	1	+	13, 1	13, 1	13, 1	I ₁₃ , I ₁	

843 **843**
 $N = 843 = 3 \cdot 281$ (1 isogeny class)

A1	1	1	0	5	4	1	1	-	3, 1	3, 1	1, 1	I ₃ , I ₁	
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845 **845**
 $N = 845 = 5 \cdot 13^2$ (1 isogeny class)

A1	1	0	1	-173	171	0	2	+	1, 7	1, 1	1, 4	I ₁ , I ₁ [*]	2 : 2
A2	1	0	1	672	1523	0	2	-	2, 8	2, 2	2, 4	I ₂ , I ₂ [*]	2 : 1

846 **846**
 $N = 846 = 2 \cdot 3^2 \cdot 47$ (3 isogeny classes)

A1	1	-1	0	-135	-707	0	2	-	8, 8, 1	8, 2, 1	2, 4, 1	I ₈ , I ₂ [*] , I ₁	2 : 2
A2	1	-1	0	-2295	-41747	0	2	+	4, 7, 2	4, 1, 2	2, 2, 2	I ₄ , I ₁ [*] , I ₂	2 : 1
B1	1	-1	0	3	17	1	2	-	2, 6, 1	2, 0, 1	2, 4, 1	I ₂ , I ₀ [*] , I ₁	2 : 2
B2	1	-1	0	-87	323	1	2	+	1, 6, 2	1, 0, 2	1, 2, 2	I ₁ , I ₀ [*] , I ₂	2 : 1
C1	1	-1	0	522	2164	1	2	-	12, 10, 1	12, 4, 1	2, 4, 1	I ₁₂ , I ₄ [*] , I ₁	2 : 2
C2	1	-1	0	-2358	20020	1	4	+	6, 14, 2	6, 8, 2	2, 4, 2	I ₆ , I ₈ [*] , I ₂	2 : 1, 3, 4
C3	1	-1	0	-19278	-1012100	1	2	+	3, 22, 1	3, 16, 1	1, 4, 1	I ₃ , I ₁₆ [*] , I ₁	2 : 2
C4	1	-1	0	-31518	2160364	1	2	+	3, 10, 4	3, 4, 4	1, 2, 4	I ₃ , I ₄ [*] , I ₄	2 : 2

847 **847**
 $N = 847 = 7 \cdot 11^2$ (3 isogeny classes)

A1	0	1	1	-10809	-436166	0	1	-	2, 7	2, 1	2, 4	I ₂ , I ₁ [*]	3 : 2
A2	0	1	1	-5969	-822761	0	1	-	6, 9	6, 3	2, 4	I ₆ , I ₃ [*]	3 : 1, 3
A3	0	1	1	53321	21262764	0	1	-	2, 15	2, 9	2, 4	I ₂ , I ₉ [*]	3 : 2
B1	0	0	1	242	-333	1	1	-	2, 7	2, 1	2, 4	I ₂ , I ₁ [*]	
C1	1	1	1	421	-12440	1	2	-	3, 8	3, 2	3, 4	I ₃ , I ₂ [*]	2 : 2
C2	1	1	1	-6234	-177484	1	2	+	6, 7	6, 1	6, 4	I ₆ , I ₁ [*]	2 : 1

848 **848**
 $N = 848 = 2^4 \cdot 53$ (7 isogeny classes)

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
848 848 $N = 848 = 2^4 \cdot 53$ (continued)													
B1	0	-1	0	-4528	150464	0	1	-	36, 1	24, 1	2, 1	I_{28}^*, I_1	3 : 2
B2	0	-1	0	-393648	95194048	0	1	-	20, 3	8, 3	2, 1	I_{12}^*, I_3	3 : 1
C1	0	-1	0	16	-64	0	1	-	15, 1	3, 1	2, 1	I_7^*, I_1	3 : 2
C2	0	-1	0	-144	1856	0	1	-	13, 3	1, 3	2, 1	I_5^*, I_3	3 : 1
D1	0	1	0	-12	40	0	2	-	8, 2	0, 2	1, 2	I_0^*, I_2	2 : 2
D2	0	1	0	-17	22	0	2	+	4, 1	0, 1	1, 1	II, I_1	2 : 1
E1	0	0	0	5	-22	0	1	-	12, 1	0, 1	2, 1	I_4^*, I_1	
F1	0	1	0	-4	-8	1	1	-	8, 1	0, 1	1, 1	I_0^*, I_1	
G1	0	1	0	-440	3412	1	1	-	17, 1	5, 1	4, 1	I_9^*, I_1	
849 849 $N = 849 = 3 \cdot 283$ (1 isogeny class)													
A1	1	1	1	5	-4	1	1	-	4, 1	4, 1	2, 1	I_4, I_1	
850 850 $N = 850 = 2 \cdot 5^2 \cdot 17$ (12 isogeny classes)													
A1	1	1	0	9975	-114875	0	1	-	21, 9, 1	21, 3, 1	1, 2, 1	I_{21}, I_3^*, I_1	3 : 2
A2	1	1	0	-166025	-26946875	0	1	-	7, 15, 3	7, 9, 3	1, 2, 3	I_7, I_9^*, I_3	3 : 1
B1	1	1	0	-75	125	0	2	+	6, 6, 1	6, 0, 1	2, 2, 1	I_6, I_0^*, I_1	2 : 2; 3 : 3
B2	1	1	0	-1075	13125	0	2	+	3, 6, 2	3, 0, 2	1, 2, 2	I_3, I_0^*, I_2	2 : 1; 3 : 4
B3	1	1	0	-2575	-51375	0	2	+	2, 6, 3	2, 0, 3	2, 2, 3	I_2, I_0^*, I_3	2 : 4; 3 : 1
B4	1	1	0	-2825	-41125	0	2	+	1, 6, 6	1, 0, 6	1, 2, 6	I_1, I_0^*, I_6	2 : 3; 3 : 2
C1	1	0	1	-451	4798	1	1	-	7, 9, 1	7, 0, 1	1, 2, 1	I_7, III^*, I_1	
D1	1	0	1	33924	-387702	1	1	-	4, 8, 7	4, 0, 7	2, 1, 7	I_4, IV^*, I_7	
E1	1	-1	0	8	16	1	1	-	4, 4, 1	4, 0, 1	2, 3, 1	I_4, IV, I_1	
F1	1	1	1	1357	-2559	0	1	-	4, 2, 7	4, 0, 7	4, 1, 1	I_4, II, I_7	
G1	1	1	1	-188	781	0	2	+	4, 8, 1	4, 2, 1	4, 2, 1	I_4, I_2^*, I_1	2 : 2
G2	1	1	1	312	4781	0	2	-	2, 10, 2	2, 4, 2	2, 4, 2	I_2, I_4^*, I_2	2 : 1
H1	1	1	1	-63838	6181531	0	2	+	8, 8, 3	8, 2, 3	8, 2, 1	I_8, I_2^*, I_3	2 : 2; 3 : 3
H2	1	1	1	-61838	6589531	0	2	-	4, 10, 6	4, 4, 6	4, 4, 2	I_4, I_4^*, I_6	2 : 1; 3 : 4
H3	1	1	1	-104213	-2590469	0	2	+	24, 12, 1	24, 6, 1	24, 2, 1	I_{24}, I_6^*, I_1	2 : 4; 3 : 1
H4	1	1	1	407787	-19998469	0	2	-	12, 18, 2	12, 12, 2	12, 4, 2	I_{12}, I_{12}^*, I_2	2 : 3; 3 : 2
I1	1	-1	1	195	2197	0	1	-	4, 10, 1	4, 0, 1	4, 1, 1	I_4, II^*, I_1	
J1	1	-1	1	-255	-1503	0	1	-	1, 7, 1	1, 1, 1	1, 2, 1	I_1, I_1^*, I_1	
K1	1	1	1	-63	781	1	1	-	3, 9, 1	3, 3, 1	3, 4, 1	I_3, I_3^*, I_1	3 : 2
K2	1	1	1	562	-20469	1	1	-	9, 7, 3	9, 1, 3	9, 4, 3	I_9, I_1^*, I_3	3 : 1
L1	1	1	1	-18	31	1	1	-	7, 3, 1	7, 0, 1	7, 2, 1	I_7, III, I_1	
851 851 $N = 851 = 23 \cdot 37$ (1 isogeny class)													
A1	0	1	1	-28	48	1	1	+	2, 1	2, 1	2, 1	I_2, I_1	
854 854 $N = 854 = 2 \cdot 7 \cdot 61$ (4 isogeny classes)													
A1	1	0	1	-722	7396	1	1	+	10, 3, 1	10, 3, 1	2, 1, 1	I_{10}, I_3, I_1	
B1	1	0	1	-2706	53940	1	3	+	4, 1, 1	4, 1, 1	2, 1, 1	I_4, I_1, I_1	3 : 2
B2	1	0	1	-2801	49924	1	3	+	12, 3, 3	12, 3, 3	2, 3, 3	I_{12}, I_2, I_2	3 : 1, 3

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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854

$N = 854 = 2 \cdot 7 \cdot 61$ (continued)

854

C1	1	1	1	-13	3	1	1	+	8, 1, 1	8, 1, 1	8, 1, 1	I_8, I_1, I_1	
D1	1	1	1	-399	1237	1	1	+	6, 7, 1	6, 7, 1	6, 7, 1	I_6, I_7, I_1	

855

$N = 855 = 3^2 \cdot 5 \cdot 19$ (3 isogeny classes)

855

A1	1	-1	1	202	4956	0	2	-	14, 3, 1	8, 3, 1	4, 1, 1	I_8^*, I_3, I_1	$2 : 2$
A2	1	-1	1	-3443	73482	0	4	+	10, 6, 2	4, 6, 2	4, 2, 2	I_4^*, I_6, I_2	$2 : 1, 3, 4$
A3	1	-1	1	-11138	-363594	0	2	+	8, 12, 1	2, 12, 1	4, 2, 1	I_2^*, I_{12}, I_1	$2 : 2$
A4	1	-1	1	-54068	4852482	0	2	+	8, 3, 4	2, 3, 4	2, 1, 2	I_2^*, I_3, I_4	$2 : 2$
B1	1	-1	1	13	474	1	2	-	7, 3, 2	1, 3, 2	2, 3, 2	I_1^*, I_3, I_2	$2 : 2$
B2	1	-1	1	-842	9366	1	2	+	8, 6, 1	2, 6, 1	4, 6, 1	I_2^*, I_6, I_1	$2 : 1$
C1	1	-1	0	171	0	0	2	-	11, 1, 2	5, 1, 2	4, 1, 2	I_5^*, I_1, I_2	$2 : 2$
C2	1	-1	0	-684	513	0	2	+	16, 2, 1	10, 2, 1	4, 2, 1	I_{10}^*, I_2, I_1	$2 : 1$

856

$N = 856 = 2^3 \cdot 107$ (4 isogeny classes)

856

A1	0	1	0	-3	2	1	1	-	4, 1	0, 1	2, 1	III, I_1	
B1	0	1	0	0	-16	1	1	-	10, 1	0, 1	2, 1	III^*, I_1	
C1	0	-1	0	-28	68	1	1	-	8, 1	0, 1	4, 1	I_1^*, I_1	
D1	0	-1	0	-432	-3316	1	1	-	11, 1	0, 1	1, 1	II^*, I_1	

858

$N = 858 = 2 \cdot 3 \cdot 11 \cdot 13$ (13 isogeny classes)

858

A1	1	1	0	6	-108	0	2	-	12, 2, 1, 1	12, 2, 1, 1	2, 2, 1, 1	I_{12}, I_2, I_1, I_1	$2 : 2$
A2	1	1	0	-314	-2220	0	4	+	6, 4, 2, 2	6, 4, 2, 2	2, 2, 2, 2	I_6, I_4, I_2, I_2	$2 : 1, 3, 4$
A3	1	1	0	-4994	-137940	0	2	+	3, 2, 4, 1	3, 2, 4, 1	1, 2, 4, 1	I_3, I_2, I_4, I_1	$2 : 2$
A4	1	1	0	-754	4732	0	2	+	3, 8, 1, 4	3, 8, 1, 4	1, 2, 1, 2	I_3, I_8, I_1, I_4	$2 : 2$
B1	1	0	1	359	1916	1	6	-	8, 6, 1, 3	8, 6, 1, 3	2, 6, 1, 3	I_8, I_6, I_1, I_3	$2 : 2; 3 : 3$
B2	1	0	1	-1801	16604	1	6	+	4, 3, 2, 6	4, 3, 2, 6	2, 3, 2, 6	I_4, I_3, I_2, I_6	$2 : 1; 3 : 4$
B3	1	0	1	-3736	-117658	1	2	-	24, 2, 3, 1	24, 2, 3, 1	2, 2, 1, 1	I_{24}, I_2, I_3, I_1	$2 : 4; 3 : 1$
B4	1	0	1	-65176	-6409114	1	2	+	12, 1, 6, 2	12, 1, 6, 2	2, 1, 2, 2	I_{12}, I_1, I_6, I_2	$2 : 3; 3 : 2$
C1	1	0	1	-7	-10	0	2	-	2, 1, 2, 1	2, 1, 2, 1	2, 1, 2, 1	I_2, I_1, I_2, I_1	$2 : 2$
C2	1	0	1	-117	-494	0	2	+	1, 2, 1, 2	1, 2, 1, 2	1, 2, 1, 2	I_1, I_2, I_1, I_2	$2 : 1$
D1	1	0	1	-103987	12897998	0	3	-	13, 6, 3, 1	13, 6, 3, 1	1, 6, 3, 1	I_{13}, I_6, I_3, I_1	$3 : 2$
D2	1	0	1	-80722	18827108	0	1	-	39, 2, 1, 3	39, 2, 1, 3	1, 2, 1, 3	I_{39}, I_2, I_1, I_3	$3 : 1$
E1	1	1	1	-1067	12953	0	4	+	12, 3, 1, 2	12, 3, 1, 2	12, 1, 1, 2	I_{12}, I_3, I_1, I_2	$2 : 2$
E2	1	1	1	-1387	4121	0	4	+	6, 6, 2, 4	6, 6, 2, 4	6, 2, 2, 2	I_6, I_6, I_2, I_4	$2 : 1, 3, 4$
E3	1	1	1	-13267	-589879	0	2	+	3, 3, 1, 8	3, 3, 1, 8	3, 1, 1, 2	I_3, I_3, I_1, I_8	$2 : 2$
E4	1	1	1	5373	39273	0	2	-	3, 12, 4, 2	3, 12, 4, 2	3, 2, 2, 2	I_3, I_{12}, I_4, I_2	$2 : 2$
F1	1	1	1	-572	118685	1	1	-	11, 6, 1, 5	11, 6, 1, 5	11, 2, 1, 5	I_{11}, I_6, I_1, I_5	
G1	1	1	1	-46	107	1	1	-	9, 2, 1, 1	9, 2, 1, 1	9, 2, 1, 1	I_9, I_2, I_1, I_1	
H1	1	1	1	-154	791	0	4	-	4, 3, 4, 1	4, 3, 4, 1	4, 1, 4, 1	I_4, I_3, I_4, I_1	$2 : 2$
H2	1	1	1	-2574	49191	0	4	+	2, 6, 2, 2	2, 6, 2, 2	2, 2, 2, 2	I_2, I_6, I_2, I_2	$2 : 1, 3, 4$
H3	1	1	1	-2684	44615	0	2	+	1, 12, 1, 4	1, 12, 1, 4	1, 2, 1, 4	I_1, I_{12}, I_1, I_4	$2 : 2$
H4	1	1	1	-41184	3199767	0	2	+	1, 3, 1, 1	1, 3, 1, 1	1, 1, 1, 1	I_1, I_3, I_1, I_1	$2 : 2$
I1	1	1	1	-2301	-43629	0	2	-	16, 6, 1, 1	16, 6, 1, 1	16, 2, 1, 1	I_{16}, I_6, I_1, I_1	$2 : 2$

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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858 $N = 858 = 2 \cdot 3 \cdot 11 \cdot 13$ (continued)**858**

J1	1	0	0	13	-39	0	3	-	3, 6, 1, 1	3, 6, 1, 1	3, 6, 1, 1	I_3, I_6, I_1, I_1	3 : 2
J2	1	0	0	-617	-5961	0	1	-	1, 2, 3, 3	1, 2, 3, 3	1, 2, 1, 3	I_1, I_2, I_3, I_3	3 : 1
K1	1	0	0	-5774401	5346023177	0	7	-	7, 14, 7, 3	7, 14, 7, 3	7, 14, 7, 1	I_7, I_{14}, I_7, I_3	7 : 2
K2	1	0	0	16353089	-335543012233	0	1	-	1, 2, 1, 21	1, 2, 1, 21	1, 2, 1, 1	I_1, I_2, I_1, I_{21}	7 : 1
L1	1	0	0	-332	-6000	0	2	-	14, 1, 2, 3	14, 1, 2, 3	14, 1, 2, 1	I_{14}, I_1, I_2, I_3	2 : 2
L2	1	0	0	-7372	-243952	0	2	+	7, 2, 1, 6	7, 2, 1, 6	7, 2, 1, 2	I_7, I_2, I_1, I_6	2 : 1
M1	1	0	0	-1	-7	0	2	-	4, 2, 1, 1	4, 2, 1, 1	4, 2, 1, 1	I_4, I_2, I_1, I_1	2 : 2
M2	1	0	0	-61	-187	0	2	+	2, 1, 2, 2	2, 1, 2, 2	2, 1, 2, 2	I_2, I_1, I_2, I_2	2 : 1

861 $N = 861 = 3 \cdot 7 \cdot 41$ (4 isogeny classes)**861**

A1	1	1	1	3	-6	0	2	-	4, 1, 1	4, 1, 1	2, 1, 1	I_4, I_1, I_1	2 : 2
A2	1	1	1	-42	-114	0	4	+	2, 2, 2	2, 2, 2	2, 2, 2	I_2, I_2, I_2	2 : 1, 3, 4
A3	1	1	1	-657	-6756	0	2	+	1, 4, 1	1, 4, 1	1, 4, 1	I_1, I_4, I_1	2 : 2
A4	1	1	1	-147	516	0	2	+	1, 1, 4	1, 1, 4	1, 1, 2	I_1, I_1, I_4	2 : 2
B1	1	0	1	706	-64375	1	1	-	17, 3, 1	17, 3, 1	17, 1, 1	I_{17}, I_3, I_1	
C1	1	0	0	2941	18606	1	1	-	7, 1, 5	7, 1, 5	7, 1, 5	I_7, I_1, I_5	
D1	1	0	0	-7	14	1	1	-	5, 1, 1	5, 1, 1	5, 1, 1	I_5, I_1, I_1	

862 $N = 862 = 2 \cdot 431$ (6 isogeny classes)**862**

A1	1	0	1	1	-2	1	1	-	2, 1	2, 1	2, 1	I_2, I_1	
B1	1	-1	0	-70	244	1	1	-	6, 1	6, 1	2, 1	I_6, I_1	
C1	1	-1	1	6	-7	0	2	-	6, 1	6, 1	6, 1	I_6, I_1	2 : 2
C2	1	-1	1	-34	-39	0	2	+	3, 2	3, 2	3, 2	I_3, I_2	2 : 1
D1	1	0	0	8	64	0	3	-	12, 1	12, 1	12, 1	I_{12}, I_1	3 : 2
D2	1	0	0	-72	-1744	0	1	-	4, 3	4, 3	4, 1	I_4, I_3	3 : 1
E1	1	1	1	-2460	45949	1	5	-	20, 1	20, 1	20, 1	I_{20}, I_1	5 : 2
E2	1	1	1	15380	-102531	1	1	-	4, 5	4, 5	4, 5	I_4, I_5	5 : 1
F1	1	1	1	-2	15	1	1	-	8, 1	8, 1	8, 1	I_8, I_1	

864 $N = 864 = 2^5 \cdot 3^3$ (12 isogeny classes)**864**

A1	0	0	0	-3	6	1	1	-	9, 3	0, 0	2, 1	I_0^*, II	
B1	0	0	0	-24	48	1	1	-	12, 3	0, 0	2, 1	III^*, II	
C1	0	0	0	24	-16	1	1	-	12, 5	0, 0	2, 3	III^*, IV	
D1	0	0	0	-3	-6	0	1	-	9, 3	0, 0	1, 1	I_0^*, II	
E1	0	0	0	216	-432	0	1	-	12, 11	0, 0	2, 1	III^*, II^*	
F1	0	0	0	-24	-48	0	1	-	12, 3	0, 0	2, 1	III^*, II	
G1	0	0	0	-27	162	0	1	-	9, 9	0, 0	1, 1	I_0^*, IV^*	
H1	0	0	0	216	432	0	1	-	12, 11	0, 0	2, 1	III^*, II^*	
I1	0	0	0	-216	-1296	0	1	-	12, 9	0, 0	2, 1	III^*, IV^*	
J1	0	0	0	-27	-162	1	1	-	9, 9	0, 0	2, 3	I_0^*, IV^*	
K1	0	0	0	24	16	1	1	-	12, 5	0, 0	2, 1	III^*, IV	

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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866

$N = 866 = 2 \cdot 433$ (1 isogeny class)

866

A1	1	0	0	-8	64	1	3	-	12, 1	12, 1	12, 1	I_{12}, I_1	3 : 2
A2	1	0	0	72	-1712	1	1	-	4, 3	4, 3	4, 3	I_4, I_3	3 : 1

867

$N = 867 = 3 \cdot 17^2$ (5 isogeny classes)

867

A1	0	-1	1	193	-5023	1	1	-	3, 7	3, 1	1, 4	I_3, I_1^*	3 : 2
A2	0	-1	1	-17147	-859018	1	1	-	1, 9	1, 3	1, 4	I_1, I_3^*	3 : 1
B1	1	1	1	-23	20	1	2	+	4, 3	4, 0	2, 2	I_4, III	2 : 2
B2	1	1	1	62	224	1	2	-	8, 3	8, 0	2, 2	I_8, III	2 : 1
C1	0	-1	1	1638	-13693	1	1	-	1, 9	1, 0	1, 2	I_1, III^*	5 : 2
C2	0	-1	1	-244012	-46313805	1	1	-	5, 9	5, 0	1, 2	I_5, III^*	5 : 1
D1	1	0	0	-6653	145704	0	2	+	4, 9	4, 0	4, 2	I_4, III^*	2 : 2
D2	1	0	0	17912	976001	0	2	-	8, 9	8, 0	8, 2	I_8, III^*	2 : 1
E1	0	1	1	6	-1	0	1	-	1, 3	1, 0	1, 2	I_1, III	5 : 2
E2	0	1	1	-844	-9725	0	1	-	5, 3	5, 0	5, 2	I_5, III	5 : 1

869

$N = 869 = 11 \cdot 79$ (4 isogeny classes)

869

A1	1	0	1	-138	609	1	1	+	2, 1	2, 1	2, 1	I_2, I_1	
B1	0	1	1	10	-2	1	1	-	1, 2	1, 2	1, 2	I_1, I_2	
C1	1	0	0	-2	-5	0	2	-	2, 1	2, 1	2, 1	I_2, I_1	2 : 2
C2	1	0	0	-57	-170	0	2	+	1, 2	1, 2	1, 2	I_1, I_2	2 : 1
D1	1	1	0	-512	4237	1	1	+	2, 3	2, 3	2, 3	I_2, I_3	

870

$N = 870 = 2 \cdot 3 \cdot 5 \cdot 29$ (9 isogeny classes)

870

A1	1	1	0	-87	261	1	2	+	4, 4, 3, 1	4, 4, 3, 1	2, 2, 3, 1	I_4, I_4, I_3, I_1	2 : 2
A2	1	1	0	-267	-1431	1	4	+	2, 2, 6, 2	2, 2, 6, 2	2, 2, 6, 2	I_2, I_2, I_6, I_2	2 : 1, 3, 4
A3	1	1	0	-4017	-99681	1	2	+	1, 1, 3, 4	1, 1, 3, 4	1, 1, 3, 4	I_1, I_1, I_3, I_4	2 : 2
A4	1	1	0	603	-7869	1	2	-	1, 1, 12, 1	1, 1, 12, 1	1, 1, 12, 1	I_1, I_1, I_{12}, I_1	2 : 2
B1	1	0	1	-2829	55816	1	6	+	10, 6, 1, 3	10, 6, 1, 3	2, 6, 1, 3	I_{10}, I_6, I_1, I_3	2 : 2; 3 : 3
B2	1	0	1	-7149	-156728	1	6	+	5, 3, 2, 6	5, 3, 2, 6	1, 3, 2, 6	I_5, I_3, I_2, I_6	2 : 1; 3 : 4
B3	1	0	1	-32844	-2275958	1	2	+	30, 2, 3, 1	30, 2, 3, 1	2, 2, 1, 1	I_{30}, I_2, I_3, I_1	2 : 4; 3 : 1
B4	1	0	1	-524364	-146193014	1	2	+	15, 1, 6, 2	15, 1, 6, 2	1, 1, 2, 2	I_{15}, I_1, I_6, I_2	2 : 3; 3 : 2
C1	1	0	1	-58	56	1	6	+	2, 6, 3, 1	2, 6, 3, 1	2, 6, 3, 1	I_2, I_6, I_3, I_1	2 : 2; 3 : 3
C2	1	0	1	212	488	1	6	-	1, 3, 6, 2	1, 3, 6, 2	1, 3, 6, 2	I_1, I_3, I_6, I_2	2 : 1; 3 : 4
C3	1	0	1	-2533	-49264	1	2	+	6, 2, 1, 3	6, 2, 1, 3	2, 2, 1, 1	I_6, I_2, I_1, I_3	2 : 4; 3 : 1
C4	1	0	1	-2413	-54112	1	2	-	3, 1, 2, 6	3, 1, 2, 6	1, 1, 2, 2	I_3, I_1, I_2, I_6	2 : 3; 3 : 2
D1	1	0	1	-113	-124	0	2	+	16, 2, 1, 1	16, 2, 1, 1	2, 2, 1, 1	I_{16}, I_2, I_1, I_1	2 : 2
D2	1	0	1	-1393	-20092	0	4	+	8, 4, 2, 2	8, 4, 2, 2	2, 4, 2, 2	I_8, I_4, I_2, I_2	2 : 1, 3, 4
D3	1	0	1	-22273	-1281244	0	2	+	4, 2, 4, 1	4, 2, 4, 1	2, 2, 4, 1	I_4, I_2, I_4, I_1	2 : 2
D4	1	0	1	-993	-31772	0	4	-	4, 8, 1, 4	4, 8, 1, 4	2, 8, 1, 4	I_4, I_8, I_1, I_4	2 : 2
E1	1	1	1	-11	-7	1	2	+	6, 2, 1, 1	6, 2, 1, 1	6, 2, 1, 1	I_6, I_2, I_1, I_1	2 : 2
E2	1	1	1	-131	-631	1	2	+	3, 1, 2, 2	3, 1, 2, 2	3, 1, 2, 2	I_3, I_1, I_2, I_2	2 : 1
F1	1	1	1	-1760	27137	1	2	+	14, 2, 5, 1	14, 2, 5, 1	14, 2, 5, 1	I_{14}, I_2, I_5, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
870 870 $N = 870 = 2 \cdot 3 \cdot 5 \cdot 29$ (continued)													
G1	1	1	1	-250	1415	0	4	+	8, 4, 1, 1	8, 4, 1, 1	8, 2, 1, 1	I_8, I_4, I_1, I_1	2 : 2
G2	1	1	1	-330	327	0	4	+	4, 8, 2, 2	4, 8, 2, 2	4, 2, 2, 2	I_4, I_8, I_2, I_2	2 : 1, 3, 4
G3	1	1	1	-3230	-71593	0	2	+	2, 16, 1, 1	2, 16, 1, 1	2, 2, 1, 1	I_2, I_{16}, I_1, I_1	2 : 2
G4	1	1	1	1290	4215	0	4	-	2, 4, 4, 4	2, 4, 4, 4	2, 2, 4, 4	I_2, I_4, I_4, I_4	2 : 2
H1	1	0	0	-5	-3	0	2	+	2, 2, 1, 1	2, 2, 1, 1	2, 2, 1, 1	I_2, I_2, I_1, I_1	2 : 2
H2	1	0	0	-35	75	0	2	+	1, 1, 2, 2	1, 1, 2, 2	1, 1, 2, 2	I_1, I_1, I_2, I_2	2 : 1
I1	1	0	0	-4480	-25600	0	10	+	10, 10, 5, 1	10, 10, 5, 1	10, 10, 5, 1	I_{10}, I_{10}, I_5, I_1	2 : 2; 5 : 3
I2	1	0	0	-43360	3450272	0	10	+	5, 5, 10, 2	5, 5, 10, 2	5, 5, 10, 2	I_5, I_5, I_{10}, I_2	2 : 1; 5 : 4
I3	1	0	0	-2136580	-1202240020	0	2	+	2, 2, 1, 5	2, 2, 1, 5	2, 2, 1, 1	I_2, I_2, I_1, I_5	2 : 4; 5 : 1
I4	1	0	0	-2136610	-1202204578	0	2	+	1, 1, 2, 10	1, 1, 2, 10	1, 1, 2, 2	I_1, I_1, I_2, I_{10}	2 : 3; 5 : 2
871 871 $N = 871 = 13 \cdot 67$ (1 isogeny class)													
A1	0	-1	1	-42	139	0	1	-	4, 1	4, 1	2, 1	I_4, I_1	
872 872 $N = 872 = 2^3 \cdot 109$ (1 isogeny class)													
A1	0	1	0	0	16	1	1	-	10, 1	0, 1	2, 1	III^*, I_1	
873 873 $N = 873 = 3^2 \cdot 97$ (4 isogeny classes)													
A1	1	-1	0	-27	-32	0	2	+	8, 1	2, 1	4, 1	I_2^*, I_1	2 : 2
A2	1	-1	0	-162	805	0	2	+	7, 2	1, 2	2, 2	I_1^*, I_2	2 : 1
B1	1	-1	0	-1476	-21461	1	2	+	10, 1	4, 1	4, 1	I_4^*, I_1	2 : 2
B2	1	-1	0	-1521	-20048	1	4	+	14, 2	8, 2	4, 2	I_8^*, I_2	2 : 1, 3, 4
B3	1	-1	0	-5886	153679	1	2	+	22, 1	16, 1	4, 1	I_{16}^*, I_1	2 : 2
B4	1	-1	0	2124	-103883	1	2	-	10, 4	4, 4	2, 4	I_4^*, I_4	2 : 2
C1	0	0	1	-19569	-4064513	1	1	-	29, 1	23, 1	4, 1	I_{23}^*, I_1	
D1	0	0	1	-3	22	1	1	-	7, 1	1, 1	4, 1	I_1^*, I_1	
874 874 $N = 874 = 2 \cdot 19 \cdot 23$ (6 isogeny classes)													
A1	1	-1	0	-19	-13	0	1	+	1, 3, 1	1, 3, 1	1, 1, 1	I_1, I_3, I_1	
B1	1	-1	0	-13189	575701	0	1	+	25, 3, 1	25, 3, 1	1, 1, 1	I_{25}, I_3, I_1	
C1	1	1	0	-38	76	1	1	+	3, 1, 1	3, 1, 1	1, 1, 1	I_3, I_1, I_1	
D1	1	0	0	-12	-16	1	1	+	5, 1, 1	5, 1, 1	5, 1, 1	I_5, I_1, I_1	
E1	1	1	1	-410	903	1	5	+	5, 1, 5	5, 1, 5	5, 1, 5	I_5, I_1, I_5	5 : 2
E2	1	1	1	-142320	-20724857	1	1	+	1, 5, 1	1, 5, 1	1, 1, 1	I_1, I_5, I_1	5 : 1
F1	1	0	0	-7929	-270343	0	3	+	21, 1, 3	21, 1, 3	21, 1, 3	I_{21}, I_1, I_3	3 : 2
F2	1	0	0	-640889	-197533063	0	1	+	7, 3, 1	7, 3, 1	7, 3, 1	I_7, I_3, I_1	3 : 1
876 876 $N = 876 = 2^2 \cdot 3 \cdot 73$ (2 isogeny classes)													
A1	0	-1	0	-48885	4176513	1	1	-	8, 11, 1	0, 11, 1	1, 1, 1	IV^*, I_{11}, I_1	
B1	0	1	0	-61	191	1	1	-	8, 5, 1	0, 5, 1	3, 5, 1	IV^*, I_5, I_1	
880 880 $N = 880 = 2^4 \cdot 5 \cdot 11$ (10 isogeny classes)													
A1	0	0	0	2	3	1	2	-	4, 2, 1	0, 2, 1	1, 2, 1	II, I_2, I_1	2 : 2
A2	0	0	0	-23	38	1	2	+	8, 1, 2	0, 1, 2	2, 1, 2	I_0^*, I_1, I_2	2 : 1
B1	0	0	0	-38	87	0	2	+	4, 3, 2	0, 3, 2	1, 1, 2	II, I_2, I_2	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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880

$N = 880 = 2^4 \cdot 5 \cdot 11$ (continued)

880

C1	0	0	0	-5042	-137801	1	2	+	4, 3, 2	0, 3, 2	1, 3, 2	II, I ₃ , I ₂	2 : 2
C2	0	0	0	-5047	-137514	1	4	+	8, 6, 4	0, 6, 4	2, 6, 4	I ₀ [*] , I ₆ , I ₄	2 : 1, 3, 4
C3	0	0	0	-7547	12986	1	4	+	10, 3, 8	0, 3, 8	4, 3, 8	I ₂ [*] , I ₃ , I ₈	2 : 2
C4	0	0	0	-2627	-269646	1	4	-	10, 12, 2	0, 12, 2	2, 12, 2	I ₂ [*] , I ₁₂ , I ₂	2 : 2
D1	0	0	0	-67	226	1	1	-	11, 3, 1	0, 3, 1	4, 3, 1	I ₃ [*] , I ₃ , I ₁	
E1	0	-1	0	-1416	-20240	0	1	-	19, 1, 3	7, 1, 3	2, 1, 1	I ₁₁ [*] , I ₁ , I ₃	3 : 2
E2	0	-1	0	4744	-108944	0	1	-	33, 3, 1	21, 3, 1	2, 1, 1	I ₂₅ [*] , I ₃ , I ₁	3 : 1
F1	0	-1	0	-16	-64	1	1	-	15, 1, 1	3, 1, 1	4, 1, 1	I ₇ [*] , I ₁ , I ₁	3 : 2
F2	0	-1	0	144	1600	1	1	-	13, 3, 3	1, 3, 3	4, 1, 3	I ₅ [*] , I ₃ , I ₃	3 : 1
G1	0	1	0	160	3188	1	1	-	17, 5, 1	5, 5, 1	4, 5, 1	I ₉ [*] , I ₅ , I ₁	5 : 2
G2	0	1	0	-95040	11245748	1	1	-	13, 1, 5	1, 1, 5	4, 1, 1	I ₅ [*] , I ₁ , I ₅	5 : 1
H1	0	1	0	-5	-2	1	2	+	4, 1, 2	0, 1, 2	1, 1, 2	II, I ₁ , I ₂	2 : 2
H2	0	1	0	-60	-200	1	2	+	8, 2, 1	0, 2, 1	1, 2, 1	I ₀ [*] , I ₂ , I ₁	2 : 1
I1	0	0	0	13	-14	0	2	-	12, 1, 1	0, 1, 1	4, 1, 1	I ₄ [*] , I ₁ , I ₁	2 : 2
I2	0	0	0	-67	-126	0	4	+	12, 2, 2	0, 2, 2	4, 2, 2	I ₄ [*] , I ₂ , I ₂	2 : 1, 3, 4
I3	0	0	0	-947	-11214	0	2	+	12, 4, 1	0, 4, 1	2, 4, 1	I ₄ [*] , I ₄ , I ₁	2 : 2
I4	0	0	0	-467	3794	0	4	+	12, 1, 4	0, 1, 4	4, 1, 4	I ₄ [*] , I ₁ , I ₄	2 : 2
J1	0	-1	0	-45	-100	0	2	+	4, 3, 2	0, 3, 2	1, 3, 2	II, I ₃ , I ₂	2 : 2; 3 : 3
J2	0	-1	0	-100	252	0	2	+	8, 6, 1	0, 6, 1	1, 6, 1	I ₀ [*] , I ₆ , I ₁	2 : 1; 3 : 4
J3	0	-1	0	-445	3720	0	2	+	4, 1, 6	0, 1, 6	1, 1, 6	II, I ₁ , I ₆	2 : 4; 3 : 1
J4	0	-1	0	-7100	232652	0	2	+	8, 2, 3	0, 2, 3	1, 2, 3	I ₀ [*] , I ₂ , I ₃	2 : 3; 3 : 2

882

$N = 882 = 2 \cdot 3^2 \cdot 7^2$ (12 isogeny classes)

882

A1	1	-1	0	-4566	119916	1	3	-	3, 3, 8	3, 0, 0	1, 2, 3	I ₃ , III, IV [*]	3 : 2
A2	1	-1	0	579	366533	1	1	-	9, 9, 8	9, 0, 0	1, 2, 3	I ₉ , III [*] , IV [*]	3 : 1
B1	1	-1	0	-93	-323	0	1	-	3, 3, 2	3, 0, 0	1, 2, 1	I ₃ , III, II	3 : 2
B2	1	-1	0	12	-1072	0	1	-	9, 9, 2	9, 0, 0	1, 2, 1	I ₉ , III [*] , II	3 : 1
C1	1	-1	0	-450	-8366	0	1	-	1, 7, 8	1, 1, 0	1, 2, 1	I ₁ , I ₁ [*] , IV [*]	7 : 2
C2	1	-1	0	-62190	6208852	0	1	-	7, 13, 8	7, 7, 0	1, 2, 1	I ₇ , I ₇ [*] , IV [*]	7 : 1
D1	1	-1	0	-9	27	1	1	-	1, 7, 2	1, 1, 0	1, 4, 1	I ₁ , I ₁ [*] , II	7 : 2
D2	1	-1	0	-1269	-17739	1	1	-	7, 13, 2	7, 7, 0	1, 4, 1	I ₇ , I ₇ [*] , II	7 : 1
E1	1	-1	0	-1773	63909	1	2	-	8, 8, 7	8, 2, 1	2, 2, 4	I ₈ , I ₂ [*] , I ₁ [*]	2 : 2
E2	1	-1	0	-37053	2752245	1	4	+	4, 10, 8	4, 4, 2	2, 4, 4	I ₄ , I ₄ [*] , I ₂ [*]	2 : 1, 3, 4
E3	1	-1	0	-45873	1349865	1	4	+	2, 14, 10	2, 8, 4	2, 4, 4	I ₂ , I ₈ [*] , I ₄ [*]	2 : 2, 5, 6
E4	1	-1	0	-592713	175784769	1	2	+	2, 8, 7	2, 2, 1	2, 4, 2	I ₂ , I ₂ [*] , I ₁ [*]	2 : 2
E5	1	-1	0	-403083	-97454421	1	2	+	1, 10, 14	1, 4, 8	1, 2, 4	I ₁ , I ₄ [*] , I ₈ [*]	2 : 3
E6	1	-1	0	170217	10295991	1	2	-	1, 22, 8	1, 16, 2	1, 4, 2	I ₁ , I ₁₆ [*] , I ₂ [*]	2 : 3
F1	1	-1	1	64	-13597	0	3	-	9, 3, 8	9, 0, 0	9, 2, 3	I ₉ , III, IV [*]	3 : 2
F2	1	-1	1	-41096	-3196637	0	1	-	3, 9, 8	3, 0, 0	3, 2, 3	I ₃ , III [*] , IV [*]	3 : 1
G1	1	-1	1	1	39	1	1	-	9, 3, 2	9, 0, 0	9, 2, 1	I ₉ , III, II	3 : 2
G2	1	-1	1	-839	9559	1	1	-	3, 9, 2	3, 0, 0	3, 2, 1	I ₃ , III [*] , II	3 : 1
H1	1	-1	1	211	1397	1	1	-	5, 9, 4	5, 3, 0	5, 4, 3	I ₅ , I ₉ [*] , IV [*]	3 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
882 882													
$N = 882 = 2 \cdot 3^2 \cdot 7^2$ (continued)													
I1	1	-1	1	-230	2769	0	2	-	2, 6, 7	2, 0, 1	2, 2, 2	I_2, I_0^*, I_1^*	2 : 2; 3 : 3
I2	1	-1	1	-4640	122721	0	2	+	1, 6, 8	1, 0, 2	1, 2, 4	I_1, I_0^*, I_2^*	2 : 1; 3 : 4
I3	1	-1	1	1975	-57207	0	2	-	6, 6, 9	6, 0, 3	6, 2, 2	I_6, I_0^*, I_3^*	2 : 4; 3 : 1, 5
I4	1	-1	1	-15665	-614631	0	2	+	3, 6, 12	3, 0, 6	3, 2, 4	I_3, I_0^*, I_6^*	2 : 3; 3 : 2, 6
I5	1	-1	1	-75200	-7941405	0	2	-	18, 6, 7	18, 0, 1	18, 2, 2	I_{18}, I_0^*, I_1^*	2 : 6; 3 : 3
I6	1	-1	1	-1204160	-508296477	0	2	+	9, 6, 8	9, 0, 2	9, 2, 4	I_9, I_0^*, I_2^*	2 : 5; 3 : 4
J1	1	-1	1	10354	-499971	0	1	-	5, 9, 10	5, 3, 0	5, 2, 1	I_5, I_3^*, II^*	3 : 2
J2	1	-1	1	-313781	-67920051	0	1	-	15, 7, 10	15, 1, 0	15, 2, 1	I_{15}, I_1^*, II^*	3 : 1
K1	1	-1	1	22	-871	0	2	-	4, 10, 3	4, 4, 0	4, 2, 2	I_4, I_4^*, III	2 : 2
K2	1	-1	1	-1238	-15991	0	2	+	2, 14, 3	2, 8, 0	2, 4, 2	I_2, I_8^*, III	2 : 1
L1	1	-1	1	1093	296475	0	2	-	4, 10, 9	4, 4, 0	4, 2, 2	I_4, I_4^*, III^*	2 : 2
L2	1	-1	1	-60647	5606115	0	2	+	2, 14, 9	2, 8, 0	2, 4, 2	I_2, I_8^*, III^*	2 : 1
885 885													
$N = 885 = 3 \cdot 5 \cdot 59$ (4 isogeny classes)													
A1	0	-1	1	-126	587	0	1	+	7, 1, 1	7, 1, 1	1, 1, 1	I_7, I_1, I_1	
B1	1	1	0	-92	-381	1	2	+	1, 2, 1	1, 2, 1	1, 2, 1	I_1, I_2, I_1	2 : 2
B2	1	1	0	-97	-344	1	4	+	2, 4, 2	2, 4, 2	2, 4, 2	I_2, I_4, I_2	2 : 1, 3, 4
B3	1	1	0	-472	3481	1	4	+	1, 2, 4	1, 2, 4	1, 2, 4	I_1, I_2, I_4	2 : 2
B4	1	1	0	198	-1701	1	2	-	4, 8, 1	4, 8, 1	2, 8, 1	I_4, I_8, I_1	2 : 2
C1	0	1	1	-5	-4	1	1	+	3, 1, 1	3, 1, 1	3, 1, 1	I_3, I_1, I_1	
D1	0	1	1	-280	1684	1	5	+	5, 5, 1	5, 5, 1	5, 5, 1	I_5, I_5, I_1	5 : 2
D2	0	1	1	-19330	-1040876	1	1	+	1, 1, 5	1, 1, 5	1, 1, 1	I_1, I_1, I_5	5 : 1
886 886													
$N = 886 = 2 \cdot 443$ (5 isogeny classes)													
A1	1	-1	0	-14	24	1	1	+	2, 1	2, 1	2, 1	I_2, I_1	
B1	1	0	1	-1203	15950	1	1	-	9, 1	9, 1	1, 1	I_9, I_1	
C1	1	1	0	-283	-1635	0	1	+	20, 1	20, 1	2, 1	I_{20}, I_1	
D1	1	-1	1	-241390	45705725	1	1	+	38, 1	38, 1	38, 1	I_{38}, I_1	
E1	1	-1	1	-4	7	1	1	-	5, 1	5, 1	5, 1	I_5, I_1	
888 888													
$N = 888 = 2^3 \cdot 3 \cdot 37$ (4 isogeny classes)													
A1	0	-1	0	-200	-1044	0	1	-	11, 5, 1	0, 5, 1	1, 1, 1	II^*, I_5, I_1	
B1	0	1	0	-39	-18	1	4	+	4, 8, 1	0, 8, 1	2, 8, 1	III, I_8, I_1	2 : 2
B2	0	1	0	-444	-3744	1	4	+	8, 4, 2	0, 4, 2	2, 4, 2	I_1^*, I_4, I_2	2 : 1, 3, 4
B3	0	1	0	-7104	-232848	1	2	+	10, 2, 1	0, 2, 1	2, 2, 1	III^*, I_2, I_1	2 : 2
B4	0	1	0	-264	-6624	1	4	-	10, 2, 4	0, 2, 4	2, 2, 4	III^*, I_2, I_4	2 : 2
C1	0	-1	0	-3	-36	1	2	-	4, 3, 2	0, 3, 2	2, 1, 2	III, I_3, I_2	2 : 2
C2	0	-1	0	-188	-924	1	2	+	8, 6, 1	0, 6, 1	4, 2, 1	I_1^*, I_6, I_1	2 : 1
D1	0	1	0	-11	-18	0	2	+	4, 2, 1	0, 2, 1	2, 2, 1	III, I_2, I_1	2 : 2
D2	0	1	0	4	-48	0	2	-	8, 1, 2	0, 1, 2	4, 1, 2	I_1^*, I_1, I_2	2 : 1
890 890													
$N = 890 = 2 \cdot 5 \cdot 89$ (8 isogeny classes)													
A1	1	-1	0	-5	1	1	2	+	2, 2, 1	2, 2, 1	2, 2, 1	I_2, I_2, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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890 $N = 890 = 2 \cdot 5 \cdot 89$ (continued) **890**

B1	1	0	1	-9	-4	1	2	+	4, 2, 1	4, 2, 1	2, 2, 1	I_4, I_2, I_1	2 : 2
B2	1	0	1	-109	-444	1	2	+	2, 1, 2	2, 1, 2	2, 1, 2	I_2, I_1, I_2	2 : 1
C1	1	1	0	-418	3072	0	2	+	2, 8, 1	2, 8, 1	2, 2, 1	I_2, I_8, I_1	2 : 2
C2	1	1	0	-6668	206822	0	2	+	1, 4, 2	1, 4, 2	1, 2, 2	I_1, I_4, I_2	2 : 1
D1	1	0	1	-13	16	1	1	-	3, 1, 1	3, 1, 1	1, 1, 1	I_3, I_1, I_1	
E1	1	0	1	-1138	-14844	1	2	+	12, 4, 1	12, 4, 1	2, 4, 1	I_{12}, I_4, I_1	2 : 2
E2	1	0	1	-818	-23292	1	2	-	6, 8, 2	6, 8, 2	2, 8, 2	I_6, I_8, I_2	2 : 1
F1	1	-1	1	12	87	1	1	-	13, 1, 1	13, 1, 1	13, 1, 1	I_{13}, I_1, I_1	
G1	1	1	1	10	147	1	5	-	5, 5, 1	5, 5, 1	5, 5, 1	I_5, I_5, I_1	5 : 2
G2	1	1	1	-2040	-38093	1	1	-	1, 1, 5	1, 1, 5	1, 1, 1	I_1, I_1, I_5	5 : 1
H1	1	-1	1	-52	151	0	4	+	8, 2, 1	8, 2, 1	8, 2, 1	I_8, I_2, I_1	2 : 2
H2	1	-1	1	-132	-361	0	4	+	4, 4, 2	4, 4, 2	4, 4, 2	I_4, I_4, I_2	2 : 1, 3, 4
H3	1	-1	1	-1912	-31689	0	2	+	2, 8, 1	2, 8, 1	2, 8, 1	I_2, I_8, I_1	2 : 2
H4	1	-1	1	368	-2761	0	4	-	2, 2, 4	2, 2, 4	2, 2, 4	I_2, I_2, I_4	2 : 2

891 $N = 891 = 3^4 \cdot 11$ (8 isogeny classes) **891**

A1	1	-1	1	7	10	1	1	-	6, 2	0, 2	1, 2	IV, I_2	
B1	0	0	1	6	-15	0	3	-	4, 3	0, 3	1, 3	II, I_3	3 : 2
B2	0	0	1	-324	-2248	0	1	-	12, 1	0, 1	1, 1	II^*, I_1	3 : 1
C1	1	-1	0	66	-343	0	1	-	12, 2	0, 2	1, 2	II^*, I_2	
D1	1	-1	0	-339	2492	0	1	-	12, 1	0, 1	1, 1	II^*, I_1	7 : 2
D2	1	-1	0	876	-154729	0	1	-	12, 7	0, 7	1, 7	II^*, I_7	7 : 1
E1	0	0	1	-81	-304	0	1	-	12, 1	0, 1	1, 1	II^*, I_1	
F1	0	0	1	-36	83	0	3	-	6, 1	0, 1	3, 1	IV, I_1	3 : 2
F2	0	0	1	54	398	0	1	-	10, 3	0, 3	1, 1	IV^*, I_3	3 : 1
G1	1	-1	1	-38	-80	0	1	-	6, 1	0, 1	1, 1	IV, I_1	7 : 2
G2	1	-1	1	97	5698	0	1	-	6, 7	0, 7	1, 1	IV, I_7	7 : 1
H1	0	0	1	-9	11	0	1	-	6, 1	0, 1	1, 1	IV, I_1	

892 $N = 892 = 2^2 \cdot 223$ (3 isogeny classes) **892**

A1	0	0	0	-415	3254	0	1	+	8, 1	0, 1	1, 1	IV^*, I_1	
B1	0	1	0	-188	932	1	3	+	8, 1	0, 1	3, 1	IV^*, I_1	3 : 2
B2	0	1	0	-388	-1580	1	1	+	8, 3	0, 3	1, 3	IV^*, I_3	3 : 1
C1	0	-1	0	-12	-8	1	1	+	8, 1	0, 1	3, 1	IV^*, I_1	

894 $N = 894 = 2 \cdot 3 \cdot 149$ (7 isogeny classes) **894**

A1	1	1	0	-18630	971028	1	1	-	13, 8, 1	13, 8, 1	1, 2, 1	I_{13}, I_8, I_1	
B1	1	1	0	-59	-201	1	1	+	1, 5, 1	1, 5, 1	1, 1, 1	I_1, I_5, I_1	
C1	1	0	1	-407	-268	0	3	+	1, 15, 1	1, 15, 1	1, 15, 1	I_1, I_{15}, I_1	3 : 2
C2	1	0	1	-23492	-1387798	0	1	+	3, 5, 3	3, 5, 3	1, 5, 1	I_3, I_5, I_3	3 : 1
D1	1	0	1	-13	-16	1	1	+	3, 3, 1	3, 3, 1	1, 3, 1	I_3, I_3, I_1	
E1	1	1	1	-38	-15325	1	1	-	23, 4, 1	23, 4, 1	23, 2, 1	I_{23}, I_4, I_1	
F1	1	1	1	-42	87	1	1	+	5, 1, 1	5, 1, 1	5, 1, 1	I_5, I_1, I_1	

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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895 $N = 895 = 5 \cdot 179$ (2 isogeny classes)**895**

A1	1	0	0	-6	5	1	1	+	1, 1	1, 1	1, 1	I_1, I_1	
B1	1	-1	1	-183	352	0	1	+	9, 1	9, 1	1, 1	I_9, I_1	

896 $N = 896 = 2^7 \cdot 7$ (4 isogeny classes)**896**

A1	0	0	0	-10	-12	1	2	+	8, 1	0, 1	2, 1	III, I_1	2 : 2
A2	0	0	0	-20	16	1	2	+	13, 2	0, 2	2, 2	I_2^*, I_2	2 : 1
B1	0	0	0	-5	2	1	2	+	7, 2	0, 2	1, 2	II, I_2	2 : 2
B2	0	0	0	-40	-96	1	2	+	14, 1	0, 1	2, 1	III^*, I_1	2 : 1
C1	0	0	0	-5	-2	0	2	+	7, 2	0, 2	1, 2	II, I_2	2 : 2
C2	0	0	0	-40	96	0	2	+	14, 1	0, 1	2, 1	III^*, I_1	2 : 1
D1	0	0	0	-10	12	1	2	+	8, 1	0, 1	2, 1	III, I_1	2 : 2
D2	0	0	0	-20	-16	1	2	+	13, 2	0, 2	4, 2	I_2^*, I_2	2 : 1

897 $N = 897 = 3 \cdot 13 \cdot 23$ (6 isogeny classes)**897**

A1	1	1	0	-97	5560	0	2	-	8, 2, 3	8, 2, 3	2, 2, 1	I_8, I_2, I_3	2 : 2
A2	1	1	0	-5362	147715	0	2	+	4, 1, 6	4, 1, 6	2, 1, 2	I_4, I_1, I_6	2 : 1
B1	1	1	1	-52	164	0	4	-	2, 4, 1	2, 4, 1	2, 4, 1	I_2, I_4, I_1	2 : 2
B2	1	1	1	-897	9966	0	4	+	4, 2, 2	4, 2, 2	2, 2, 2	I_4, I_2, I_2	2 : 1, 3, 4
B3	1	1	1	-962	8354	0	2	+	8, 1, 4	8, 1, 4	2, 1, 2	I_8, I_1, I_4	2 : 2
B4	1	1	1	-14352	655806	0	2	+	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	2 : 2
C1	1	1	1	-19	-40	1	2	+	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	2 : 2
C2	1	1	1	-24	-24	1	4	+	2, 2, 2	2, 2, 2	2, 2, 2	I_2, I_2, I_2	2 : 1, 3, 4
C3	1	1	1	-219	1146	1	4	+	1, 1, 4	1, 1, 4	1, 1, 4	I_1, I_1, I_4	2 : 2
C4	1	1	1	91	-70	1	2	-	4, 4, 1	4, 4, 1	2, 4, 1	I_4, I_4, I_1	2 : 2
D1	1	0	1	130884	-59725523	1	2	-	12, 10, 1	12, 10, 1	12, 10, 1	I_{12}, I_{10}, I_1	2 : 2
D2	1	0	1	-1725581	-795628249	1	2	+	24, 5, 2	24, 5, 2	24, 5, 2	I_{24}, I_5, I_2	2 : 1
E1	1	0	0	-19602	1069443	1	4	-	20, 2, 1	20, 2, 1	20, 2, 1	I_{20}, I_2, I_1	2 : 2
E2	1	0	0	-314847	67971960	1	4	+	10, 4, 2	10, 4, 2	10, 4, 2	I_{10}, I_4, I_2	2 : 1, 3, 4
E3	1	0	0	-316062	67420593	1	2	+	5, 8, 4	5, 8, 4	5, 8, 2	I_5, I_8, I_4	2 : 2
E4	1	0	0	-5037552	4351465395	1	2	+	5, 2, 1	5, 2, 1	5, 2, 1	I_5, I_2, I_1	2 : 2
F1	1	0	0	0	-9	1	2	-	2, 2, 1	2, 2, 1	2, 2, 1	I_2, I_2, I_1	2 : 2
F2	1	0	0	-65	-204	1	2	+	4, 1, 2	4, 1, 2	4, 1, 2	I_4, I_1, I_2	2 : 1

898 $N = 898 = 2 \cdot 449$ (4 isogeny classes)**898**

A1	1	0	1	-202	1084	1	1	+	7, 1	7, 1	1, 1	I_7, I_1	
B1	1	1	0	-451	3789	0	1	-	21, 1	21, 1	1, 1	I_{21}, I_1	
C1	1	1	1	-12	-19	0	2	+	6, 1	6, 1	6, 1	I_6, I_1	2 : 2
C2	1	1	1	-52	109	0	2	+	3, 2	3, 2	3, 2	I_3, I_2	2 : 1
D1	1	1	1	-4	-3	1	1	+	3, 1	3, 1	3, 1	I_3, I_1	

899 $N = 899 = 29 \cdot 31$ (2 isogeny classes)**899**

A1	1	0	1	-3	-1	1	1	+	1, 1	1, 1	1, 1	I_1, I_1	
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	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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900

$N = 900 = 2^2 \cdot 3^2 \cdot 5^2$ (8 isogeny classes)

900

A1	0	0	0	0	12500	0	1	-	8, 3, 10	0, 0, 0	1, 2, 1	IV*, III, II*	3 : 2
A2	0	0	0	0	-337500	0	1	-	8, 9, 10	0, 0, 0	3, 2, 1	IV*, III*, II*	3 : 1
B1	0	0	0	0	125	0	2	-	4, 3, 6	0, 0, 0	1, 2, 2	IV, III, I ₀ *	2 : 2; 3 : 3
B2	0	0	0	-375	2750	0	2	+	8, 3, 6	0, 0, 0	1, 2, 2	IV*, III, I ₀ *	2 : 1; 3 : 4
B3	0	0	0	0	-3375	0	2	-	4, 9, 6	0, 0, 0	3, 2, 2	IV, III*, I ₀ *	2 : 4; 3 : 1
B4	0	0	0	-3375	-74250	0	2	+	8, 9, 6	0, 0, 0	3, 2, 2	IV*, III*, I ₀ *	2 : 3; 3 : 2
C1	0	0	0	0	100	1	3	-	8, 3, 4	0, 0, 0	3, 2, 3	IV*, III, IV	3 : 2
C2	0	0	0	0	-2700	1	1	-	8, 9, 4	0, 0, 0	1, 2, 1	IV*, III*, IV	3 : 1
D1	0	0	0	-120	740	1	1	-	8, 9, 2	0, 3, 0	3, 4, 1	IV*, I ₃ *, II	3 : 2
D2	0	0	0	-10920	439220	1	1	-	8, 7, 2	0, 1, 0	1, 4, 1	IV*, I ₁ *, II	3 : 1
E1	0	0	0	-300	-1375	1	2	+	4, 6, 7	0, 0, 1	3, 2, 4	IV, I ₀ *, I ₁ *	2 : 2; 3 : 3
E2	0	0	0	825	-9250	1	2	-	8, 6, 8	0, 0, 2	3, 2, 4	IV*, I ₀ *, I ₂ *	2 : 1; 3 : 4
E3	0	0	0	-9300	345125	1	2	+	4, 6, 9	0, 0, 3	1, 2, 4	IV, I ₀ *, I ₃ *	2 : 4; 3 : 1
E4	0	0	0	-8175	431750	1	2	-	8, 6, 12	0, 0, 6	1, 2, 4	IV*, I ₀ *, I ₆ *	2 : 3; 3 : 2
F1	0	0	0	-3000	92500	0	1	-	8, 9, 8	0, 3, 0	1, 2, 1	IV*, I ₃ *, IV*	3 : 2
F2	0	0	0	-273000	54902500	0	3	-	8, 7, 8	0, 1, 0	3, 2, 3	IV*, I ₁ *, IV*	3 : 1
G1	0	0	0	-3000	-59375	0	2	+	4, 8, 9	0, 2, 0	3, 2, 2	IV, I ₂ *, III*	2 : 2
G2	0	0	0	2625	-256250	0	2	-	8, 10, 9	0, 4, 0	3, 4, 2	IV*, I ₄ *, III*	2 : 1
H1	0	0	0	-120	-475	0	2	+	4, 8, 3	0, 2, 0	1, 2, 2	IV, I ₂ *, III	2 : 2
H2	0	0	0	105	-2050	0	2	-	8, 10, 3	0, 4, 0	1, 4, 2	IV*, I ₄ *, III	2 : 1

901

$N = 901 = 17 \cdot 53$ (6 isogeny classes)

901

A1	1	-1	1	-85	-220	1	2	+	3, 2	3, 2	1, 2	I ₃ , I ₂	2 : 2
A2	1	-1	1	180	-1492	1	2	-	6, 1	6, 1	2, 1	I ₆ , I ₁	2 : 1
B1	1	1	1	-29598	1947602	1	2	+	5, 2	5, 2	1, 2	I ₅ , I ₂	2 : 2
B2	1	1	1	-29863	1910608	1	2	+	10, 1	10, 1	2, 1	I ₁₀ , I ₁	2 : 1
C1	0	1	1	-17	7	0	3	+	3, 1	3, 1	3, 1	I ₃ , I ₁	3 : 2
C2	0	1	1	-697	-7320	0	1	+	1, 3	1, 3	1, 1	I ₁ , I ₃	3 : 1
D1	1	-1	1	-346	-68922	0	1	-	3, 5	3, 5	3, 1	I ₃ , I ₅	
E1	0	0	1	-1507	4209	1	1	+	5, 3	5, 3	5, 3	I ₅ , I ₃	
F1	0	-1	1	-4	-2	1	1	+	1, 1	1, 1	1, 1	I ₁ , I ₁	

902

$N = 902 = 2 \cdot 11 \cdot 41$ (2 isogeny classes)

902

A1	1	0	1	-2382	77312	1	1	-	18, 5, 1	18, 5, 1	2, 1, 1	I ₁₈ , I ₅ , I ₁	
B1	1	0	0	-64	192	0	3	-	6, 1, 1	6, 1, 1	6, 1, 1	I ₆ , I ₁ , I ₁	3 : 2
B2	1	0	0	76	892	0	1	-	2, 3, 3	2, 3, 3	2, 1, 1	I ₂ , I ₃ , I ₃	3 : 1

903

$N = 903 = 3 \cdot 7 \cdot 43$ (2 isogeny classes)

903

A1	0	1	1	7	2	1	1	-	2, 2, 1	2, 2, 1	2, 2, 1	I ₂ , I ₂ , I ₁	
B1	0	1	1	-43	-43484	0	3	-	18, 2, 1	18, 2, 1	18, 2, 1	I ₁₈ , I ₂ , I ₁	3 : 2
B2	0	1	1	-94813	-11269355	0	3	-	6, 6, 3	6, 6, 3	6, 6, 3	I ₆ , I ₆ , I ₃	3 : 1, 3

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
904 904 $N = 904 = 2^3 \cdot 113$ (1 isogeny class)													
A1	0	0	0	-35	78	1	2	+	10, 1	0, 1	2, 1	III*, I ₁	2 : 2
A2	0	0	0	5	246	1	2	-	11, 2	0, 2	1, 2	II*, I ₂	2 : 1
905 905 $N = 905 = 5 \cdot 181$ (2 isogeny classes)													
A1	1	1	0	-18	23	1	2	+	1, 1	1, 1	1, 1	I ₁ , I ₁	2 : 2
A2	1	1	0	-13	42	1	2	-	2, 2	2, 2	2, 2	I ₂ , I ₂	2 : 1
B1	1	0	1	-388	-2969	0	1	-	5, 1	5, 1	5, 1	I ₅ , I ₁	
906 906 $N = 906 = 2 \cdot 3 \cdot 151$ (9 isogeny classes)													
A1	1	1	0	3395	-211907	1	1	-	26, 7, 1	26, 7, 1	2, 1, 1	I ₂₆ , I ₇ , I ₁	
B1	1	1	0	-16	-32	1	1	+	5, 1, 1	5, 1, 1	1, 1, 1	I ₅ , I ₁ , I ₁	
C1	1	0	1	54	64	1	3	-	2, 9, 1	2, 9, 1	2, 9, 1	I ₂ , I ₉ , I ₁	3 : 2
C2	1	0	1	-621	-7064	1	3	-	6, 3, 3	6, 3, 3	2, 3, 3	I ₆ , I ₃ , I ₃	3 : 1, 3
C3	1	0	1	-52716	-4662998	1	1	-	18, 1, 1	18, 1, 1	2, 1, 1	I ₁₈ , I ₁ , I ₁	3 : 2
D1	1	0	1	-1715	27182	1	3	+	5, 3, 1	5, 3, 1	1, 3, 1	I ₅ , I ₃ , I ₁	3 : 2
D2	1	0	1	-1970	18500	1	1	+	15, 1, 3	15, 1, 3	1, 1, 3	I ₁₅ , I ₁ , I ₃	3 : 1
E1	1	1	1	-40466325	99063769563	0	1	+	5, 7, 1	5, 7, 1	5, 1, 1	I ₅ , I ₇ , I ₁	
F1	1	1	1	-11	-19	0	1	-	2, 1, 1	2, 1, 1	2, 1, 1	I ₂ , I ₁ , I ₁	
G1	1	1	1	-21	-45	1	1	+	3, 3, 1	3, 3, 1	3, 1, 1	I ₃ , I ₃ , I ₁	
H1	1	0	0	-152	576	1	1	+	11, 5, 1	11, 5, 1	11, 5, 1	I ₁₁ , I ₅ , I ₁	
I1	1	0	0	-6	-6	0	1	+	1, 1, 1	1, 1, 1	1, 1, 1	I ₁ , I ₁ , I ₁	
909 909 $N = 909 = 3^2 \cdot 101$ (3 isogeny classes)													
A1	0	0	1	-1776	3834	0	1	+	20, 1	14, 1	2, 1	I ₁₄ *, I ₁	
B1	0	0	1	-57	-117	0	1	+	10, 1	4, 1	2, 1	I ₄ *, I ₁	
C1	0	0	1	-12	9	1	1	+	6, 1	0, 1	2, 1	I ₀ *, I ₁	
910 910 $N = 910 = 2 \cdot 5 \cdot 7 \cdot 13$ (11 isogeny classes)													
A1	1	-1	0	-2000	32000	0	2	+	20, 3, 2, 1	20, 3, 2, 1	2, 1, 2, 1	I ₂₀ , I ₃ , I ₂ , I ₁	2 : 2
A2	1	-1	0	-7120	-194304	0	4	+	10, 6, 4, 2	10, 6, 4, 2	2, 2, 4, 2	I ₁₀ , I ₆ , I ₄ , I ₂	2 : 1, 3, 4
A3	1	-1	0	-109040	-13831200	0	2	+	5, 12, 2, 1	5, 12, 2, 1	1, 2, 2, 1	I ₅ , I ₁₂ , I ₂ , I ₁	2 : 2
A4	1	-1	0	12880	-1102304	0	2	-	5, 3, 8, 4	5, 3, 8, 4	1, 1, 8, 2	I ₅ , I ₃ , I ₈ , I ₄	2 : 2
B1	1	0	1	6	42	1	3	-	1, 2, 1, 3	1, 2, 1, 3	1, 2, 1, 3	I ₁ , I ₂ , I ₁ , I ₃	3 : 2
B2	1	0	1	-59	-1154	1	1	-	3, 6, 3, 1	3, 6, 3, 1	1, 2, 3, 1	I ₃ , I ₆ , I ₃ , I ₁	3 : 1
C1	1	0	1	-234	1352	1	6	+	2, 1, 2, 3	2, 1, 2, 3	2, 1, 2, 3	I ₂ , I ₁ , I ₂ , I ₃	2 : 2; 3 : 3
C2	1	0	1	-304	456	1	6	+	1, 2, 1, 6	1, 2, 1, 6	1, 2, 1, 6	I ₁ , I ₂ , I ₁ , I ₆	2 : 1; 3 : 4
C3	1	0	1	-949	-9984	1	2	+	6, 3, 6, 1	6, 3, 6, 1	2, 1, 6, 1	I ₆ , I ₃ , I ₆ , I ₁	2 : 4; 3 : 1
C4	1	0	1	-14669	-685008	1	2	+	3, 6, 3, 2	3, 6, 3, 2	1, 2, 3, 2	I ₃ , I ₆ , I ₃ , I ₂	2 : 3; 3 : 2
D1	1	-1	0	-29	-47	1	2	+	2, 3, 2, 1	2, 3, 2, 1	2, 3, 2, 1	I ₂ , I ₃ , I ₂ , I ₁	2 : 2
D2	1	-1	0	41	-285	1	2	-	1, 6, 1, 2	1, 6, 1, 2	1, 6, 1, 2	I ₁ , I ₆ , I ₁ , I ₂	2 : 1
E1	1	0	1	-578448	183565278	0	3	-	7, 18, 3, 1	7, 18, 3, 1	1, 18, 3, 1	I ₇ , I ₁₈ , I ₃ , I ₁	3 : 2
E2	1	0	1	3562177	-168122222	0	3	-	21, 6, 9, 3	21, 6, 9, 3	1, 6, 9, 3	I ₂₁ , I ₆ , I ₉ , I ₃	3 : 1, 3
E3	1	0	1	-50503198	-146507820272	0	1	-	63, 2, 3, 1	63, 2, 3, 1	1, 2, 3, 1	I ₆₃ , I ₂ , I ₃ , I ₁	3 : 2
F1	1	-1	1	-33898	2219177	1	2	+	22, 1, 2, 5	22, 1, 2, 5	22, 1, 2, 5	I ₂₂ , I ₁ , I ₂ , I ₅	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
910 $N = 910 = 2 \cdot 5 \cdot 7 \cdot 13$ (continued) 910													
G1	1	-1	1	-33	81	1	1	-	5, 2, 1, 1	5, 2, 1, 1	5, 2, 1, 1	I_5, I_2, I_1, I_1	
H1	1	1	1	-196	5829	1	1	-	17, 2, 3, 1	17, 2, 3, 1	17, 2, 3, 1	I_{17}, I_2, I_3, I_1	
I1	1	1	1	-6	-1	0	2	+	2, 1, 2, 1	2, 1, 2, 1	2, 1, 2, 1	I_2, I_1, I_2, I_1	2 : 2
I2	1	1	1	-76	223	0	2	+	1, 2, 1, 2	1, 2, 1, 2	1, 2, 1, 2	I_1, I_2, I_1, I_2	2 : 1
J1	1	0	0	-1196	15760	0	6	+	18, 1, 2, 1	18, 1, 2, 1	18, 1, 2, 1	I_{18}, I_1, I_2, I_1	2 : 2 ; 3 : 3
J2	1	0	0	-19116	1015696	0	6	+	9, 2, 1, 2	9, 2, 1, 2	9, 2, 1, 2	I_9, I_2, I_1, I_2	2 : 1 ; 3 : 4
J3	1	0	0	-6636	-196784	0	6	+	6, 3, 6, 3	6, 3, 6, 3	6, 1, 6, 3	I_6, I_3, I_6, I_3	2 : 4 ; 3 : 1, 5
J4	1	0	0	-20356	876120	0	6	+	3, 6, 3, 6	3, 6, 3, 6	3, 2, 3, 6	I_3, I_6, I_3, I_6	2 : 3 ; 3 : 2, 6
J5	1	0	0	-528976	-148126020	0	2	+	2, 9, 2, 1	2, 9, 2, 1	2, 1, 2, 1	I_2, I_9, I_2, I_1	2 : 6 ; 3 : 3
J6	1	0	0	-529046	-148084874	0	2	+	1, 18, 1, 2	1, 18, 1, 2	1, 2, 1, 2	I_1, I_{18}, I_1, I_2	2 : 5 ; 3 : 4
K1	1	0	0	-1145	12025	1	2	+	14, 5, 2, 1	14, 5, 2, 1	14, 5, 2, 1	I_{14}, I_5, I_2, I_1	2 : 2
K2	1	0	0	-5625	-151943	1	2	+	7, 10, 1, 2	7, 10, 1, 2	7, 10, 1, 2	I_7, I_{10}, I_1, I_2	2 : 1

912 $N = 912 = 2^4 \cdot 3 \cdot 19$ (12 isogeny classes) 912													
A1	0	-1	0	-57	-171	1	1	-	8, 6, 1	0, 6, 1	1, 2, 1	I_0^*, I_6, I_1	
B1	0	-1	0	-172	928	0	2	+	8, 3, 1	0, 3, 1	2, 1, 1	I_0^*, I_3, I_1	2 : 2
B2	0	-1	0	-192	720	0	4	+	10, 6, 2	0, 6, 2	4, 2, 2	I_2^*, I_6, I_2	2 : 1, 3, 4
B3	0	-1	0	-1272	-16560	0	2	+	11, 3, 4	0, 3, 4	2, 1, 4	I_3^*, I_3, I_4	2 : 2
B4	0	-1	0	568	4368	0	2	-	11, 12, 1	0, 12, 1	4, 2, 1	I_3^*, I_{12}, I_1	2 : 2
C1	0	1	0	55	-93	0	1	-	8, 2, 3	0, 2, 3	1, 2, 1	I_0^*, I_2, I_3	
D1	0	1	0	-16	-28	0	2	+	10, 1, 1	0, 1, 1	4, 1, 1	I_2^*, I_1, I_1	2 : 2
D2	0	1	0	24	-108	0	2	-	11, 2, 2	0, 2, 2	2, 2, 2	I_3^*, I_2, I_2	2 : 1
E1	0	-1	0	-128	0	0	2	+	18, 3, 1	6, 3, 1	4, 1, 1	I_{10}^*, I_3, I_1	2 : 2 ; 3 : 3
E2	0	-1	0	512	-512	0	2	-	15, 6, 2	3, 6, 2	2, 2, 2	I_7^*, I_6, I_2	2 : 1 ; 3 : 4
E3	0	-1	0	-6848	220416	0	2	+	14, 1, 3	2, 1, 3	4, 1, 1	I_6^*, I_1, I_3	2 : 4 ; 3 : 1
E4	0	-1	0	-6688	231040	0	2	-	13, 2, 6	1, 2, 6	2, 2, 2	I_5^*, I_2, I_6	2 : 3 ; 3 : 2
F1	0	-1	0	315	2349	1	1	-	12, 10, 1	0, 10, 1	1, 2, 1	Π^*, I_{10}, I_1	5 : 2
F2	0	-1	0	-70245	7189389	1	1	-	12, 2, 5	0, 2, 5	1, 2, 5	Π^*, I_2, I_5	5 : 1
G1	0	-1	0	-24	48	1	2	+	12, 1, 1	0, 1, 1	4, 1, 1	I_4^*, I_1, I_1	2 : 2
G2	0	-1	0	-104	-336	1	4	+	12, 2, 2	0, 2, 2	4, 2, 2	I_4^*, I_2, I_2	2 : 1, 3, 4
G3	0	-1	0	-1624	-24656	1	2	+	12, 4, 1	0, 4, 1	2, 2, 1	I_4^*, I_4, I_1	2 : 2
G4	0	-1	0	136	-1872	1	4	-	12, 1, 4	0, 1, 4	4, 1, 4	I_4^*, I_1, I_4	2 : 2
H1	0	1	0	-1528	22484	1	2	+	14, 5, 1	2, 5, 1	4, 5, 1	I_6^*, I_5, I_1	2 : 2
H2	0	1	0	-1368	27540	1	2	-	13, 10, 2	1, 10, 2	4, 10, 2	I_5^*, I_{10}, I_2	2 : 1
I1	0	1	0	3	-9	1	1	-	8, 2, 1	0, 2, 1	2, 2, 1	I_0^*, I_2, I_1	
J1	0	1	0	3	-18	0	2	-	4, 3, 2	0, 3, 2	1, 3, 2	Π, I_3, I_2	2 : 2
J2	0	1	0	-92	-360	0	2	+	8, 6, 1	0, 6, 1	1, 6, 1	I_0^*, I_6, I_1	2 : 1
K1	0	1	0	-5632	144308	0	2	+	32, 3, 1	20, 3, 1	4, 3, 1	I_{24}^*, I_3, I_1	2 : 2
K2	0	1	0	-87552	9941940	0	4	+	22, 6, 2	10, 6, 2	4, 6, 2	I_{14}^*, I_6, I_2	2 : 1, 3, 4
K3	0	1	0	-1400832	637689780	0	2	+	17, 3, 1	5, 3, 1	4, 3, 1	I_9^*, I_3, I_1	2 : 2
K4	0	1	0	-84992	10553268	0	4	-	17, 12, 4	5, 12, 4	2, 12, 4	I_9^*, I_{12}, I_4	2 : 2
L1	0	1	0	-37	-109	0	1	-	12, 2, 1	0, 2, 1	1, 2, 1	Π^*, I_2, I_1	

913 $N = 913 = 11 \cdot 83$ (2 isogeny classes) 913												
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	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
913	$N = 913 = 11 \cdot 83$ (continued)											913	
B1	0	0	1	-1	13	0	1	-	1, 2	1, 2	1, 2	I_1, I_2	
914	$N = 914 = 2 \cdot 457$ (2 isogeny classes)											914	
A1	1	-1	0	-52	-48	1	2	+	14, 1	14, 1	2, 1	I_{14}, I_1	2 : 2
A2	1	-1	0	-692	-6832	1	2	+	7, 2	7, 2	1, 2	I_7, I_2	2 : 1
B1	1	0	1	-2	-2	0	1	-	1, 1	1, 1	1, 1	I_1, I_1	
915	$N = 915 = 3 \cdot 5 \cdot 61$ (4 isogeny classes)											915	
A1	0	-1	1	-460	-11577	0	1	-	1, 7, 3	1, 7, 3	1, 7, 1	I_1, I_7, I_3	
B1	1	1	0	-57	144	1	2	+	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	2 : 2
B2	1	1	0	-62	111	1	4	+	4, 2, 2	4, 2, 2	2, 2, 2	I_4, I_2, I_2	2 : 1, 3, 4
B3	1	1	0	-367	-2756	1	2	+	8, 4, 1	8, 4, 1	2, 4, 1	I_8, I_4, I_1	2 : 2
B4	1	1	0	163	966	1	4	-	2, 1, 4	2, 1, 4	2, 1, 4	I_2, I_1, I_4	2 : 2
C1	0	1	1	-6	-25	0	1	-	3, 3, 1	3, 3, 1	3, 1, 1	I_3, I_3, I_1	
D1	1	0	0	50	107	1	2	-	3, 3, 2	3, 3, 2	3, 3, 2	I_3, I_3, I_2	2 : 2
D2	1	0	0	-255	900	1	2	+	6, 6, 1	6, 6, 1	6, 6, 1	I_6, I_6, I_1	2 : 1
916	$N = 916 = 2^2 \cdot 229$ (5 isogeny classes)											916	
A1	0	0	0	-71	-290	0	2	-	8, 2	0, 2	3, 2	IV^*, I_2	2 : 2
A2	0	0	0	-76	-255	0	2	+	4, 1	0, 1	3, 1	IV, I_1	2 : 1
B1	0	0	0	-1013692	392832257	0	1	+	4, 1	0, 1	3, 1	IV, I_1	
C1	0	0	0	-4	1	2	1	+	4, 1	0, 1	3, 1	IV, I_1	
D1	0	1	0	-77	236	1	3	+	4, 1	0, 1	3, 1	IV, I_1	3 : 2
D2	0	1	0	-157	-416	1	1	+	4, 3	0, 3	1, 3	IV, I_3	3 : 1
E1	0	-1	0	-5	-2	1	1	+	4, 1	0, 1	3, 1	IV, I_1	
918	$N = 918 = 2 \cdot 3^3 \cdot 17$ (12 isogeny classes)											918	
A1	1	-1	0	-24990	1526804	1	1	-	8, 11, 1	8, 0, 1	2, 1, 1	I_8, II^*, I_1	
B1	1	-1	0	0	-18	1	1	-	1, 5, 2	1, 0, 2	1, 3, 2	I_1, IV, I_2	
C1	1	-1	0	-771	-8875	1	1	-	11, 11, 1	11, 0, 1	1, 1, 1	I_{11}, II^*, I_1	
D1	1	-1	0	-48	-768	0	1	-	15, 3, 2	15, 0, 2	1, 1, 2	I_{15}, II, I_2	3 : 2
D2	1	-1	0	432	20448	0	3	-	5, 5, 6	5, 0, 6	1, 3, 6	I_5, IV, I_6	3 : 1
E1	1	-1	0	3	-3	1	1	-	3, 3, 1	3, 0, 1	1, 1, 1	I_3, II, I_1	3 : 2
E2	1	-1	0	-27	99	1	3	-	1, 5, 3	1, 0, 3	1, 1, 3	I_1, IV, I_3	3 : 1
F1	1	-1	0	24	48	1	3	-	4, 3, 3	4, 0, 3	2, 1, 3	I_4, II, I_3	3 : 2
F2	1	-1	0	-231	-2179	1	1	-	12, 9, 1	12, 0, 1	2, 3, 1	I_{12}, IV^*, I_1	3 : 1
G1	1	-1	1	-26	89	0	3	-	12, 3, 1	12, 0, 1	12, 1, 1	I_{12}, II, I_1	3 : 2
G2	1	-1	1	214	-1511	0	1	-	4, 9, 3	4, 0, 3	4, 1, 1	I_4, IV^*, I_3	3 : 1
H1	1	-1	1	-86	357	1	1	-	11, 5, 1	11, 0, 1	11, 3, 1	I_{11}, IV, I_1	
I1	1	-1	1	25	55	1	3	-	3, 9, 1	3, 0, 1	3, 3, 1	I_3, IV^*, I_1	3 : 2
I2	1	-1	1	-245	-2429	1	1	-	1, 11, 3	1, 0, 3	1, 1, 1	I_1, II^*, I_3	3 : 1
J1	1	-1	1	-434	21169	1	3	-	15, 9, 2	15, 0, 2	15, 3, 2	I_{15}, IV^*, I_2	3 : 2
J2	1	-1	1	3886	-555983	1	1	-	5, 11, 6	5, 0, 6	5, 1, 2	I_5, II^*, I_6	3 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
918 918 $N = 918 = 2 \cdot 3^3 \cdot 17$ (continued)													
L1	1	-1	1	-2	487	0	1	-	1, 11, 2	1, 0, 2	1, 1, 2	I_1, II^*, I_2	
920 920 $N = 920 = 2^3 \cdot 5 \cdot 23$ (4 isogeny classes)													
A1	0	0	0	1468	-2844	1	1	-	8, 3, 5	0, 3, 5	4, 3, 5	I_1^*, I_3, I_5	
B1	0	0	0	-187	991	1	1	-	4, 6, 1	0, 6, 1	2, 6, 1	III, I_6, I_1	
C1	0	1	0	4	5	1	1	-	4, 2, 1	0, 2, 1	2, 2, 1	III, I_2, I_1	
D1	0	-1	0	0	-23	1	1	-	4, 4, 1	0, 4, 1	2, 4, 1	III, I_4, I_1	
921 921 $N = 921 = 3 \cdot 307$ (2 isogeny classes)													
A1	0	-1	1	-3058	-64080	0	1	-	6, 1	6, 1	2, 1	I_6, I_1	
B1	0	1	1	-23	41	1	3	-	6, 1	6, 1	6, 1	I_6, I_1	3 : 2
B2	0	1	1	157	-130	1	1	-	2, 3	2, 3	2, 3	I_2, I_3	3 : 1
922 922 $N = 922 = 2 \cdot 461$ (1 isogeny class)													
A1	1	0	0	-2	-2	0	1	-	1, 1	1, 1	1, 1	I_1, I_1	
923 923 $N = 923 = 13 \cdot 71$ (1 isogeny class)													
A1	0	0	1	-4	19	0	1	-	3, 1	3, 1	1, 1	I_3, I_1	
924 924 $N = 924 = 2^2 \cdot 3 \cdot 7 \cdot 11$ (8 isogeny classes)													
A1	0	-1	0	25158	-775719	0	1	-	4, 5, 5, 7	0, 5, 5, 7	3, 1, 1, 1	IV, I_5, I_5, I_7	
B1	0	-1	0	14	1057	1	1	-	4, 3, 1, 5	0, 3, 1, 5	3, 1, 1, 5	IV, I_3, I_1, I_5	
C1	0	-1	0	14	-11	1	1	-	4, 1, 3, 1	0, 1, 3, 1	1, 1, 3, 1	IV, I_1, I_3, I_1	
D1	0	-1	0	-470	-4311	0	1	-	4, 13, 1, 1	0, 13, 1, 1	3, 1, 1, 1	IV, I_{13}, I_1, I_1	
E1	0	1	0	-22	41	1	1	-	4, 5, 1, 1	0, 5, 1, 1	3, 5, 1, 1	IV, I_5, I_1, I_1	
F1	0	1	0	-1706	-27699	0	1	-	4, 5, 3, 1	0, 5, 3, 1	1, 5, 1, 1	IV, I_5, I_3, I_1	
G1	0	1	0	6	9	0	3	-	4, 3, 1, 1	0, 3, 1, 1	3, 3, 1, 1	IV, I_3, I_1, I_1	3 : 2
G2	0	1	0	-54	-291	0	1	-	4, 1, 3, 3	0, 1, 3, 3	1, 1, 3, 1	IV, I_1, I_3, I_3	3 : 1
H1	0	1	0	-17242	875009	1	3	-	4, 9, 5, 3	0, 9, 5, 3	3, 9, 5, 3	IV, I_9, I_5, I_3	3 : 2
H2	0	1	0	59978	4520981	1	1	-	4, 3, 15, 1	0, 3, 15, 1	1, 3, 15, 1	IV, I_3, I_{15}, I_1	3 : 1
925 925 $N = 925 = 5^2 \cdot 37$ (5 isogeny classes)													
A1	0	1	1	-133	519	1	1	+	8, 1	2, 1	2, 1	I_2^*, I_1	
B1	0	-1	1	-83	318	1	1	+	6, 1	0, 1	2, 1	I_0^*, I_1	3 : 2
B2	0	-1	1	-583	-5057	1	1	+	6, 3	0, 3	2, 1	I_0^*, I_3	3 : 1, 3
B3	0	-1	1	-46833	-3885432	1	1	+	6, 1	0, 1	2, 1	I_0^*, I_1	3 : 2
C1	1	1	1	-88	-344	0	2	+	7, 1	1, 1	4, 1	I_1^*, I_1	2 : 2
C2	1	1	1	37	-1094	0	2	-	8, 2	2, 2	4, 2	I_2^*, I_2	2 : 1
D1	0	-1	1	-3908	95343	0	1	+	10, 1	4, 1	2, 1	I_4^*, I_1	
E1	0	0	1	-25	31	0	1	+	6, 1	0, 1	2, 1	I_0^*, I_1	
926 926 $N = 926 = 2 \cdot 463$ (1 isogeny class)													
A1	1	1	1	7	7	0	2	-	6, 1	6, 1	6, 1	I_c, I_1	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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927 $N = 927 = 3^2 \cdot 103$ (1 isogeny class) **927**

A1	1	-1	0	-54	-243	1	1	-	11, 1	5, 1	2, 1	I_5^*, I_1	
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928 $N = 928 = 2^5 \cdot 29$ (2 isogeny classes) **928**

A1	0	1	0	-1	-17	1	1	-	12, 1	0, 1	4, 1	I_3^*, I_1	
B1	0	-1	0	-1	17	1	1	-	12, 1	0, 1	4, 1	I_3^*, I_1	

930 $N = 930 = 2 \cdot 3 \cdot 5 \cdot 31$ (15 isogeny classes) **930**

A1	1	1	0	-108	-432	1	2	+	12, 3, 1, 1	12, 3, 1, 1	2, 1, 1, 1	I_{12}, I_3, I_1, I_1	2 : 2
A2	1	1	0	-428	2832	1	4	+	6, 6, 2, 2	6, 6, 2, 2	2, 2, 2, 2	I_6, I_6, I_2, I_2	2 : 1, 3, 4
A3	1	1	0	-6628	204952	1	2	+	3, 12, 1, 1	3, 12, 1, 1	1, 2, 1, 1	I_3, I_{12}, I_1, I_1	2 : 2
A4	1	1	0	652	16008	1	2	-	3, 3, 4, 4	3, 3, 4, 4	1, 1, 2, 2	I_3, I_3, I_4, I_4	2 : 2
B1	1	1	0	-203	-1347	0	1	-	9, 1, 5, 1	9, 1, 5, 1	1, 1, 1, 1	I_9, I_1, I_5, I_1	
C1	1	1	0	98	244	0	1	-	11, 5, 1, 1	11, 5, 1, 1	1, 1, 1, 1	I_{11}, I_5, I_1, I_1	
D1	1	1	0	2238	181236	1	2	-	16, 1, 7, 2	16, 1, 7, 2	2, 1, 7, 2	I_{16}, I_1, I_7, I_2	2 : 2
D2	1	1	0	-37442	2585844	1	2	+	8, 2, 14, 1	8, 2, 14, 1	2, 2, 14, 1	I_8, I_2, I_{14}, I_1	2 : 1
E1	1	1	0	3	9	1	2	-	2, 2, 2, 1	2, 2, 2, 1	2, 2, 2, 1	I_2, I_2, I_2, I_1	2 : 2
E2	1	1	0	-47	99	1	2	+	1, 4, 1, 2	1, 4, 1, 2	1, 2, 1, 2	I_1, I_4, I_1, I_2	2 : 1
F1	1	0	1	-10400749	13377941672	0	1	-	23, 11, 3, 5	23, 11, 3, 5	1, 11, 1, 1	I_{23}, I_{11}, I_3, I_5	
G1	1	0	1	-244	1442	0	2	+	4, 1, 3, 1	4, 1, 3, 1	2, 1, 1, 1	I_4, I_1, I_3, I_1	2 : 2
G2	1	0	1	-264	1186	0	4	+	2, 2, 6, 2	2, 2, 6, 2	2, 2, 2, 2	I_2, I_2, I_6, I_2	2 : 1, 3, 4
G3	1	0	1	-1514	-21814	0	2	+	1, 4, 3, 4	1, 4, 3, 4	1, 4, 1, 2	I_1, I_4, I_3, I_4	2 : 2
G4	1	0	1	666	7882	0	2	-	1, 1, 12, 1	1, 1, 12, 1	1, 1, 2, 1	I_1, I_1, I_{12}, I_1	2 : 2
H1	1	0	1	467	-1432	1	2	-	8, 5, 3, 2	8, 5, 3, 2	2, 5, 3, 2	I_8, I_5, I_3, I_2	2 : 2
H2	1	0	1	-2013	-12344	1	2	+	4, 10, 6, 1	4, 10, 6, 1	2, 10, 6, 1	I_4, I_{10}, I_6, I_1	2 : 1
I1	1	0	1	2	-22	0	3	-	1, 3, 3, 1	1, 3, 3, 1	1, 3, 3, 1	I_1, I_3, I_3, I_1	3 : 2
I2	1	0	1	-523	-4642	0	1	-	3, 1, 1, 3	3, 1, 1, 3	1, 1, 1, 3	I_3, I_1, I_1, I_3	3 : 1
J1	1	0	1	-13648	613406	0	2	-	26, 2, 2, 1	26, 2, 2, 1	2, 2, 2, 1	I_{26}, I_2, I_2, I_1	2 : 2
J2	1	0	1	-218448	39279646	0	2	+	13, 4, 1, 2	13, 4, 1, 2	1, 4, 1, 2	I_{13}, I_4, I_1, I_2	2 : 1
K1	1	1	1	-41	-121	0	2	-	4, 1, 1, 2	4, 1, 1, 2	4, 1, 1, 2	I_4, I_1, I_1, I_2	2 : 2
K2	1	1	1	-661	-6817	0	2	+	2, 2, 2, 1	2, 2, 2, 1	2, 2, 2, 1	I_2, I_2, I_2, I_1	2 : 1
L1	1	1	1	-23051	1344449	0	1	-	3, 3, 13, 1	3, 3, 13, 1	3, 1, 1, 1	I_3, I_3, I_{13}, I_1	
M1	1	1	1	39	39	1	2	-	6, 4, 2, 1	6, 4, 2, 1	6, 2, 2, 1	I_6, I_4, I_2, I_1	2 : 2
M2	1	1	1	-161	119	1	2	+	3, 8, 1, 2	3, 8, 1, 2	3, 2, 1, 2	I_3, I_8, I_1, I_2	2 : 1
N1	1	0	0	1389	-22239	0	6	-	12, 9, 1, 2	12, 9, 1, 2	12, 9, 1, 2	I_{12}, I_9, I_1, I_2	2 : 2; 3 : 3
N2	1	0	0	-8531	-218655	0	6	+	6, 18, 2, 1	6, 18, 2, 1	6, 18, 2, 1	I_6, I_{18}, I_2, I_1	2 : 1; 3 : 4
N3	1	0	0	-39651	-3060495	0	2	-	4, 3, 3, 6	4, 3, 3, 6	4, 3, 1, 6	I_4, I_3, I_3, I_6	2 : 4; 3 : 1
N4	1	0	0	-635471	-195033699	0	2	+	2, 6, 6, 3	2, 6, 6, 3	2, 6, 2, 3	I_2, I_6, I_6, I_3	2 : 3; 3 : 2
O1	1	0	0	60	-1008	0	4	-	16, 2, 2, 1	16, 2, 2, 1	16, 2, 2, 1	I_{16}, I_2, I_2, I_1	2 : 2
O2	1	0	0	-1220	-15600	0	8	+	8, 4, 4, 2	8, 4, 4, 2	8, 4, 4, 2	I_8, I_4, I_4, I_2	2 : 1, 3, 4
O3	1	0	0	-19220	-1027200	0	4	+	4, 2, 2, 4	4, 2, 2, 4	4, 2, 2, 2	I_4, I_2, I_2, I_4	2 : 2, 5, 6
O4	1	0	0	-3700	67232	0	8	+	4, 8, 8, 1	4, 8, 8, 1	4, 8, 8, 1	I_4, I_8, I_8, I_1	2 : 2
O5	1	0	0	-307520	-65664060	0	2	+	2, 1, 1, 2	2, 1, 1, 2	2, 1, 1, 2	I_2, I_1, I_1, I_2	2 : 3

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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931 $N = 931 = 7^2 \cdot 19$ (3 isogeny classes) **931**

A1	0	-1	1	-114	727	0	1	-	8, 1	0, 1	3, 1	IV*, I ₁	
B1	0	-1	1	33	-8	0	1	-	6, 1	0, 1	1, 1	I ₀ *, I ₁	3 : 2
B2	0	-1	1	-457	4157	0	1	-	6, 3	0, 3	1, 1	I ₀ *, I ₃	3 : 1, 3
B3	0	-1	1	-37697	2829742	0	1	-	6, 1	0, 1	1, 1	I ₀ *, I ₁	3 : 2
C1	0	1	1	-2	-3	0	1	-	2, 1	0, 1	1, 1	II, I ₁	

933 $N = 933 = 3 \cdot 311$ (2 isogeny classes) **933**

A1	0	-1	1	-3	-1	1	1	+	1, 1	1, 1	1, 1	I ₁ , I ₁	
B1	0	1	1	-399	-3184	1	1	+	11, 1	11, 1	11, 1	I ₁₁ , I ₁	

934 $N = 934 = 2 \cdot 467$ (3 isogeny classes) **934**

A1	1	0	1	-3	0	1	1	+	1, 1	1, 1	1, 1	I ₁ , I ₁	
B1	1	0	0	-129	521	0	3	+	15, 1	15, 1	15, 1	I ₁₅ , I ₁	3 : 2
B2	1	0	0	-1889	-31639	0	1	+	5, 3	5, 3	5, 1	I ₅ , I ₃	3 : 1
C1	1	-1	1	-183	-905	0	1	+	3, 1	3, 1	3, 1	I ₃ , I ₁	

935 $N = 935 = 5 \cdot 11 \cdot 17$ (2 isogeny classes) **935**

A1	0	1	1	-1	-4	1	1	-	2, 1, 1	2, 1, 1	2, 1, 1	I ₂ , I ₁ , I ₁	
B1	0	1	1	-13155	576381	0	3	-	6, 3, 1	6, 3, 1	6, 1, 1	I ₆ , I ₃ , I ₁	3 : 2
B2	0	1	1	-9655	893306	0	1	-	2, 9, 3	2, 9, 3	2, 1, 1	I ₂ , I ₉ , I ₃	3 : 1

936 $N = 936 = 2^3 \cdot 3^2 \cdot 13$ (9 isogeny classes) **936**

A1	0	0	0	9	10	1	2	-	8, 3, 1	0, 0, 1	2, 2, 1	I ₁ *, III, I ₁	2 : 2
A2	0	0	0	-51	94	1	2	+	10, 3, 2	0, 0, 2	2, 2, 2	III*, III, I ₂	2 : 1
B1	0	0	0	-147	718	0	1	-	11, 6, 1	0, 0, 1	1, 1, 1	II*, I ₀ *, I ₁	
C1	0	0	0	42	-335	0	2	-	4, 9, 2	0, 3, 2	2, 2, 2	III, I ₃ *, I ₂	2 : 2
C2	0	0	0	-543	-4430	0	2	+	8, 12, 1	0, 6, 1	2, 4, 1	I ₁ *, I ₆ *, I ₁	2 : 1
D1	0	0	0	-5862	-162295	0	2	+	4, 16, 3	0, 10, 3	2, 4, 1	III, I ₁₀ *, I ₃	2 : 2
D2	0	0	0	5073	-698110	0	2	-	8, 11, 6	0, 5, 6	2, 4, 2	I ₁ *, I ₅ *, I ₆	2 : 1
E1	0	0	0	-66	-119	1	2	+	4, 10, 1	0, 4, 1	2, 4, 1	III, I ₄ *, I ₁	2 : 2
E2	0	0	0	-471	3850	1	4	+	8, 8, 2	0, 2, 2	2, 4, 2	I ₁ *, I ₂ *, I ₂	2 : 1, 3, 4
E3	0	0	0	-7491	249550	1	4	+	10, 7, 1	0, 1, 1	2, 4, 1	III*, I ₁ *, I ₁	2 : 2
E4	0	0	0	69	12166	1	2	-	10, 7, 4	0, 1, 4	2, 2, 4	III*, I ₁ *, I ₄	2 : 2
F1	0	0	0	81	-270	0	2	-	8, 9, 1	0, 0, 1	4, 2, 1	I ₁ *, III*, I ₁	2 : 2
F2	0	0	0	-459	-2538	0	2	+	10, 9, 2	0, 0, 2	2, 2, 2	III*, III*, I ₂	2 : 1
G1	0	0	0	-30	133	1	2	-	4, 7, 2	0, 1, 2	2, 2, 2	III, I ₁ *, I ₂	2 : 2
G2	0	0	0	-615	5866	1	2	+	8, 8, 1	0, 2, 1	4, 4, 1	I ₁ *, I ₂ *, I ₁	2 : 1
H1	0	0	0	-30	29	1	2	+	4, 8, 1	0, 2, 1	2, 4, 1	III, I ₂ *, I ₁	2 : 2
H2	0	0	0	105	218	1	2	-	8, 7, 2	0, 1, 2	4, 4, 2	I ₁ *, I ₁ *, I ₂	2 : 1
I1	0	0	0	-354	-2563	0	2	+	4, 8, 1	0, 2, 1	2, 4, 1	III, I ₂ *, I ₁	2 : 2
I2	0	0	0	-399	-1870	0	4	+	8, 10, 2	0, 4, 2	4, 4, 2	I ₁ *, I ₄ *, I ₂	2 : 1, 3, 4
I3	0	0	0	-2739	53822	0	2	+	10, 14, 1	0, 8, 1	2, 4, 1	III*, I ₁ *, I ₁	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
938	$N = 938 = 2 \cdot 7 \cdot 67$ (4 isogeny classes)											938	
A1	1	0	1	-4	-2	1	1	+	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	
B1	1	0	1	-365	13608	1	2	-	10, 5, 2	10, 5, 2	2, 5, 2	I_{10}, I_5, I_2	2 : 2
B2	1	0	1	-11085	446696	1	2	+	5, 10, 1	5, 10, 1	1, 10, 1	I_5, I_{10}, I_1	2 : 1
C1	1	1	1	-56	-135	1	1	+	8, 3, 1	8, 3, 1	8, 3, 1	I_8, I_3, I_1	
D1	1	0	0	-179	737	0	3	+	18, 1, 1	18, 1, 1	18, 1, 1	I_{18}, I_1, I_1	3 : 2
D2	1	0	0	-4339	-110303	0	3	+	6, 3, 3	6, 3, 3	6, 3, 3	I_6, I_3, I_3	3 : 1, 3
D3	1	0	0	-351399	-80206123	0	1	+	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	3 : 2
939	$N = 939 = 3 \cdot 313$ (3 isogeny classes)											939	
A1	0	-1	1	-321	-9817	1	1	-	17, 1	17, 1	1, 1	I_{17}, I_1	
B1	1	0	1	-6	-5	1	2	+	2, 1	2, 1	2, 1	I_2, I_1	2 : 2
B2	1	0	1	9	-23	1	2	-	1, 2	1, 2	1, 2	I_1, I_2	2 : 1
C1	0	1	1	4	14	1	1	-	5, 1	5, 1	5, 1	I_5, I_1	
940	$N = 940 = 2^2 \cdot 5 \cdot 47$ (5 isogeny classes)											940	
A1	0	1	0	21619	-57905	0	1	-	8, 1, 7	0, 1, 7	3, 1, 1	IV^*, I_1, I_7	
B1	0	0	0	-103	398	0	1	+	8, 3, 1	0, 3, 1	1, 1, 1	IV^*, I_3, I_1	
C1	0	1	0	-7076	226340	1	3	+	8, 5, 3	0, 5, 3	3, 1, 3	IV^*, I_5, I_3	3 : 2
C2	0	1	0	-31516	-1956716	1	1	+	8, 15, 1	0, 15, 1	1, 1, 1	IV^*, I_{15}, I_1	3 : 1
D1	0	-1	0	-20	40	1	1	+	8, 1, 1	0, 1, 1	3, 1, 1	IV^*, I_1, I_1	
E1	0	-1	0	-45	-103	0	1	-	8, 1, 1	0, 1, 1	3, 1, 1	IV^*, I_1, I_1	
942	$N = 942 = 2 \cdot 3 \cdot 157$ (4 isogeny classes)											942	
A1	1	0	1	15	4	0	1	-	9, 1, 1	9, 1, 1	1, 1, 1	I_9, I_1, I_1	
B1	1	1	1	-215539	-38605903	0	1	-	8, 18, 1	8, 18, 1	8, 2, 1	I_8, I_{18}, I_1	
C1	1	0	0	146	37508	1	1	-	16, 10, 1	16, 10, 1	16, 10, 1	I_{16}, I_{10}, I_1	
D1	1	0	0	-65	201	1	1	-	6, 4, 1	6, 4, 1	6, 4, 1	I_6, I_4, I_1	
943	$N = 943 = 23 \cdot 41$ (1 isogeny class)											943	
A1	1	-1	0	-13	24	0	2	-	1, 2	1, 2	1, 2	I_1, I_2	2 : 2
A2	1	-1	0	-218	1295	0	2	+	2, 1	2, 1	2, 1	I_2, I_1	2 : 1
944	$N = 944 = 2^4 \cdot 59$ (11 isogeny classes)											944	
A1	0	1	0	4	-4	1	1	-	8, 1	0, 1	2, 1	I_0^*, I_1	
B1	0	1	0	-276	1676	1	1	-	8, 1	0, 1	2, 1	I_0^*, I_1	
C1	0	1	0	8	-12	1	1	-	11, 1	0, 1	4, 1	I_3^*, I_1	
D1	0	0	0	2	-1	0	1	-	4, 1	0, 1	1, 1	II, I_1	
E1	0	0	0	-19	34	2	1	-	10, 1	0, 1	4, 1	I_2^*, I_1	
F1	0	1	0	-1	-2	0	1	-	4, 1	0, 1	1, 1	II, I_1	
G1	0	1	0	888	14068	0	1	-	31, 1	19, 1	2, 1	I_{23}^*, I_1	
H1	0	1	0	-400	-3308	1	1	-	22, 1	10, 1	4, 1	I_{14}^*, I_1	5 : 2
H2	0	1	0	1840	162452	1	1	-	14, 5	2, 5	4, 5	I_6^*, I_5	5 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
944 944 $N = 944 = 2^4 \cdot 59$ (continued)													
J1	0	-1	0	-9	-8	1	1	-	4, 1	0, 1	1, 1	II, I ₁	3 : 2
J2	0	-1	0	31	-68	1	1	-	4, 3	0, 3	1, 3	II, I ₃	3 : 1
K1	0	1	0	-64	180	1	1	-	13, 1	1, 1	4, 1	I ₅ [*] , I ₁	
946 946 $N = 946 = 2 \cdot 11 \cdot 43$ (3 isogeny classes)													
A1	1	-1	0	-11	-11	0	2	+	4, 1, 1	4, 1, 1	2, 1, 1	I ₄ , I ₁ , I ₁	2 : 2
A2	1	-1	0	-31	57	0	4	+	2, 2, 2	2, 2, 2	2, 2, 2	I ₂ , I ₂ , I ₂	2 : 1, 3, 4
A3	1	-1	0	-461	3927	0	2	+	1, 4, 1	1, 4, 1	1, 2, 1	I ₁ , I ₄ , I ₁	2 : 2
A4	1	-1	0	79	299	0	2	-	1, 1, 4	1, 1, 4	1, 1, 4	I ₁ , I ₁ , I ₄	2 : 2
B1	1	0	1	14	-8	1	3	-	2, 3, 1	2, 3, 1	2, 3, 1	I ₂ , I ₃ , I ₁	3 : 2
B2	1	0	1	-261	-1680	1	1	-	6, 1, 3	6, 1, 3	2, 1, 3	I ₆ , I ₁ , I ₃	3 : 1
C1	1	0	0	-1806	-29692	0	1	-	10, 1, 1	10, 1, 1	10, 1, 1	I ₁₀ , I ₁ , I ₁	
948 948 $N = 948 = 2^2 \cdot 3 \cdot 79$ (3 isogeny classes)													
A1	0	-1	0	-17	-78	0	2	-	4, 3, 2	0, 3, 2	3, 1, 2	IV, I ₃ , I ₂	2 : 2
A2	0	-1	0	-412	-3080	0	2	+	8, 6, 1	0, 6, 1	3, 2, 1	IV [*] , I ₆ , I ₁	2 : 1
B1	0	-1	0	-796	8968	0	1	-	8, 9, 1	0, 9, 1	1, 1, 1	IV [*] , I ₉ , I ₁	
C1	0	1	0	12	36	0	3	-	8, 3, 1	0, 3, 1	3, 3, 1	IV [*] , I ₃ , I ₁	3 : 2
C2	0	1	0	-108	-1068	0	1	-	8, 1, 3	0, 1, 3	1, 1, 3	IV [*] , I ₁ , I ₃	3 : 1
950 950 $N = 950 = 2 \cdot 5^2 \cdot 19$ (5 isogeny classes)													
A1	1	0	1	-1	148	1	1	-	5, 6, 1	5, 0, 1	1, 2, 1	I ₅ , I ₀ [*] , I ₁	5 : 2
A2	1	0	1	-1751	-31352	1	1	-	1, 6, 5	1, 0, 5	1, 2, 1	I ₁ , I ₀ [*] , I ₅	5 : 1
B1	1	1	0	-750	-12500	0	1	-	3, 12, 1	3, 6, 1	1, 2, 1	I ₃ , I ₆ [*] , I ₁	3 : 2
B2	1	1	0	-69500	-7081250	0	1	-	1, 8, 3	1, 2, 3	1, 2, 3	I ₁ , I ₂ [*] , I ₃	3 : 1
C1	1	-1	0	-1192	17216	0	1	-	11, 8, 1	11, 2, 1	1, 2, 1	I ₁₁ , I ₂ [*] , I ₁	
D1	1	0	0	37	167	0	1	-	1, 8, 1	1, 2, 1	1, 2, 1	I ₁ , I ₂ [*] , I ₁	
E1	1	1	1	-388	2781	1	1	-	3, 6, 1	3, 0, 1	3, 2, 1	I ₃ , I ₀ [*] , I ₁	3 : 2
E2	1	1	1	237	11281	1	1	-	9, 6, 3	9, 0, 3	9, 2, 3	I ₉ , I ₀ [*] , I ₃	3 : 1, 3
E3	1	1	1	-2138	-306969	1	1	-	27, 6, 1	27, 0, 1	27, 2, 1	I ₂₇ , I ₀ [*] , I ₁	3 : 2
954 954 $N = 954 = 2 \cdot 3^2 \cdot 53$ (13 isogeny classes)													
A1	1	-1	0	-96	-640	1	1	-	7, 9, 1	7, 0, 1	1, 2, 1	I ₇ , III [*] , I ₁	
B1	1	-1	0	12	-100	0	2	-	2, 9, 1	2, 0, 1	2, 2, 1	I ₂ , III [*] , I ₁	2 : 2
B2	1	-1	0	-258	-1450	0	2	+	1, 9, 2	1, 0, 2	1, 2, 2	I ₁ , III [*] , I ₂	2 : 1
C1	1	-1	0	-108	-1328	0	1	-	11, 8, 1	11, 2, 1	1, 2, 1	I ₁₁ , I ₂ [*] , I ₁	
D1	1	-1	0	18	202	1	1	-	1, 11, 1	1, 5, 1	1, 4, 1	I ₁ , I ₅ [*] , I ₁	
E1	1	-1	0	-2547	63477	1	1	-	24, 6, 1	24, 0, 1	2, 1, 1	I ₂₄ , I ₀ [*] , I ₁	3 : 2
E2	1	-1	0	-221427	40159989	1	3	-	8, 6, 3	8, 0, 3	2, 1, 3	I ₈ , I ₀ [*] , I ₃	3 : 1
F1	1	-1	0	9	-27	1	1	-	3, 6, 1	3, 0, 1	1, 2, 1	I ₃ , I ₀ [*] , I ₁	3 : 2
F2	1	-1	0	-81	783	1	3	-	1, 6, 3	1, 0, 3	1, 2, 3	I ₁ , I ₀ [*] , I ₃	3 : 1
G1	1	-1	1	1	3	0	2	-	2, 3, 1	2, 0, 1	2, 2, 1	I ₂ , III, I ₁	2 : 2
G2	1	-1	1	-29	63	0	2	+	1, 3, 2	1, 0, 2	1, 2, 2	I ₁ , III, I ₂	2 : 1
H1	1	-1	1	-11	27	1	1	-	7, 3, 1	7, 0, 1	7, 2, 1	I ₇ , III, I ₁	

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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954 $N = 954 = 2 \cdot 3^2 \cdot 53$ (continued)**954**

J1	1	-1	1	1273	-3585	1	1	-	17, 9, 1	17, 3, 1	17, 4, 1	I_{17}, I_3^*, I_1	
K1	1	-1	1	-545	-4759	0	1	-	3, 9, 1	3, 3, 1	3, 2, 1	I_3, I_3^*, I_1	3 : 2
K2	1	-1	1	400	-19501	0	3	-	9, 7, 3	9, 1, 3	9, 2, 3	I_9, I_1^*, I_3	3 : 1
L1	1	-1	1	58	303	0	1	-	1, 12, 1	1, 6, 1	1, 2, 1	I_1, I_6^*, I_1	
M1	1	-1	1	-68	-201	0	1	-	4, 6, 1	4, 0, 1	4, 1, 1	I_4, I_0^*, I_1	

955 $N = 955 = 5 \cdot 191$ (1 isogeny class)**955**

A1	1	-1	1	-1038	13292	0	2	-	10, 1	10, 1	2, 1	I_{10}, I_1	2 : 2
A2	1	-1	1	-16663	832042	0	2	+	5, 2	5, 2	1, 2	I_5, I_2	2 : 1

956 $N = 956 = 2^2 \cdot 239$ (1 isogeny class)**956**

A1	0	0	0	-1	-3	0	1	-	4, 1	0, 1	1, 1	IV, I_1	
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957 $N = 957 = 3 \cdot 11 \cdot 29$ (1 isogeny class)**957**

A1	1	1	0	-491	3984	0	2	+	7, 1, 2	7, 1, 2	1, 1, 2	I_7, I_1, I_2	2 : 2
A2	1	1	0	-346	6565	0	2	-	14, 2, 1	14, 2, 1	2, 2, 1	I_{14}, I_2, I_1	2 : 1

960 $N = 960 = 2^6 \cdot 3 \cdot 5$ (16 isogeny classes)**960**

A1	0	-1	0	4	6	1	2	-	6, 4, 1	0, 4, 1	1, 2, 1	II, I_4, I_1	2 : 2
A2	0	-1	0	-41	105	1	4	+	12, 2, 2	0, 2, 2	4, 2, 2	I_2^*, I_2, I_2	2 : 1, 3, 4
A3	0	-1	0	-161	-639	1	2	+	15, 1, 4	0, 1, 4	2, 1, 2	I_5^*, I_1, I_4	2 : 2
A4	0	-1	0	-641	6465	1	2	+	15, 1, 1	0, 1, 1	2, 1, 1	I_5^*, I_1, I_1	2 : 2
B1	0	-1	0	-61	205	1	2	+	10, 2, 1	0, 2, 1	2, 2, 1	I_0^*, I_2, I_1	2 : 2
B2	0	-1	0	-81	81	1	4	+	14, 4, 2	0, 4, 2	4, 2, 2	I_4^*, I_4, I_2	2 : 1, 3, 4
B3	0	-1	0	-801	-8415	1	4	+	16, 2, 4	0, 2, 4	4, 2, 2	I_6^*, I_2, I_4	2 : 2, 5, 6
B4	0	-1	0	319	321	1	2	-	16, 8, 1	0, 8, 1	2, 2, 1	I_6^*, I_8, I_1	2 : 2
B5	0	-1	0	-12801	-553215	1	2	+	17, 1, 2	0, 1, 2	2, 1, 2	I_7^*, I_1, I_2	2 : 3
B6	0	-1	0	-321	-18879	1	2	-	17, 1, 8	0, 1, 8	2, 1, 2	I_7^*, I_1, I_8	2 : 3
C1	0	-1	0	15	-15	0	2	-	14, 1, 1	0, 1, 1	4, 1, 1	I_4^*, I_1, I_1	2 : 2
C2	0	-1	0	-65	-63	0	4	+	16, 2, 2	0, 2, 2	4, 2, 2	I_6^*, I_2, I_2	2 : 1, 3, 4
C3	0	-1	0	-865	-9503	0	2	+	17, 4, 1	0, 4, 1	4, 2, 1	I_7^*, I_4, I_1	2 : 2
C4	0	-1	0	-545	5025	0	4	+	17, 1, 4	0, 1, 4	4, 1, 4	I_7^*, I_1, I_4	2 : 2
D1	0	-1	0	-900	-10098	0	2	+	6, 3, 2	0, 3, 2	1, 1, 2	II, I_3, I_2	2 : 2
D2	0	-1	0	-905	-9975	0	4	+	12, 6, 4	0, 6, 4	4, 2, 4	I_2^*, I_6, I_4	2 : 1, 3, 4
D3	0	-1	0	-1985	19617	0	4	+	15, 3, 8	0, 3, 8	4, 1, 8	I_5^*, I_3, I_8	2 : 2
D4	0	-1	0	95	-31775	0	2	-	15, 12, 2	0, 12, 2	4, 2, 2	I_5^*, I_{12}, I_2	2 : 2
E1	0	-1	0	95	1057	0	2	-	22, 3, 1	4, 3, 1	4, 1, 1	I_{12}^*, I_3, I_1	2 : 2; 3 : 3
E2	0	-1	0	-1185	14625	0	4	+	20, 6, 2	2, 6, 2	4, 2, 2	I_{10}^*, I_6, I_2	2 : 1, 4, 5; 3 : 6
E3	0	-1	0	-865	-31775	0	2	-	30, 1, 3	12, 1, 3	4, 1, 3	I_{20}^*, I_1, I_3	2 : 6; 3 : 1
E4	0	-1	0	-4385	-94815	0	2	+	19, 12, 1	1, 12, 1	4, 2, 1	I_9^*, I_{12}, I_1	2 : 2; 3 : 7
E5	0	-1	0	-18465	971937	0	4	+	19, 3, 4	1, 3, 4	4, 1, 4	I_9^*, I_3, I_4	2 : 2; 3 : 8
E6	0	-1	0	-21345	-1190943	0	4	+	24, 2, 6	6, 2, 6	4, 2, 6	I_{14}^*, I_2, I_6	2 : 3, 7, 8; 3 : 2
E7	0	-1	0	-341345	-76646943	0	2	+	21, 4, 3	3, 4, 3	4, 2, 3	I_{12}^*, I_4, I_3	2 : 6; 3 : 4

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
960													
$N = 960 = 2^6 \cdot 3 \cdot 5$ (continued)													
960													
F1	0	1	0	4	-6	0	2	-	6, 4, 1	0, 4, 1	1, 4, 1	II, I ₄ , I ₁	2 : 2
F2	0	1	0	-41	-105	0	4	+	12, 2, 2	0, 2, 2	4, 2, 2	I ₂ [*] , I ₂ , I ₂	2 : 1, 3, 4
F3	0	1	0	-641	-6465	0	2	+	15, 1, 1	0, 1, 1	2, 1, 1	I ₅ [*] , I ₁ , I ₁	2 : 2
F4	0	1	0	-161	639	0	2	+	15, 1, 4	0, 1, 4	2, 1, 2	I ₅ [*] , I ₁ , I ₄	2 : 2

G1	0	1	0	-1	95	0	2	-	18, 1, 1	0, 1, 1	4, 1, 1	I ₈ [*] , I ₁ , I ₁	2 : 2
G2	0	1	0	-321	2079	0	4	+	18, 2, 2	0, 2, 2	4, 2, 2	I ₈ [*] , I ₂ , I ₂	2 : 1, 3, 4
G3	0	1	0	-641	-3105	0	4	+	18, 4, 4	0, 4, 4	4, 4, 2	I ₈ [*] , I ₄ , I ₄	2 : 2, 5, 6
G4	0	1	0	-5121	139359	0	2	+	18, 1, 1	0, 1, 1	4, 1, 1	I ₈ [*] , I ₁ , I ₁	2 : 2
G5	0	1	0	-8641	-311905	0	4	+	18, 8, 2	0, 8, 2	4, 8, 2	I ₈ [*] , I ₈ , I ₂	2 : 3, 7, 8
G6	0	1	0	2239	-20961	0	2	-	18, 2, 8	0, 2, 8	4, 2, 2	I ₈ [*] , I ₂ , I ₈	2 : 3
G7	0	1	0	-138241	-19829665	0	2	+	18, 4, 1	0, 4, 1	2, 4, 1	I ₈ [*] , I ₄ , I ₁	2 : 5
G8	0	1	0	-7041	-429345	0	2	-	18, 16, 1	0, 16, 1	2, 16, 1	I ₈ [*] , I ₁₆ , I ₁	2 : 5

H1	0	1	0	-900	10098	1	2	+	6, 3, 2	0, 3, 2	1, 3, 2	II, I ₃ , I ₂	2 : 2
H2	0	1	0	-905	9975	1	4	+	12, 6, 4	0, 6, 4	4, 6, 4	I ₂ [*] , I ₆ , I ₄	2 : 1, 3, 4
H3	0	1	0	-1985	-19617	1	2	+	15, 3, 8	0, 3, 8	4, 3, 8	I ₅ [*] , I ₃ , I ₈	2 : 2
H4	0	1	0	95	31775	1	4	-	15, 12, 2	0, 12, 2	4, 12, 2	I ₅ [*] , I ₁₂ , I ₂	2 : 2

I1	0	-1	0	-1	-95	0	2	-	18, 1, 1	0, 1, 1	4, 1, 1	I ₈ [*] , I ₁ , I ₁	2 : 2
I2	0	-1	0	-321	-2079	0	4	+	18, 2, 2	0, 2, 2	4, 2, 2	I ₈ [*] , I ₂ , I ₂	2 : 1, 3, 4
I3	0	-1	0	-5121	-139359	0	2	+	18, 1, 1	0, 1, 1	2, 1, 1	I ₈ [*] , I ₁ , I ₁	2 : 2
I4	0	-1	0	-641	3105	0	4	+	18, 4, 4	0, 4, 4	4, 2, 2	I ₈ [*] , I ₄ , I ₄	2 : 2, 5, 6
I5	0	-1	0	-8641	311905	0	4	+	18, 8, 2	0, 8, 2	4, 2, 2	I ₈ [*] , I ₈ , I ₂	2 : 4, 7, 8
I6	0	-1	0	2239	20961	0	2	-	18, 2, 8	0, 2, 8	2, 2, 2	I ₈ [*] , I ₂ , I ₈	2 : 4
I7	0	-1	0	-138241	19829665	0	2	+	18, 4, 1	0, 4, 1	2, 2, 1	I ₈ [*] , I ₄ , I ₁	2 : 5
I8	0	-1	0	-7041	429345	0	2	-	18, 16, 1	0, 16, 1	4, 2, 1	I ₈ [*] , I ₁₆ , I ₁	2 : 5

J1	0	-1	0	4	-30	0	2	-	6, 2, 4	0, 2, 4	1, 2, 2	II, I ₂ , I ₄	2 : 2
J2	0	-1	0	-121	-455	0	4	+	12, 4, 2	0, 4, 2	4, 2, 2	I ₂ [*] , I ₄ , I ₂	2 : 1, 3, 4
J3	0	-1	0	-1921	-31775	0	2	+	15, 2, 1	0, 2, 1	4, 2, 1	I ₅ [*] , I ₂ , I ₁	2 : 2
J4	0	-1	0	-321	1665	0	2	+	15, 8, 1	0, 8, 1	2, 2, 1	I ₅ [*] , I ₈ , I ₁	2 : 2

K1	0	-1	0	-20	42	1	2	+	6, 1, 1	0, 1, 1	1, 1, 1	II, I ₁ , I ₁	2 : 2
K2	0	-1	0	-25	25	1	4	+	12, 2, 2	0, 2, 2	4, 2, 2	I ₂ [*] , I ₂ , I ₂	2 : 1, 3, 4
K3	0	-1	0	-225	-1215	1	2	+	15, 4, 1	0, 4, 1	2, 2, 1	I ₅ [*] , I ₄ , I ₁	2 : 2
K4	0	-1	0	95	97	1	4	-	15, 1, 4	0, 1, 4	4, 1, 4	I ₅ [*] , I ₁ , I ₄	2 : 2

L1	0	1	0	-61	-205	1	2	+	10, 2, 1	0, 2, 1	2, 2, 1	I ₀ [*] , I ₂ , I ₁	2 : 2
L2	0	1	0	-81	-81	1	4	+	14, 4, 2	0, 4, 2	4, 4, 2	I ₄ [*] , I ₄ , I ₂	2 : 1, 3, 4
L3	0	1	0	-801	8415	1	4	+	16, 2, 4	0, 2, 4	4, 2, 2	I ₆ [*] , I ₂ , I ₄	2 : 2, 5, 6
L4	0	1	0	319	-321	1	2	-	16, 8, 1	0, 8, 1	4, 8, 1	I ₆ [*] , I ₈ , I ₁	2 : 2
L5	0	1	0	-12801	553215	1	2	+	17, 1, 2	0, 1, 2	4, 1, 2	I ₇ [*] , I ₁ , I ₂	2 : 3
L6	0	1	0	-321	18879	1	2	-	17, 1, 8	0, 1, 8	2, 1, 2	I ₇ [*] , I ₁ , I ₈	2 : 3

M1	0	1	0	4	30	1	2	-	6, 2, 4	0, 2, 4	1, 2, 2	II, I ₂ , I ₄	2 : 2
M2	0	1	0	-121	455	1	4	+	12, 4, 2	0, 4, 2	4, 4, 2	I ₂ [*] , I ₄ , I ₂	2 : 1, 3, 4
M3	0	1	0	-321	-1665	1	2	+	15, 8, 1	0, 8, 1	4, 8, 1	I ₅ [*] , I ₈ , I ₁	2 : 2
M4	0	1	0	-1921	31775	1	2	+	15, 2, 1	0, 2, 1	2, 2, 1	I ₅ [*] , I ₂ , I ₁	2 : 2

N1	0	1	0	-20	-42	0	2	+	6, 1, 1	0, 1, 1	1, 1, 1	II, I ₁ , I ₁	2 : 2
N2	0	1	0	-25	-25	0	4	+	12, 2, 2	0, 2, 2	4, 2, 2	I ₂ [*] , I ₂ , I ₂	2 : 1, 3, 4
N3	0	1	0	-225	1215	0	4	+	15, 4, 1	0, 4, 1	4, 4, 1	I ₅ [*] , I ₄ , I ₁	2 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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960 $N = 960 = 2^6 \cdot 3 \cdot 5$ (continued)**960**

O1	0	1	0	95	-1057	0	2	-	22, 3, 1	4, 3, 1	4, 3, 1	I_{12}^*, I_3, I_1	2 : 2; 3 : 3
O2	0	1	0	-1185	-14625	0	4	+	20, 6, 2	2, 6, 2	4, 6, 2	I_{10}^*, I_6, I_2	2 : 1, 4, 5; 3 : 6
O3	0	1	0	-865	31775	0	2	-	30, 1, 3	12, 1, 3	4, 1, 3	I_{20}^*, I_1, I_3	2 : 6; 3 : 1
O4	0	1	0	-18465	-971937	0	2	+	19, 3, 4	1, 3, 4	2, 3, 4	I_9^*, I_3, I_4	2 : 2; 3 : 7
O5	0	1	0	-4385	94815	0	4	+	19, 12, 1	1, 12, 1	4, 12, 1	I_9^*, I_{12}, I_1	2 : 2; 3 : 8
O6	0	1	0	-21345	1190943	0	4	+	24, 2, 6	6, 2, 6	4, 2, 6	I_{14}^*, I_2, I_6	2 : 3, 7, 8; 3 : 2
O7	0	1	0	-29025	249375	0	2	+	21, 1, 12	3, 1, 12	2, 1, 12	I_{11}^*, I_1, I_{12}	2 : 6; 3 : 4
O8	0	1	0	-341345	76646943	0	4	+	21, 4, 3	3, 4, 3	4, 4, 3	I_{11}^*, I_4, I_3	2 : 6; 3 : 5
P1	0	1	0	15	15	0	2	-	14, 1, 1	0, 1, 1	4, 1, 1	I_4^*, I_1, I_1	2 : 2
P2	0	1	0	-65	63	0	4	+	16, 2, 2	0, 2, 2	4, 2, 2	I_6^*, I_2, I_2	2 : 1, 3, 4
P3	0	1	0	-545	-5025	0	2	+	17, 1, 4	0, 1, 4	2, 1, 4	I_7^*, I_1, I_4	2 : 2
P4	0	1	0	-865	9503	0	4	+	17, 4, 1	0, 4, 1	4, 4, 1	I_7^*, I_4, I_1	2 : 2

962 $N = 962 = 2 \cdot 13 \cdot 37$ (1 isogeny class)**962**

A1	1	-1	1	-9	-7	0	2	+	4, 1, 1	4, 1, 1	4, 1, 1	I_4, I_1, I_1	2 : 2
A2	1	-1	1	11	-47	0	2	-	2, 2, 2	2, 2, 2	2, 2, 2	I_2, I_2, I_2	2 : 1

964 $N = 964 = 2^2 \cdot 241$ (1 isogeny class)**964**

A1	0	1	0	-20	-44	0	1	-	8, 1	0, 1	1, 1	IV^*, I_1	
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965 $N = 965 = 5 \cdot 193$ (1 isogeny class)**965**

A1	1	-1	0	-100	411	0	2	+	2, 1	2, 1	2, 1	I_2, I_1	2 : 2
A2	1	-1	0	-95	450	0	2	-	4, 2	4, 2	2, 2	I_4, I_2	2 : 1

966 $N = 966 = 2 \cdot 3 \cdot 7 \cdot 23$ (11 isogeny classes)**966**

A1	1	1	0	334	5556	1	2	-	10, 4, 3, 2	10, 4, 3, 2	2, 2, 1, 2	I_{10}, I_4, I_3, I_2	2 : 2
A2	1	1	0	-3346	63700	1	2	+	5, 8, 6, 1	5, 8, 6, 1	1, 2, 2, 1	I_5, I_8, I_6, I_1	2 : 1
B1	1	1	0	-5131	-144323	0	1	-	13, 3, 5, 1	13, 3, 5, 1	1, 1, 5, 1	I_{13}, I_3, I_5, I_1	
C1	1	1	0	-14744	836928	1	2	-	22, 8, 1, 2	22, 8, 1, 2	2, 2, 1, 2	I_{22}, I_8, I_1, I_2	2 : 2
C2	1	1	0	-250264	48082240	1	2	+	11, 16, 2, 1	11, 16, 2, 1	1, 2, 2, 1	I_{11}, I_{16}, I_2, I_1	2 : 1
D1	1	1	0	18	0	1	2	-	2, 4, 2, 1	2, 4, 2, 1	2, 2, 2, 1	I_2, I_4, I_2, I_1	2 : 2
D2	1	1	0	-72	-90	1	2	+	1, 2, 4, 2	1, 2, 4, 2	1, 2, 4, 2	I_1, I_2, I_4, I_2	2 : 1
E1	1	0	1	-1	116	1	2	-	6, 4, 2, 1	6, 4, 2, 1	2, 4, 2, 1	I_6, I_4, I_2, I_1	2 : 2
E2	1	0	1	-361	2564	1	2	+	3, 2, 4, 2	3, 2, 4, 2	1, 2, 2, 2	I_3, I_2, I_4, I_2	2 : 1
F1	1	0	1	4644	858394	0	6	-	10, 12, 2, 3	10, 12, 2, 3	2, 12, 2, 3	I_{10}, I_{12}, I_2, I_3	2 : 2; 3 : 3
F2	1	0	1	-111996	13735450	0	6	+	5, 6, 4, 6	5, 6, 4, 6	1, 6, 4, 6	I_5, I_6, I_4, I_6	2 : 1; 3 : 4
F3	1	0	1	-41931	-23576714	0	2	-	30, 4, 6, 1	30, 4, 6, 1	2, 4, 6, 1	I_{30}, I_4, I_6, I_1	2 : 4; 3 : 1
F4	1	0	1	-1516491	-715440266	0	2	+	15, 2, 12, 2	15, 2, 12, 2	1, 2, 12, 2	I_{15}, I_2, I_{12}, I_2	2 : 3; 3 : 2
G1	1	1	1	126	1167	1	4	-	16, 2, 2, 1	16, 2, 2, 1	16, 2, 2, 1	I_{16}, I_2, I_2, I_1	2 : 2
G2	1	1	1	-1154	12431	1	8	+	8, 4, 4, 2	8, 4, 4, 2	8, 2, 4, 2	I_8, I_4, I_4, I_2	2 : 1, 3, 4
G3	1	1	1	-5074	-128689	1	4	+	4, 8, 2, 4	4, 8, 2, 4	4, 2, 2, 2	I_4, I_8, I_2, I_4	2 : 2, 5, 6
G4	1	1	1	-17714	900047	1	8	+	4, 2, 8, 1	4, 2, 8, 1	4, 2, 8, 1	I_4, I_2, I_8, I_1	2 : 2
G5	1	1	1	-79134	-8601153	1	2	+	2, 16, 1, 2	2, 16, 1, 2	2, 2, 1, 2	I_2, I_{16}, I_1, I_2	2 : 3
G6	1	1	1	6266	-609505	1	2	-	2, 4, 1, 8	2, 4, 1, 8	2, 2, 1, 2	I_2, I_4, I_1, I_8	2 : 3

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
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966 **966**
 $N = 966 = 2 \cdot 3 \cdot 7 \cdot 23$ (continued)

I1	1	0	0	-599	-9255	0	4	-	4, 6, 1, 4	4, 6, 1, 4	4, 6, 1, 4	I_4, I_6, I_1, I_4	2 : 2
I2	1	0	0	-11179	-455731	0	4	+	2, 12, 2, 2	2, 12, 2, 2	2, 12, 2, 2	I_2, I_{12}, I_2, I_2	2 : 1, 3, 4
I3	1	0	0	-178849	-29127301	0	2	+	1, 6, 4, 1	1, 6, 4, 1	1, 6, 2, 1	I_1, I_6, I_4, I_1	2 : 2
I4	1	0	0	-12789	-316305	0	2	+	1, 24, 1, 1	1, 24, 1, 1	1, 24, 1, 1	I_1, I_{24}, I_1, I_1	2 : 2
J1	1	0	0	9096	224832	0	1	-	9, 1, 11, 1	9, 1, 11, 1	9, 1, 1, 1	I_9, I_1, I_{11}, I_1	
K1	1	0	0	3	9	0	3	-	3, 3, 1, 1	3, 3, 1, 1	3, 3, 1, 1	I_3, I_3, I_1, I_1	3 : 2
K2	1	0	0	-27	-249	0	1	-	1, 1, 3, 3	1, 1, 3, 3	1, 1, 3, 1	I_1, I_1, I_3, I_3	3 : 1

968 **968**
 $N = 968 = 2^3 \cdot 11^2$ (5 isogeny classes)

A1	0	1	0	15	-13	1	1	-	8, 3	0, 0	4, 2	I_1^*, III	
B1	0	0	0	-1331	-29282	0	1	-	10, 8	0, 0	2, 3	III^*, IV^*	
C1	0	1	0	1775	24451	0	1	-	8, 9	0, 0	2, 2	I_1^*, III^*	
D1	0	0	0	-11	22	1	1	-	10, 2	0, 0	2, 1	III^*, II	
E1	0	0	0	-484	-5324	1	1	-	8, 7	0, 1	2, 4	I_1^*, I_1^*	

969 **969**
 $N = 969 = 3 \cdot 17 \cdot 19$ (1 isogeny class)

A1	1	0	1	-10	-1	0	2	+	2, 1, 2	2, 1, 2	2, 1, 2	I_2, I_1, I_2	2 : 2
A2	1	0	1	-105	-419	0	2	+	4, 2, 1	4, 2, 1	4, 2, 1	I_4, I_2, I_1	2 : 1

970 **970**
 $N = 970 = 2 \cdot 5 \cdot 97$ (2 isogeny classes)

A1	1	0	1	-21444	1420226	0	1	-	11, 13, 1	11, 13, 1	1, 1, 1	I_{11}, I_{13}, I_1	
B1	1	0	0	-5	-5	0	1	-	1, 1, 1	1, 1, 1	1, 1, 1	I_1, I_1, I_1	

972 **972**
 $N = 972 = 2^2 \cdot 3^5$ (4 isogeny classes)

A1	0	0	0	0	-12	0	1	-	8, 5	0, 0	1, 1	IV^*, II	3 : 2
A2	0	0	0	0	324	0	3	-	8, 11	0, 0	3, 3	IV^*, IV^*	3 : 1
B1	0	0	0	0	-3	0	1	-	4, 5	0, 0	1, 1	IV, II	3 : 2
B2	0	0	0	0	81	0	3	-	4, 11	0, 0	3, 3	IV, IV^*	3 : 1
C1	0	0	0	0	9	1	3	-	4, 7	0, 0	3, 3	IV, IV	3 : 2
C2	0	0	0	0	-243	1	1	-	4, 13	0, 0	1, 1	IV, II^*	3 : 1
D1	0	0	0	0	36	1	3	-	8, 7	0, 0	3, 3	IV^*, IV	3 : 2
D2	0	0	0	0	-972	1	1	-	8, 13	0, 0	1, 1	IV^*, II^*	3 : 1

973 **973**
 $N = 973 = 7 \cdot 139$ (2 isogeny classes)

A1	0	1	1	-26	43	0	1	+	1, 1	1, 1	1, 1	I_1, I_1	
B1	0	1	1	-203	1048	1	3	+	1, 1	1, 1	1, 1	I_1, I_1	3 : 2
B2	0	1	1	-253	441	1	3	+	3, 3	3, 3	3, 3	I_3, I_3	3 : 1, 3
B3	0	1	1	-11373	-470630	1	1	+	9, 1	9, 1	9, 1	I_9, I_1	3 : 2

974 **974**
 $N = 974 = 2 \cdot 487$ (8 isogeny classes)

A1	1	-1	0	-13	-27	0	1	-	9, 1	9, 1	1, 1	I_9, I_1	
B1	1	1	0	-9421	-355915	0	1	-	3, 1	3, 1	1, 1	I_3, I_1	
C1	1	1	0	8	0	0	2	-	6, 1	6, 1	2, 1	I_6, I_1	2 : 2
C2	1	1	0	-32	-40	0	2	+	3, 2	3, 2	1, 2	I_3, I_2	2 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
974	$N = 974 = 2 \cdot 487$ (continued)												974
E1	1	1	1	-5	3	1	1	-	3, 1	3, 1	3, 1	I_3, I_1	
F1	1	1	1	-91	297	1	1	-	9, 1	9, 1	9, 1	I_9, I_1	
G1	1	-1	1	3	-3	1	1	-	3, 1	3, 1	3, 1	I_3, I_1	
H1	1	-1	1	51	117	1	1	-	15, 1	15, 1	15, 1	I_{15}, I_1	
975	$N = 975 = 3 \cdot 5^2 \cdot 13$ (11 isogeny classes)												975
A1	1	1	0	-2750	54375	1	2	+	4, 7, 1	4, 1, 1	2, 2, 1	I_4, I_1^*, I_1	2 : 2
A2	1	1	0	-2875	49000	1	4	+	8, 8, 2	8, 2, 2	2, 4, 2	I_8, I_2^*, I_2	2 : 1, 3, 4
A3	1	1	0	-13000	-528125	1	4	+	4, 10, 4	4, 4, 4	2, 4, 2	I_4, I_4^*, I_4	2 : 2, 5, 6
A4	1	1	0	5250	284625	1	2	-	16, 7, 1	16, 1, 1	2, 2, 1	I_{16}, I_1^*, I_1	2 : 2
A5	1	1	0	-203125	-35321000	1	4	+	2, 14, 2	2, 8, 2	2, 4, 2	I_2, I_8^*, I_2	2 : 3, 7, 8
A6	1	1	0	15125	-2468750	1	2	-	2, 8, 8	2, 2, 8	2, 4, 2	I_2, I_2^*, I_8	2 : 3
A7	1	1	0	-3250000	-2256492875	1	2	+	1, 10, 1	1, 4, 1	1, 4, 1	I_1, I_4^*, I_1	2 : 5
A8	1	1	0	-198250	-37090625	1	2	-	1, 22, 1	1, 16, 1	1, 4, 1	I_1, I_{16}^*, I_1	2 : 5
B1	0	-1	1	-8	-82	1	1	-	1, 7, 1	1, 1, 1	1, 4, 1	I_1, I_1^*, I_1	
C1	1	1	0	300	14625	0	1	-	6, 10, 1	6, 0, 1	2, 1, 1	I_6, II^*, I_1	
D1	0	-1	1	-1658	-40282	0	1	-	3, 13, 1	3, 7, 1	1, 2, 1	I_3, I_7^*, I_1	
E1	1	1	1	-1138	-15844	0	1	-	2, 8, 3	2, 0, 3	2, 1, 1	I_2, IV^*, I_3	
F1	0	-1	1	-83	3818	1	1	-	5, 9, 1	5, 0, 1	1, 2, 1	I_5, III^*, I_1	
G1	1	0	0	12	-33	0	2	-	1, 6, 1	1, 0, 1	1, 4, 1	I_1, I_0^*, I_1	2 : 2
G2	1	0	0	-113	-408	0	4	+	2, 6, 2	2, 0, 2	2, 4, 2	I_2, I_0^*, I_2	2 : 1, 3, 4
G3	1	0	0	-1738	-28033	0	2	+	4, 6, 1	4, 0, 1	4, 2, 1	I_4, I_0^*, I_1	2 : 2
G4	1	0	0	-488	3717	0	2	+	1, 6, 4	1, 0, 4	1, 2, 2	I_1, I_0^*, I_4	2 : 2
H1	1	0	1	-46	-127	1	1	-	2, 2, 3	2, 0, 3	2, 1, 3	I_2, II, I_3	
I1	0	1	1	-4758	128144	1	1	-	7, 7, 3	7, 1, 3	7, 4, 3	I_7, I_1^*, I_3	
J1	0	1	1	-3	29	1	1	-	5, 3, 1	5, 0, 1	5, 2, 1	I_5, III, I_1	
K1	1	0	0	12	117	1	1	-	6, 4, 1	6, 0, 1	6, 3, 1	I_6, IV, I_1	
976	$N = 976 = 2^4 \cdot 61$ (3 isogeny classes)												976
A1	0	-1	0	40	-16	0	1	-	16, 1	4, 1	2, 1	I_8^*, I_1	
B1	0	-1	0	-32	-64	0	1	-	12, 1	0, 1	2, 1	I_4^*, I_1	
C1	0	0	0	1	-6	1	1	-	8, 1	0, 1	1, 1	I_0^*, I_1	
978	$N = 978 = 2 \cdot 3 \cdot 163$ (8 isogeny classes)												978
A1	1	1	0	-37670	2798484	0	1	-	19, 5, 1	19, 5, 1	1, 1, 1	I_{19}, I_5, I_1	
B1	1	1	0	-9	-15	0	2	+	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	2 : 2
B2	1	1	0	1	-33	0	2	-	1, 2, 2	1, 2, 2	1, 2, 2	I_1, I_2, I_2	2 : 1
C1	1	1	0	-2188119	-1243572651	0	1	+	13, 26, 1	13, 26, 1	1, 2, 1	I_{13}, I_{26}, I_1	
D1	1	1	0	458	-2060	0	1	-	5, 13, 1	5, 13, 1	1, 1, 1	I_5, I_{13}, I_1	
E1	1	0	1	-5	2	1	1	+	1, 2, 1	1, 2, 1	1, 2, 1	I_1, I_2, I_1	
F1	1	1	1	-121	455	1	1	+	11, 2, 1	11, 2, 1	11, 2, 1	I_{11}, I_2, I_1	
G1	1	0	0	-132	144	1	1	+	7, 8, 1	7, 8, 1	7, 8, 1	I_7, I_8, I_1	
H1	1	0	0	-3	9	0	3	-	3 3 1	3 3 1	3 3 1	I_3, I_3, I_3	3 : 2

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
979 $N = 979 = 11 \cdot 89$ (2 isogeny classes) 979													
A1	0	-1	1	1	-2	0	1	-	1, 1	1, 1	1, 1	I_1, I_1	
B1	1	1	0	-14646	-688345	1	2	+	4, 3	4, 3	4, 3	I_4, I_3	2 : 2
B2	1	1	0	-14041	-747030	1	2	-	2, 6	2, 6	2, 6	I_2, I_6	2 : 1
980 $N = 980 = 2^2 \cdot 5 \cdot 7^2$ (9 isogeny classes) 980													
A1	0	1	0	-996	11780	0	3	-	8, 3, 4	0, 3, 0	3, 1, 3	IV^*, I_3, IV	3 : 2
A2	0	1	0	964	51764	0	1	-	8, 9, 4	0, 9, 0	1, 1, 3	IV^*, I_9, IV	3 : 1
B1	0	0	0	-343	-4802	0	1	-	8, 1, 8	0, 1, 0	1, 1, 1	IV^*, I_1, IV^*	
C1	0	1	0	19	-1	1	1	-	8, 1, 3	0, 1, 0	1, 1, 2	IV^*, I_1, III	
D1	0	-1	0	-261	8065	1	1	-	8, 3, 7	0, 3, 1	3, 1, 4	IV^*, I_3, I_1^*	3 : 2
D2	0	-1	0	-39461	3030385	1	1	-	8, 1, 9	0, 1, 3	1, 1, 4	IV^*, I_1, I_3^*	3 : 1
E1	0	-1	0	915	2185	0	1	-	8, 1, 9	0, 1, 0	1, 1, 2	IV^*, I_1, III^*	
F1	0	-1	0	-48820	-4138168	0	1	-	8, 3, 10	0, 3, 0	3, 3, 1	IV^*, I_3, II^*	3 : 2
F2	0	-1	0	47220	-17660600	0	1	-	8, 9, 10	0, 9, 0	1, 9, 1	IV^*, I_9, II^*	3 : 1
G1	0	-1	0	-65	-118	0	2	+	4, 1, 6	0, 1, 0	3, 1, 2	IV, I_1, I_0^*	2 : 2; 3 : 3
G2	0	-1	0	180	-1000	0	2	-	8, 2, 6	0, 2, 0	3, 2, 2	IV^*, I_2, I_0^*	2 : 1; 3 : 4
G3	0	-1	0	-2025	35750	0	2	+	4, 3, 6	0, 3, 0	1, 3, 2	IV, I_3, I_0^*	2 : 4; 3 : 1
G4	0	-1	0	-1780	44472	0	2	-	8, 6, 6	0, 6, 0	1, 6, 2	IV^*, I_6, I_0^*	2 : 3; 3 : 2
H1	0	0	0	-7	14	0	1	-	8, 1, 2	0, 1, 0	1, 1, 1	IV^*, I_1, II	
I1	0	0	0	1568	-72716	0	1	-	8, 1, 11	0, 1, 5	1, 1, 2	IV^*, I_1, I_5^*	
981 $N = 981 = 3^2 \cdot 109$ (2 isogeny classes) 981													
A1	1	-1	0	36	81	1	1	-	10, 1	4, 1	2, 1	I_4^*, I_1	
B1	1	-1	1	-74	262	1	1	-	6, 1	0, 1	2, 1	I_0^*, I_1	
982 $N = 982 = 2 \cdot 491$ (1 isogeny class) 982													
A1	1	0	1	-22	40	1	1	-	8, 1	8, 1	2, 1	I_8, I_1	
984 $N = 984 = 2^3 \cdot 3 \cdot 41$ (4 isogeny classes) 984													
A1	0	-1	0	184	1644	0	1	-	11, 9, 1	0, 9, 1	1, 1, 1	II^*, I_9, I_1	
B1	0	-1	0	-577	-5147	0	1	-	8, 3, 1	0, 3, 1	4, 1, 1	I_1^*, I_3, I_1	
C1	0	-1	0	-369	4293	1	1	-	8, 5, 3	0, 5, 3	2, 1, 3	I_1^*, I_5, I_3	
D1	0	1	0	7	27	1	1	-	8, 3, 1	0, 3, 1	2, 3, 1	I_1^*, I_3, I_1	
985 $N = 985 = 5 \cdot 197$ (2 isogeny classes) 985													
A1	1	-1	0	-89	-302	0	1	-	3, 1	3, 1	3, 1	I_3, I_1	
B1	0	1	1	-20	24	1	1	+	4, 1	4, 1	4, 1	I_4, I_1	
986 $N = 986 = 2 \cdot 17 \cdot 29$ (6 isogeny classes) 986													
A1	1	0	1	9	-34	0	3	-	2, 3, 1	2, 3, 1	2, 3, 1	I_2, I_3, I_1	3 : 2
A2	1	0	1	-586	-5508	0	1	-	6, 1, 3	6, 1, 3	2, 1, 1	I_6, I_1, I_3	3 : 1
B1	1	1	0	-10407	-413003	1	1	-	12, 2, 1	12, 2, 1	2, 2, 1	I_{12}, I_2, I_1	
C1	1	1	0	-276	1616	1	2	+	12, 1, 2	12, 1, 2	2, 1, 2	I_{12}, I_1, I_2	2 : 2
C2	1	1	0	44	5520	1	2	-	6, 2, 4	6, 2, 4	2, 2, 4	I_6, I_2, I_4	2 : 1

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
986 986 $N = 986 = 2 \cdot 17 \cdot 29$ (continued)													
E1	1	0	0	3467	-83679	1	1	-	14, 1, 5	14, 1, 5	14, 1, 5	I_{14}, I_1, I_5	
F1	1	-1	1	-1	17	1	1	-	8, 1, 1	8, 1, 1	8, 1, 1	I_8, I_1, I_1	
987 987 $N = 987 = 3 \cdot 7 \cdot 47$ (5 isogeny classes)													
A1	1	1	0	7	0	0	2	-	2, 2, 1	2, 2, 1	2, 2, 1	I_2, I_2, I_1	2 : 2
A2	1	1	0	-28	-35	0	2	+	4, 1, 2	4, 1, 2	2, 1, 2	I_4, I_1, I_2	2 : 1
B1	1	1	1	-62	-214	0	2	+	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	2 : 2
B2	1	1	1	-67	-184	0	4	+	4, 2, 2	4, 2, 2	2, 2, 2	I_4, I_2, I_2	2 : 1, 3, 4
B3	1	1	1	-382	2588	0	4	+	2, 1, 4	2, 1, 4	2, 1, 4	I_2, I_1, I_4	2 : 2
B4	1	1	1	168	-936	0	2	-	8, 4, 1	8, 4, 1	2, 2, 1	I_8, I_4, I_1	2 : 2
C1	0	-1	1	-208	1227	0	1	-	3, 3, 1	3, 3, 1	1, 1, 1	I_3, I_3, I_1	
D1	0	1	1	-2066	100013	0	1	-	7, 5, 3	7, 5, 3	7, 1, 1	I_7, I_5, I_3	
E1	1	0	0	1596	9783	1	2	-	10, 2, 3	10, 2, 3	10, 2, 3	I_{10}, I_2, I_3	2 : 2
E2	1	0	0	-6909	79524	1	2	+	5, 1, 6	5, 1, 6	5, 1, 6	I_5, I_1, I_6	2 : 1
988 988 $N = 988 = 2^2 \cdot 13 \cdot 19$ (4 isogeny classes)													
A1	0	-1	0	114	-247	0	1	-	4, 5, 1	0, 5, 1	3, 1, 1	IV, I_5, I_1	
B1	0	0	0	-362249	165197113	1	1	-	4, 1, 13	0, 1, 13	3, 1, 13	IV, I_1, I_{13}	
C1	0	0	0	16	36	1	1	-	8, 2, 1	0, 2, 1	3, 2, 1	IV^*, I_2, I_1	
D1	0	1	0	-18	-71	0	3	-	4, 1, 3	0, 1, 3	3, 1, 3	IV, I_1, I_3	3 : 2
D2	0	1	0	-1918	-32979	0	1	-	4, 3, 1	0, 3, 1	1, 3, 1	IV, I_3, I_1	3 : 1
989 989 $N = 989 = 23 \cdot 43$ (1 isogeny class)													
A1	1	-1	0	-241	1502	0	1	-	1, 1	1, 1	1, 1	I_1, I_1	
990 990 $N = 990 = 2 \cdot 3^2 \cdot 5 \cdot 11$ (12 isogeny classes)													
A1	1	-1	0	-15	25	1	2	+	2, 3, 2, 1	2, 0, 2, 1	2, 2, 2, 1	I_2, III, I_2, I_1	2 : 2
A2	1	-1	0	15	91	1	2	-	1, 3, 4, 2	1, 0, 4, 2	1, 2, 2, 2	I_1, III, I_4, I_2	2 : 1
B1	1	-1	0	-10734	430740	0	6	+	6, 3, 6, 1	6, 0, 6, 1	2, 2, 6, 1	I_6, III, I_6, I_1	2 : 2; 3 : 3
B2	1	-1	0	-10614	440748	0	6	-	3, 3, 12, 2	3, 0, 12, 2	1, 2, 12, 2	I_3, III, I_{12}, I_2	2 : 1; 3 : 4
B3	1	-1	0	-14109	140165	0	2	+	18, 9, 2, 3	18, 0, 2, 3	2, 2, 2, 1	I_{18}, III^*, I_2, I_3	2 : 4; 3 : 1
B4	1	-1	0	55011	1066373	0	2	-	9, 9, 4, 6	9, 0, 4, 6	1, 2, 4, 2	I_9, III^*, I_4, I_6	2 : 3; 3 : 2
C1	1	-1	0	2295	-4595	0	2	-	16, 9, 1, 2	16, 3, 1, 2	2, 2, 1, 2	I_{16}, I_3^*, I_1, I_2	2 : 2
C2	1	-1	0	-9225	-29939	0	4	+	8, 12, 2, 4	8, 6, 2, 4	2, 4, 2, 2	I_8, I_6^*, I_2, I_4	2 : 1, 3, 4
C3	1	-1	0	-106425	-13307459	0	2	+	4, 9, 1, 8	4, 3, 1, 8	2, 4, 1, 2	I_4, I_3^*, I_1, I_8	2 : 2
C4	1	-1	0	-96345	11487325	0	4	+	4, 18, 4, 2	4, 12, 4, 2	2, 4, 2, 2	I_4, I_{12}^*, I_4, I_2	2 : 2, 5, 6
C5	1	-1	0	-1539765	735795481	0	2	+	2, 12, 8, 1	2, 6, 8, 1	2, 2, 2, 1	I_2, I_6^*, I_8, I_1	2 : 4
C6	1	-1	0	-46845	23238625	0	2	-	2, 30, 2, 1	2, 24, 2, 1	2, 4, 2, 1	I_2, I_{24}^*, I_2, I_1	2 : 4
D1	1	-1	0	90	1300	0	1	-	5, 6, 5, 1	5, 0, 5, 1	1, 1, 1, 1	I_5, I_0^*, I_5, I_1	5 : 2
D2	1	-1	0	-53460	4771030	0	1	-	1, 6, 1, 5	1, 0, 1, 5	1, 1, 1, 1	I_1, I_0^*, I_1, I_5	5 : 1
E1	1	-1	0	45	-459	1	2	-	8, 8, 1, 1	8, 2, 1, 1	2, 2, 1, 1	I_8, I_2^*, I_1, I_1	2 : 2
E2	1	-1	0	-675	-6075	1	4	+	4, 10, 2, 2	4, 4, 2, 2	2, 4, 2, 2	I_4, I_4^*, I_2, I_2	2 : 1, 3, 4
E3	1	-1	0	-10575	-415935	1	2	+	2, 14, 1, 1	2, 8, 1, 1	2, 4, 1, 1	I_2, I_8^*, I_1, I_1	2 : 2
E4	1	-1	0	-2295	35721	1	4	+	2, 8, 4, 4	2, 2, 4, 4	2, 4, 2, 4	I_2, I_2^*, I_4, I_4	2 : 2, 5, 6
E5	1	-1	0	-34965	2525175	1	2	+	1, 7, 8, 2	1, 1, 8, 2	1, 2, 2, 2	I_1, I_8^*, I_8, I_2	2 : 4

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
990 $N = 990 = 2 \cdot 3^2 \cdot 5 \cdot 11$ (continued) 990													
F1	1	-1	0	-9	-27	0	1	-	3, 6, 1, 1	3, 0, 1, 1	1, 1, 1, 1	I_3, I_0^*, I_1, I_1	3 : 2
F2	1	-1	0	81	675	0	3	-	1, 6, 3, 3	1, 0, 3, 3	1, 1, 3, 3	I_1, I_0^*, I_3, I_3	3 : 1
G1	1	-1	0	-362394	-79244492	0	2	+	28, 11, 4, 1	28, 5, 4, 1	2, 4, 4, 1	I_{28}, I_5^*, I_4, I_1	2 : 2
G2	1	-1	0	-1099674	346460980	0	4	+	14, 16, 8, 2	14, 10, 8, 2	2, 4, 8, 2	$I_{14}, I_{10}^*, I_8, I_2$	2 : 1, 3, 4
G3	1	-1	0	-16496154	25790683828	0	2	+	7, 11, 16, 1	7, 5, 16, 1	1, 2, 16, 1	I_7, I_5^*, I_{16}, I_1	2 : 2
G4	1	-1	0	2500326	2138540980	0	2	-	7, 26, 4, 4	7, 20, 4, 4	1, 4, 4, 4	I_7, I_{20}^*, I_4, I_4	2 : 2
H1	1	-1	1	-96608	-11533373	1	2	+	6, 9, 6, 1	6, 0, 6, 1	6, 2, 2, 1	I_6, III^*, I_6, I_1	2 : 2; 3 : 3
H2	1	-1	1	-95528	-11804669	1	2	-	3, 9, 12, 2	3, 0, 12, 2	3, 2, 2, 2	I_3, III^*, I_{12}, I_2	2 : 1; 3 : 4
H3	1	-1	1	-1568	-4669	1	6	+	18, 3, 2, 3	18, 0, 2, 3	18, 2, 2, 3	I_{18}, III, I_2, I_3	2 : 4; 3 : 1
H4	1	-1	1	6112	-41533	1	6	-	9, 3, 4, 6	9, 0, 4, 6	9, 2, 2, 6	I_9, III, I_4, I_6	2 : 3; 3 : 2
I1	1	-1	1	-137	-539	0	2	+	2, 9, 2, 1	2, 0, 2, 1	2, 2, 2, 1	I_2, III^*, I_2, I_1	2 : 2
I2	1	-1	1	133	-2591	0	2	-	1, 9, 4, 2	1, 0, 4, 2	1, 2, 4, 2	I_1, III^*, I_4, I_2	2 : 1
J1	1	-1	1	-203	987	1	4	+	8, 7, 2, 1	8, 1, 2, 1	8, 4, 2, 1	I_8, I_1^*, I_2, I_1	2 : 2
J2	1	-1	1	-923	-9669	1	4	+	4, 8, 4, 2	4, 2, 4, 2	4, 4, 2, 2	I_4, I_2^*, I_4, I_2	2 : 1, 3, 4
J3	1	-1	1	-14423	-663069	1	2	+	2, 7, 2, 4	2, 1, 2, 4	2, 2, 2, 2	I_2, I_1^*, I_2, I_4	2 : 2
J4	1	-1	1	1057	-46893	1	2	-	2, 10, 8, 1	2, 4, 8, 1	2, 4, 2, 1	I_2, I_4^*, I_8, I_1	2 : 2
K1	1	-1	1	-12542	543741	0	4	+	4, 11, 2, 1	4, 5, 2, 1	4, 4, 2, 1	I_4, I_5^*, I_2, I_1	2 : 2
K2	1	-1	1	-12722	527469	0	4	+	2, 16, 4, 2	2, 10, 4, 2	2, 4, 4, 2	I_2, I_{10}^*, I_4, I_2	2 : 1, 3, 4
K3	1	-1	1	-37472	-2125731	0	2	+	1, 26, 2, 1	1, 20, 2, 1	1, 4, 2, 1	I_1, I_{20}^*, I_2, I_1	2 : 2
K4	1	-1	1	9148	2137101	0	2	-	1, 11, 8, 4	1, 5, 8, 4	1, 2, 8, 2	I_1, I_5^*, I_8, I_4	2 : 2
L1	1	-1	1	-797	-8539	0	1	-	7, 6, 1, 3	7, 0, 1, 3	7, 1, 1, 1	I_7, I_0^*, I_1, I_3	3 : 2
L2	1	-1	1	2668	-45961	0	3	-	21, 6, 3, 1	21, 0, 3, 1	21, 1, 3, 1	I_{21}, I_0^*, I_3, I_1	3 : 1

994 $N = 994 = 2 \cdot 7 \cdot 71$ (7 isogeny classes) 994													
A1	1	0	1	-1	4	1	1	-	4, 1, 1	4, 1, 1	2, 1, 1	I_4, I_1, I_1	
B1	1	0	1	255	-796	0	2	-	4, 5, 2	4, 5, 2	2, 1, 2	I_4, I_5, I_2	2 : 2
B2	1	0	1	-1165	-7044	0	2	+	2, 10, 1	2, 10, 1	2, 2, 1	I_2, I_{10}, I_1	2 : 1
C1	1	1	0	-371	-3091	0	2	-	8, 3, 2	8, 3, 2	2, 3, 2	I_8, I_3, I_2	2 : 2
C2	1	1	0	-6051	-183715	0	2	+	4, 6, 1	4, 6, 1	2, 6, 1	I_4, I_6, I_1	2 : 1
D1	1	0	1	164	922	1	3	-	8, 1, 3	8, 1, 3	2, 1, 3	I_8, I_1, I_3	3 : 2
D2	1	0	1	-1611	-39690	1	1	-	24, 3, 1	24, 3, 1	2, 3, 1	I_{24}, I_3, I_1	3 : 1
E1	1	0	0	-11	13	0	2	+	2, 1, 1	2, 1, 1	2, 1, 1	I_2, I_1, I_1	2 : 2
E2	1	0	0	-21	-17	0	2	+	1, 2, 2	1, 2, 2	1, 2, 2	I_1, I_2, I_2	2 : 1
F1	1	-1	1	-16	-13	1	2	+	8, 1, 1	8, 1, 1	8, 1, 1	I_8, I_1, I_1	2 : 2
F2	1	-1	1	-96	371	1	4	+	4, 2, 2	4, 2, 2	4, 2, 2	I_4, I_2, I_2	2 : 1, 3, 4
F3	1	-1	1	-1516	23091	1	4	+	2, 4, 1	2, 4, 1	2, 4, 1	I_2, I_4, I_1	2 : 2
F4	1	-1	1	44	1267	1	2	-	2, 1, 4	2, 1, 4	2, 1, 2	I_2, I_1, I_4	2 : 2
G1	1	0	0	-678	-5660	1	6	+	18, 3, 1	18, 3, 1	18, 3, 1	I_{18}, I_3, I_1	2 : 2; 3 : 3
G2	1	0	0	-3238	65508	1	6	+	9, 6, 2	9, 6, 2	9, 6, 2	I_9, I_6, I_2	2 : 1; 3 : 4
G3	1	0	0	-52198	-4594524	1	2	+	6, 1, 3	6, 1, 3	6, 1, 1	I_6, I_1, I_3	2 : 4; 3 : 1
G4	1	0	0	-52238	-4587140	1	2	+	3, 2, 6	3, 2, 6	3, 2, 2	I_3, I_2, I_6	2 : 3; 3 : 2

995 $N = 995 = 5 \cdot 199$ (2 isogeny classes) 995													
A1	1	0	1	2	3	0	2	-	2, 1	2, 1	2, 1	I_2, I_1	2 : 2

TABLE 1: ELLIPTIC CURVES 995B–999B

	a_1	a_2	a_3	a_4	a_6	r	$ T $	s	$\text{ord}(\Delta)$	$\text{ord}_-(j)$	c_p	Kodaira	Isogenies
995 995													
$N = 995 = 5 \cdot 199$ (continued)													
B1	0	1	1	-15	19	1	3	-	3, 1	3, 1	3, 1	I_3, I_1	3 : 2
B2	0	1	1	85	64	1	1	-	1, 3	1, 3	1, 3	I_1, I_3	3 : 1
996 996													
$N = 996 = 2^2 \cdot 3 \cdot 83$ (3 isogeny classes)													
A1	0	-1	0	19	-42	0	2	-	4, 6, 1	0, 6, 1	3, 2, 1	IV, I_6, I_1	2 : 2
A2	0	-1	0	-116	-312	0	2	+	8, 3, 2	0, 3, 2	3, 1, 2	IV^*, I_3, I_2	2 : 1
B1	0	1	0	164	-8764	1	1	-	8, 13, 1	0, 13, 1	3, 13, 1	IV^*, I_{13}, I_1	
C1	0	1	0	-12	36	1	3	-	8, 3, 1	0, 3, 1	3, 3, 1	IV^*, I_3, I_1	3 : 2
C2	0	1	0	108	-876	1	1	-	8, 1, 3	0, 1, 3	1, 1, 1	IV^*, I_1, I_3	3 : 1
997 997													
$N = 997 = 997$ (3 isogeny classes)													
A1	0	-1	1	-18	36	1	1	+	1	1	1	I_1	
B1	0	-1	1	-5	-3	2	1	+	1	1	1	I_1	
C1	0	-1	1	-24	54	2	1	+	1	1	1	I_1	
999 999													
$N = 999 = 3^3 \cdot 37$ (2 isogeny classes)													
A1	1	-1	0	-69	-208	1	1	-	9, 1	0, 1	1, 1	IV^*, I_1	
B1	1	-1	1	-8	10	1	1	-	3, 1	0, 1	1, 1	II, I_1	

TABLE 2

MORDELL–WEIL GENERATORS

This table contains an entry for the strong Weil curve in each isogeny class¹ of positive rank. For each we give the (x, y) coordinates of generators of the Mordell–Weil group (modulo torsion) with respect to the minimal equation of Table 1. In a few cases the coordinates are not integral, in which case we give them in the form $(a/c^2, b/c^3)$ with $a, b, c \in \mathbb{Z}$ and $\gcd(a, b, c) = 1$.

¹This is the first curve in the class except for class 990H, where we give a generator for the curve 990H3.

Curve	x	y	Curve	x	y	Curve	x	y
37A1 (A)	0	0	156A1 (E)	1	1	224A1	1	2
43A1 (A)	0	0	158A1 (E)	-1	4	225A1	1	1
53A1 (A)	0	0	158B1 (D)	0	1	225E1	-5	22
57A1 (E)	2	1	160A1 (D)	0	2	226A1	0	1
58A1 (A)	0	1	162A1 (K)	2	0	228B1	3	6
61A1 (A)	1	0	163A1 (A)	1	0	229A1	-1	1
65A1 (A)	1	0	166A1 (A)	0	2	232A1	2	4
77A1 (F)	2	3	170A1 (A)	0	2	234C1	1	1
79A1 (A)	0	0	171B1 (A)	2	4	235A1	-2	3
82A1 (A)	0	0	172A1 (A)	2	1	236A1	1	1
83A1 (A)	0	0	175A1 (B)	2	-3	238A1	24	100
88A1 (A)	2	2	175B1 (C)	-3	12	238B1	1	1
89A1 (C)	0	0	176C1 (A)	1	2	240C1	1	2
91A1 (A)	0	0	184A1 (C)	0	1	242A1	0	1
91B1 (B)	-1	3	184B1 (B)	2	1	243A1	1	0
92B1 (C)	1	1	185A1 (D)	4	12	244A1	-1	2
99A1 (A)	0	0	185B1 (A)	0	2	245A1	7	17
101A1 (A)	-1	0	185C1 (B)	3	2	245C1	12	24
102A1 (E)	-1	2	189A1 (A)	-1	1	246D1	3	3
106B1 (A)	2	1	189B1 (C)	-3	9	248A1	0	1
112A1 (K)	0	2	190A1 (D)	13	33	248C1	1	1
117A1 (A)	0	2	190B1 (C)	1	2	249A1	4	-2
118A1 (A)	0	1	192A1 (Q)	3	2	249B1	0	1
121B1 (D)	4	5	196A1 (A)	0	1	252B1	-2	9
122A1 (A)	1	1	197A1 (A)	1	0	254A1	2	0
123A1 (A)	1	1	198A1 (I)	-1	5	254C1	-1	1
123B1 (C)	1	0	200B1 (C)	-1	1	256A1	0	1
124A1 (B)	1	1	201A1	1	1	256B1	-1	1
128A1 (C)	0	1	201B1	-1	2	258A1	2	3
129A1 (E)	1	4	201C1	16	-7	258C1	5	6
130A1 (E)	2	2	203B1	2	2	262A1	-2	5
131A1 (A)	0	0	205A1	-1	8	262B1	1	0
135A1 (A)	4	7	207A1	0	4	265A1	8	0
136A1 (A)	-2	2	208A1	4	8	269A1	-1	0
138A1 (E)	0	1	208B1	4	4	272A1	0	2
141A1 (E)	-3	4	209A1	-5	9	272B1	-1	2
141D1 (I)	0	0	210D1	-1	1	273A1	11	31
142A1 (F)	1	1	212A1	2	2	274A1	2	1
142B1 (E)	-1	1	214A1	0	4	274B1	31	-15
143A1 (A)	4	6	214B1	0	0	274C1	-1	1
145A1 (A)	0	1	214C1	11	10	275A1	8	21
148A1 (A)	-1	2	215A1	6	12	277A1	1	0
152A1 (A)	-1	2	216A1	-2	6	278A1	2	3
153A1 (C)	0	1	218A1	-2	2	280A1	1	2
153B1 (A)	5	13	219A1	2	0	280B1	-18	70
154A1 (C)	2	3	219B1	2	4	282B1	3	2
155A1 (D)	2	5	219C1	-6	7	285A1	1	4
155C1 (C)	1	0	220A1	3	1	285B1	6	13

Curve	x	y	Curve	x	y	Curve	x	y
286 B1	19	78	333 B1	2	7	372 D1	-2	3
286 C1	1	5	333 C1	2	1	373 A1	-1	0
288 A1	1	2	335 A1	2	2	374 A1	-1	6
288 B1	-3	4	336 E1	2	6	377 A1	-2	5
289 A1	-12	38	338 A1	0	1	378 D1	2	2
290 A1	-5	4	338 E1	5	10	378 F1	4	11
291 C1	0	0	338 F1	23	73	380 A1	-1	2
294 G1	1	5	339 A1	18	40	381 A1	-2	1
296 A1	1	2	339 C1	1	1	384 D1	2	1
296 B1	3	2	340 A1	4	3	384 H1	4	3
297 A1	15	49	342 C1	-3	15	385 A1	2	-9
297 B1	0	0	342 E1	0	1	385 B1	-1	3
297 C1	4	7	344 A1	0	2	387 B1	10	22
298 A1	2	1	345 B1	-1	1	387 C1	0	1
298 B1	1	0	345 F1	5	7	389 A1	$\begin{cases} 0 \\ 1 \end{cases}$	$\begin{cases} 0 \\ 0 \end{cases}$
300 D1	1	3	346 B1	-1	2	390 A1	1	1
302 A1	7	3	347 A1	0	0	392 A1	9	22
302 C1	1	1	348 A1	0	1	392 C1	-2	7
303 A1	-2	13	348 D1	10	27	392 F1	1	1
303 B1	0	1	350 C1	1	3	396 B1	2	9
304 A1	10	32	350 F1	-1	35	399 A1	-10	33
304 C1	0	4	352 B1	1	4	399 B1	-2	1
304 F1	3	2	352 C1	3	4	400 A1	15	50
306 B1	-2	5	352 D1	3	4	400 C1	12	40
308 A1	7	14	352 F1	12	44	400 H1	1	4
309 A1	3	3	354 C1	13	7	402 A1	4	6
310 B1	6	0	354 F1	3	4	402 D1	7	2
312 B1	-1	1	356 A1	2	2	404 A1	0	2
312 F1	-1	3	357 B1	4	3	405 B1	1	3
314 A1	6	13	357 D1	0	10	405 C1	0	1
315 B1	-2	1	359 A1	3	-1	405 D1	4	2
316 B1	-1	2	359 B1	2	-1	405 F1	-1	0
318 C1	5	11	360 E1	-2	1	406 A1	9	10
318 D1	1	5	361 A1	0	9	406 B1	3	12
320 B1	1	1	362 A1	1	0	406 C1	7	3
320 F1	-2	1	362 B1	1	3	408 D1	7	18
322 A1	-2	8	364 A1	-8	98	410 A1	1	2
322 D1	0	2	364 B1	1	2	410 D1	8	16
324 C1	1	1	366 F1	3	4	414 C1	5	11
325 A1	2	9	366 G1	-3	13	414 D1	1	17
325 B1	1	0	368 A1	3	6	416 B1	0	2
326 A1	-5	3	368 D1	1	1	418 B1	5	19
326 B1	0	2	368 E1	1	1	422 A1	2	1
327 A1	1	1	368 G1	4	1	423 A1	-2	4
328 A1	-2	2	369 A1	1	4	423 C1	18	63
330 E1	-3	4	370 A1	1	0	423 F1	8	4
331 A1	1	0	371 A1	14	42	423 G1	1	1
333 A1	-3	0	372 A1	0	3			

Curve	x	y	Curve	x	y	Curve	x	y
425 A1	0	4	455 B1	14	36	493 B1	40	226
425 B1	10	20	456 C1	3	6	494 A1	3	8
425 C1	1	0	456 D1	23	114	494 D1	45	224
425 D1	-9	5	458 A1	2	1	495 A1	2	2
426 A1	7	10	458 B1	-3	5	496 A1	0	1
427 B1	1	0	459 A1	2	1	496 E1	2	1
427 C1	-3	1	459 B1	4	8	496 F1	7	14
428 B1	1	1	459 H1	-2	5	497 A1	2	6
429 A1	0	1	460 C1	-6	25	498 B1	-1	6
429 B1	6	-15	460 D1	4	5	503 A1	7	4
430 A1	3	-1	462 A1	4	7	504 A1	0	3
430 B1	1	2	462 C1	1	2	504 E1	2	1
430 C1	-2	0	462 E1	-17	92	504 F1	6	5
430 D1	-26	213	464 A1	0	2	505 A1	6	9
431 A1	1	0	464 B1	6	2	506 A1	-4	2
432 B1	2	2	465 A1	0	-4	506 D1	17	-3
432 D1	5	12	465 B1	7	13	506 E1	-1	1
432 F1	5	16	467 A1	1	0	506 F1	4	2
433 A1	$\begin{cases} -1 \\ 0 \end{cases}$	$\begin{cases} 0 \\ 1 \end{cases}$	468 C1	0	9	507 A1	70	472
434 A1	-1	2	469 A1	-5	4	507 B1	2	0
434 D1	0	7	469 B1	2	-1	507 C1	$94/3^2$	$913/3^3$
437 A1	10	34	470 A1	1	7	510 D1	3	4
438 C1	-1	2	470 C1	-8	29	513 A1	8	-3
438 D1	24	-20	470 E1	1	0	513 B1	2	-3
438 F1	1	0	470 F1	-9	24	514 A1	-7	6
438 G1	0	1	471 A1	0	1	514 B1	2	0
440 A1	-4	1	472 A1	0	1	516 B1	7	-18
440 B1	2	3	472 E1	0	2	517 C1	$85/2^2$	$513/2^3$
441 B1	2	4	473 A1	15	21	520 A1	-1	8
441 C1	30	-211	474 A1	14	57	522 A1	7	10
441 D1	2	2	474 B1	1	2	522 E1	-1	14
441 F1	4	4	475 B1	10	31	522 F1	6	13
442 B1	-9	21	475 C1	0	1	522 I1	1	2
443 A1	-1	0	477 A1	2	0	522 J1	11	-24
443 B1	-1	1	480 A1	-1	2	524 A1	10	1
444 B1	3	3	480 F1	-1	10	525 A1	6	3
446 A1	4	2	481 A1	$87/2^2$	$63/2^3$	525 C1	14	1
446 B1	-5	10	482 A1	17	55	525 D1	3	0
446 D1	$\begin{cases} 0 \\ 1 \end{cases}$	$\begin{cases} 2 \\ 0 \end{cases}$	484 A1	18	121	528 A1	-2	2
448 A1	0	4	485 B1	0	0	528 G1	-6	2
448 B1	4	8	486 A1	2	3	528 H1	-2	24
448 G1	1	4	486 B1	-1	1	530 B1	-1	2
450 C1	9	18	486 F1	1	2	530 C1	156	1922
450 F1	-1	38	490 A1	1	12	530 D1	1	4
451 A1	7	20	490 D1	0	1	534 A1	3	-5
455 A1	2	-5	490 G1	-2	21	539 C1	123	1310
			492 A1	3	2	539 D1	9	-25
			492 B1	-7	18	540 B1	0	1

Curve	x	y	Curve	x	y	Curve	x	y
540 C1	16	10	574 H1	1	2	608 E1	128	1444
540 D1	1	1	574 I1	61	18	608 F1	1	2
542 B1	1	1	575 A1	0	1	609 A1	$-1/2^2$	$15/2^3$
544 A1	0	2	575 B1	45	312	609 B1	$211/2^2$	$2529/2^3$
545 A1	6	17	575 D1	-6	15	610 B1	7	-1
546 C1	-4	3	575 E1	3	-3	612 B1	8	6
549 A1	4	6	576 A1	1	3	612 C1	-4	18
549 B1	2	4	576 H1	4	10	614 A1	5	-2
550 A1	5	10	576 I1	1	9	614 B1	0	1
550 F1	52	286	579 B1	-1	2	615 A1	-2	2
550 G1	1	1	580 A1	-2	1	615 B1	22	112
550 I1	-35	-258	580 B1	-2	5	616 A1	10	44
550 J1	-1	10	582 A1	-3	3	616 D1	29	154
551 A1	7	15	582 C1	1	3	616 E1	6	3
551 B1	5	7	585 A1	8	8	618 A1	0	2
551 C1	$509/2^2$	$10465/2^3$	585 D1	5	4	618 B1	61	-1
551 D1	9	14	585 F1	238	3513	618 C1	$11/2^2$	$-9/2^3$
552 A1	18	64	585 G1	-1	4	618 D1	15	28
552 D1	17	5	585 H1	4	2	618 E1	-1	2
552 E1	-1	6	585 I1	8	-68	618 F1	10	19
556 A1	2	1	586 B1	-7	19	620 A1	2	13
557 A1	0	1	586 C1	1	0	620 B1	18	5
558 A1	1	1	588 B1	5	49	620 C1	0	2
558 D1	9	-45	588 C1	-1	1	621 B1	-2	1
558 F1	-3	7	590 C1	1	2	622 A1	1	1
558 G1	9	4	590 D1	10	-10	623 A1	12	43
560 D1	-1	10	591 A1	0	1	624 A1	2	2
560 E1	6	14	592 A1	-2	1	624 B1	6	14
560 F1	5	10	592 D1	-1	2	624 F1	12	36
561 B1	34	181	592 E1	-4	1	624 G1	0	2
561 C1	1	1	593 A1	1	0	626 A1	1	1
563 A1	$\left\{ \begin{matrix} 2 \\ 4 \end{matrix} \right.$	$\left. \begin{matrix} -1 \\ 4 \end{matrix} \right\}$	594 A1	0	6	629 A1	2	2
564 A1	-8	1	594 D1	18	79	629 C1	-6	8
564 B1	1	6	598 A1	1	19	629 D1	16	47
566 A1	0	2	598 B1	-5	14	630 D1	12	50
567 A1	4	5	598 D1	21	-103	630 E1	2	9
567 B1	10	26	600 A1	11	3	632 A1	0	4
570 A1	4	-10	600 B1	1	2	633 A1	12	34
570 C1	-2	11	600 E1	43	270	635 A1	$-3/2^2$	$9/2^3$
570 E1	2	3	603 E1	6	6	635 B1	2	0
571 B1	$\left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right.$	$\left. \begin{matrix} 1 \\ 0 \end{matrix} \right\}$	603 F1	5	4	637 A1	6	-2
573 C1	-1	1	605 A1	212	2919	637 C1	$4776/7^2$	$158761/7^3$
574 A1	-1	1	605 B1	$84/5^2$	$563/5^3$	637 D1	7	24
574 B1	17	65	605 C1	6	9	639 A1	6	10
574 F1	-2	19	606 B1	0	1	640 A1	$17/2^2$	$15/2^3$
574 G1	-1	4	606 E1	0	24	640 B1	-2	6
			608 A1	4	4	640 G1	0	2
			608 D1	0	4	640 H1	0	5

Curve	x	y	Curve	x	y	Curve	x	y
642 C1	15	64	670 B1	$-5/2^2$	$11/2^3$	702 M1	-29	86
643 A1	$\left\{ \begin{array}{l} 1 \\ 2 \end{array} \right\}$	$\left\{ \begin{array}{l} 0 \\ 1 \end{array} \right\}$	670 C1	3	0	703 B1	7	18
644 A1	13	49	670 D1	8	12	704 A1	0	1
644 B1	4	7	672 A1	0	2	704 B1	0	1
645 E1	51	607	672 B1	0	42	704 J1	2	1
645 F1	1	7	672 E1	-1	6	704 K1	2	1
646 D1	9	11	672 F1	6	12	704 L1	5	11
648 A1	-1	4	674 A1	0	0	705 A1	120	1093
648 B1	-1	1	674 B1	3	1	705 B1	312	5366
648 D1	-3	9	674 C1	157	1969	705 D1	1	2
649 A1	3	4	675 A1	5	12	705 E1	3	-3
650 A1	-2	9	675 B1	0	1	706 A1	1	1
650 B1	84	726	675 I1	-6	28	706 B1	41	-277
650 C1	5	4	677 A1	0	0	706 C1	7	12
650 G1	3	-1	678 A1	2	0	706 D1	0	2
650 K1	25	117	678 B1	5	9	707 A1	$\left\{ \begin{array}{l} 3 \\ 0 \\ 0 \end{array} \right\}$	$\left\{ \begin{array}{l} 3 \\ 3 \\ 0 \end{array} \right\}$
651 C1	$11/2^2$	$27/2^3$	678 C1	29	129	709 A1	$\left\{ \begin{array}{l} 0 \\ -1 \end{array} \right\}$	$\left\{ \begin{array}{l} 0 \\ 0 \end{array} \right\}$
651 D1	$15/2^2$	$69/2^3$	680 A1	6	4	710 A1	-3	4
654 A1	17	45	681 A1	4	4	710 B1	-11	85
654 B1	-3	37	681 C1	$\left\{ \begin{array}{l} -1 \\ 0 \end{array} \right\}$	$\left\{ \begin{array}{l} 0 \\ 1 \end{array} \right\}$	710 C1	3	3
655 A1	$\left\{ \begin{array}{l} 1 \\ 3 \end{array} \right\}$	$\left\{ \begin{array}{l} 2 \\ 2 \end{array} \right\}$	681 E1	7	4	711 A1	2	2
656 A1	3	2	682 A1	-6	11	711 B1	4	-16
657 C1	2	4	682 B1	15	36	713 A1	-1	1
657 D1	4	2	684 A1	4	18	714 A1	13	196
658 D1	5	13	684 B1	10	27	714 D1	2	3
658 E1	5	165	685 A1	2	0	714 F1	-1	10
658 F1	3	-1	688 A1	1	1	715 A1	-2	3
659 A1	$-50/3^2$	$76/3^3$	688 C1	4	3	715 B1	87	812
660 B1	1	3	689 A1	3	1	718 B1	$\left\{ \begin{array}{l} 0 \\ -1 \end{array} \right\}$	$\left\{ \begin{array}{l} 0 \\ 2 \end{array} \right\}$
660 C1	-3	15	690 A1	11	32	718 C1	13	-5
662 A1	25	115	690 E1	-14	11	720 A1	1	4
663 B1	$51/2^2$	$-43/2^3$	690 H1	-1	5	720 E1	5	16
663 C1	-3	3	693 B1	1	3	720 G1	7	-30
664 A1	$\left\{ \begin{array}{l} 2 \\ 1 \end{array} \right\}$	$\left\{ \begin{array}{l} 2 \\ 2 \end{array} \right\}$	696 A1	6	1	720 H1	-1	18
664 B1	-1	1	696 C1	0	3	722 A1	$27444/13^2$	$4423160/13^3$
664 C1	1	1	696 F1	12	29	722 B1	5	-12
665 A1	4	22	696 G1	2	3	722 E1	93	314
665 B1	$-119/8^2$	$527/8^3$	700 C1	1	1	722 F1	-1	1
665 C1	0	1	700 D1	180	2450	723 A1	2	0
665 D1	-18	66	700 E1	-26	7	723 B1	2	1
666 C1	3	12	700 F1	0	25	725 A1	8	8
666 D1	5	3	700 G1	0	10	726 A1	-2	5
666 E1	27	130	702 A1	5	4	726 D1	3	1
669 A1	2	2	702 B1	-1	1	726 E1	-34	198
670 A1	31	47	702 H1	$7/2^2$	$151/2^3$	726 G1	17	-108
			702 K1	-1	12			
			702 L1	25	104			

Curve	x	y	Curve	x	y	Curve	x	y
728 C1	12	26	763 A1	-2	3	798 A1	0	2
728 D1	5	14	765 C1	-4	24	798 C1	3	7
730 F1	17	-1	768 A1	2	3	798 D1	-4	12
730 G1	-1	1	768 B1	1	2	798 G1	-9	23
730 I1	1	3	768 G1	0	3	798 H1	8	-67
730 J1	-7	-22	768 H1	3	6	799 B1	-2	26
731 A1	13	-5	770 D1	4	25	800 A1	-4	6
732 B1	10	18	770 E1	19	64	800 B1	2	2
732 C1	1	-3	770 F1	6	52	800 C1	-8	2
735 C1	0	2	774 D1	66	-609	800 H1	-1	2
735 E1	13	40	774 E1	-3	-3	800 I1	-8	50
735 F1	-19	-53	774 F1	9	-2	801 C1	8	13
737 A1	106	1105	774 G1	-1	9	801 D1	6	8
738 A1	1	13	775 A1	-2	12	804 B1	71	-486
738 D1	41	101	776 A1	1	6	804 C1	2	2
738 E1	3	-8	777 D1	4	5	804 D1	12	54
738 F1	-7	75	777 E1	43	50	805 A1	$181/2^2$	$15015/2^3$
740 B1	-3	10	777 F1	0	1	806 A1	6	12
740 C1	-5	10	777 G1	-6	10	806 B1	5	13
741 E1	7	19	780 A1	5	5	806 C1	12	25
742 A1	3	2	780 C1	-3	-15	806 D1	137	1543
742 E1	277	-4034	781 B1	14	16	810 D1	4	4
742 G1	9	23	782 A1	0	2	810 H1	5	7
744 A1	2	-3	784 A1	-3	1	811 A1	2	1
744 C1	8	27	784 B1	0	49	812 B1	-6	14
744 F1	44	279	784 H1	1	8	813 B1	2	7
744 G1	6	3	784 I1	-1	1	814 A1	-2	4
747 A1	-4	5	784 J1	-12	98	814 B1	3	-3
747 C1	26	-4	786 A1	1	0	815 A1	3	3
747 D1	0	2	786 B1	10	-1	816 A1	0	12
747 E1	2	3	786 C1	12085	1322560	816 G1	1	-2
749 A1	3	2	786 G1	-6	4	816 H1	-14	18
752 A1	-1	6	786 H1	-3	-8	816 I1	-1	162
753 C1	-1	1	786 J1	-9	-124	816 J1	-4	6
754 B1	60	-16	786 K1	3	7	817 A1	$\left\{ \begin{array}{l} 4 \\ 2 \end{array} \right\}$	$\left\{ \begin{array}{l} 9 \\ 4 \end{array} \right\}$
754 C1	-2	1	786 L1	0	6	817 B1	-2	924
754 D1	14	51	790 A1	2	3	819 A1	14	37
755 A1	1	1	791 C1	68	522	819 B1	2	-1
755 B1	1	1	792 A1	-5	8	822 A1	3	3
756 B1	-3	1	792 C1	1	2	822 D1	3	-14
756 C1	-4	2	792 D1	5	2	825 A1	1	5
758 A1	3	-10	793 A1	$82/3^2$	$497/3^3$	825 B1	-7	5
759 A1	10	11	794 A1	$\left\{ \begin{array}{l} 0 \\ 1 \end{array} \right\}$	$\left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\}$	825 C1	14	16
759 B1	7	16	794 B1	-8	15	827 A1	2	0
760 D1	1	5	794 C1	1	1	828 B1	-8	1
760 E1	6	15	794 D1	6	3	828 C1	4	1
762 C1	-2	2	795 A1	-2	5	829 A1	-1	0
762 D1	1	2	797 A1	0	1	830 B1	63	48
762 E1	6	-15						

Curve	x	y	Curve	x	y	Curve	x	y
830 C1	7	16	862 B1	4	2	888 C1	5	7
831 A1	13	-47	862 E1	-11	273	890 A1	0	1
832 A1	3	8	862 F1	1	3	890 B1	-1	2
832 B1	5	8	864 A1	1	2	890 D1	2	-1
832 C1	9	32	864 B1	4	4	890 E1	-20	12
832 H1	1	3	864 C1	4	-12	890 F1	5	-19
832 I1	-1	16	864 J1	9	18	890 G1	-5	3
832 J1	42	256	864 K1	0	4	891 A1	2	-7
834 C1	-1	3	864 L1	0	36	892 B1	-16	10
834 E1	1	1	866 A1	4	8	892 C1	-2	2
834 F1	35	126	867 A1	57	433	894 A1	81	0
834 G1	2	14	867 B1	0	4	894 B1	-5	3
836 A1	8	19	867 C1	$301/6^2$	$4805/6^3$	894 D1	-2	2
840 A1	54	368	869 A1	9	6	894 E1	33	127
840 E1	1	3	869 B1	11	39	894 F1	3	-1
840 F1	17	51	869 D1	-4	81	894 G1	2	17
840 H1	-5	3	870 A1	2	9	895 A1	1	0
842 A1	-2	1	870 B1	-14	311	896 A1	6	12
842 B1	-2	-15	870 C1	0	7	896 B1	-2	2
843 A1	0	2	870 E1	-1	2	896 D1	3	3
846 B1	1	4	870 F1	17	41	897 C1	6	7
846 C1	-1	41	872 A1	0	4	897 D1	2987	-165762
847 B1	55	423	873 B1	$275/2^2$	$3289/2^3$	897 E1	63	261
847 C1	116	1212	873 C1	$227473/16^2$	$106817593/16^3$	897 F1	3	3
848 F1	3	4	873 D1	1	4	898 A1	8	-4
848 G1	6	32	874 C1	3	-1	898 D1	-1	1
849 A1	2	-6	874 D1	-2	2	899 A1	-1	1
850 C1	2	61	874 E1	-3	47	900 C1	-4	6
850 D1	21	567	876 A1	128	1	900 D1	16	54
850 E1	4	-12	876 B1	5	-6	900 E1	-10	25
850 K1	25	112	880 A1	3	6	901 A1	-4	8
850 L1	5	7	880 C1	103	660	901 B1	90	106
851 A1	6	11	880 D1	-3	-20	901 E1	$-23/2^2$	$897/2^3$
854 A1	13	9	880 F1	8	16	901 F1	-1	0
854 B1	-15	309	880 G1	26	160	902 A1	5	253
854 C1	-1	4	880 H1	-2	2	903 A1	4	10
854 D1	-19	-40	882 A1	39	-15	904 A1	2	4
855 B1	2	21	882 D1	3	3	905 A1	$-1/2^2$	$43/2^3$
856 A1	1	1	882 E1	9	-225	906 A1	$394/3^2$	$3505/3^3$
856 B1	4	8	882 G1	-1	6	906 B1	-3	2
856 C1	4	2	882 H1	51	352	906 C1	-1	3
856 D1	137	1578	885 B1	58	411	906 D1	12	85
858 B1	-2	35	885 C1	-2	1	906 G1	-3	2
858 F1	-9	355	885 D1	1	37	906 H1	-8	-32
858 G1	5	3	886 A1	2	0	909 C1	-1	4
861 B1	45	220	886 B1	20	-10	910 B1	6	14
861 C1	-5	64	886 D1	281	-77	910 C1	-5	51
861 D1	-1	5	886 E1	1	1	910 D1	-3	4
862 A1	1	0	888 B1	-6	6	910 F1	137	295

Curve	x	y	Curve	x	y	Curve	x	y
910 G1	5	-8	936 H1	-2	9	974 G1	1	0
910 H1	9	-75	938 A1	-1	1	974 H1	-1	-8
910 K1	10	35	938 B1	3	110	975 A1	26	23
912 A1	12	27	938 C1	-5	-5	975 B1	7	12
912 F1	36	243	939 A1	29	66	975 F1	-8	62
912 G1	2	2	939 B1	11	30	975 H1	13	32
912 H1	20	18	939 C1	1	4	975 I1	273	4387
912 I1	3	-6	940 C1	44	46	975 J1	3	7
914 A1	-1	2	940 D1	2	2	975 K1	-3	9
915 B1	0	12	942 C1	-28	122	976 C1	2	2
915 D1	-1	8	942 D1	4	-5	978 E1	0	1
916 C1	$\begin{Bmatrix} 0 \\ -1 \end{Bmatrix}$	$\begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$	944 A1	2	4	978 F1	-1	24
916 D1	8	14	944 B1	10	4	978 G1	-6	-24
916 E1	-1	1	944 C1	2	4	979 B1	1514	57983
918 A1	92	-38	944 E1	$\begin{Bmatrix} 3 \\ 1 \end{Bmatrix}$	$\begin{Bmatrix} 2 \\ -4 \end{Bmatrix}$	980 C1	2	7
918 B1	9	21	944 H1	66	512	980 D1	-9	98
918 C1	53	285	944 I1	6	-16	981 A1	0	9
918 E1	1	0	944 J1	4	2	981 B1	4	2
918 F1	4	12	944 K1	2	8	982 A1	5	5
918 H1	5	3	946 B1	5	11	984 C1	-13	82
918 I1	5	14	950 A1	2	11	984 D1	1	6
918 J1	17	127	950 E1	15	17	985 B1	1	2
920 A1	302	5290	954 A1	13	7	986 B1	126	481
920 B1	7	-5	954 D1	11	35	986 C1	-1	44
920 C1	2	5	954 E1	166	1965	986 D1	6	14
920 D1	4	5	954 F1	3	3	986 E1	80	801
921 B1	-5	7	954 H1	-1	6	986 F1	-1	4
924 B1	24	121	954 I1	11	-15	987 E1	-3	72
924 C1	3	7	954 J1	11	102	988 B1	18309	2476099
924 E1	2	3	960 A1	3	6	988 C1	8	26
924 H1	89	-231	960 B1	1	12	990 A1	0	5
925 A1	3	12	960 H1	21	30	990 E1	15	51
925 B1	2	12	960 K1	7	14	990 H3	-35	97
927 A1	12	21	960 L1	11	24	990 J1	3	18
928 A1	3	4	960 M1	1	6	994 A1	-1	2
928 B1	1	4	966 A1	11	98	994 D1	65	503
930 A1	-7	10	966 C1	-121	992	994 F1	-1	1
930 D1	-13	394	966 D1	2	-8	994 G1	-16	42
930 E1	0	3	966 E1	-2	11	995 B1	7	17
930 H1	19	-130	966 G1	1	35	996 B1	20	54
930 M1	1	8	968 A1	7	22	996 C1	3	6
933 A1	-1	0	968 D1	3	4	997 A1	3	0
933 B1	-12	4	968 E1	44	242	997 B1	$\begin{Bmatrix} -1 \\ 5 \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ 8 \end{Bmatrix}$
934 A1	0	0	972 C1	-2	1	997 C1	$\begin{Bmatrix} 3 \\ 1 \end{Bmatrix}$	$\begin{Bmatrix} 0 \\ -6 \end{Bmatrix}$
935 A1	2	2	972 D1	-3	3	999 A1	32	156
936 A1	2	6	973 B1	$74/5^2$	$2682/5^3$	999 B1	2	-1
936 E1	-4	9	974 E1	1	0			
936 G1	2	9	974 F1	3	6			

TABLE 3

HECKE EIGENVALUES

This table is largely self-explanatory. There is one row for each rational newform f for $\Gamma_0(N)$ for $N \leq 1000$; forms at the same level are given an identifying letter as in Table 1, together with the Antwerp code for $N \leq 200$. The other columns contain the Hecke eigenvalues of f for primes up to 100: either T_p , when $p \nmid N$, or W_q when $q \mid N$. The latter are indicated in the table simply as $+$ or $-$ to distinguish them. Where the largest prime divisor q of N is greater than 100, the extra value ε_q is entered in the right-most column: there is only ever at most one such prime $q > 100$.

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
11A(B)	-2	-1	1	-2	-	4	-2	0	-1	0	7	3	-8	-6	8	-6	5	12	-7	-3	4	-10	-6	15	-7	
14A(C)	+	-2	0	-	0	-4	6	2	0	-6	-4	2	6	8	-12	6	-6	8	-4	0	2	8	-6	-6	-10	
15A(C)	-1	+	-	0	-4	-2	2	4	0	-2	0	-10	10	4	8	-10	-4	-2	12	-8	10	0	12	-6	2	
17A(C)	-1	0	-2	4	0	-2	-	-4	4	6	4	-2	-6	4	0	6	-12	-10	4	-4	-6	12	-4	10	2	
19A(B)	0	-2	3	-1	3	-4	-3	-	0	6	-4	2	-6	-1	-3	12	-6	-1	-4	6	-7	8	12	12	8	
20A(B)	-	-2	+	2	0	2	-6	-4	6	6	-4	2	6	-10	-6	-6	12	2	2	-12	2	8	6	-6	2	
21A(B)	-1	-	-2	+	4	-2	-6	4	0	-2	0	6	2	-4	0	6	12	-2	4	0	-6	-16	-12	-14	18	
24A(B)	-	+	-2	0	4	-2	2	-4	-8	6	8	6	-6	4	0	-2	4	-2	-4	8	10	-8	-4	-6	2	
26A(B)	+	1	-3	-1	6	-	-3	2	0	6	-4	-7	0	-1	3	0	-6	8	14	-3	2	8	12	-6	-10	
26B(D)	-	-3	-1	1	-2	+	-3	6	-4	2	4	3	0	-5	13	12	-10	-8	-2	-5	-10	-4	0	6	14	
27A(B)	0	-	0	-1	0	5	0	-7	0	0	-4	11	0	8	0	0	0	-1	5	0	-7	17	0	0	-19	
30A(A)	+	-	+	-4	0	2	6	-4	0	-6	8	2	-6	-4	0	-6	0	-10	-4	0	2	8	12	18	2	
32A(B)	-	0	-2	0	0	6	2	0	0	-10	0	-2	10	0	0	14	0	-10	0	0	-6	0	0	10	18	
33A(B)	1	+	-2	4	-	-2	-2	0	8	-6	-8	6	-2	0	8	6	-4	6	-4	0	-14	-4	12	-6	2	
34A(A)	-	-2	0	-4	6	2	+	-4	0	0	-4	-4	6	8	0	-6	0	-4	8	0	2	8	0	-6	14	
35A(B)	0	1	+	-	-3	5	3	2	-6	3	-4	2	-12	-10	9	12	0	8	-4	0	2	-1	12	-12	-1	
36A(A)	-	+	0	-4	0	2	0	8	0	0	-4	-10	0	8	0	0	0	14	-16	0	-10	-4	0	0	14	
37A(A)	-2	-3	-2	-1	-5	-2	0	0	2	6	-4	+	-9	2	-9	1	8	-8	8	9	-1	4	-15	4	4	
37B(C)	0	1	0	-1	3	-4	6	2	6	-6	-4	-	-9	8	3	-3	12	8	-4	-15	11	-10	9	6	8	
38A(D)	+	1	0	-1	-6	5	3	-	3	9	-4	2	0	8	0	-3	9	-10	5	-6	-7	-10	-6	-12	-10	
38B(A)	-	-1	-4	3	2	-1	3	+	-1	-5	-8	-2	-8	4	8	-1	15	2	3	2	9	-10	-6	0	-2	
39A(B)	1	+	2	-4	4	-	2	0	0	-10	4	-2	6	-12	0	6	12	-2	-8	0	2	8	4	-2	10	
40A(B)	+	0	-	-4	4	-2	2	4	4	-2	-8	6	-6	-8	4	6	-4	-2	8	0	-6	0	-16	-6	-14	
42A(A)	-	+	-2	+	-4	6	2	-4	8	-2	0	-10	-6	-4	0	6	4	6	4	8	10	0	-4	-6	-14	
43A(A)	-2	-2	-4	0	3	-5	-3	-2	-1	-6	-1	0	5	+	4	-5	-12	2	-3	2	2	-8	15	-4	7	
44A(A)	-	1	-3	2	+	-4	6	8	-3	0	5	-1	0	-10	0	-6	3	-4	-1	15	-4	2	6	-9	-7	
45A(A)	1	-	+	0	4	-2	-2	4	0	2	0	-10	-10	4	-8	10	4	-2	12	8	10	0	-12	6	2	
46A(A)	+	0	4	-4	2	-2	-2	-2	-	2	0	-4	6	10	0	-4	12	-8	-10	0	6	-12	14	-6	6	
48A(B)	+	-	-2	0	-4	-2	2	4	8	6	-8	6	-6	-4	0	-2	-4	-2	4	-8	10	8	4	-6	2	
49A(A)	1	0	0	-	4	0	0	0	8	2	0	-6	0	-12	0	-10	0	0	4	16	0	8	0	0	0	
50A(E)	+	1	-	2	-3	-4	-3	5	6	0	2	2	-3	-4	12	6	0	2	-13	12	11	-10	-9	15	2	
50B(A)	-	-1	+	-2	-3	4	3	5	-6	0	2	-2	-3	4	-12	-6	0	2	13	12	-11	-10	9	15	-2	
51A(A)	0	-	3	-4	-3	-1	+	-1	9	6	2	-4	-3	-7	-6	-6	6	8	-4	12	2	-10	-6	0	-16	
52A(B)	-	0	2	-2	-2	+	6	-6	8	2	10	-6	-6	4	-2	6	-10	-2	10	10	2	-4	-6	-6	2	
53A(A)	-1	-3	0	-4	0	-3	-3	-5	7	-7	4	5	6	-2	-2	+	-2	-8	-12	1	-4	-1	-1	-14	1	
54A(E)	+	-	3	-1	-3	-4	0	2	-6	6	5	2	-6	-10	6	9	12	8	14	0	-7	8	-3	-18	-1	
54B(A)	-	+	-3	-1	3	-4	0	2	6	-6	5	2	6	-10	-6	-9	-12	8	14	0	-7	8	3	18	-1	
55A(B)	1	0	-	0	+	2	6	-4	4	6	-8	-2	2	4	-12	-2	4	-10	-16	8	14	8	-4	10	10	
56A(C)	-	0	2	+	-4	2	-6	8	0	6	8	-2	2	-4	-8	6	0	-6	-4	-8	10	16	8	-6	-6	
56B(A)	+	2	-4	-	0	0	-2	-2	8	2	4	-6	-2	8	-4	-10	6	4	-12	0	-14	-8	6	10	-2	
57A(E)	-2	+	-3	-5	1	2	-1	+	-4	-2	-6	0	0	-1	-9	10	-8	-1	8	-12	-11	16	12	-6	-10	
57B(B)	1	-	-2	0	0	6	-6	+	4	2	8	-10	-2	-4	12	-6	-12	-2	-4	0	10	0	16	-2	10	
57C(F)	-2	-	1	3	-3	-6	3	+	4	-10	2	8	-8	-1	3	-6	0	7	8	12	-11	0	4	10	-2	
58A(A)	+	-3	-3	-2	-1	3	-4	-8	0	+	3	-8	-2	7	11	1	-4	4	-4	-2	-12	-7	0	-6	-6	
58B(B)	-	-1	1	-2	-3	-1	8	0	4	+	-3	8	2	-11	13	-11	0	-8	-12	2	4	15	4	-10	-2	
61A(A)	-1	-2	-3	1	-5	1	4	-4	-9	-6	0	8	5	-8	4	6	9	+	-7	-8	-11	3	4	-4	-14	
62A(A)	-	0	-2	0	0	2	-6	4	8	2	+	10	-6	8	-8	-6	-12	-6	-12	8	10	-8	8	-6	2	
63A(A)	1	-	2	+	-4	-2	6	4	0	2	0	6	-2	-4	0	-6	-12	-2	4	0	-6	-16	12	14	18	
64A(B)	-	0	2	0	0	-6	2	0	0	10	0	2	10	0	0	-14	0	10	0	0	-6	0	0	10	18	
65A(A)	-1	-2	+	-4	2	+	2	-6	-6	2	-10	-2	-6	10	4	2	6	2	-4	6	-6	-12	-16	2	-2	
66A(A)	+	-	0	2	+	-4	-6	-4	6	6	8	-10	6	8	-6	0	0	8	-4	6	2	14	-12	-6	14	
66B(E)	-	+	2	-4	+	-6	2	4	4	6	0	6	-6	4	-12	2	12	-14	4	-12	-6	-4	4	10	-14	

TABLE 3: HECKE EIGENVALUES 66C–108A

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
66C (I)	-	-	-4	-2	-	4	-2	0	-6	10	-8	-2	2	4	-2	4	0	-8	-12	2	-6	10	4	10	-2	
67A (A)	2	-2	2	-2	-4	2	3	7	9	-5	-10	-1	0	-2	-1	10	9	-2	-	0	-7	-8	4	7	0	
69A (A)	1	-	0	-2	4	-6	4	2	+	2	4	2	2	10	0	-12	-12	-6	-10	8	-14	10	12	-16	-10	
70A (A)	-	0	+	+	4	-6	2	0	0	6	8	-10	2	4	8	-2	-8	-14	-12	-16	2	-8	8	10	2	
72A (A)	+	-	2	0	-4	-2	-2	-4	8	-6	8	6	6	4	0	2	-4	-2	-4	-8	10	-8	4	6	2	
73A (B)	1	0	2	2	-2	-6	2	8	4	2	-2	-6	6	-2	6	10	-6	-14	8	0	-	-4	-14	-6	-10	
75A (A)	2	+	-	-3	2	1	2	-5	6	10	-3	2	-8	1	2	-4	-10	7	-3	-8	-14	0	6	0	17	
75B (E)	1	-	+	0	-4	2	-2	4	0	-2	0	10	10	-4	-8	10	-4	-2	-12	-8	-10	0	-12	-6	-2	
75C (C)	-2	-	+	3	2	-1	-2	-5	-6	10	-3	-2	-8	-1	-2	4	-10	7	3	-8	14	0	-6	0	-17	
76A (A)	-	2	-1	-3	5	-4	-3	+	8	-2	4	10	10	1	-1	-4	6	-13	-12	2	9	8	-12	12	-8	
77A (F)	0	-3	-1	+	+	-4	2	-6	-5	10	1	-5	-2	-8	8	-6	3	-2	-3	1	10	6	12	-15	-5	
77B (D)	0	1	3	-	+	-4	-6	2	3	-6	5	11	6	8	0	-6	-9	-10	5	9	2	-10	12	-3	-1	
77C (A)	1	2	-2	+	-	4	4	0	-4	-6	10	-6	4	12	-10	-6	2	0	8	-12	-8	8	0	-6	-10	
78A (A)	+	+	2	4	-4	-	2	-8	0	6	-4	-2	-10	4	8	-10	4	-2	-16	-8	2	8	12	14	10	
79A (A)	-1	-1	-3	-1	-2	3	-6	4	2	-6	-10	-2	-10	4	7	8	-3	-4	8	15	2	+	-6	-7	-19	
80A (F)	+	0	-	4	-4	-2	2	-4	-4	-2	8	6	-6	8	-4	6	4	-2	-8	0	-6	0	16	-6	-14	
80B (B)	-	2	+	-2	0	2	-6	4	-6	6	4	2	6	10	6	-6	-12	2	-2	12	2	-8	-6	-6	2	
82A (A)	+	-2	-2	-4	-2	4	-2	6	-8	0	-8	2	+	-12	4	-4	8	-14	-2	8	10	4	12	-14	6	
83A (A)	-1	-1	-2	-3	3	-6	5	2	-4	-7	5	-11	-2	-8	0	6	5	5	-2	2	0	14	+	0	-8	
84A (C)	-	-	0	-	-6	2	0	-4	-6	6	8	2	12	-4	12	-6	0	-10	8	6	-10	-4	-12	12	-10	
84B (A)	-	+	4	+	2	-6	-4	-4	2	-2	0	2	0	-4	12	-6	-8	6	-8	14	-2	12	-4	0	-2	
85A (A)	1	2	+	-2	2	2	-	0	6	-6	-10	2	10	4	12	-10	8	-14	8	-2	-14	-14	4	6	2	
88A (A)	+	-3	-3	-2	+	0	-6	4	1	-8	-7	-1	4	6	-8	2	-1	4	-5	3	16	2	-2	15	-7	
89A (C)	-1	-1	-1	-4	-2	2	3	-5	7	0	-9	-2	0	-7	-12	-3	4	6	12	-10	7	-6	12	+	9	
89B (A)	1	2	-2	2	-4	2	6	-2	2	-6	6	10	-6	2	12	-6	-10	-6	12	4	10	-12	-6	-	-18	
90A (M)	+	+	-	2	6	-4	-6	-4	0	-6	-4	8	0	8	0	-6	6	2	-4	-12	-10	-4	12	12	2	
90B (A)	-	+	+	2	-6	-4	6	-4	0	6	-4	8	0	8	0	6	-6	2	-4	12	-10	-4	-12	-12	2	
90C (E)	-	-	-	-4	0	2	-6	-4	0	6	8	2	6	-4	0	6	0	-10	-4	0	2	8	-12	-18	2	
91A (A)	-2	0	-3	+	-6	+	4	5	3	-5	-3	-4	-6	-1	7	-9	8	-10	-6	-8	-13	3	15	3	7	
91B (B)	0	-2	-3	-	0	-	-6	-7	3	-9	5	2	-6	-1	3	-9	0	-10	14	-6	11	-1	3	15	-1	
92A (A)	-	1	0	2	0	-1	-6	2	+	-3	5	8	3	8	9	6	-12	14	8	-15	-7	-10	6	0	-10	
92B (C)	-	-3	-2	-4	2	-5	4	-2	-	-7	-3	2	-9	-8	9	2	0	-2	14	-3	-3	-6	8	12	0	
94A (A)	-	0	0	0	2	-4	-2	-2	4	4	4	2	6	6	+	2	12	2	2	8	-14	-16	-16	-10	-14	
96A (E)	+	-	2	-4	4	-2	-6	-4	0	2	4	-2	2	4	8	10	-4	6	4	-16	-6	4	12	10	-14	
96B (A)	-	+	2	4	-4	-2	-6	4	0	2	-4	-2	2	-4	-8	10	4	6	-4	16	-6	-4	-12	10	-14	
98A (B)	+	2	0	-	0	4	-6	-2	0	-6	4	2	-6	8	12	6	6	-8	-4	0	-2	8	6	6	10	
99A (A)	-1	+	-4	-2	+	-2	2	-6	4	-6	4	-6	-10	6	-8	0	4	-6	8	0	-2	-10	12	0	2	
99B (H)	-1	-	2	4	+	-2	2	0	-8	6	-8	6	2	0	-8	-6	4	6	-4	0	-14	-4	-12	6	2	
99C (F)	1	+	4	-2	-	-2	-2	-6	-4	6	4	-6	10	6	8	0	-4	-6	8	0	-2	-10	-12	0	2	
99D (C)	2	-	-1	-2	+	4	2	0	1	0	7	3	8	-6	-8	6	-5	12	-7	3	4	-10	6	-15	-7	
100A (A)	-	2	+	-2	0	-2	6	-4	-6	6	-4	-2	6	10	6	6	12	2	-2	-12	-2	8	-6	-6	-2	
101A (A)	0	-2	-1	-2	-2	1	3	-5	1	-4	-9	-2	8	-8	7	-2	-14	4	2	13	8	-9	-4	14	2	
102A (E)	+	+	-4	-2	0	-6	+	4	6	-4	-6	-4	-10	-4	4	-2	12	-4	-12	-6	2	10	-12	-2	6	
102B (G)	-	-	-2	0	-4	-2	-	4	0	-10	8	-2	10	12	0	6	12	-10	-12	0	10	-8	4	-6	-14	
102C (A)	+	-	0	2	0	2	+	-4	-6	0	-10	8	6	-4	12	6	-12	8	-4	6	2	-10	12	-18	14	
104A (A)	-	1	-1	5	-2	+	-3	-2	4	-6	-4	11	8	-1	9	-12	6	0	6	7	-2	12	-16	-10	-10	
105A (A)	1	-	-	-	0	-6	2	-8	8	-2	4	-2	-6	4	8	10	4	-2	4	-12	-2	8	-4	-6	-18	
106A (B)	-	-2	3	2	-3	-4	3	-4	-9	6	5	-10	6	-1	0	+	15	-10	-4	12	8	11	-6	9	-13	
106B (A)	+	-1	-4	0	-4	1	5	-7	1	5	-4	1	-10	-10	-6	+	-6	4	4	15	-8	1	-3	2	17	
106C (E)	-	1	0	-4	0	5	-3	-1	3	9	-4	5	6	-10	6	+	6	8	-4	-3	-4	-13	3	18	-7	
106D (D)	+	2	1	-2	5	-4	3	-4	-3	-6	7	-6	2	7	4	-	7	2	16	12	-12	-7	-14	17	3	
108A (A)	-	+	0	5	0	-7	0	-1	0	0	-4	-1	0	8	0	0	0	-13	11	0	17	-13	0	0	5	

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
109A(A)	1	0	3	2	1	0	-8	-5	7	-5	6	2	2	-4	9	12	12	-5	-12	-6	-5	8	-2	1	1	-
110A(C)	-	-1	-	3	-	-6	-7	5	-6	5	-3	3	2	4	-2	-1	-10	7	8	7	14	10	-6	-15	-12	-
110B(A)	-	1	+	-1	+	2	-3	-1	6	-9	5	5	-6	8	6	9	6	5	8	-9	-10	14	-6	-15	8	-
110C(E)	+	1	+	5	-	2	3	-7	-6	-3	-7	-7	6	8	6	-3	-6	-1	8	3	2	-10	-6	9	-4	-
112A(K)	+	-2	-4	+	0	0	-2	2	-8	2	-4	-6	-2	-8	4	-10	-6	4	12	0	-14	8	-6	10	-2	-
112B(A)	+	0	2	-	4	2	-6	-8	0	6	-8	-2	2	4	8	6	0	-6	4	8	10	-16	-8	-6	-6	-
112C(E)	-	2	0	+	0	-4	6	-2	0	-6	4	2	6	-8	12	6	6	8	4	0	2	-8	6	-6	-10	-
113A(B)	-1	2	2	0	0	2	-6	6	-6	-6	-4	2	-2	6	6	10	6	6	2	-6	2	10	-4	-14	-14	-
114A(A)	-	-	0	-4	0	-4	6	-	-6	6	2	-4	6	-4	6	6	-12	14	8	0	14	-10	-12	-6	-10	-
114B(E)	+	+	0	4	4	0	-2	-	-2	-6	6	-8	10	-12	10	2	4	-10	0	-16	-2	10	-16	-2	-10	-
114C(G)	-	+	2	0	-4	2	-6	+	-4	-2	4	10	10	4	-4	-10	12	14	-12	8	-6	-4	12	-6	10	-
115A(A)	2	0	+	1	2	-2	3	-2	-	7	-5	11	1	0	0	11	-13	-8	5	5	6	-12	9	4	-14	-
116A(E)	-	-3	3	4	-1	-3	2	4	-6	+	9	-8	-8	-5	-7	-5	-10	10	8	-2	0	-1	6	12	0	-
116B(A)	-	1	3	-4	3	5	-6	-4	-6	+	5	8	0	-1	-3	3	6	2	8	6	-16	11	6	-12	8	-
116C(D)	-	2	-2	4	-6	2	2	-6	4	+	-6	2	2	10	-2	10	0	10	-12	8	10	-6	16	2	10	-
117A(A)	-1	-	-2	-4	-4	-	-2	0	0	10	4	-2	-6	-12	0	-6	-12	-2	-8	0	2	8	-4	2	10	-
118A(A)	+	-1	-3	-1	-2	-2	-2	3	0	-1	10	-12	7	-6	-6	-11	+	-12	10	4	12	-15	-14	4	0	-
118B(B)	-	-1	1	3	2	-6	-2	-5	4	-5	2	8	7	-6	-2	9	+	-8	-2	12	4	5	14	0	8	-
118C(D)	-	2	-2	-3	-1	-3	7	4	4	4	-4	-7	-11	9	10	0	+	-2	4	9	-14	11	-13	18	2	-
118D(E)	+	2	2	-3	1	3	-1	-8	8	-4	-4	-1	5	-9	2	12	-	10	4	-15	10	11	-11	-6	14	-
120A(E)	-	-	-	0	-4	6	-6	-4	0	-2	-8	-2	-6	12	8	6	12	14	4	8	-6	-8	-12	10	2	-
120B(A)	+	-	+	4	0	-6	-2	4	-8	-6	0	-6	10	-4	8	10	0	6	-4	0	-14	16	12	2	2	-
121A(H)	-1	2	1	2	-	-1	5	-6	2	-9	-2	-3	5	0	2	9	8	-6	2	12	2	10	-6	-9	-13	-
121B(D)	0	-1	-3	0	+	0	0	0	-9	0	-5	7	0	0	-12	6	-15	0	13	-3	0	0	0	-9	17	-
121C(F)	1	2	1	-2	-	1	-5	6	2	9	-2	-3	-5	0	2	9	8	6	2	12	-2	-10	6	-9	-13	-
121D(A)	2	-1	1	2	-	-4	2	0	-1	0	7	3	8	6	8	-6	5	-12	-7	-3	-4	10	6	15	-7	-
122A(A)	+	-2	1	-5	-3	-3	0	0	5	6	0	-12	-3	-8	12	-2	-9	+	7	-16	-3	1	-12	12	2	-
123A(A)	-2	-	-4	-2	-3	-6	3	0	-6	5	7	-7	-	-1	3	-6	0	-3	-2	-3	-11	10	-16	-10	-12	-
123B(C)	0	+	-2	-4	5	-4	-5	-2	4	1	-5	-7	+	7	7	-14	-12	-3	-2	-3	13	-2	-2	18	-14	-
124A(B)	-	-2	-3	-1	-6	2	6	-1	-6	0	-	-10	-9	8	0	0	-3	-10	-4	-15	14	8	6	12	-7	-
124B(A)	-	0	1	3	6	-4	0	-5	-4	2	+	-2	-9	2	4	12	9	12	-12	5	-14	10	2	6	-7	-
126A(A)	-	-	0	-	0	-4	-6	2	0	6	-4	2	-6	8	12	-6	6	8	-4	0	2	8	6	6	-10	-
126B(G)	+	-	2	+	4	6	-2	-4	-8	2	0	-10	6	-4	0	-6	-4	6	4	-8	10	0	4	6	-14	-
128A(C)	+	-2	-2	-4	2	-2	-2	-2	4	6	0	-10	-6	-6	-8	6	-14	-2	-10	12	14	-8	6	-2	-2	-
128B(F)	-	-2	2	4	2	2	-2	-2	-4	-6	0	10	-6	-6	8	-6	-14	2	-10	-12	14	8	6	-2	-2	-
128C(A)	-	2	-2	4	-2	-2	-2	2	-4	6	0	-10	-6	6	8	6	14	-2	10	-12	14	8	-6	-2	-2	-
128D(G)	-	2	2	-4	-2	2	-2	2	4	-6	0	10	-6	6	-8	-6	14	2	10	12	14	-8	-6	-2	-2	-
129A(E)	0	+	-2	-2	-5	3	-3	2	-1	0	-5	8	-7	+	-8	3	12	-8	-15	-14	12	-16	15	10	11	-
129B(B)	1	-	2	0	0	-2	-6	4	-4	-6	8	6	2	+	4	-2	0	14	12	8	2	-8	0	14	-14	-
130A(E)	+	-2	-	-4	-6	-	-6	2	6	-6	2	2	-6	2	-12	6	6	2	-4	-6	-10	-4	0	-6	2	-
130B(A)	-	0	-	0	0	-	2	-8	-4	-2	-4	6	10	0	8	6	8	-2	4	-12	10	-8	12	10	-14	-
130C(J)	-	2	+	-4	-2	+	2	6	6	2	-6	-2	10	-10	-12	2	10	2	-12	10	10	-4	0	-14	14	-
131A(A)	0	-1	-2	-1	0	-3	4	-2	-2	0	-2	-8	-3	3	10	-9	1	-15	-6	10	4	-8	4	-11	12	+
132A(A)	-	-	2	-2	-	-2	4	-6	0	-8	-8	10	8	-2	-8	-2	12	10	12	8	6	-2	16	-14	-2	-
132B(C)	-	+	2	2	+	6	-4	-2	-8	0	0	-6	0	10	0	14	-12	-14	4	0	6	2	16	-14	-2	-
135A(A)	-2	+	+	-3	-2	-5	-8	1	6	2	0	5	-10	4	4	-2	-8	7	-9	2	-5	-3	6	-12	-13	-
135B(B)	2	+	-	-3	2	-5	8	1	-6	-2	0	5	10	4	-4	2	8	7	-9	-2	-5	-3	-6	12	-13	-
136A(A)	-	-2	-2	-2	-6	2	-	0	6	-10	2	6	-6	-8	0	-10	-8	14	4	2	-14	-10	8	-10	2	-
136B(C)	-	2	0	0	2	-6	+	4	4	0	-8	-4	6	8	-8	10	0	12	8	12	2	-4	16	10	-18	-
138A(E)	+	+	-2	-2	-6	-2	0	0	+	6	8	0	10	-12	-8	2	-12	4	-12	0	-10	-6	14	0	-6	-
138B(G)	+	-	0	2	0	2	0	2	+	-6	-4	-10	-6	2	0	12	12	-10	14	0	2	-10	0	12	-10	-
138C(A)	-	+	2	0	0	-2	2	-8	+	-2	-8	2	10	8	8	2	-4	2	8	0	-6	8	-16	18	10	-

TABLE 3: HECKE EIGENVALUES 139A–166A

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
139A(A)	1	2	-1	3	5	-7	-6	-2	2	9	9	2	-6	-4	8	0	6	4	5	5	-6	-5	7	7	-12	-
140A(A)	-	1	-	-	3	-1	-3	2	-6	-9	8	-10	0	2	-3	0	12	8	8	0	14	5	-12	12	17	-
140B(C)	-	3	+	+	-5	-3	-1	6	6	-9	-4	2	-4	10	-1	4	-8	-8	12	8	2	13	-4	4	-13	-
141A(E)	-2	-	-3	-3	-5	2	-6	-6	9	1	-2	1	6	2	-	0	-12	-2	2	-2	-2	-15	-4	10	1	-
141B(G)	-1	+	0	4	0	6	-6	2	4	8	6	-6	-8	-6	-	2	12	2	-2	0	-10	-4	4	-10	-18	-
141C(A)	-1	-	2	0	4	-2	2	0	0	-6	-4	-10	-2	8	+	-2	-4	14	-8	16	2	8	-4	18	-14	-
141D(I)	0	+	-1	-3	-3	-4	8	-6	3	-1	4	1	-10	-8	+	10	-10	2	4	-6	-8	-3	-18	-2	5	-
141E(H)	2	-	-1	-3	1	-2	2	6	3	3	2	-7	10	-10	+	4	8	-10	10	-14	-10	17	8	6	1	-
142A(F)	-	-3	-4	-3	0	1	0	-5	-7	-8	7	4	4	-5	-13	-6	10	-2	-4	-	7	0	-4	-3	-4	-
142B(E)	+	-1	-2	-1	-2	-3	-6	5	-1	6	1	6	-6	5	-3	-6	2	-6	-14	+	-17	10	4	9	-6	-
142C(A)	+	0	2	0	6	4	6	-8	-4	-2	-8	10	-2	-8	-4	0	10	-8	2	-	-2	0	-4	6	14	-
142D(C)	-	1	0	-1	0	-1	0	-1	3	0	5	-4	0	-1	9	6	6	2	8	+	-1	8	12	-3	-16	-
142E(G)	+	3	2	-3	-6	-5	6	1	5	-2	-5	-2	10	1	-1	6	-2	-2	2	-	7	-6	-4	9	2	-
143A(A)	0	-1	-1	-2	+	+	-4	2	7	-2	-3	-11	10	-4	-4	2	-1	-2	-1	-9	-16	8	0	-7	-13	-
144A(A)	-	+	0	4	0	2	0	-8	0	0	4	-10	0	-8	0	0	0	14	16	0	-10	4	0	0	14	-
144B(E)	+	-	2	0	4	-2	-2	4	-8	-6	-8	6	6	-4	0	2	4	-2	4	8	10	8	-4	6	2	-
145A(A)	-1	0	+	-2	-6	2	-2	-2	2	+	2	10	2	8	-12	-6	-8	-6	2	-12	-6	-10	-14	18	2	-
147A(C)	-1	+	2	-	4	2	6	-4	0	-2	0	6	-2	-4	0	6	-12	2	4	0	6	-16	12	14	-18	-
147B(I)	2	-	-2	+	-2	1	0	1	0	4	9	3	-10	5	-6	12	-12	10	-5	-6	-3	-1	6	16	-6	-
147C(A)	2	+	2	-	-2	-1	0	-1	0	4	-9	3	10	5	6	12	12	-10	-5	-6	3	-1	-6	-16	6	-
148A(A)	-	-1	-4	-3	5	0	-6	2	-6	-6	4	-	-9	4	-7	9	-4	-8	-12	3	-5	6	-1	2	0	-
150A(A)	-	-	-	-2	2	-6	-2	0	4	0	-8	-2	2	4	8	-6	10	2	8	12	4	0	4	-10	8	-
150B(G)	+	+	-	2	2	6	2	0	-4	0	-8	2	2	-4	-8	6	10	2	-8	12	-4	0	-4	-10	-8	-
150C(I)	-	+	+	4	0	-2	-6	-4	0	-6	8	-2	-6	4	0	6	0	-10	4	0	-2	8	-12	18	-2	-
152A(A)	+	-2	-1	-3	-3	-4	5	+	0	2	8	-10	6	-7	-9	-8	14	-5	0	-6	-15	-4	4	0	16	-
152B(B)	+	1	0	3	2	1	-5	-	-1	-3	4	2	-8	-8	-8	9	1	14	13	10	9	-10	10	-12	14	-
153A(C)	-2	+	-1	-2	-3	-5	+	-1	-7	6	4	10	9	1	-12	-12	6	2	4	-8	0	-6	4	2	8	-
153B(A)	0	-	-3	-4	3	-1	-	-1	-9	-6	2	-4	3	-7	6	6	-6	8	-4	-12	2	-10	6	0	-16	-
153C(E)	1	-	2	4	0	-2	+	-4	-4	-6	4	-2	6	4	0	-6	12	-10	4	4	-6	12	4	-10	2	-
153D(D)	2	+	1	-2	3	-5	-	-1	7	-6	4	10	-9	1	12	12	-6	2	4	8	0	-6	-4	-2	8	-
154A(C)	+	0	-4	+	+	2	-4	-6	4	-2	-2	10	4	-8	2	6	-12	-14	-12	-8	4	0	-6	-6	-14	-
154B(E)	-	0	2	+	+	2	2	0	-8	-2	-8	-2	10	4	8	6	0	10	-12	16	-14	0	0	-6	10	-
154C(A)	+	2	2	+	-	-4	0	4	4	2	-10	-6	0	-4	10	-14	10	-8	8	-4	4	16	4	10	6	-
155A(D)	-2	-1	-	-2	2	-6	-7	-5	4	0	-	-7	-3	9	-2	9	-5	-8	8	-3	-1	0	-11	10	18	-
155B(A)	-1	2	+	4	4	0	-8	4	2	-6	-	-4	-6	-6	8	-12	-4	10	8	0	-4	0	2	14	-18	-
155C(C)	0	-1	+	0	-4	-6	5	-1	8	-10	+	1	-3	-7	-6	5	11	-12	-2	9	-9	-10	9	0	-14	-
156A(E)	-	+	-4	-2	-4	-	2	-2	0	-6	-10	10	8	4	-4	-10	-8	-14	2	16	-10	-16	0	-4	-2	-
156B(A)	-	-	0	2	0	-	-6	2	0	-6	2	2	-12	-4	0	6	12	2	-10	12	14	8	12	0	-10	-
158A(E)	-	-3	-3	-3	-2	-5	6	0	-2	6	-10	-10	2	4	-3	-12	-1	12	-8	-3	-6	-	14	-7	-11	-
158B(D)	+	-1	-1	-3	4	-7	-4	-6	6	4	8	10	-8	-8	-3	2	1	0	-4	-11	-6	+	6	-15	1	-
158C(H)	-	-1	1	3	2	-1	-2	0	-6	-10	2	-2	2	4	3	4	5	12	8	-13	-6	+	-6	-15	13	-
158D(B)	+	1	3	-1	0	5	0	2	-6	0	-4	2	-12	8	-9	6	-9	8	-4	-9	2	-	18	9	17	-
158E(F)	-	2	-2	0	-4	2	-2	0	0	8	8	4	-10	-2	0	-8	14	0	8	8	6	+	12	6	10	-
160A(D)	+	-2	+	-2	-4	-6	2	8	-6	-2	4	2	-10	-2	-2	2	0	2	-6	-12	10	-8	-10	-6	10	-
160B(A)	-	2	+	2	4	-6	2	-8	6	-2	-4	2	-10	2	2	2	0	2	6	12	10	8	10	-6	10	-
161A(B)	-1	0	2	-	4	6	-2	4	+	-2	-4	-2	-6	12	-12	-10	0	2	12	8	-14	8	-4	6	-10	-
162A(K)	+	+	-3	-4	0	-1	-3	-4	0	9	-4	-1	6	8	-12	-6	0	-1	-4	-12	11	-16	-12	-3	2	-
162B(G)	-	+	0	2	-3	2	-3	-1	-6	6	-4	-4	9	-1	-6	12	3	8	5	-12	11	-4	12	6	5	-
162C(A)	+	-	0	2	3	2	3	-1	6	-6	-4	-4	-9	-1	6	-12	-3	8	5	12	11	-4	-12	-6	5	-
162D(E)	-	+	3	-4	0	-1	3	-4	0	-9	-4	-1	-6	8	12	6	0	-1	-4	12	11	-16	12	3	2	-
163A(A)	0	0	-4	2	-6	4	0	-6	6	-4	-6	-8	3	7	1	-9	-2	3	-2	-5	-2	-8	5	-14	-11	+
166A(A)	+	-1	-2	1	-5	-2	-3	-2	4	-3	1	1	6	8	12	-14	-3	-7	2	-14	-4	-6	+	4	12	-

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
168A(B)	+	-	2	+	0	-2	6	-4	-4	6	-8	-10	-10	12	-8	6	4	-10	12	4	2	8	4	6	10	
168B(E)	+	+	2	-	0	6	-2	4	-4	-10	-8	6	-2	-4	8	-10	12	-2	12	-12	-14	-8	12	-2	10	
170A(A)	+	-2	-	-2	-2	-6	-	-8	-2	6	-2	6	2	-4	4	-10	0	-10	8	14	10	-14	-4	6	-14	
170B(H)	+	-2	+	2	6	2	-	8	-6	-6	2	2	-6	-4	12	6	0	2	8	-6	2	-10	12	6	2	
170C(F)	-	1	+	2	0	-1	+	-1	-6	-3	5	8	6	-10	-3	-3	3	11	2	9	11	8	-12	15	-7	
170D(D)	+	1	-	2	0	5	+	-1	6	-9	-1	-4	-6	2	-9	-9	3	-7	14	3	11	8	0	-9	-7	
170E(C)	+	3	+	2	-4	-3	-	3	-6	9	-3	-8	-6	6	-13	-9	15	7	-2	9	-3	0	12	-9	7	
171A(D)	-1	-	2	0	0	6	6	+	-4	-2	8	-10	2	-4	-12	6	12	-2	-4	0	10	0	-16	2	10	
171B(A)	0	-	-3	-1	-3	-4	3	-	0	-6	-4	2	6	-1	3	-12	6	-1	-4	-6	-7	8	-12	-12	8	
171C(I)	2	-	-1	3	3	-6	-3	+	-4	10	2	8	8	-1	-3	6	0	7	8	-12	-11	0	-4	-10	-2	
171D(H)	2	-	3	-5	-1	2	1	+	4	2	-6	0	0	-1	9	-10	8	-1	8	12	-11	16	-12	6	-10	
172A(A)	-	-2	0	-4	-3	-1	-3	2	-3	6	5	8	-3	-	-12	-9	-12	-10	11	6	-10	8	-15	0	-1	
174A(I)	+	-	-3	5	6	-4	3	-1	0	+	-4	-1	-9	-7	-3	-6	3	-10	-4	12	2	14	0	-6	8	
174B(G)	-	-	-1	1	-2	0	-3	-1	-4	-	4	3	-7	9	-1	-2	-3	6	12	16	-10	10	0	6	0	
174C(F)	-	+	1	1	6	-4	-7	-3	4	+	0	-7	5	-5	-5	10	3	10	0	-4	10	-6	16	-10	-8	
174D(A)	+	-	2	0	-4	6	-2	4	0	+	-4	-6	6	-12	-8	-6	8	10	-4	-8	2	4	0	14	18	
174E(E)	+	+	3	-3	6	0	7	5	-8	-	-8	-3	-5	3	9	-2	-11	-6	0	0	-10	-2	0	10	0	
175A(B)	-2	-1	-	-	-3	-1	-7	0	-6	-5	2	-2	2	4	3	-6	10	-8	-2	-8	-6	-5	4	0	-7	
175B(C)	0	-1	+	+	-3	-5	-3	2	6	3	-4	-2	-12	10	-9	-12	0	8	4	0	-2	-1	-12	-12	1	
175C(F)	2	1	-	+	-3	1	7	0	6	-5	2	2	2	-4	-3	6	10	-8	2	-8	6	-5	-4	0	7	
176A(C)	+	3	-3	2	-	0	-6	-4	-1	-8	7	-1	4	-6	8	2	1	4	5	-3	16	-2	2	15	-7	
176B(D)	-	1	1	2	+	4	-2	0	1	0	-7	3	-8	6	-8	-6	-5	12	7	3	4	10	6	15	-7	
176C(A)	-	-1	-3	-2	-	-4	6	-8	3	0	-5	-1	0	10	0	-6	-3	-4	1	-15	-4	-2	-6	-9	-7	
178A(A)	-	1	3	-4	-6	2	3	5	-3	0	5	-10	0	-1	12	9	12	-10	-4	-6	-1	-10	-12	+	17	
178B(C)	+	2	2	0	0	-4	2	-2	8	0	0	0	-10	-2	-8	6	10	-4	-8	8	-2	8	14	-	-2	
179A(A)	2	0	3	-4	4	-1	1	-3	6	3	-8	2	12	-11	1	0	-5	14	-9	0	10	10	17	-1	-14	
180A(A)	-	-	-	2	0	2	6	-4	-6	-6	-4	2	-6	-10	6	6	-12	2	2	12	2	8	-6	6	2	
182A(E)	-	0	2	+	4	+	-6	0	8	-10	-8	6	-6	4	-8	6	8	10	4	-8	2	8	0	18	2	
182B(A)	-	1	0	-	-3	-	0	2	-3	0	5	-7	3	8	-3	-12	6	-1	5	12	11	-1	12	-18	17	
182C(J)	+	1	4	+	-1	-	4	2	-7	-8	3	7	-7	-8	3	0	-6	-13	7	4	9	-13	-16	-6	11	
182D(D)	-	3	-4	+	1	+	0	-6	-7	-4	7	9	-3	4	7	0	-10	1	1	16	5	11	0	-6	-1	
182E(I)	+	3	0	-	-5	+	-4	2	5	4	1	7	-9	-12	-7	-4	-6	13	11	0	7	-17	4	14	5	
184A(C)	-	-1	-4	2	-4	-5	-2	6	-	1	-9	-4	3	8	-5	6	-4	-10	-4	-5	-15	-6	6	-8	10	
184B(B)	+	-1	-2	-4	-2	7	-4	-6	+	5	3	2	-9	8	-1	-6	-8	-10	2	-13	-3	6	0	-4	-8	
184C(D)	+	0	0	4	6	-2	6	-6	-	-6	0	-8	6	-2	-8	-8	4	-4	2	-8	6	12	10	10	-18	
184D(A)	+	3	0	-2	0	-5	-6	6	-	9	3	-8	3	-8	7	-2	4	-10	8	7	9	-6	-14	16	6	
185A(D)	-2	1	+	-5	3	-2	-4	-4	-2	2	0	+	7	-10	11	-3	0	-4	16	-15	11	-12	-3	-4	8	
185B(A)	0	-1	-	-3	-5	4	-4	-8	4	4	2	-	-5	-6	9	3	-8	-10	-4	5	-15	-14	11	-2	10	
185C(B)	1	-2	+	-2	0	-2	2	2	-8	2	-6	+	10	-4	-10	-6	-6	2	-14	0	2	-6	18	2	-10	
186A(D)	+	+	-1	2	3	3	1	7	0	4	-	-10	-6	6	-5	-2	6	3	-3	7	-10	-1	17	6	5	
186B(B)	-	-	1	-2	-3	-1	3	-5	4	0	-	-2	2	-6	-7	14	10	7	-7	-3	-6	15	-1	10	13	
186C(A)	+	-	3	-2	5	-7	-1	7	4	-8	+	-6	-2	-10	-1	6	-10	1	-3	3	14	-11	7	-6	-3	
187A(A)	0	1	3	2	-	2	+	2	-3	-6	-7	-7	12	-10	0	6	-3	8	-7	-9	2	8	6	15	11	
187B(C)	2	0	4	-5	+	4	-	2	-2	-3	4	-2	-3	-2	3	9	-3	-10	7	2	-3	0	14	1	-10	
189A(A)	-2	+	-1	+	-4	-2	3	-8	-6	-4	6	-3	1	11	9	6	-15	4	-8	-12	6	-1	-9	2	12	
189B(C)	0	-	-3	-	-6	-4	-3	2	6	6	-4	-7	3	-1	-9	6	-9	-10	-4	0	2	-1	-3	-6	-10	
189C(F)	0	+	3	-	6	-4	3	2	-6	-6	-4	-7	-3	-1	9	-6	9	-10	-4	0	2	-1	3	6	-10	
189D(B)	2	-	1	+	4	-2	-3	-8	6	4	6	-3	-1	11	-9	-6	15	4	-8	12	6	-1	9	-2	12	
190A(D)	-	-3	+	-5	-4	-1	-3	-	7	-3	-2	-2	-6	6	0	-13	-9	-12	-3	0	11	-2	-10	2	-2	
190B(C)	+	-1	+	-1	0	-3	-7	+	-5	-5	10	2	2	6	0	9	-7	-4	7	0	-9	-10	-2	-10	-18	
190C(A)	-	1	-	-1	0	-1	-3	-	3	-3	2	-10	6	2	0	3	3	8	-7	12	-13	14	6	6	-10	
192A(Q)	+	+	-2	-4	-4	2	-6	4	0	-2	4	2	2	-4	8	-10	4	-6	-4	-16	-6	4	-12	10	-14	

TABLE 3: HECKE EIGENVALUES 192B–214D

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
192B(A)	+	-	-2	4	4	2	-6	-4	0	-2	-4	2	2	4	-8	-10	-4	-6	4	16	-6	-4	12	10	-14	
192C(K)	+	-	2	0	-4	2	2	4	-8	-6	8	-6	-6	-4	0	2	-4	2	4	8	10	-8	4	-6	2	
192D(E)	-	+	2	0	4	2	2	-4	8	-6	-8	-6	-6	4	0	2	4	2	-4	-8	10	8	-4	-6	2	
194A(A)	-	0	4	-4	4	-4	6	-6	-4	0	0	-8	-2	-8	0	6	6	10	6	0	-10	8	-2	14	+	
195A(A)	-1	-	-	0	4	-	2	-4	8	-2	-8	6	-6	-4	-8	6	-12	-2	-4	0	-6	16	-4	10	18	
195B(I)	2	-	-	-3	-5	-	5	2	-1	10	-2	-3	-9	-4	10	9	0	-11	-4	15	6	-11	8	-11	-9	
195C(K)	2	-	+	-1	5	+	-7	-6	3	2	2	7	9	-8	10	5	0	5	-4	9	-6	-3	-4	11	-11	
195D(J)	2	+	-	3	-1	+	-1	-2	-3	-2	-6	11	-5	4	-10	11	8	13	12	-5	10	-3	-12	-15	17	
196A(A)	-	-1	-3	-	-3	-2	-3	1	3	-6	7	-1	-6	-4	9	3	-9	1	-7	0	1	-13	-12	-15	10	
196B(C)	-	1	3	+	-3	2	3	-1	3	-6	-7	-1	6	-4	-9	3	9	-1	-7	0	-1	-13	12	15	-10	
197A(A)	-2	0	0	-3	4	-2	-8	-3	-3	7	-10	7	9	1	-11	10	0	5	-10	8	6	2	-7	-8	-2	+
198A(I)	+	-	-2	-4	-	-6	-2	4	-4	-6	0	6	6	4	12	-2	-12	-14	4	12	-6	-4	-4	-10	-14	
198B(E)	-	-	0	2	-	-4	6	-4	-6	-6	8	-10	-6	8	6	0	0	8	-4	-6	2	14	12	6	14	
198C(M)	-	+	0	2	+	2	-6	2	0	-6	-4	2	6	-10	12	-12	12	-10	8	12	14	2	-12	0	2	
198D(A)	+	+	0	2	-	2	6	2	0	6	-4	2	-6	-10	-12	12	-12	-10	8	-12	14	2	12	0	2	
198E(Q)	+	-	4	-2	+	4	2	0	6	-10	-8	-2	-2	4	2	-4	0	-8	-12	-2	-6	10	-4	-10	-2	
200A(B)	+	-3	-	2	1	4	5	1	-2	-8	10	-6	-3	4	4	6	8	10	-1	-12	3	6	-13	-9	-14	
200B(C)	-	-2	-	-2	-4	-4	0	-4	2	2	0	-4	2	6	6	4	-12	-10	-14	8	-8	16	-2	6	-16	
200C(G)	-	0	+	4	4	2	-2	4	-4	-2	-8	-6	-6	8	-4	-6	-4	-2	-8	0	6	0	16	-6	14	
200D(E)	+	2	-	2	-4	4	0	-4	-2	2	0	4	2	-6	-6	-4	-12	-10	14	8	8	16	2	6	16	
200E(A)	-	3	+	-2	1	-4	-5	1	2	-8	10	6	-3	-4	-4	-6	8	10	1	-12	-3	6	13	-9	14	
201A	-2	+	0	0	-6	4	-7	-5	-1	1	-4	3	0	-6	9	10	3	2	+	-16	-7	8	-4	-15	4	
201B	-1	-	-1	-5	-4	-4	6	-2	-3	4	-7	5	-3	7	8	-5	3	-2	-	-12	-13	-8	1	4	-12	
201C	1	+	-3	-3	0	4	2	-2	-7	-8	-1	-3	-9	9	0	1	-9	14	+	-4	11	-16	5	0	16	
202A	+	0	2	1	4	0	5	1	6	-5	0	-8	-4	-5	6	3	-12	-1	2	-10	-16	-2	16	0	13	-
203A	-2	-1	-4	-	2	4	-2	5	9	+	-8	8	-3	-6	-7	9	0	2	3	7	-1	0	14	15	3	
203B	-1	-1	1	-	-5	-5	-4	-4	6	-	7	-10	0	-9	7	3	0	14	-6	8	-16	-9	16	-6	0	
203C	1	2	2	-	-4	-2	4	2	0	+	-2	2	0	0	-10	6	12	-4	12	-8	-4	12	-16	12	12	
204A	-	+	-1	4	3	3	+	1	3	-10	6	-4	5	-1	-2	-14	-6	8	-12	12	2	-14	6	16	0	
204B	-	-	1	0	5	-5	-	1	-3	2	2	-8	-5	-9	6	-6	6	-4	12	-12	-2	10	-2	12	16	
205A	-1	0	-	-4	0	-2	-6	0	-8	6	0	6	-	4	-4	6	-4	14	-8	-12	-6	-4	4	-6	-6	
205B	-1	2	+	2	6	2	2	-6	-4	10	0	-6	-	-4	-2	-14	12	-10	-2	-2	6	-2	0	10	10	
205C	1	2	-	2	0	-4	4	0	-8	2	0	-6	+	8	2	8	-12	2	10	8	-6	-8	12	14	-8	
206A	+	2	4	0	-6	-2	2	-4	0	-6	8	8	2	2	-8	-12	12	10	-2	0	10	0	-4	2	14	-
207A	-1	-	0	-2	-4	-6	-4	2	-	-2	4	2	-2	10	0	12	12	-6	-10	-8	-14	10	-12	16	-10	
208A	-	-1	-3	1	-6	-	-3	-2	0	6	4	-7	0	1	-3	0	6	8	-14	3	2	-8	-12	-6	-10	
208B	+	-1	-1	-5	2	+	-3	2	-4	-6	4	11	8	1	-9	-12	-6	0	-6	-7	-2	-12	16	-10	-10	
208C	-	0	2	2	2	+	6	6	-8	2	-10	-6	-6	-4	2	6	10	-2	-10	-10	2	4	6	-6	2	
208D	-	3	-1	-1	2	+	-3	-6	4	2	-4	3	0	5	-13	12	10	-8	2	5	-10	4	0	6	14	
209A	0	1	-3	-4	-	2	0	-	3	-6	-7	-7	0	-10	0	6	3	-10	11	15	8	-16	0	9	-1	
210A	-	-	+	-	0	2	-6	-4	0	-6	-4	2	6	8	-12	6	-12	2	8	0	14	-16	12	6	14	
210B	+	-	-	-	0	2	-6	8	0	6	-4	-10	-6	-4	0	-6	-12	-10	-4	12	-10	8	12	-6	-10	
210C	-	+	-	-	4	-2	2	-4	-8	6	-8	-2	2	-12	-8	6	4	-2	12	8	-14	0	12	2	10	
210D	+	+	+	+	-4	-2	-6	0	-8	10	-8	2	-2	8	4	10	4	-6	0	-12	-6	-8	-4	14	2	
210E	-	-	-	+	-4	-2	2	4	-8	-2	0	6	-6	-4	0	-10	12	14	-12	-8	10	16	-12	10	2	
212A	-	-1	-2	-2	2	-7	-3	5	-3	9	-8	-3	2	4	10	-	-2	-10	4	-9	-6	5	-11	-10	-3	
212B	-	2	2	0	-4	-2	2	2	-2	2	2	10	2	-4	-12	+	-12	10	-2	6	10	10	-6	-10	14	
213A	1	-	2	2	0	-2	0	0	0	-2	-10	-6	0	-4	12	-4	12	10	2	+	-10	4	-4	6	-2	
214A	-	-2	-3	-4	-2	4	-2	-2	1	-4	-10	12	-11	1	-1	6	-5	4	-5	-12	-16	7	-16	9	12	-
214B	+	-2	-1	4	-6	-4	-6	-2	5	0	-2	0	-11	-9	11	10	-3	-8	5	0	8	11	4	-15	-12	+
214C	+	1	-4	-2	-3	-1	6	1	-7	-6	4	-9	-5	12	8	7	-6	1	-10	6	-4	-7	4	-15	-6	+
214D	-	1	0	2	-3	-1	6	-7	9	-6	-4	-1	3	8	0	-9	6	-7	14	6	-4	-7	12	9	14	+

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
215A	0	0	+	-2	-1	-1	-3	-2	-1	4	3	-8	5	+	0	-5	12	-4	-3	6	-8	0	-9	-6	-17	
216A	+	+	-4	-3	-4	1	4	-1	-4	0	-4	-9	0	-8	12	8	-4	-5	11	-8	1	-5	-8	-12	5	
216B	+	-	-1	3	5	4	-8	2	2	6	-7	-6	-6	-2	6	5	-4	-8	-10	-8	1	16	-11	6	-1	
216C	-	+	1	3	-5	4	8	2	-2	-6	-7	-6	6	-2	-6	-5	4	-8	-10	8	1	16	11	-6	-1	
216D	-	+	4	-3	4	1	-4	-1	4	0	-4	-9	0	-8	-12	-8	4	-5	11	8	1	-5	8	12	5	
218A	-	-2	-3	-4	3	-4	-6	5	3	-3	-4	-4	0	-10	-3	12	12	-7	-4	-12	-1	-16	6	-3	-19	-
219A	-2	+	-1	2	-4	-2	-3	-1	0	-10	-6	1	2	6	7	3	1	-5	-13	10	+	-1	-11	-2	-11	
219B	0	-	-3	-4	0	-4	3	-1	6	-6	-10	-7	0	2	-3	9	-9	-1	-13	12	-	11	15	-18	5	
219C	1	+	-4	2	-4	-2	0	-4	0	8	6	-2	-10	-6	-8	-12	4	-14	8	-8	+	8	16	-14	-2	
220A	-	-2	-	-4	+	-4	0	-4	-6	-6	8	2	6	8	6	-6	-12	2	-10	-12	-16	8	0	6	14	
220B	-	2	-	0	-	0	-4	-4	6	2	0	-6	-10	4	10	2	-4	-14	2	4	-4	-8	12	6	6	
221A	-1	0	4	-2	6	+	-	8	4	-6	-2	-8	0	4	0	-6	0	-10	-8	2	0	0	-4	-2	-4	
221B	1	2	2	2	-6	+	-	4	6	-6	-2	2	-6	0	-4	14	4	2	0	-10	10	14	12	-18	2	
222A	-	-	0	-1	3	-1	-3	-7	3	0	2	-	-6	-4	6	9	0	-10	2	12	5	2	3	-3	2	
222B	-	+	0	3	1	1	-3	3	-1	-4	-6	+	-10	12	-6	-1	0	2	2	0	-3	14	9	-3	-10	
222C	+	+	2	0	-4	6	6	8	0	-6	4	-	-6	-8	8	6	-4	-2	-12	0	10	-12	-4	-10	-6	
222D	+	-	4	-1	-1	-3	3	-5	5	4	-10	+	-6	4	2	-11	-12	10	14	0	-11	-10	-9	11	10	
222E	+	+	-4	3	5	3	3	-7	9	0	-2	-	6	4	-10	3	-4	-2	6	-12	13	-6	5	11	6	
224A	+	-2	0	+	-4	-4	-2	-6	8	2	-4	10	-10	4	4	-2	10	-8	-8	0	-6	-16	2	18	-2	
224B	+	2	0	-	4	-4	-2	6	-8	2	4	10	-10	-4	-4	-2	-10	-8	8	0	-6	16	-2	18	-2	
225A	0	+	+	-5	0	-5	0	-1	0	0	-7	10	0	-5	0	0	0	-13	-5	0	10	-4	0	0	-5	
225B	0	+	-	5	0	5	0	-1	0	0	-7	-10	0	5	0	0	0	-13	5	0	-10	-4	0	0	5	
225C	-1	-	+	0	4	2	2	4	0	2	0	10	-10	-4	8	-10	4	-2	-12	8	-10	0	12	6	-2	
225D	2	-	+	3	-2	-1	2	-5	6	-10	-3	-2	8	-1	2	-4	10	7	3	8	14	0	6	0	-17	
225E	-2	-	-	-3	-2	1	-2	-5	-6	-10	-3	2	8	1	-2	4	10	7	-3	8	-14	0	-6	0	17	
226A	-	-2	-4	0	-4	-2	-2	-2	4	-4	8	-8	-6	6	-12	10	-6	-6	2	-8	-14	8	16	-14	-2	-
228A	-	+	2	0	2	2	6	+	2	4	-8	-2	-8	-8	2	-4	0	2	12	-4	6	-16	6	0	-2	
228B	-	+	-3	1	-5	-6	-5	-	4	6	6	-8	-8	9	1	2	-8	11	0	-4	-11	-8	-4	10	-10	
229A	-1	1	-3	2	-3	-6	-7	3	4	-6	4	2	6	7	6	-10	4	5	-10	-9	-2	6	11	-18	-5	+
231A	-1	+	-2	-	+	6	2	4	0	-2	8	6	10	-4	-8	6	4	-10	-12	0	2	16	4	18	2	
232A	+	-1	-3	2	-3	-5	-4	0	0	+	9	8	-2	-11	-7	9	4	-12	12	2	-4	3	-16	2	-14	
232B	-	1	1	2	3	-1	0	0	4	+	3	-8	-6	-5	3	5	-8	0	-12	6	-4	1	-12	6	14	
233A	1	-2	2	4	6	6	-6	-4	0	-2	4	-6	2	-2	2	-6	-10	-6	10	-8	-14	2	2	10	10	-
234A	+	-	1	1	2	+	3	6	4	-2	4	3	0	-5	-13	-12	10	-8	-2	5	-10	-4	0	-6	14	
234B	-	+	2	-2	4	+	0	-6	-4	8	-2	6	-6	-8	-8	-12	-4	10	-2	16	14	-4	12	6	-10	
234C	+	+	-2	-2	-4	+	0	-6	4	-8	-2	6	6	-8	8	12	4	10	-2	-16	14	-4	-12	-6	-10	
234D	-	-	-2	4	4	-	-2	-8	0	-6	-4	-2	10	4	-8	10	-4	-2	-16	8	2	8	-12	-14	10	
234E	-	-	3	-1	-6	-	3	2	0	-6	-4	-7	0	-1	-3	0	6	8	14	3	2	8	-12	6	-10	
235A	-1	-1	-	1	-3	-3	-6	-7	4	-10	3	12	-8	0	-	-4	6	5	-8	12	5	14	-17	-10	0	
235B	-1	-1	+	1	3	3	6	-1	4	2	-3	0	4	0	-	8	-6	5	4	0	-13	-10	7	14	12	
235C	2	2	+	-2	0	3	0	-4	1	8	6	-6	-2	9	-	8	3	-1	-8	3	5	-13	-14	-1	12	
236A	-	-1	-1	-3	-2	0	2	-5	-4	5	-4	8	-1	0	8	3	-	-2	-14	0	-2	-13	4	-18	2	
236B	-	1	3	-1	6	-4	-6	5	0	9	-4	-4	-9	8	-12	-9	+	2	2	0	14	-7	0	-6	2	
238A	-	-2	-4	-	-6	-2	+	0	-4	8	0	4	-2	-8	-8	-6	-4	-8	-16	4	10	-12	12	10	6	
238B	+	0	-2	+	-2	0	+	-2	-8	0	8	-4	-6	4	8	-6	10	10	8	4	-10	-4	-6	-6	-14	
238C	-	0	2	-	0	-2	-	4	0	-6	0	-6	-6	-12	8	-2	4	2	12	0	2	-8	12	10	-14	
238D	-	2	0	+	-2	-2	+	0	4	4	0	8	-2	0	0	2	4	-12	-8	12	-14	12	4	-6	6	
238E	+	2	4	-	-4	-4	+	-6	0	6	4	-10	6	0	4	14	-6	-12	4	-8	2	0	10	10	6	
240A	+	+	-	0	4	6	-6	4	0	-2	8	-2	-6	-12	-8	6	-12	14	-4	-8	-6	8	12	10	2	
240B	-	+	+	4	0	2	6	4	0	-6	-8	2	-6	4	0	-6	0	-10	4	0	2	-8	-12	18	2	
240C	+	+	+	-4	0	-6	-2	-4	8	-6	0	-6	10	4	-8	10	0	6	4	0	-14	-16	-12	2	2	
240D	-	-	-	0	4	-2	2	-4	0	-2	0	-10	10	-4	-8	-10	4	-2	-12	8	10	0	-12	-6	2	

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
270C	-	-	-	2	-3	-1	-3	8	3	-9	-7	2	-12	-7	-3	12	12	-10	-4	0	2	-1	18	0	14	
270D	+	+	-	2	-3	5	3	-4	9	3	5	-10	0	-1	-9	12	-12	2	-4	-12	-10	-13	-6	12	2	
272A	+	-2	0	0	-2	-6	+	-4	-4	0	8	-4	6	-8	8	10	0	12	-8	-12	2	4	-16	10	-18	
272B	-	0	-2	-4	0	-2	-	4	-4	6	-4	-2	-6	-4	0	6	12	-10	-4	4	-6	-12	4	10	2	
272C	+	2	-2	2	6	2	-	0	-6	-10	-2	6	-6	8	0	-10	8	14	-4	-2	-14	10	-8	-10	2	
272D	-	2	0	4	-6	2	+	4	0	0	4	-4	6	-8	0	-6	0	-4	-8	0	2	-8	0	-6	14	
273A	-2	+	-1	-	-2	-	-4	3	-9	-1	-5	-8	6	-9	-3	3	0	10	-2	12	5	-13	-11	1	1	
273B	2	-	1	+	-2	+	0	1	3	-5	9	0	2	-1	3	-9	0	-2	10	-12	15	11	3	-17	3	
274A	-	-2	-3	0	-3	-6	1	-3	0	-3	7	10	-10	6	3	-11	-5	-8	2	-1	7	5	-14	-14	-10	-
274B	+	0	-3	2	-1	-2	-7	-1	0	1	-11	4	0	6	-7	9	9	0	2	5	11	-5	6	-8	12	+
274C	+	0	0	-4	-4	4	2	-4	-6	-8	10	-2	6	0	2	0	-12	6	8	-10	14	-14	12	-14	6	+
275A	-1	0	+	0	+	-2	-6	-4	-4	6	-8	2	2	-4	12	2	4	-10	16	8	-14	8	4	10	-10	
275B	2	1	+	2	-	-4	2	0	1	0	7	-3	-8	6	-8	6	5	12	7	-3	-4	-10	6	15	7	
277A	1	-2	2	-4	1	-5	2	-6	0	5	-3	-4	7	-1	-2	2	4	6	-12	6	-8	-16	-16	-15	4	+
278A	-	-2	-1	-5	-3	1	2	-2	-6	1	9	-6	-6	-4	0	12	10	-4	-11	-3	-10	-5	-1	-9	-16	-
278B	+	-2	3	-1	-3	5	6	2	6	-3	5	2	-6	8	0	-12	6	8	5	-15	2	-1	-9	15	8	-
280A	+	-1	+	+	-5	1	3	-6	-6	-9	0	6	8	6	3	-12	8	-4	-4	8	10	-3	-12	-16	7	
280B	+	-3	-	-	-5	-5	-7	-2	-2	7	4	-6	-12	-2	1	0	-4	4	8	0	6	-3	-4	0	13	
282A	-	+	2	0	0	2	2	0	0	2	-8	-2	2	-8	+	-2	-4	-10	-8	0	10	0	12	10	2	
282B	-	+	-4	-4	0	-2	-6	6	-4	4	2	-6	-12	-2	-	-6	-4	2	10	8	-2	-12	12	-18	14	
285A	-1	-	+	-2	-6	0	-6	-	-8	4	0	4	0	-2	-8	2	12	2	-8	16	14	8	0	0	-12	
285B	1	+	+	-2	-2	-4	2	+	-4	4	0	0	0	-10	12	-2	4	2	-16	0	-2	-8	-12	0	-16	
285C	1	+	-	4	4	2	2	+	-4	-2	0	-6	-6	8	-12	-14	4	14	-4	0	-14	16	0	-6	-10	
286A	+	-2	3	-1	+	-	6	8	-3	9	2	-10	9	-1	0	6	-3	-7	-7	12	-1	-4	0	12	-4	
286B	-	-1	-3	-5	+	-	7	0	-4	-8	0	-3	-8	-5	-3	2	-14	8	0	-5	16	-6	-4	0	0	
286C	+	-1	-1	1	+	+	-1	-4	-8	-8	0	7	-8	11	-1	2	14	-8	8	9	-4	2	0	-4	8	
286D	-	-1	1	3	-	-	3	0	4	0	-8	-7	-8	-1	-7	-6	10	-8	8	7	-16	10	4	0	8	
286E	-	2	-1	1	+	+	2	-4	1	7	-6	-2	-5	5	8	2	5	7	-7	0	5	-4	0	-4	-16	
286F	-	2	1	-3	-	-	-6	0	1	-3	10	2	7	-1	-4	6	-5	-11	-1	16	-7	4	4	12	-16	
288A	+	+	-4	0	0	-6	-8	0	0	4	0	-2	8	0	0	4	0	-10	0	0	6	0	0	-16	-18	
288B	-	-	-2	-4	-4	-2	6	-4	0	-2	4	-2	-2	4	-8	-10	4	6	4	16	-6	4	-12	-10	-14	
288C	+	-	-2	4	4	-2	6	4	0	-2	-4	-2	-2	-4	8	-10	-4	6	-4	-16	-6	-4	12	-10	-14	
288D	+	-	2	0	0	6	-2	0	0	10	0	-2	-10	0	0	-14	0	-10	0	0	-6	0	0	-10	18	
288E	-	+	4	0	0	-6	8	0	0	-4	0	-2	-8	0	0	-4	0	-10	0	0	6	0	0	16	-18	
289A	-1	0	2	-4	0	-2	+	-4	-4	-6	-4	2	6	4	0	6	-12	10	4	4	6	-12	-4	10	-2	
290A	+	0	+	-2	2	-6	2	-2	-6	+	-6	-2	10	-8	-4	10	8	10	2	4	6	-10	-6	-6	6	
291A	-2	+	3	-2	0	-4	6	6	0	7	7	4	5	1	-10	10	-5	5	-14	15	7	-5	-9	-8	-	
291B	-1	+	-2	-4	4	6	2	-8	4	6	8	-2	10	-4	0	-10	8	14	8	-4	-6	-8	8	10	-	
291C	-1	+	0	2	-4	-2	-8	-2	-4	0	8	10	-12	-8	8	-2	-8	-10	2	8	6	4	8	10	+	
291D	2	+	1	2	4	0	2	-2	-8	-3	-1	4	7	-7	6	2	-7	5	-10	5	-9	-5	5	16	-	
294A	-	+	1	+	5	0	-4	8	-4	-5	3	-4	0	2	-6	-9	-11	-6	-2	2	10	3	-7	-6	7	
294B	-	-	-1	-	5	0	4	-8	-4	-5	-3	-4	0	2	6	-9	11	6	-2	2	-10	3	7	6	-7	
294C	-	-	2	-	-4	-6	-2	4	8	-2	0	-10	6	-4	0	6	-4	-6	4	8	-10	0	4	6	14	
294D	+	-	3	+	3	-4	0	-4	0	9	-1	8	0	-10	-6	-3	3	-10	-10	-6	2	-1	-9	6	-1	
294E	+	+	-3	-	3	4	0	4	0	9	1	8	0	-10	6	-3	-3	10	-10	-6	-2	-1	9	-6	1	
294F	+	+	4	-	-4	4	0	4	0	2	8	-6	0	4	-8	-10	4	-4	4	8	-16	-8	-12	8	8	
294G	+	-	-4	-	-4	-4	0	-4	0	2	-8	-6	0	4	8	-10	-4	4	4	8	16	-8	12	-8	-8	
296A	+	-1	-2	1	1	-6	-4	-8	6	2	-4	+	7	2	9	-3	-12	4	0	7	7	0	3	-12	-8	
296B	-	-1	0	-3	-3	0	2	-2	-6	-2	-4	-	7	4	1	9	8	-4	12	-5	-13	-10	-1	-2	-12	
297A	-2	-	-2	1	-	-5	-2	3	-4	-6	-8	-9	4	0	-10	6	14	9	5	-12	7	11	-12	-6	-7	
297B	-1	+	2	-5	+	-2	-7	0	1	-3	-8	-3	11	-9	1	12	-5	6	-4	0	4	5	6	6	11	
297C	1	-	-2	-5	-	-2	7	0	-1	3	-8	-3	-11	-9	-1	-12	5	6	-4	0	4	5	-6	-6	11	

TABLE 3: HECKE EIGENVALUES 297D–320E

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
297D	2	-	2	1	+	-5	2	3	4	6	-8	-9	-4	0	10	-6	-14	9	5	12	7	11	12	6	-7	
298A	-	-2	-2	-2	0	-5	-7	1	-1	8	4	0	-6	8	-6	-10	4	6	3	-15	9	1	0	2	-8	-
298B	+	0	-4	4	2	-5	-7	-7	3	-8	2	-4	0	4	-6	4	10	2	-5	13	-7	1	-4	-2	-10	+
300A	-	+	+	1	6	-5	6	5	6	-6	-1	-2	0	1	-6	12	-6	-13	-11	0	-2	8	6	0	7	
300B	-	-	-	-1	6	5	-6	5	-6	-6	-1	2	0	-1	6	-12	-6	-13	11	0	2	8	-6	0	-7	
300C	-	-	-	4	-4	0	4	0	4	-6	4	-8	-10	4	-4	-12	4	2	-4	0	-8	-12	4	-10	8	
300D	-	+	-	-4	-4	0	-4	0	-4	-6	4	8	-10	-4	4	12	4	2	4	0	8	-12	-4	-10	-8	
302A	-	-1	-4	-2	2	-6	3	0	-6	0	-3	-2	12	-6	-7	9	-10	-13	-7	12	4	10	-11	0	-7	-
302B	+	2	2	4	-4	0	-6	0	0	6	0	-2	6	0	8	-12	-4	8	2	-12	10	-8	-14	-6	2	-
302C	-	-3	0	-2	-6	-2	-5	-8	6	8	9	2	0	-6	-3	-9	2	5	3	4	-8	10	-1	8	-15	-
303A	0	-	-3	0	-2	-3	-7	-5	-5	6	7	10	6	4	-7	-4	-10	-2	10	-9	-8	7	2	-8	-10	-
303B	-2	-	-1	-2	-6	1	-5	7	-3	-6	-1	-10	-2	-12	11	4	4	10	-2	1	2	11	8	14	-10	-
304A	-	1	-4	-3	-2	-1	3	-	1	-5	8	-2	-8	-4	-8	-1	-15	2	-3	-2	9	10	6	0	-2	
304B	-	-1	0	1	6	5	3	+	-3	9	4	2	0	-8	0	-3	-9	-10	-5	6	-7	10	6	-12	-10	
304C	+	-1	0	-3	-2	1	-5	+	1	-3	-4	2	-8	8	8	9	-1	14	-13	-10	9	10	-10	-12	14	
304D	+	2	-1	3	3	-4	5	-	0	2	-8	-10	6	7	9	-8	-14	-5	0	6	-15	4	-4	0	16	
304E	-	2	3	1	-3	-4	-3	+	0	6	4	2	-6	1	3	12	6	-1	4	-6	-7	-8	-12	12	8	
304F	-	-2	-1	3	-5	-4	-3	-	-8	-2	-4	10	10	-1	1	-4	-6	-13	12	-2	9	-8	12	12	-8	
306A	-	-	0	2	0	2	-	-4	6	0	-10	8	-6	-4	-12	-6	12	8	-4	-6	2	-10	-12	18	14	
306B	+	-	0	-4	-6	2	-	-4	0	0	-4	-4	-6	8	0	6	0	-4	8	0	2	8	0	6	14	
306C	+	-	2	0	4	-2	+	4	0	10	8	-2	-10	12	0	-6	-12	-10	-12	0	10	-8	-4	6	-14	
306D	-	-	4	-2	0	-6	-	4	-6	4	-6	-4	10	-4	-4	2	-12	-4	-12	6	2	10	12	2	6	
307A	0	0	4	0	3	6	-1	-1	-2	0	4	3	5	-10	-6	-10	4	-8	-8	-15	2	-13	5	9	7	-
307B	1	2	0	3	5	0	-5	-1	6	-6	-4	-9	-3	10	-4	5	6	-10	2	13	8	8	-16	6	-2	-
307C	2	0	2	3	-4	0	3	1	2	6	-4	-6	2	-4	-10	-3	10	4	-4	-1	8	11	9	-3	11	-
307D	2	2	0	-3	1	6	2	-4	-6	0	2	3	9	4	4	1	-12	14	2	8	-10	11	13	9	-5	-
308A	-	-1	-1	+	-	-4	-6	-2	1	2	-1	-9	6	8	-8	10	1	-2	11	11	-14	-14	4	13	-9	
309A	-1	-	-1	-2	-2	-5	0	-8	1	-2	5	2	8	-11	-2	10	-11	-5	11	16	12	6	1	-6	-7	-
310A	-	2	+	0	2	0	2	-4	-4	-4	+	-8	6	2	0	8	8	0	4	0	6	-4	6	-6	-2	
310B	-	-2	+	-4	0	-4	0	-4	-6	6	-	8	-6	-10	0	0	-12	14	8	0	-4	8	6	-18	-10	
312A	+	-	0	0	6	+	2	0	4	-6	-4	-2	0	4	10	-10	-6	-6	-12	2	6	-16	6	4	14	
312B	+	+	0	-4	-2	+	-6	-4	4	10	-8	-2	0	-4	2	-2	10	10	8	2	-10	8	6	-12	-2	
312C	-	-	2	0	0	-	2	-4	0	6	0	-2	6	-12	-4	6	-8	-2	4	-12	-14	0	8	-18	-6	
312D	+	+	-2	4	0	-	2	8	8	-2	4	-10	2	-4	-12	6	0	-2	8	-12	10	-8	0	-14	2	
312E	-	+	4	0	-2	+	2	8	4	-6	-4	6	-12	4	-6	-2	-14	10	-4	2	-2	-8	14	0	-10	
312F	-	-	-4	-4	-2	+	-6	4	4	-6	8	-10	-4	-4	-6	6	-6	-6	0	10	-2	0	-10	8	-10	
314A	+	0	0	-3	-2	-1	3	-4	-1	0	-6	-1	0	1	0	12	-7	0	-2	10	12	-8	0	-3	-2	+
315A	0	-	-	-	3	5	-3	2	6	-3	-4	2	12	-10	-9	-12	0	8	-4	0	2	-1	-12	12	-1	
315B	-1	-	+	-	0	-6	-2	-8	-8	2	4	-2	6	4	-8	-10	-4	-2	4	12	-2	8	4	6	-18	
316A	-	-1	1	3	2	-1	4	6	6	8	-4	-8	-10	4	-9	-2	5	-6	-10	-1	6	+	0	9	-11	
316B	-	-3	1	1	-6	-1	-4	-6	2	-8	4	4	-6	4	-3	14	-9	6	-10	5	6	-	4	1	-11	
318A	-	+	0	1	5	0	2	-1	3	-1	-4	0	-9	0	6	+	-4	-7	1	7	-14	-8	8	-12	13	
318B	+	-	0	5	-3	-4	6	5	-3	3	8	-4	-3	-4	6	+	-12	-1	-13	-15	2	-16	0	0	5	
318C	+	+	-1	0	-1	-2	-7	2	-5	-4	-1	-2	-4	-1	6	+	9	10	-2	0	10	1	6	-1	-13	
318D	-	+	-3	-4	-5	-2	5	6	-7	-8	1	2	4	-1	-6	-	-3	-2	-10	0	-6	15	-10	-5	19	
318E	+	+	4	1	-1	-4	6	-1	9	-3	-8	12	5	-8	-2	-	4	-7	1	-3	6	-4	-8	-4	-3	
319A	2	-3	1	4	+	6	4	-2	3	-	-7	-11	4	-4	8	2	-3	2	-15	-7	2	6	-6	9	-17	
320A	-	0	+	4	4	2	2	4	-4	2	8	-6	-6	-8	-4	-6	-4	2	8	0	-6	0	-16	-6	-14	
320B	+	0	+	-4	-4	2	2	-4	4	2	-8	-6	-6	8	4	-6	4	2	-8	0	-6	0	16	-6	-14	
320C	+	2	-	2	0	-2	-6	4	6	-6	-4	-2	6	10	-6	6	-12	-2	-2	-12	2	8	-6	-6	2	
320D	+	2	-	-2	4	6	2	-8	-6	2	4	-2	-10	2	-2	-2	0	-2	6	-12	10	-8	10	-6	10	
320E	+	-2	-	2	-4	6	2	8	6	2	-4	-2	-10	-2	2	-2	0	-2	-6	12	10	8	-10	-6	10	

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
320F	--2	--2	0-2	-6-4	-6-6	-6	4-2	6-10	6	6	12-2	2	12	2	-8	6	-6	2	12	2	-8	6	-6	2		
322A	+ 0-2	--4	4-8	-2	-	2	-6-10	6-8	6	2	0	10	8-12	6	0	2	12	12								
322B	+ 2 0	- 4	0 6-6	+ 10	4	-2-10	-4	12	-6	-2	0	0	-8	-6	-8	-14	-14	-2								
322C	- 2-2	- 6-4	-2	4	-10	-8	-8	-2	6	12	12	-6	-6	-2	16	2	0	4	-6	2						
322D	--2-2	+ -2-4	-6	0	-	-2	4	0	6	6	0-12	-10	2	-2	8	2	8	-16	6	-2						
323A	0 3-2	4-2	6+	-	0-9	-9	2	-6	-1	-3	2	14	-6-14	16	-2	8	-3	2	-7							
324A	- + 3-1	3-1	6-4	-3	3	5	2	3	-1	-9	-6	-3	-13	-7	-12	-10	11	-9	6	11						
324B	- + 3 2-6	5-3	2	6	3	-4	5	-6	-10	0	-6	-12	5	2	6	-1	-10	0	-3	-10						
324C	- - -3-1	-3-1	-6-4	3	-3	5	2	-3	-1	9	6	3	-13	-7	12	-10	11	9	-6	11						
324D	- + -3 2	6	5	3	2-6	-3	-4	5	6	-10	0	6	12	5	2	-6	-1	-10	0	3	-10					
325A	0 1	-4-6	-	6-4	3	-3	-4	2	6	-7	0	-9	-6	-1	14	-6	-4	11	-6	0	-10					
325B	0-1	+ 4-6	+ -6-4	-3	-3	-4	-2	6	7	0	9	-6	-1	-14	-6	4	11	6	0	10						
325C	1 2	+ 4	2	-2-6	6	2	-10	2	-6	-10	-4	-2	6	2	4	6	6	-12	16	2	2					
325D	2 1	- 2	2	+ 2	0-9	5	2	-8	12	1	-8	11	0	-13	2	12	6	15	-4	-10	-8					
325E	-2-1	+ -2	2	-2	0	9	5	2	8	12	-1	8	-11	0	-13	-2	12	-6	15	4	-10	8				
326A	+ 0-1	-1	0-5	6-6	-3	-1	-3	-2	-3	1	10	-6	10	-12	10	-2	16	16	-1	-2	-5	+				
326B	--2-1	-3-4	-1	0-2	-1	3	-9	6	1	7	-4	8	-6	-4	4	12	2	-16	5	0	-17	-				
326C	+ -2-3	-1	0	5	0	2-3	9	5	2	9	-1	-12	0	6	8	-4	-12	2	8	-3	12	-1	-			
327A	-1	- -1-2	-1-4	-4-7	1	7	-2	-6	-2	4	7	-4	4	11	-12	-10	11	8	14	5	-7	-				
328A	+ 0-2	-2	0-4	-2	4-4	0	4	-6	+ 12	-6	-4	-4	10	12	-6	-2	-2	-4	-6	14						
328B	+ 2 2-2	2	6-6	-2	0	6	-8	10	-	0	-6	-2	-4	-2	-10	-2	-2	-2	12	10	-6					
329A	-1-1	3	+ 3-6	6	8	4	2	6	9	-5	-9	-	-1	3	-4	4	0	-13	8	7	-4	-6				
330A	+ + +	0	- 2-2	8	4	2	8	-2	6	8	-4	2	4	-6	-12	-12	2	0	4	-6	-14					
330B	- - -	0	+ -2	2-4	0	-2	0	-2	2	-12	8	6	-12	6	4	0	-6	-16	4	10	2					
330C	- + -	0	- 6	2-4	0	-10	0	6	2	4	-8	-10	-4	-2	-4	-8	2	-8	-12	-6	18					
330D	- + +	4	+ 2	2	4-4	6	0	-10	-6	-12	-4	-6	-4	10	-12	-4	10	4	4	10	18					
330E	+ + -	-4	- -2	-2-8	0	2	-8	-10	-10	0	0	14	-4	14	-4	8	10	12	4	-6	-14					
331A	-1-2	1	2	0-4	1-3	-8	-10	7	-8	0	11	-4	1	-10	-8	7	1	8	-9	-12	6	8	+			
333A	0	- 0-1	-3-4	-6	2-6	6	-4	-	9	8	-3	3	-12	8	-4	15	11	-10	-9	-6	8					
333B	1	+ -2-4	4-2	-6-6	8	6	2	+	0	-10	-12	4	-4	10	-4	-12	-10	10	0	-2	-2					
333C	-1	+ 2-4	-4-2	6-6	-8	-6	2	+	0	-10	12	-4	4	10	-4	12	-10	10	0	2	-2					
333D	2	- 2-1	5-2	0	0-2	-6	-4	+	9	2	9	-1	-8	-8	8	-9	-1	4	15	-4	4					
334A	- 0	3	1	0-2	-2	2	2	-4	1	-3	2	4	-1	-11	-3	-4	-3	6	-4	-12	7	3	7	+		
335A	0	0	- -2	-2-3	-1-1	-9	0	-3	-2	6	9	12	5	0	-	-4	-1	-4	-4	3	-14					
336A	- + 0	+ 6	2	0	4	6	6	-8	2	12	4	-12	-6	0	-10	-8	-6	-10	4	12	12	-10				
336B	+ + 2	- 0-2	6	4	4	6	8	-10	-10	-12	8	6	-4	-10	-12	-4	2	-8	-4	6	10					
336C	+ - 2	+ 0	6-2	-4	4	-10	8	6	-2	4	-8	-10	-12	-2	-12	12	-14	8	-12	-2	10					
336D	- - -2	- 4	6	2	4-8	-2	0	-10	-6	4	0	6	-4	6	-4	-8	10	0	4	-6	-14					
336E	- + -2	- -4	-2-6	-4	0	-2	0	6	2	4	0	6	-12	-2	-4	0	-6	16	12	-14	18					
336F	- - 4	- -2	-6-4	4-2	-2	0	2	0	4	-12	-6	8	6	8	-14	-2	-12	4	0	-2						
338A	+ 0	1-4	-4	+ 3	0-4	-1	-4	-3	9	-8	8	-9	4	7	-4	8	-11	-4	0	6	-2					
338B	- 0-1	4	4	+ 3	0-4	-1	4	3	-9	-8	-8	-9	-4	7	4	-8	11	-4	0	-6	2					
338C	- 1	3	1-6	+ -3	-2	0	6	4	7	0	-1	-3	0	6	8	-14	3	-2	8	-12	6	10				
338D	+ -1	3	3	0	- -3	6	6	0	0	3	0	1	3	-6	-6	-8	12	-15	6	10	-6	-6	12			
338E	--1-3	-3	0	- -3	-6	6	0	0	-3	0	1	-3	-6	6	-8	-12	15	-6	10	6	6	-12				
338F	+ -3	1-1	2	+ -3	-6-4	2	-4	-3	0	-5	-13	12	10	-8	2	5	10	-4	0	-6	-14					
339A	0	- -1-3	-4-2	-2-2	1	-7	8	4	0	12	-9	-8	-3	-3	2	7	4	-4	-2	13	1	-				
339B	2	+ 2	3-6	5	3	0	3	-3	-7	2	-8	0	-9	4	-9	6	14	0	-10	-14	14	1	13	-		
339C	-2	- -3	1-2	-2-2	0-5	-5	-4	-4	4	-12	-3	6	-9	-3	16	1	14	2	6	3	1	-				
340A	- 0	+ -4	2-6	- 0	0	-6	6	-2	-6	6	-10	-6	0	10	-2	6	6	6	6	-18	-14					
342A	- - 0-1	6	5-3	- -3	-9	-4	2	0	8	0	3	-9	-10	5	6	-7	-10	6	12	-10						
342B	- - 0	4-4	0	2	- 2	6	6	-8	-10	-12	-10	-2	-4	-10	0	16	-2	10	16	2	-10					

TABLE 3: HECKE EIGENVALUES 342C–360D

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
342C	+	-	0	-4	0	-4	-6	-	6	-6	2	-4	-6	-4	-6	-6	12	14	8	0	14	-10	12	6	-10	
342D	-	+	2	0	2	-4	0	+	8	-2	-2	-8	-2	4	-4	2	0	-10	0	-16	6	14	-6	-18	10	
342E	+	+	-2	0	-2	-4	0	+	-8	2	-2	-8	2	4	4	-2	0	-10	0	16	6	14	6	18	10	
342F	+	-	-2	0	4	2	6	+	4	2	4	10	-10	4	4	10	-12	14	-12	-8	-6	-4	-12	6	10	
342G	+	-	4	3	-2	-1	-3	+	1	5	-8	-2	8	4	-8	1	-15	2	3	-2	9	-10	6	0	-2	
344A	-	0	-2	-2	1	-1	-7	-6	9	4	1	-4	-11	-	0	11	12	0	7	-10	-4	-8	-3	6	3	
345A	0	+	+	1	4	0	5	0	-	5	3	-5	3	-4	6	-3	9	10	-7	7	-12	8	-1	16	-6	
345B	0	-	+	-3	-4	0	-3	-8	-	9	-5	-9	7	4	-2	13	-3	-14	13	-13	-4	0	-1	-8	10	
345C	1	-	+	4	4	6	-2	-4	+	-10	-8	2	2	-8	0	-6	0	6	8	-4	10	16	-12	-10	-10	
345D	-1	-	+	4	-4	-2	6	8	+	6	8	6	-6	-8	-8	2	-4	-10	8	0	-6	-4	-12	6	-14	
345E	2	+	-	3	2	-2	5	-2	+	-5	3	-7	-11	-8	8	5	-1	-8	-9	1	10	0	15	0	-10	
345F	-2	-	-	-5	-2	-6	1	2	+	-1	-5	-7	-7	-8	-12	9	3	12	-1	5	-2	-8	3	8	14	
346A	-	1	-1	4	4	-6	-4	5	5	8	-7	-2	-5	-10	-3	-1	9	-15	-8	4	1	16	6	-6	-8	+
346B	-	-1	-3	-2	-4	0	-2	7	-3	-4	-7	-4	3	6	9	-3	-9	3	2	-12	-7	10	6	-10	-6	-
347A	-2	1	0	-2	-3	-2	4	-4	4	-9	8	-12	8	-7	-10	-6	8	5	-11	12	7	10	9	1	16	+
348A	-	+	0	-3	-3	-3	1	-4	-2	-	-2	-6	10	0	-3	4	10	-6	3	6	14	4	-18	7	0	
348B	-	-	2	1	1	-3	-3	2	8	-	-8	0	2	0	5	-2	-6	-12	3	4	-16	-2	-6	3	-6	
348C	-	+	-2	1	3	5	-1	6	4	+	0	8	-10	4	7	-2	6	-8	3	-4	4	6	-14	-7	-2	
348D	-	-	-4	-3	-1	-3	-5	4	-6	+	2	6	6	-12	7	-12	-10	10	-13	-2	14	-8	6	5	0	
350A	+	0	+	-	4	6	-2	0	0	6	8	10	2	-4	-8	2	-8	-14	12	-16	-2	-8	-8	10	-2	
350B	-	1	-	-	3	2	3	-7	0	-6	-4	8	-9	8	-6	-12	12	-10	-7	6	5	14	-9	-15	-10	
350C	+	-1	+	+	3	-2	-3	-7	0	-6	-4	-8	-9	-8	6	12	12	-10	7	6	-5	14	9	-15	10	
350D	-	2	+	+	0	4	-6	2	0	-6	-4	-2	6	-8	12	-6	-6	8	4	0	-2	8	6	-6	10	
350E	+	3	+	-	-5	6	1	-3	0	-6	-4	-8	11	8	-2	-4	4	-2	-9	-10	7	-2	-11	-11	10	
350F	-	-3	-	+	-5	-6	-1	-3	0	-6	-4	8	11	-8	2	4	4	-2	9	-10	-7	-2	11	-11	-10	
352A	+	1	1	4	-	-2	0	-2	9	4	5	-9	2	-6	-4	-6	-5	0	-13	-1	14	-10	14	-13	-19	
352B	-	1	-3	-4	-	-2	-8	6	5	4	1	3	-6	-6	-12	-6	3	0	11	-5	-10	-2	-2	-5	13	
352C	+	-1	1	-4	+	-2	0	2	-9	4	-5	-9	2	6	4	-6	5	0	13	1	14	10	-14	-13	-19	
352D	+	-1	-3	4	+	-2	-8	-6	-5	4	-1	3	-6	6	12	-6	-3	0	-11	5	-10	2	2	-5	13	
352E	-	3	1	0	+	-6	-4	6	3	-4	-9	7	-2	6	12	2	9	8	-15	-3	-6	-6	-6	-5	-3	
352F	-	-3	1	0	-	-6	-4	-6	-3	-4	9	7	-2	-6	-12	2	-9	8	15	3	-6	6	6	-5	-3	
353A	-1	2	2	-2	4	2	2	0	4	2	2	2	-2	8	-4	-6	-2	2	2	6	-10	-10	-12	-14	-14	-
354A	-	+	0	0	4	4	6	-4	-4	0	2	-8	6	4	-4	4	+	0	-16	-14	2	-8	-4	14	-10	
354B	+	-	0	-1	3	5	-3	8	-6	6	8	5	-9	-1	0	12	+	-10	-4	-3	-16	5	-9	0	-4	
354C	+	+	0	-1	-5	1	1	0	-6	-10	-8	9	-5	3	0	4	+	6	4	1	0	-3	7	16	-12	
354D	+	+	2	0	4	-6	2	4	8	2	8	2	2	0	8	-6	-	10	-8	-12	-14	-16	4	6	2	
354E	-	+	4	0	-4	0	-2	4	4	4	-10	-4	-2	-12	4	0	+	4	-8	6	-14	8	-4	-18	14	
354F	-	+	-4	-1	-3	-1	-7	-4	2	-2	0	7	3	5	12	-8	-	-14	-4	-15	-4	5	1	4	-4	
355A	0	-2	-	-1	0	5	6	-1	0	-3	2	8	6	2	3	-3	-6	2	-4	+	-4	-1	6	15	-7	
356A	-	-1	-1	0	0	-4	-1	-5	-1	-6	3	-6	2	1	10	9	4	-4	-2	2	7	2	-4	-	1	
357A	0	+	1	+	3	3	-	3	7	-6	10	4	-9	9	6	-10	-2	0	-12	-12	6	10	10	-4	8	
357B	0	+	1	-	-5	-5	-	-5	-1	-6	-6	4	7	-7	6	6	14	0	-12	4	6	-6	-6	12	8	
357C	2	-	1	+	1	1	+	1	-3	-2	0	-6	-1	5	12	0	0	-2	-8	0	6	-4	6	16	-12	
357D	-2	-	-3	-	-3	1	+	-7	1	-10	4	-10	3	-11	-8	-4	4	10	-8	8	-2	16	6	-8	-4	
358A	+	2	0	-2	5	6	3	-2	2	2	5	-1	-6	-10	5	11	-12	-10	-8	12	8	-10	-2	-1	-2	-
358B	-	-2	0	2	3	2	3	2	6	-6	5	-7	6	-10	-3	-3	0	2	-4	0	-4	-10	6	-9	2	+
359A	1	-2	1	1	-2	-6	-3	-1	0	-4	-1	7	-2	1	0	4	11	2	12	-9	-7	-4	9	-6	-8	+
359B	-1	0	1	-1	-2	0	-3	1	-6	-6	1	-9	6	-5	8	6	5	-4	-4	13	1	-14	15	-2	10	+
360A	+	-	+	0	4	6	6	-4	0	2	-8	-2	6	12	-8	-6	-12	14	4	-8	-6	-8	12	-10	2	
360B	-	+	+	2	2	4	-2	4	8	-10	4	0	0	-8	8	6	-14	-14	-4	12	6	-12	4	-12	-14	
360C	+	+	-	2	-2	4	2	4	-8	10	4	0	0	-8	-8	-6	14	-14	-4	-12	6	-12	-4	12	-14	
360D	-	-	-	4	0	-6	2	4	8	6	0	-6	-10	-4	-8	-10	0	6	-4	0	-14	16	-12	-2	2	

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
360E	-	-	+	-4	-4	-2	-2	4	-4	2	-8	6	6	-8	-4	-6	4	-2	8	0	-6	0	16	6	-14	
361A	0	0	-1	3	-5	0	-7	+	-4	0	0	0	0	-1	13	0	0	15	0	0	-11	0	-16	0	0	
361B	0	2	3	-1	3	4	-3	-	0	-6	4	-2	6	-1	-3	-12	6	-1	4	-6	-7	-8	12	-12	-8	
362A	+	-1	2	-4	-1	4	-6	-2	-3	4	-11	-12	4	-1	-11	6	9	5	12	3	-15	0	-2	16	-10	+
362B	-	-1	-2	-4	-1	-4	2	6	-1	-8	-1	0	0	-1	-1	-6	9	-1	-12	9	-7	-8	10	0	-14	-
363A	-1	+	-2	-4	-	2	2	0	8	6	-8	6	2	0	8	6	-4	-6	-4	0	14	4	-12	-6	2	
363B	2	+	4	-1	-	2	-4	3	2	-6	-5	3	2	-12	2	6	-10	-3	-1	0	11	-11	-6	12	5	
363C	-2	+	4	1	-	-2	4	-3	2	6	-5	3	-2	12	2	6	-10	3	-1	0	-11	11	6	12	5	
364A	-	0	-3	-	-2	+	-4	-1	-7	7	-5	4	-6	9	-7	11	0	-2	-10	0	7	1	-11	-1	-13	
364B	-	-2	1	+	-4	-	-2	-1	-7	-5	-9	-2	2	1	9	3	0	14	10	-14	3	5	5	-9	-1	
366A	-	-	1	1	-1	-5	2	0	-3	8	4	-4	-9	-4	2	0	9	-	-9	0	-7	-5	14	-4	-2	
366B	-	-	1	-2	2	4	-7	0	9	-10	-8	-7	12	-1	8	-6	0	-	-12	-3	-1	10	-1	5	-17	
366C	+	-	1	-2	6	0	3	0	-1	6	0	3	12	1	-12	-2	0	+	4	-13	-9	-14	3	-9	-1	
366D	-	+	-1	2	2	4	1	4	-3	-2	4	-1	-4	-3	0	-6	4	+	4	-15	-9	10	-3	-3	-1	
366E	+	+	-2	4	-4	-2	6	4	8	10	4	6	2	-8	-8	-6	12	-	0	0	10	-12	-12	6	2	
366F	+	-	-3	-1	-3	-1	-6	-4	3	0	-4	8	-9	-4	-6	12	3	-	5	0	-7	5	6	12	-10	
366G	-	+	-3	-3	-1	-5	2	-8	5	0	-4	4	3	4	2	0	-7	-	-13	-16	9	-1	14	-4	14	
368A	+	0	0	-4	-6	-2	6	6	+	-6	0	-8	6	2	8	-8	-4	-4	-2	8	6	-12	-10	10	-18	
368B	-	0	4	4	-2	-2	-2	2	+	2	0	-4	6	-10	0	-4	-12	-8	10	0	6	12	-14	-6	6	
368C	+	1	-2	4	2	7	-4	6	-	5	-3	2	-9	-8	1	-6	8	-10	-2	13	-3	-6	0	-4	-8	
368D	+	1	-4	-2	4	-5	-2	-6	+	1	9	-4	3	-8	5	6	4	-10	4	5	-15	6	-6	-8	10	
368E	-	-1	0	-2	0	-1	-6	-2	-	-3	-5	8	3	-8	-9	6	12	14	-8	15	-7	10	-6	0	-10	
368F	-	3	-2	4	-2	-5	4	2	+	-7	3	2	-9	8	-9	2	0	-2	-14	3	-3	6	-8	12	0	
368G	+	-3	0	2	0	-5	-6	-6	+	9	-3	-8	3	8	-7	-2	-4	-10	-8	-7	9	6	14	16	6	
369A	0	-	2	-4	-5	-4	5	-2	-4	-1	-5	-7	-	7	-7	14	12	-3	-2	3	13	-2	2	-18	-14	
369B	2	-	4	-2	3	-6	-3	0	6	-5	7	-7	+	-1	-3	6	0	-3	-2	3	-11	10	16	10	-12	
370A	+	0	+	0	-4	2	-2	-4	0	-6	-4	+	-6	4	-8	10	4	10	-8	0	10	-4	0	2	6	
370B	+	2	-	1	3	0	3	-6	2	-3	3	+	3	-1	4	13	0	-15	0	-2	0	-8	-4	-18	-7	
370C	+	-2	+	-1	3	-4	3	2	6	3	5	-	3	-1	12	3	0	-1	-4	6	-16	8	-12	-6	17	
370D	-	-2	-	2	0	2	6	2	0	6	-10	-	-6	-4	-6	6	-6	-10	2	0	2	-10	-6	-6	2	
371A	1	-1	0	+	0	1	-7	-7	1	9	4	-3	-10	6	6	+	-14	4	4	7	-8	1	-11	-6	-11	
371B	2	0	3	-	3	-6	6	-5	4	5	-11	5	-9	4	4	+	-2	1	-12	4	-10	-10	5	16	10	
372A	-	+	-1	-1	0	-6	-8	7	-6	-8	-	8	9	0	-8	4	3	0	12	-5	-4	14	2	-6	-7	
372B	-	-	-2	4	0	2	0	4	4	0	-	-2	2	-12	-10	8	-14	-2	-4	6	6	0	-16	4	-2	
372C	-	-	3	-1	0	2	0	-1	-6	0	-	8	-3	8	0	-12	-9	8	-4	-9	-4	-10	-6	-6	-7	
372D	-	-	-3	-5	2	-4	-4	-5	4	10	+	-6	-5	2	-4	-12	5	-8	12	9	-10	-2	10	-6	-15	
373A	-2	1	2	-4	-6	-1	-1	6	-4	2	-3	5	-5	2	-12	-8	-1	10	-2	5	3	-8	1	-3	14	+
374A	+	0	0	-2	+	-2	+	-4	6	-4	-2	-4	-2	-4	0	2	4	0	12	2	2	-14	12	6	-2	
377A	1	0	-2	0	-4	-	2	-4	8	-	-8	2	-10	-8	8	6	12	6	12	-16	-10	-12	-12	-10	14	
378A	-	-	0	-	0	5	3	2	-9	-3	5	2	-6	-1	-6	3	-3	-10	-13	9	2	-10	-12	15	8	
378B	+	+	0	-	0	5	-3	2	9	3	5	2	6	-1	6	-3	3	-10	-13	-9	2	-10	12	-15	8	
378C	-	+	1	+	5	0	2	-1	-1	4	-9	5	-9	-10	6	12	-14	0	-8	-13	-2	6	-4	-9	16	
378D	+	+	-1	+	-5	0	-2	-1	1	-4	-9	5	9	-10	-6	-12	14	0	-8	13	-2	6	4	9	16	
378E	-	-	3	-	-3	-4	6	-7	3	0	5	-7	9	-10	-6	-12	6	8	-4	-9	2	-10	0	-15	8	
378F	+	-	-3	-	3	-4	-6	-7	-3	0	5	-7	-9	-10	6	12	-6	8	-4	9	2	-10	0	15	8	
378G	-	+	4	+	-4	3	-7	2	-1	1	-9	2	6	11	-6	-9	-5	-6	7	-7	-14	-6	-4	-3	-8	
378H	+	-	-4	+	4	3	7	2	1	-1	-9	2	-6	11	6	9	5	-6	7	7	-14	-6	4	3	-8	
380A	-	0	+	-2	-4	-4	6	-	-2	-6	-8	4	6	-6	6	8	-12	6	0	0	-10	-8	14	14	16	
380B	-	2	+	2	0	6	2	+	-2	-2	4	-10	-10	6	-6	6	-4	2	-2	12	-6	8	-2	2	-18	
381A	0	-	-1	-2	-4	-3	0	-4	-3	5	-5	5	4	-4	12	-1	5	-5	-8	-6	-1	8	-3	7	4	-
381B	2	-	3	-4	6	-7	-2	0	1	9	-5	-3	-6	4	2	-1	13	-5	-2	6	-1	0	-7	15	2	+
384A	+	-	0	2	4	-6	6	0	4	-4	10	-2	-2	-8	-12	12	4	-2	-4	-4	-10	-6	-12	2	-6	

TABLE 3: HECKE EIGENVALUES 384B–404B

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
384B	-	+	0	2	-4	6	6	0	4	4	10	2	-2	8	-12	-12	-4	2	4	-4	-10	-6	12	2	-6	
384C	+	-	0	-2	4	6	6	0	-4	4	-10	2	-2	-8	12	-12	4	2	-4	4	-10	6	-12	2	-6	
384D	+	+	0	-2	-4	-6	6	0	-4	-4	-10	-2	-2	8	12	12	-4	-2	4	4	-10	6	12	2	-6	
384E	+	-	4	2	-4	-2	-2	-8	4	0	-6	2	6	0	4	0	4	-14	-4	12	-10	10	12	-14	10	
384F	-	+	4	-2	4	-2	-2	8	-4	0	6	2	6	0	-4	0	-4	-14	4	-12	-10	-10	-12	-14	10	
384G	-	+	-4	2	4	2	-2	8	4	0	-6	-2	6	0	4	0	-4	14	4	12	-10	10	-12	-14	10	
384H	-	-	-4	-2	-4	2	-2	-8	-4	0	6	-2	6	0	-4	0	4	14	-4	-12	-10	-10	12	-14	10	
385A	-1	0	-	+	-	-6	6	-4	-8	-10	-4	6	-10	4	-4	6	0	-6	4	0	6	-8	12	10	10	
385B	-1	-2	-	-	+	4	-4	-8	0	-6	-6	-6	0	-4	-6	10	-14	12	12	-12	-8	8	-16	-14	-2	
387A	0	-	2	-2	5	3	3	2	1	0	-5	8	7	+	8	-3	-12	-8	-15	14	12	-16	-15	-10	11	
387B	1	+	-1	-3	-3	-5	6	1	8	-9	-4	-6	8	+	1	-8	0	-10	12	2	2	10	-15	2	-11	
387C	-1	+	1	-3	3	-5	-6	1	-8	9	-4	-6	-8	+	-1	8	0	-10	12	-2	2	10	15	-2	-11	
387D	-1	-	-2	0	0	-2	6	4	4	6	8	6	-2	+	-4	2	0	14	12	-8	2	-8	0	-14	-14	
387E	2	-	4	0	-3	-5	3	-2	1	6	-1	0	-5	+	-4	5	12	2	-3	-2	2	-8	-15	4	7	
389A	-2	-2	-3	-5	-4	-3	-6	5	-4	-6	4	-8	-3	12	-2	-6	3	-8	-5	-10	-7	-13	-12	-8	-9	-
390A	+	+	+	0	0	+	-6	0	-4	-10	0	-6	2	-4	0	-6	0	6	4	16	-2	0	4	-6	14	
390B	-	+	-	0	4	-	-6	4	8	6	-8	-10	-6	4	0	-10	4	-2	-12	16	2	-16	-12	10	-6	
390C	-	-	+	2	0	-	0	2	-6	0	-4	2	-6	-4	0	-6	0	-10	8	0	8	8	-12	6	8	
390D	+	-	-	2	0	-	0	2	-6	0	8	2	6	-4	0	-6	0	14	-4	0	-4	-16	-12	-6	-4	
390E	-	+	+	2	4	+	8	-6	6	-4	0	-2	-2	-4	0	-10	4	-10	12	-8	-8	8	12	-14	-16	
390F	+	+	-	-2	4	+	4	-2	2	8	4	6	10	4	0	6	-12	-2	-8	0	0	-8	-12	-10	-8	
390G	+	-	+	4	0	+	-2	4	8	2	-8	2	-6	12	0	10	0	-10	-4	-16	-6	-8	-4	-14	-6	
392A	-	0	-2	-	-4	-2	6	-8	0	6	-8	-2	-2	-4	8	6	0	6	-4	-8	-10	16	-8	6	6	
392B	+	1	1	-	3	6	5	-1	-7	2	5	3	2	-4	-5	-1	-15	5	-9	0	-7	1	-12	-7	2	
392C	+	-1	-1	+	3	-6	-5	1	-7	2	-5	3	-2	-4	5	-1	15	-5	-9	0	7	1	12	7	-2	
392D	+	-2	4	-	0	0	2	2	8	2	-4	-6	2	8	4	-10	-6	-4	-12	0	14	-8	-6	-10	2	
392E	-	3	-1	+	-1	2	3	5	-3	-6	-1	-5	-10	-4	1	-9	3	3	11	16	7	-11	-4	-9	6	
392F	-	-3	1	-	-1	-2	-3	-5	-3	-6	1	-5	10	-4	-1	-9	-3	-3	11	16	-7	-11	4	9	-6	
395A	-1	0	-	-4	4	6	6	-4	0	6	0	10	2	8	12	-14	-4	-10	-4	-8	2	+	4	-6	10	
395B	-1	2	-	2	4	-6	0	4	8	-6	8	4	-10	10	-2	8	-12	2	-4	0	-10	+	0	-10	-10	
395C	-2	-1	-	3	-3	4	-2	0	4	0	7	3	12	4	-12	9	0	12	-2	-8	14	+	4	-10	8	
396A	-	-	-2	2	-	6	4	-2	8	0	0	-6	0	10	0	-14	12	-14	4	0	6	2	-16	14	-2	
396B	-	-	-2	-2	+	-2	-4	-6	0	8	-8	10	-8	-2	8	2	-12	10	12	-8	6	-2	-16	14	-2	
396C	-	-	3	2	-	-4	-6	8	3	0	5	-1	0	-10	0	6	-3	-4	-1	-15	-4	2	-6	9	-7	
398A	+	2	-2	0	2	6	6	6	0	-6	8	-8	-2	0	-8	-2	10	10	2	-8	10	-16	-6	-6	14	-
399A	1	+	0	+	-2	-4	-4	+	2	-2	0	6	-6	8	0	-2	4	-10	14	-12	10	10	-12	-6	-4	
399B	-1	+	0	-	-2	0	-4	-	-6	-6	0	-2	-10	8	4	-6	-4	-2	-10	4	10	-6	0	-2	-8	
399C	-1	-	4	+	-2	4	0	+	-6	10	0	6	-10	8	12	-6	-12	-2	-2	-12	-6	2	0	-2	-12	
400A	+	0	+	-4	-4	2	-2	-4	4	-2	8	-6	-6	-8	4	-6	4	-2	8	0	6	0	-16	-6	14	
400B	-	1	+	2	3	4	3	-5	6	0	-2	-2	-3	-4	12	-6	0	2	-13	-12	-11	10	-9	15	-2	
400C	-	-1	-	-2	3	-4	-3	-5	-6	0	-2	2	-3	4	-12	6	0	2	13	-12	11	10	9	15	2	
400D	+	2	-	2	4	-4	0	4	-2	2	0	-4	2	-6	-6	4	12	-10	14	-8	-8	-16	2	6	-16	
400E	-	-2	+	2	0	-2	6	4	6	6	4	-2	6	-10	-6	6	-12	2	2	12	-2	-8	6	-6	-2	
400F	+	-2	-	-2	4	4	0	4	2	2	0	4	2	6	6	-4	12	-10	-14	-8	8	-16	-2	6	16	
400G	+	3	-	-2	-1	4	5	-1	2	-8	-10	-6	-3	-4	-4	6	-8	10	1	12	3	-6	13	-9	-14	
400H	+	-3	+	2	-1	-4	-5	-1	-2	-8	-10	6	-3	4	4	-6	-8	10	-1	12	-3	-6	-13	-9	14	
402A	+	+	1	-3	0	-4	2	-2	-3	0	-9	-3	3	-7	-8	-3	3	6	+	4	11	0	9	16	0	
402B	+	-	2	0	4	-2	2	-4	4	-2	0	6	-2	4	12	2	0	-10	+	-4	-6	0	-16	-6	-6	
402C	-	+	2	2	-4	0	6	4	-6	8	2	-2	-10	4	-6	-6	-8	8	+	-14	-6	-2	-12	-6	-2	
402D	+	-	-3	-1	0	-4	-6	2	-9	0	5	-7	3	-1	0	9	-3	-10	-	-12	11	8	15	0	8	
404A	-	0	-1	-2	-2	-3	-1	1	3	-2	-3	-2	2	4	-3	0	12	-10	2	-1	2	1	4	-6	-2	-
404B	-	-2	3	2	-6	5	3	5	3	0	5	-10	12	8	-3	-6	-6	8	-10	-9	-4	5	-12	6	2	+

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
405A	0	+	-	2	3	-4	6	-1	6	9	-1	8	-3	-4	-12	-6	-3	-10	14	3	2	-16	12	-15	-4	
405B	0	+	+	2	-3	-4	-6	-1	-6	-9	-1	8	3	-4	12	6	3	-10	14	-3	2	-16	-12	15	-4	
405C	1	+	+	-3	-2	-2	4	-8	3	-1	0	-4	5	-8	7	-2	-14	7	-3	2	4	-6	9	-15	2	
405D	-1	-	-	-3	2	-2	-4	-8	-3	1	0	-4	-5	-8	-7	2	14	7	-3	-2	4	-6	-9	15	2	
405E	2	+	-	0	5	4	-4	-5	6	-5	-9	-10	7	-2	2	8	-1	-2	6	1	-8	12	6	-9	14	
405F	-2	+	+	0	-5	4	4	-5	-6	5	-9	-10	-7	-2	-2	-8	1	-2	6	-1	-8	12	-6	9	14	
406A	+	0	0	+	-4	0	-4	4	0	+	-6	-2	-8	4	2	-2	-10	-2	8	16	0	-4	-6	0	12	
406B	+	1	-3	-	-3	-1	0	-4	-6	-	5	2	0	-7	-3	-9	12	-10	2	-12	8	5	12	6	8	
406C	-	-1	-3	+	-1	-1	-4	-4	-2	-	-1	6	0	3	-9	3	0	6	2	-8	0	-13	0	-14	16	
406D	+	2	2	-	4	-2	-4	2	0	+	-2	2	8	-8	6	6	-4	4	-4	8	-12	-12	0	4	4	
408A	+	-	0	2	0	2	+	4	2	0	6	0	-10	4	-4	-2	-4	0	4	-2	-14	6	-12	-2	-2	
408B	-	-	2	-4	4	6	-	4	-4	-6	-4	10	-6	4	-8	6	-4	-14	-12	-12	10	-4	4	-6	-6	
408C	-	+	3	0	-1	3	+	1	7	6	-2	-4	9	-1	10	-2	-6	-12	-4	-12	-10	2	-14	4	12	
408D	+	-	-3	-4	1	-5	-	-7	1	2	-6	8	7	-1	-6	-2	-10	8	-12	12	-14	10	-14	8	12	
410A	+	0	-	-2	-6	-2	8	-6	0	-8	0	-6	-	-4	6	2	8	10	-8	-4	-6	-8	-4	-2	12	
410B	-	0	-	4	0	-2	2	0	0	-2	0	6	-	-4	-12	-10	-4	-2	-8	-4	-6	4	-4	10	18	
410C	+	-2	-	2	0	-4	0	8	0	6	8	2	+	8	-6	0	12	2	14	-12	2	-4	-12	6	-4	
410D	-	-2	+	-2	2	-6	-6	-2	-4	-6	0	10	-	4	2	-6	12	-10	2	10	-10	-6	0	10	2	
414A	-	-	0	2	0	2	0	2	-	6	-4	-10	6	2	0	-12	-12	-10	14	0	2	-10	0	-12	-10	
414B	-	-	2	-2	6	-2	0	0	-	-6	8	0	-10	-12	8	-2	12	4	-12	0	-10	-6	-14	0	-6	
414C	+	-	-2	0	0	-2	-2	-8	-	2	-8	2	-10	8	-8	-2	4	2	8	0	-6	8	16	-18	10	
414D	-	-	-4	-4	-2	-2	2	-2	+	-2	0	-4	-6	10	0	4	-12	-8	-10	0	6	-12	-14	6	6	
415A	1	3	-	1	3	-6	-7	2	4	-7	5	-7	6	4	-4	-10	-3	5	2	14	-4	-14	+	12	8	
416A	+	1	1	3	2	-	-3	2	4	2	4	5	-12	7	-9	4	6	-4	-10	-15	-2	-8	-4	2	10	
416B	-	-1	1	-3	-2	-	-3	-2	-4	2	-4	5	-12	-7	9	4	-6	-4	10	15	-2	8	4	2	10	
417A	1	+	2	0	5	5	-3	7	2	0	-6	-7	-6	11	11	9	-6	-8	-4	-16	-12	-8	4	4	-18	-
418A	-	0	2	2	-	-2	6	-	-8	-6	6	8	6	-8	-8	12	0	-8	-8	-6	-14	-12	-12	2	-2	
418B	-	-1	-2	-3	+	1	-7	-	-5	1	10	-6	6	-4	0	-1	3	-12	3	-10	3	8	8	-8	8	
418C	-	3	-2	1	-	-7	-3	-	3	1	2	-6	-2	4	0	3	7	-12	15	6	-9	-8	16	-16	8	
420A	-	+	+	+	2	4	6	6	-8	-2	10	2	10	-4	-8	4	-8	6	12	-6	-12	-8	-4	-10	8	
420B	-	+	-	-	-2	4	2	2	4	6	-2	10	-10	12	-8	0	-8	-2	-12	-10	4	0	-12	2	-8	
420C	-	-	+	-	6	-4	6	2	0	6	-10	2	-6	-4	0	-12	0	14	-4	6	-4	-16	-12	6	-16	
420D	-	-	-	+	2	4	2	-2	4	-2	-6	-6	6	-4	0	8	0	-10	-12	-14	4	-8	12	-14	8	
422A	+	0	1	-2	-3	-7	4	7	-6	-6	2	-7	2	-3	7	6	12	-8	-8	-9	-10	-3	16	16	-12	+
423A	0	-	1	-3	3	-4	-8	-6	-3	1	4	1	10	-8	-	-10	10	2	4	6	-8	-3	18	2	5	
423B	1	-	0	4	0	6	6	2	-4	-8	6	-6	8	-6	+	-2	-12	2	-2	0	-10	-4	-4	10	-18	
423C	1	-	-2	0	-4	-2	-2	0	0	6	-4	-10	2	8	-	2	4	14	-8	-16	2	8	4	-18	-14	
423D	2	+	3	1	-3	0	0	-4	7	-1	0	-3	10	-12	-	2	-6	14	-14	6	-10	5	-2	2	9	
423E	2	-	3	-3	5	2	6	-6	-9	-1	-2	1	-6	2	+	0	12	-2	2	2	-2	-15	4	-10	1	
423F	-2	-	1	-3	-1	-2	-2	6	-3	-3	2	-7	-10	-10	-	-4	-8	-10	10	14	-10	17	-8	-6	1	
423G	-2	+	-3	1	3	0	0	-4	-7	1	0	-3	-10	-12	+	-2	6	14	-14	-6	-10	5	2	-2	9	
425A	1	0	+	-4	0	2	+	-4	-4	6	4	2	-6	-4	0	-6	-12	-10	-4	-4	6	12	4	10	-2	
425B	1	-1	-	1	-4	-1	-	-6	0	0	-7	-4	-2	4	-6	11	8	10	8	7	4	-11	-8	-6	-16	
425C	-1	1	+	-1	-4	1	+	-6	0	0	-7	4	-2	-4	6	-11	8	10	-8	7	-4	-11	8	-6	16	
425D	-1	-2	+	2	2	-2	+	0	-6	-6	-10	-2	10	-4	-12	10	8	-14	-8	-2	14	-14	-4	6	-2	
426A	-	-	1	3	-3	-6	-2	5	-6	5	7	8	7	-11	-12	-6	-5	-13	8	-	9	10	-6	-10	18	
426B	+	+	-2	2	-2	0	0	-4	-4	-6	-2	-6	0	-4	0	6	-10	0	4	+	10	-8	-8	6	18	
426C	+	-	3	-1	3	2	-6	5	-6	-9	11	-4	9	5	12	-6	-3	-1	-4	+	-7	-10	-6	-6	14	
427A	0	2	4	-	-2	2	5	-8	-6	2	1	4	0	8	-8	-12	1	+	6	6	-10	-14	-2	10	-2	
427B	1	1	-4	-	-3	-4	5	1	7	-10	-8	10	-6	-1	-9	-2	-6	-	3	-1	-2	-5	-15	5	14	
427C	-1	1	0	+	-5	4	-5	-7	9	-6	0	2	-10	1	7	-6	-6	+	5	1	10	-3	1	-13	10	
428A	-	1	2	4	-3	5	-6	1	-1	6	4	-3	-5	6	8	-11	0	-5	-10	6	-16	-1	4	-3	12	+

TABLE 3: HECKE EIGENVALUES 428B–443A

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
428B	-	-1	2	-4	-5	1	2	-1	-3	-10	4	-7	3	-6	0	1	8	7	2	-6	-8	13	12	-3	-12	-
429A	-1	+	0	0	-	-	-4	-8	0	4	-6	-6	6	-2	-8	6	0	-14	14	-4	6	-10	-12	12	-2	-
429B	-1	-	-2	0	+	-	-6	-4	-8	-10	0	6	10	4	8	-10	-12	14	-12	0	-6	8	12	2	-14	-
430A	+	0	+	1	-4	-1	0	1	-4	-5	-9	4	-7	+	6	-2	0	-7	15	-6	-5	9	0	0	-2	-
430B	+	0	-	-3	0	-3	-4	-1	0	-3	7	-8	-7	-	-6	-6	-4	7	5	2	-1	9	8	4	-2	-
430C	-	-2	+	-1	-6	5	-6	-7	-6	-3	5	2	-3	-	12	6	-12	-1	-13	12	11	-1	0	6	8	-
430D	-	-2	-	-5	-2	-5	2	3	-6	-1	-11	-10	5	+	4	10	8	-3	-3	-8	7	7	0	6	12	-
431A	-1	1	1	-2	-5	-2	-2	5	-1	-3	-4	4	2	-6	6	-9	15	-14	-2	-2	2	4	4	14	-13	+
431B	-1	3	-3	2	1	-2	6	7	1	-7	4	4	2	6	-6	-13	-11	2	2	10	-6	4	12	-18	-5	-
432A	-	+	0	1	0	5	0	7	0	0	4	11	0	-8	0	0	0	-1	-5	0	-7	-17	0	0	-19	-
432B	-	-	0	-5	0	-7	0	1	0	0	4	-1	0	-8	0	0	0	-13	-11	0	17	13	0	0	5	-
432C	+	-	1	-3	5	4	8	-2	2	-6	7	-6	6	2	6	-5	-4	-8	10	-8	1	-16	-11	-6	-1	-
432D	+	+	-1	-3	-5	4	-8	-2	-2	6	7	-6	-6	2	-6	5	4	-8	10	8	1	-16	11	6	-1	-
432E	-	+	3	1	3	-4	0	-2	6	6	-5	2	-6	10	-6	9	-12	8	-14	0	-7	-8	3	-18	-1	-
432F	-	-	-3	1	-3	-4	0	-2	-6	-6	-5	2	6	10	6	-9	12	8	-14	0	-7	-8	-3	18	-1	-
432G	+	-	4	3	-4	1	-4	1	-4	0	4	-9	0	8	12	-8	-4	-5	-11	-8	1	5	-8	12	5	-
432H	+	-	-4	3	4	1	4	1	4	0	4	-9	0	8	-12	8	4	-5	-11	8	1	5	8	-12	5	-
433A	-1	-2	-4	-3	-4	-5	-3	-4	8	2	-9	-3	-9	-7	9	-5	-8	-8	-7	-9	-2	10	9	0	-12	-
434A	+	0	0	+	-2	-2	2	-6	0	8	+	-8	-10	-6	-4	4	6	6	-4	-8	14	-16	8	-6	14	-
434B	-	1	3	-	0	-4	-6	2	-3	3	-	2	12	-10	3	6	0	8	-13	-12	11	-1	-9	-9	8	-
434C	-	2	2	+	-6	4	2	-4	-4	0	+	8	-2	6	8	0	0	-8	4	-8	6	0	6	6	-2	-
434D	-	-2	-2	-	-2	-4	-2	-8	0	0	+	-8	6	2	8	0	12	-8	4	0	-14	4	2	-6	14	-
434E	-	-3	-3	+	4	4	2	6	-9	5	+	-2	8	6	-7	10	0	12	-1	-8	11	5	11	-9	8	-
435A	0	-	+	2	3	2	0	2	3	+	8	-1	-3	-1	-6	-3	-12	8	14	-6	-7	-4	9	-6	11	-
435B	0	+	+	-2	1	6	4	-2	3	-	-4	-3	7	5	6	13	0	0	-10	6	3	0	9	-10	17	-
435C	1	-	-	4	-4	6	6	-4	-4	-	-8	2	-6	4	0	-10	-12	-10	8	-8	-2	0	8	-6	-2	-
435D	-1	-	-	-4	0	6	2	8	-4	-	4	6	2	-4	0	6	-12	6	-8	16	-6	12	-16	2	-14	-
437A	0	2	-1	-5	-1	0	-7	-	-	6	4	2	-2	-5	-3	-4	6	11	-16	-10	-7	4	4	-16	-4	-
437B	2	2	1	-3	5	-2	3	+	-	4	-4	-8	0	-3	-3	12	4	5	12	12	1	-10	12	-6	10	-
438A	-	-	0	2	0	-4	6	-4	0	0	2	2	6	-4	-6	-12	0	-10	-4	12	-	-4	0	6	2	-
438B	-	-	0	-2	4	4	-2	4	0	0	-10	-6	-10	-8	6	4	12	-2	12	-12	-	-12	12	6	2	-
438C	+	+	0	-2	4	-6	0	-4	0	-4	2	-10	-2	2	-12	0	-4	-6	8	8	+	8	8	10	14	-
438D	+	-	0	-4	-6	-4	-6	8	0	0	8	2	-6	2	0	-12	6	-10	-4	0	-	-16	6	6	14	-
438E	+	-	2	-2	2	4	4	-4	0	6	-2	-6	6	8	8	6	-10	-2	-12	-8	+	0	-6	-6	2	-
438F	-	+	-2	-4	0	-2	-6	-4	0	6	-4	6	10	-8	4	-2	-8	-2	-4	8	-	-8	0	-6	-14	-
438G	+	-	-4	0	2	0	-6	-8	-8	-4	-4	2	10	-6	4	-8	14	-2	12	0	-	8	-18	6	-2	-
440A	+	0	+	-2	+	0	0	-8	-8	10	8	-10	-2	-6	-8	14	-4	10	4	0	-8	-4	10	6	-10	-
440B	-	0	+	-2	-	-4	-4	0	0	-6	0	-2	6	2	0	-10	12	-6	-12	16	4	-4	2	6	-2	-
440C	+	0	-	4	+	6	-6	4	4	-2	8	-10	10	0	4	-10	-4	-2	-8	0	-14	-16	-8	-6	2	-
440D	+	3	-	1	+	-6	3	-5	-2	-5	5	-1	-2	12	-2	-13	2	1	16	15	10	2	-14	9	-16	-
441A	0	+	0	-	0	7	0	7	0	0	7	-1	0	5	0	0	0	-14	11	0	7	-13	0	0	-14	-
441B	0	+	0	+	0	-7	0	-7	0	0	-7	-1	0	5	0	0	0	14	11	0	-7	-13	0	0	14	-
441C	1	-	-2	-	-4	2	-6	-4	0	2	0	6	2	-4	0	-6	12	2	4	0	6	-16	-12	-14	-18	-
441D	-1	-	0	-	-4	0	0	0	-8	-2	0	-6	0	-12	0	10	0	0	4	-16	0	8	0	0	0	-
441E	-2	-	2	+	2	1	0	1	0	-4	9	3	10	5	6	-12	12	10	-5	6	-3	-1	-6	-16	-6	-
441F	-2	-	-2	-	2	-1	0	-1	0	-4	-9	3	-10	5	-6	-12	-12	-10	-5	6	3	-1	6	16	6	-
442A	-	0	2	4	-2	+	+	0	2	8	-8	-6	12	4	-8	-6	-4	-8	-8	-8	8	-10	0	6	-16	-
442B	-	0	-4	-2	-2	+	-	0	-4	2	-2	0	0	4	-8	-6	8	-2	16	-14	-16	8	-12	-18	-4	-
442C	+	2	2	2	2	+	-	-4	-2	2	-2	2	2	0	4	-2	12	-6	8	6	2	-10	-12	14	-6	-
442D	-	2	-2	2	4	+	+	-4	8	-8	10	-10	-8	-12	8	2	12	0	-4	-6	-4	-4	12	-2	12	-
442E	-	2	4	-4	-2	+	+	-4	-4	-8	4	8	10	0	8	2	0	12	8	0	-10	-4	0	-14	-6	-
443A	0	1	-2	2	-2	-3	-2	-8	6	-4	-10	7	10	4	-7	12	5	-10	8	9	4	-8	-18	-1	6	+

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
443B	-1	-2	0	1	3	3	-5	-7	-3	0	7	-3	-6	-8	-2	4	6	-13	-8	16	-8	-2	-7	-1	-10	+
443C	1	-2	4	-1	5	3	3	-1	3	4	-7	-3	10	-8	6	4	-10	-13	-8	4	-4	-2	-1	-9	6	-
444A	-	+	0	0	4	-2	0	6	8	8	6	+	2	-6	0	2	0	2	8	0	-6	-10	-12	-12	-10	-
444B	-	-	-2	-4	-4	-6	6	-2	2	-2	2	+	6	-2	-4	10	-6	-14	-4	-12	-2	-10	0	-10	10	-
446A	+	-1	0	0	1	-2	1	-4	1	-3	-10	-3	-5	-6	6	-9	-1	4	9	4	-5	0	14	-5	2	+
446B	-	-1	-2	-2	-3	0	1	-6	-3	5	2	-7	3	0	2	-1	3	6	-11	0	7	-8	-6	15	12	-
446C	-	2	0	0	-2	4	-2	8	-8	-6	8	-6	10	-12	0	6	-10	4	-6	4	10	12	8	-2	-10	+
446D	+	-3	-4	-4	-5	-6	1	0	-5	-3	2	5	-5	-6	-6	-1	-11	0	11	-12	-5	-8	-6	3	-18	-
448A	+	0	-2	+	4	-2	-6	-8	0	-6	8	2	2	4	-8	-6	0	6	4	-8	10	16	-8	-6	-6	-
448B	-	0	-2	-	-4	-2	-6	8	0	-6	-8	2	2	-4	8	-6	0	6	-4	8	10	-16	8	-6	-6	-
448C	+	2	0	-	0	4	6	-2	0	6	-4	-2	6	-8	-12	-6	6	-8	4	0	2	8	6	-6	-10	-
448D	-	2	0	+	4	4	-2	6	8	-2	-4	-10	-10	-4	4	2	-10	8	8	0	-6	-16	-2	18	-2	-
448E	-	2	4	+	0	0	-2	-2	-8	-2	-4	6	-2	8	4	10	6	-4	-12	0	-14	8	6	10	-2	-
448F	-	-2	0	+	0	4	6	2	0	6	4	-2	6	8	12	-6	-6	-8	-4	0	2	-8	-6	-6	-10	-
448G	-	-2	0	-	-4	4	-2	-6	-8	-2	4	-10	-10	4	-4	2	10	8	-8	0	-6	16	2	18	-2	-
448H	+	-2	4	-	0	0	-2	2	8	-2	4	6	-2	-8	-4	10	-6	-4	12	0	-14	-8	-6	10	-2	-
450A	-	-	-	2	-2	6	-2	0	4	0	-8	2	-2	-4	8	-6	-10	2	-8	-12	-4	0	4	10	-8	-
450B	-	-	-	2	3	-4	3	5	-6	0	2	2	3	-4	-12	-6	0	2	-13	-12	11	-10	9	-15	2	-
450C	+	-	-	-2	-2	-6	2	0	-4	0	-8	-2	-2	4	-8	6	-10	2	8	-12	4	0	-4	10	8	-
450D	+	-	+	-2	3	4	-3	5	6	0	2	-2	3	4	12	6	0	2	13	-12	-11	-10	-9	-15	-2	-
450E	-	+	+	-2	6	4	6	-4	0	-6	-4	-8	0	-8	0	6	6	2	4	-12	10	-4	-12	12	-2	-
450F	+	+	+	-2	-6	4	-6	-4	0	6	-4	-8	0	-8	0	-6	-6	2	4	12	10	-4	12	-12	-2	-
450G	+	-	+	4	0	-2	6	-4	0	6	8	-2	6	4	0	-6	0	-10	4	0	-2	8	12	-18	-2	-
451A	0	1	-3	4	+	-6	2	-8	-5	-8	3	7	+	6	0	-2	9	12	-9	-13	6	10	-12	13	-5	-
455A	1	0	+	+	0	+	-2	-4	0	-2	0	2	6	-4	-8	6	-4	-10	12	4	-10	0	12	-18	-2	-
455B	-1	0	-	+	0	-	-6	0	-4	-2	-4	-10	2	-8	0	-2	0	-2	-4	12	-6	8	4	2	-14	-
456A	+	+	4	4	-4	-4	6	-	-6	2	2	4	-6	4	-2	-6	-4	-10	8	0	-2	14	-16	-18	14	-
456B	+	-	2	0	0	2	2	+	0	2	-4	2	6	-4	0	10	-4	-2	-12	0	-6	-4	-8	6	-14	-
456C	+	-	-3	-3	-1	-2	-5	-	-4	-6	-2	8	-8	13	13	-6	4	-13	4	-8	-3	-4	4	-6	2	-
456D	-	+	1	-3	-5	-2	-1	-	4	-6	-10	0	0	-11	9	10	4	-5	-4	8	13	4	-4	-6	2	-
458A	+	-3	1	-2	1	2	1	-1	-4	-2	-4	-6	-2	-5	-2	-2	0	-7	-14	15	-2	14	-9	18	3	+
458B	-	-1	-1	-4	-1	-2	-3	1	2	-6	8	-6	0	1	-2	2	-2	-1	-10	-1	-4	4	5	12	3	-
459A	1	+	-1	-2	0	-5	+	-1	-1	9	-8	-2	-3	7	6	6	0	-10	1	-11	6	0	4	2	2	-
459B	-2	+	-4	1	6	1	+	-7	-4	-6	-8	1	0	4	-6	0	-6	-7	1	4	3	-9	-14	14	-1	-
459C	0	+	3	2	-3	2	-	5	0	-3	8	8	6	-4	-6	12	-12	-10	5	-15	2	-10	-6	0	14	-
459D	2	+	-2	4	3	7	-	-4	1	-9	-2	-8	-9	7	0	6	0	2	7	-7	6	-12	14	-8	-10	-
459E	2	+	4	1	-6	1	-	-7	4	6	-8	1	0	4	6	0	6	-7	1	-4	3	-9	14	-14	-1	-
459F	0	-	-3	2	3	2	+	5	0	3	8	8	-6	-4	6	-12	12	-10	5	15	2	-10	6	0	14	-
459G	-2	-	2	4	-3	7	+	-4	-1	9	-2	-8	9	7	0	-6	0	2	7	7	6	-12	-14	8	-10	-
459H	-1	-	1	-2	0	-5	-	-1	1	-9	-8	-2	3	7	-6	-6	0	-10	1	11	6	0	-4	-2	2	-
460A	-	0	+	-1	6	6	7	2	+	-5	1	-5	-7	8	8	3	13	-8	-9	7	-2	-12	-5	-12	2	-
460B	-	3	+	2	0	-3	4	-4	+	1	1	-8	11	-10	-1	-6	-8	-8	12	13	7	-12	16	-6	2	-
460C	-	1	+	-4	-6	-1	0	2	-	9	5	2	-9	-4	-3	-6	0	2	-10	-3	-7	-10	-12	0	8	-
460D	-	-1	-	-2	-4	1	0	-4	+	-7	-7	-4	3	6	-13	10	-8	0	8	13	11	4	-4	-6	-2	-
462A	+	+	0	+	+	-2	-4	6	-4	-10	6	-6	-12	-8	2	6	-8	6	-4	0	-12	0	14	10	10	-
462B	+	+	2	+	-	2	6	-8	4	2	8	6	6	8	4	10	4	-14	-4	-4	-14	-8	4	-14	18	-
462C	+	+	-2	-	-	2	-6	-4	-4	2	-4	-2	-6	0	-8	-14	12	-14	4	12	6	0	0	-6	-14	-
462D	+	-	0	+	+	6	4	6	-4	6	-2	10	-4	8	-6	-10	0	-2	-4	16	12	-16	-2	-6	-6	-
462E	-	+	-4	-	+	-6	-4	-2	-8	-6	6	-6	12	4	6	2	0	10	4	-12	0	-16	-14	-14	-14	-
462F	-	-	2	+	-	-2	-2	0	0	-2	4	-2	-10	4	4	-2	-12	-2	12	8	6	-8	-8	-14	-14	-
462G	-	-	0	-	+	2	0	2	0	-6	2	2	0	-4	-6	-6	0	2	-4	-12	-4	8	6	-6	2	-
464A	+	1	-3	-2	3	-5	-4	0	0	+	-9	8	-2	11	7	9	-4	-12	-12	-2	-4	-3	16	2	-14	-

TABLE 3: HECKE EIGENVALUES 464B–485B

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
464B	+	-1	1	-2	-3	-1	0	0	-4	+	-3	-8	-6	5	-3	5	8	0	12	-6	-4	-1	12	6	14	
464C	-	1	1	2	3	-1	8	0	-4	+	3	8	2	11	-13	-11	0	-8	12	-2	4	-15	-4	-10	-2	
464D	-	-1	3	4	-3	5	-6	4	6	+	-5	8	0	1	3	3	-6	2	-8	-6	-16	-11	-6	-12	8	
464E	-	-2	-2	-4	6	2	2	6	-4	+	6	2	2	-10	2	10	0	10	12	-8	10	6	-16	2	10	
464F	-	3	3	-4	1	-3	2	-4	6	+	-9	-8	-8	5	7	-5	10	10	-8	2	0	1	-6	12	0	
464G	-	3	-3	2	1	3	-4	8	0	+	-3	-8	-2	-7	-11	1	4	4	4	2	-12	7	0	-6	-6	
465A	1	+	-	-2	-4	0	2	-8	-8	0	-	8	-6	0	4	6	10	-14	2	6	-16	0	4	4	6	
465B	-1	-	-	-4	-4	2	-6	-4	0	-6	+	10	-6	-12	0	-2	-8	6	8	-12	6	-8	12	6	-6	
466A	+	2	0	0	2	2	6	0	8	-2	0	2	6	-10	0	0	-10	4	14	8	-6	-4	-6	6	-10	-
466B	-	1	0	2	0	5	0	-4	6	3	-4	-7	-6	-1	9	6	3	-10	-7	-12	14	-13	-9	-3	14	+
467A	0	-3	2	1	4	-6	-7	2	-7	-8	6	-2	6	-4	4	-9	-3	-10	-4	-12	14	-10	11	-2	9	+
468A	-	+	4	4	-4	+	0	0	-8	-8	4	6	12	-8	-4	0	4	-2	-8	-4	-10	-4	12	-12	14	
468B	-	+	-4	4	4	+	0	0	8	8	4	6	-12	-8	4	0	-4	-2	-8	4	-10	-4	-12	12	14	
468C	-	-	-2	-2	2	+	-6	-6	-8	-2	10	-6	6	4	2	-6	10	-2	10	-10	2	-4	6	6	2	
468D	-	-	0	2	0	-	6	2	0	6	2	2	12	-4	0	-6	-12	2	-10	-12	14	8	-12	0	-10	
468E	-	-	4	-2	4	-	-2	-2	0	6	-10	10	-8	4	4	10	8	-14	2	-16	-10	-16	0	4	-2	
469A	1	1	-3	+	0	-1	-8	8	3	-3	-1	-3	-9	4	10	6	-14	-6	+	-9	-14	14	10	0	-14	
469B	-1	-3	1	+	0	3	0	-4	3	-3	-5	5	-5	0	-6	2	-6	-14	+	-9	14	14	-10	4	2	
470A	+	1	+	-1	-3	-5	2	-7	8	-2	-5	-4	12	8	+	-4	-10	-11	-8	0	3	10	9	-18	12	
470B	+	1	-	-1	3	5	6	-1	0	-6	5	8	0	8	+	0	-6	5	-4	-12	5	2	-15	6	-16	
470C	+	-1	-	-1	1	-5	0	5	-6	-6	-11	-8	2	-2	-	-6	8	-5	2	12	-15	0	-1	14	6	
470D	-	1	+	5	-3	5	0	-7	6	-6	5	8	-6	-10	+	-6	-12	-1	2	0	-13	-16	9	6	2	
470E	-	-1	+	-3	-5	-1	2	-1	0	2	-7	0	-8	-4	-	12	6	-7	0	-16	11	10	-9	6	-16	
470F	-	-3	-	-3	-1	-1	-8	-5	-2	-2	-5	-4	6	6	+	2	-12	11	14	-4	-11	-4	5	14	-14	
471A	-1	+	-2	3	0	1	-3	-2	-9	0	-2	1	-2	1	0	-6	-1	8	2	-12	-14	-8	4	-13	0	+
472A	+	-3	-1	3	-4	6	-6	-7	-6	-3	8	2	3	-12	-2	-5	+	-4	-8	8	-10	5	6	-4	-14	
472B	+	-1	-1	1	4	2	2	3	6	5	4	-6	3	8	-2	11	-	0	-8	-8	-6	-1	6	-16	-10	
472C	+	2	2	1	1	-1	-1	0	0	-4	4	3	-3	-1	10	-4	-	-6	4	13	-6	-1	-3	2	-10	
472D	-	3	-3	3	6	-6	-2	-1	8	-1	-2	-4	-1	-10	6	5	+	-8	2	-4	-8	-11	-10	-16	-4	
472E	-	-1	-1	1	0	-2	-6	3	-6	-3	-4	-2	-5	0	2	3	-	12	4	0	-6	15	-14	12	6	
473A	-2	1	-1	0	+	-2	6	-8	-1	6	-1	-3	-4	+	-8	-14	9	-4	9	-13	-16	16	-6	-7	13	
474A	+	+	2	-3	-5	-1	5	-6	3	-5	-4	-8	-2	-5	0	2	-2	-12	14	10	-9	+	9	-12	7	
474B	+	-	-2	-1	-5	-1	-1	-2	-5	1	0	4	-6	1	4	-2	-6	0	-10	6	7	-	-15	4	-1	
475A	0	2	+	1	3	4	3	-	0	6	-4	-2	-6	1	3	-12	-6	-1	4	6	7	8	-12	12	-8	
475B	1	0	-	-2	-4	2	-4	-	6	-6	-4	10	-10	-2	6	-10	0	2	-8	4	-4	4	18	-2	-6	
475C	-1	0	-	2	-4	-2	4	-	-6	-6	-4	-10	-10	2	-6	10	0	2	8	4	4	4	-18	-2	6	
477A	1	-	0	-4	0	-3	3	-5	-7	7	4	5	-6	-2	2	-	2	-8	-12	-1	-4	-1	1	14	1	
480A	+	+	+	0	-4	2	-2	-8	-4	-6	0	2	-6	-4	12	-6	-12	14	12	0	2	8	4	2	-14	
480B	+	+	-	0	0	2	6	-4	8	-2	4	10	2	-4	8	-2	8	-2	-12	8	-14	-12	-4	-14	2	
480C	+	-	+	0	4	2	-2	8	4	-6	0	2	-6	4	-12	-6	12	14	-12	0	2	-8	-4	2	-14	
480D	+	-	+	4	-4	6	2	-4	0	10	4	-10	2	4	-8	2	-12	-10	-12	0	10	4	-4	-6	-14	
480E	-	+	+	-4	4	6	2	4	0	10	-4	-10	2	-4	8	2	12	-10	12	0	10	-4	4	-6	-14	
480F	-	+	-	-4	0	-2	-6	0	-4	-2	-8	6	-6	12	-12	-10	8	-10	-12	8	10	16	12	-6	18	
480G	-	-	-	0	0	2	6	4	-8	-2	-4	10	2	4	-8	-2	-8	-2	12	-8	-14	12	4	-14	2	
480H	-	-	-	4	0	-2	-6	0	4	-2	8	6	-6	-12	12	-10	-8	-10	12	-8	10	-16	-12	-6	18	
481A	1	0	-2	2	-2	+	-6	0	2	-6	8	+	-6	2	-6	10	-4	10	2	6	2	-2	6	-2	-14	
482A	+	-2	-1	1	4	-2	4	-5	-9	9	-8	-8	-3	-7	-4	-11	10	7	-8	-13	14	-4	0	4	19	+
483A	2	-	4	+	-5	-2	0	-5	+	-2	6	6	5	8	-9	9	9	-5	4	12	0	-10	-18	10	-18	
483B	2	-	0	-	1	2	4	-3	-	-6	-2	-2	1	-8	-5	3	5	13	0	0	-16	-2	6	6	10	
484A	-	1	-3	-2	-	4	-6	-8	-3	0	5	-1	0	10	0	-6	3	4	-1	15	4	-2	-6	-9	-7	
485A	0	-2	+	-1	-3	5	-6	2	9	0	5	-7	-6	8	12	6	-6	-1	5	-6	2	-1	-12	9	-	
485B	0	0	-	-1	1	1	-6	-8	-7	6	-7	1	-4	10	-4	-4	2	-1	13	0	8	-5	4	-7	-	

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
486A	+	+	0	-1	-6	-1	-6	5	6	-6	-7	-7	0	5	6	6	-12	-10	-4	-6	2	11	6	12	-7	
486B	+	+	-3	2	0	-4	6	-7	-9	-9	2	-4	6	-4	-3	3	0	-10	5	-3	-7	8	6	-12	-1	
486C	+	-	3	-4	6	2	0	-1	3	-3	2	8	12	-4	-3	9	-6	8	-13	9	-7	-4	-12	0	-13	
486D	-	+	0	-1	6	-1	6	5	-6	6	-7	-7	0	5	-6	-6	12	-10	-4	6	2	11	-6	-12	-7	
486E	-	+	3	2	0	-4	-6	-7	9	9	2	-4	-6	-4	3	-3	0	-10	5	3	-7	8	-6	12	-1	
486F	-	-	-3	-4	-6	2	0	-1	-3	3	2	8	-12	-4	3	-9	6	8	-13	-9	-7	-4	12	0	-13	
490A	+	1	+	+	-6	-4	0	2	-3	-3	8	-4	9	-7	0	-6	-6	5	5	-6	-16	2	3	-15	14	
490B	+	2	+	-	3	1	6	1	9	6	-8	-7	-3	2	-9	9	0	-8	8	0	4	-10	0	-6	10	
490C	+	-2	-	+	3	-1	-6	-1	9	6	8	-7	3	2	9	9	0	8	8	0	-4	-10	0	6	-10	
490D	+	-1	-	-	-6	4	0	-2	-3	-3	-8	-4	-9	-7	0	-6	6	-5	5	-6	16	2	-3	15	-14	
490E	-	-2	+	+	3	5	6	-1	3	-6	-4	11	3	-10	3	3	0	-4	-4	12	-4	-10	-12	6	14	
490F	-	3	+	+	-2	0	-4	-6	3	9	-4	-4	-7	-5	8	-2	10	1	-9	2	-4	10	-7	1	14	
490G	-	-2	+	-	-4	-2	-8	6	-4	-6	-4	-10	-4	4	-4	10	14	10	-4	12	-4	4	2	-8	0	
490H	-	0	-	-	4	6	-2	0	0	6	-8	-10	-2	4	-8	-2	8	14	-12	-16	-2	-8	-8	-10	-2	
490I	-	2	-	-	3	-5	-6	1	3	-6	4	11	-3	-10	-3	3	0	4	-4	12	4	-10	12	-6	-14	
490J	-	2	-	-	-4	2	8	-6	-4	-6	4	-10	4	4	4	10	-14	-10	-4	12	4	4	-2	8	0	
490K	-	-3	-	-	-2	0	4	6	3	9	4	-4	7	-5	-8	-2	-10	-1	-9	2	4	10	7	-1	-14	
492A	-	+	0	-2	-1	-2	-1	-4	-6	5	-3	-3	-	-7	-3	10	0	1	2	3	-11	6	4	6	8	
492B	-	-	-2	-4	-5	4	-5	-6	4	-3	1	5	+	9	-11	2	-4	1	-14	-1	13	10	-6	-14	-6	
493A	-1	0	-2	-5	0	7	-	5	4	+	4	-11	3	-5	9	-3	6	-1	4	5	3	-6	14	-8	2	
493B	-1	-3	1	-2	3	1	-	-4	-2	-	1	-2	6	1	-9	-9	-6	8	-14	-10	0	3	8	16	2	
494A	+	-1	1	-1	0	+	-3	+	6	-8	-8	-5	-2	-1	3	-2	-10	-14	-4	3	16	4	16	8	-10	
494B	+	0	2	4	4	+	2	-	-8	2	0	10	-2	12	4	-6	-12	14	-12	8	2	-16	-12	6	-2	
494C	+	3	-3	3	0	+	5	-	6	-8	8	-5	-10	7	-1	-10	6	-6	-4	-5	8	12	-8	0	-2	
494D	-	-1	-1	-3	-4	-	-3	+	2	4	-8	1	10	-5	-7	2	-6	2	0	5	0	8	-12	0	-2	
495A	-1	-	+	0	-	2	-6	-4	-4	-6	-8	-2	-2	4	12	2	-4	-10	-16	-8	14	8	4	-10	10	
496A	+	0	-3	3	-2	-4	0	-1	-4	-6	+	-10	7	10	-12	-4	-3	12	12	13	2	-6	-6	-10	1	
496B	+	2	1	3	2	-2	-6	-1	6	4	-	-2	7	-4	-8	8	-3	-6	12	-3	-10	12	-2	-16	-7	
496C	+	2	2	0	-2	4	6	-4	0	-4	-	4	-10	2	8	4	0	0	-12	0	2	-12	14	-14	14	
496D	-	2	-3	1	6	2	6	1	6	0	+	-10	-9	-8	0	0	3	-10	4	15	14	-8	-6	12	-7	
496E	-	0	1	-3	-6	-4	0	5	4	2	-	-2	-9	-2	-4	12	-9	12	12	-5	-14	-10	-2	6	-7	
496F	-	0	-2	0	0	2	-6	-4	-8	2	-	10	-6	-8	8	-6	12	-6	12	-8	10	8	-8	-6	2	
497A	1	-1	0	-	1	-3	-2	-4	-9	-1	4	-3	9	0	0	2	-8	5	0	-	-4	-10	9	8	5	
498A	+	-	2	4	0	0	-2	0	-6	0	-4	10	-2	4	0	-14	12	-6	0	4	10	2	+	-6	-10	
498B	+	-	-1	-4	3	-6	-4	-3	-1	4	-2	3	6	-12	0	-9	-7	-1	7	0	4	-4	-	5	10	
501A	1	+	-4	4	4	6	0	4	-8	6	0	-6	0	-6	8	12	12	2	-2	-12	-2	-2	8	-6	10	-
503A	1	1	-2	-3	1	1	0	-4	-3	0	10	-4	-2	5	-5	12	-4	-7	-11	0	-6	4	-3	0	10	+
503B	1	3	-2	3	3	5	-8	4	-5	0	-2	4	-10	-1	-3	-12	12	-11	7	-8	-6	-4	15	0	-6	-
503C	-1	1	-4	-3	5	1	0	8	9	-6	-2	2	-10	5	-1	-6	4	5	13	6	6	16	9	-6	10	-
504A	+	+	-2	+	-2	2	-6	-4	-6	0	-4	10	-2	-4	-4	12	-12	6	-4	14	-2	-8	16	6	-18	
504B	+	+	2	-	6	-6	-2	4	2	8	4	-6	10	-4	-4	-4	-12	-2	12	6	-2	-8	0	-14	-2	
504C	+	-	-2	+	4	2	6	8	0	-6	8	-2	-2	-4	8	-6	0	-6	-4	8	10	16	-8	6	-6	
504D	-	+	2	+	2	2	6	-4	6	0	-4	10	2	-4	4	-12	12	6	-4	-14	-2	-8	-16	-6	-18	
504E	-	+	-2	-	-6	-6	2	4	-2	-8	4	-6	-10	-4	4	4	12	-2	12	-6	-2	-8	0	14	-2	
504F	-	-	-2	+	0	-2	-6	-4	4	-6	-8	-10	10	12	8	-6	-4	-10	12	-4	2	8	-4	-6	10	
504G	-	-	-2	-	0	6	2	4	4	10	-8	6	2	-4	-8	10	-12	-2	12	12	-14	-8	-12	2	10	
504H	-	-	4	-	0	0	2	-2	-8	-2	4	-6	2	8	4	10	-6	4	-12	0	-14	-8	-6	-10	-2	
505A	1	0	+	0	-2	2	-6	0	-6	-6	8	2	2	-6	6	6	-6	2	-4	-8	10	4	0	2	-6	+
506A	+	-2	1	-1	+	3	3	-6	+	-1	-7	-5	-2	-8	-1	-6	-10	-8	7	-5	4	11	12	6	2	
506B	+	0	-3	3	+	5	5	-2	-	9	7	3	-8	10	-7	10	12	6	-3	-7	-8	-5	14	8	-8	
506C	+	-2	3	5	-	-1	-3	2	+	3	5	-7	-6	8	3	6	-6	8	-7	-9	-16	17	12	6	-10	
506D	+	0	-1	1	-	-7	3	-2	-	-3	-5	1	-4	-10	5	6	-8	2	-5	5	-4	-7	-6	4	16	

TABLE 3: HECKE EIGENVALUES 506E–528I

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
506E	-	0	-3	-3	+	-1	-1	-2	-	3	-5	3	4	-2	5	-14	0	-6	9	5	-8	-11	2	-4	-8	
506F	-	-2	-1	-1	-	-3	-5	-6	+	-7	1	5	-6	8	-1	14	6	-8	-7	3	8	-5	12	-14	-10	
507A	1	+	-1	2	-2	+	-7	-6	-6	-1	4	1	9	6	6	-9	0	1	-2	6	11	-4	-14	-14	-2	
507B	-1	+	1	-2	2	+	-7	6	-6	-1	-4	-1	-9	6	-6	-9	0	1	2	-6	-11	-4	14	14	2	
507C	-1	+	-2	4	-4	+	2	0	0	-10	-4	2	-6	-12	0	6	-12	-2	8	0	-2	8	-4	2	-10	
510A	+	+	+	2	-4	4	-	-4	8	2	4	6	8	6	8	-2	-6	14	-2	2	4	0	-16	-2	0	
510B	+	-	-	-2	4	0	-	4	4	6	-8	-6	8	2	-8	14	6	2	2	-10	4	4	-16	6	-8	
510C	-	+	+	2	0	4	+	4	4	2	0	-2	-4	10	-8	2	-2	-14	2	-6	-4	-12	8	-10	8	
510D	-	+	+	-4	-4	-2	-	-4	-4	2	4	-6	2	-12	8	-2	12	2	4	-4	-14	-12	-4	10	18	
510E	-	+	-	0	4	-2	-	4	0	-2	8	6	-6	-4	0	-10	-4	-2	4	0	-6	8	-12	-6	-14	
510F	-	-	+	0	4	2	-	-4	4	2	-4	-6	-10	-8	0	6	-8	10	-8	8	-2	4	4	-14	-10	
510G	-	-	-	2	0	-4	+	-4	0	6	-4	2	0	2	0	-6	6	-10	2	-6	-16	8	0	6	-4	
513A	1	+	0	-2	-5	-4	2	+	8	1	-3	0	-3	-10	-9	13	4	5	-1	-6	7	-11	15	-3	8	
513B	-1	+	0	-2	5	-4	-2	+	-8	-1	-3	0	3	-10	9	-13	-4	5	-1	6	7	-11	-15	3	8	
514A	-	0	-2	-4	-4	-2	2	0	8	-2	8	-2	-6	0	-12	-2	4	-2	-12	-12	-6	8	-8	-6	2	-
514B	-	-2	-2	2	-4	-2	-2	-2	-8	-6	4	-2	2	-6	10	-10	0	10	12	6	6	4	2	-6	10	-
516A	-	+	3	-1	-1	7	-2	-5	8	3	8	-4	4	+	7	4	12	0	-10	-6	-14	-4	-9	2	-7	
516B	-	+	-2	2	-3	-1	-3	-2	-3	-8	1	-8	1	-	0	-1	4	0	7	6	4	8	1	-14	-5	
516C	-	-	0	0	-2	6	6	4	2	-8	0	6	6	-	-6	-6	-10	6	4	-16	14	-8	-6	8	-2	
516D	-	-	3	5	-3	-1	-6	-7	0	3	-4	8	-12	-	-3	12	-12	8	2	-6	-10	8	-3	-6	17	
517A	2	-1	3	4	+	0	0	2	1	2	-3	3	-2	-12	-	-10	9	-10	-5	9	-4	-10	4	-13	-15	
517B	0	3	3	-2	-	-2	4	4	-7	-6	5	3	-6	-6	+	-6	5	-14	15	5	0	-12	6	-1	1	
517C	2	-1	-3	-2	-	0	-6	8	-5	-4	3	3	4	-6	-	2	-3	8	-11	-3	-4	2	-14	-1	-15	
520A	+	0	+	0	-4	+	-6	4	0	-2	-4	-6	-6	8	0	2	4	-10	12	-4	14	-16	12	2	-2	
520B	-	2	-	0	2	-	2	2	2	-6	2	-6	2	6	-8	-2	6	-14	0	10	-2	-4	12	-6	2	
522A	+	+	3	-5	-4	-6	-1	-5	6	+	0	1	7	1	-13	-2	-13	-2	-4	10	-12	8	12	6	-12	
522B	+	+	2	4	0	2	-2	0	-4	-	6	-4	-2	4	8	14	-6	-8	-12	16	-2	-6	2	-14	-14	
522C	+	+	-3	-1	0	2	3	5	6	-	-4	11	3	-1	3	-6	9	2	8	6	8	-16	12	6	-4	
522D	+	-	1	1	2	0	3	-1	4	+	4	3	7	9	1	2	3	6	12	-16	-10	10	0	-6	0	
522E	+	-	-1	1	-6	-4	7	-3	-4	-	0	-7	-5	-5	5	-10	-3	10	0	4	10	-6	-16	10	-8	
522F	+	-	-1	-2	3	-1	-8	0	-4	-	-3	8	-2	-11	-13	11	0	-8	-12	-2	4	15	-4	10	-2	
522G	-	+	-2	4	0	2	2	0	4	+	6	-4	2	4	-8	-14	6	-8	-12	-16	-2	-6	-2	14	-14	
522H	-	+	3	-1	0	2	-3	5	-6	+	-4	11	-3	-1	-3	6	-9	2	8	-6	8	-16	-12	-6	-4	
522I	-	+	-3	-5	4	-6	1	-5	-6	-	0	1	-7	1	13	2	13	-2	-4	-10	-12	8	-12	-6	-12	
522J	-	-	-3	-3	-6	0	-7	5	8	+	-8	-3	5	3	-9	2	11	-6	0	0	-10	-2	0	-10	0	
522K	-	-	-2	0	4	6	2	4	0	-	-4	-6	-6	-12	8	6	-8	10	-4	8	2	4	0	-14	18	
522L	-	-	3	-2	1	3	4	-8	0	-	3	-8	2	7	-11	-1	4	4	-4	2	-12	-7	0	6	-6	
522M	-	-	3	5	-6	-4	-3	-1	0	-	-4	-1	9	-7	3	6	-3	-10	-4	-12	2	14	0	6	8	
524A	-	1	-2	-3	0	1	-4	-6	2	0	2	0	5	-3	6	3	-9	5	-10	-2	4	-8	12	-3	4	-
525A	-1	+	+	+	0	6	-2	-8	-8	-2	4	2	-6	-4	-8	-10	4	-2	-4	-12	2	8	4	-6	18	
525B	1	+	+	-	4	2	6	4	0	-2	0	-6	2	4	0	-6	12	-2	-4	0	6	-16	12	-14	-18	
525C	1	+	-	-	-6	2	-4	-6	0	-2	-10	4	2	4	0	-6	-8	-2	16	10	6	4	-8	6	2	
525D	-1	-	-	+	-6	-2	4	-6	0	-2	-10	-4	2	-4	0	6	-8	-2	-16	10	-6	4	8	6	-2	
528A	+	+	0	-2	+	0	-2	-8	2	-6	0	-2	2	-4	6	-8	8	-4	-12	10	-6	10	4	10	-2	
528B	+	+	-2	-4	-	6	6	8	0	-6	0	6	-10	8	0	6	-4	-2	12	8	2	4	12	-6	2	
528C	+	+	4	2	-	0	-6	-4	6	6	0	6	-10	8	-6	-12	8	4	12	-10	2	-2	-12	-6	14	
528D	+	-	2	0	+	2	6	0	-4	2	0	-10	6	8	4	-6	12	2	-4	-12	-14	-16	12	10	-14	
528E	-	+	2	2	+	-2	4	6	0	-8	8	10	8	2	8	-2	-12	10	-12	-8	6	2	-16	-14	-2	
528F	-	+	-4	2	+	4	-2	0	6	10	8	-2	2	-4	2	4	0	-8	12	-2	-6	-10	-4	10	-2	
528G	-	+	0	-2	-	-4	-6	4	-6	6	-8	-10	6	-8	6	0	0	8	4	-6	2	-14	12	-6	14	
528H	-	-	-2	-4	+	-2	-2	0	-8	-6	8	6	-2	0	-8	6	4	6	4	0	-14	4	-12	-6	2	
528I	-	-	2	-2	-	6	-4	2	8	0	0	-6	0	-10	0	14	12	-14	-4	0	6	-2	-16	-14	-2	

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
528J	-	-	2	4	-	-6	2	-4	-4	6	0	6	-6	-4	12	2	-12	-14	-4	12	-6	4	-4	10	-14	
530A	+	1	+	2	0	5	3	5	-3	3	8	-7	12	2	-6	-	0	8	-4	-3	2	-1	15	-6	-13	
530B	+	0	-	-2	0	-2	-6	-2	0	2	-2	-2	10	-10	-6	-	-12	-2	-16	-6	10	10	8	14	-6	
530C	+	-3	-	-2	0	1	3	1	-3	-1	-8	-11	-8	2	-6	-	12	-8	-4	-3	-2	-17	11	2	3	
530D	-	-1	+	-2	-4	-3	-1	-1	7	-9	0	-7	0	-6	6	-	-4	4	12	3	10	-15	9	-6	-9	
532A	-	0	-2	-	4	4	6	-	4	6	4	-10	4	-8	0	10	-4	14	-6	-6	-2	-10	-4	12	-12	
534A	-	+	-2	-2	-4	0	-2	-4	-6	0	2	4	-2	8	-8	10	0	0	12	-8	-2	-4	0	-	10	
537A	1	+	0	-1	6	7	-2	2	6	6	-2	4	-9	6	-9	-9	11	7	12	-4	10	-11	9	0	6	-
537B	0	-	1	0	0	3	3	5	4	-3	0	4	12	-7	7	-10	1	-2	3	6	-16	-8	11	-3	2	+
537C	0	-	-3	2	6	-1	3	-7	6	9	8	2	0	5	-9	12	-3	-10	-13	-6	2	8	-9	-3	-4	+
537D	1	-	4	1	2	-1	-6	2	-2	2	-2	-4	-5	6	-3	11	-7	-1	-12	0	-2	-5	11	-12	-6	+
537E	-2	-	1	-2	2	-1	3	5	4	5	-8	8	-8	9	3	14	5	2	3	12	4	10	-1	-15	-12	+
539A	0	-1	-3	-	+	4	6	-2	3	-6	-5	11	-6	8	0	-6	9	10	5	9	-2	-10	-12	3	1	
539B	0	3	1	-	+	4	-2	6	-5	10	-1	-5	2	-8	-8	-6	-3	2	-3	1	-10	6	-12	15	5	
539C	1	-2	2	-	-	-4	-4	0	-4	-6	-10	-6	-4	12	10	-6	-2	0	8	-12	8	8	0	6	10	
539D	-2	1	-1	-	-	-4	2	0	-1	0	-7	3	8	-6	-8	-6	-5	-12	-7	-3	-4	-10	6	-15	7	
540A	-	+	+	2	0	2	3	5	-3	6	5	2	-12	8	12	3	-6	-7	2	-12	-16	-1	15	12	-16	
540B	-	+	-	-4	-6	-4	3	-7	9	0	-7	2	-6	2	0	-9	12	-7	2	-6	2	-1	-9	6	8	
540C	-	-	+	-1	-6	-1	0	-1	-6	-6	8	-7	6	-4	-12	6	0	11	-7	6	11	-1	-6	12	-13	
540D	-	-	+	-4	6	-4	-3	-7	-9	0	-7	2	6	2	0	9	-12	-7	2	6	2	-1	9	-6	8	
540E	-	-	-	-1	6	-1	0	-1	6	6	8	-7	-6	-4	12	-6	0	11	-7	-6	11	-1	6	-12	-13	
540F	-	-	-	2	0	2	-3	5	3	-6	5	2	12	8	-12	-3	6	-7	2	12	-16	-1	-15	-12	-16	
542A	-	2	2	0	-4	0	-2	6	-4	-8	0	2	6	2	12	-2	6	-2	8	-8	2	0	-4	10	-2	+
542B	-	-1	0	-5	0	-1	-6	4	-8	10	-3	2	5	-7	-4	-2	-9	8	-2	-16	-4	-7	6	3	4	-
544A	+	0	0	-2	-4	2	+	-4	-6	8	2	4	-2	4	-12	-6	-4	4	-4	6	-6	10	12	-10	-10	
544B	+	2	2	2	-2	2	-	-4	2	2	-10	10	2	-4	0	6	-4	2	16	-10	-6	-6	4	6	-14	
544C	+	-2	2	-2	2	2	-	4	-2	2	10	10	2	4	0	6	4	2	-16	10	-6	6	-4	6	-14	
544D	-	0	0	2	4	2	+	4	6	8	-2	4	-2	-4	12	-6	4	4	4	-6	-6	-10	-12	-10	-10	
544E	-	2	4	-4	2	2	+	8	-8	4	-4	-8	-2	-4	0	-6	4	8	4	-8	-6	0	-4	-6	-2	
544F	-	-2	4	4	-2	2	+	-8	8	4	4	-8	-2	4	0	-6	-4	8	-4	8	-6	0	4	-6	-2	
545A	1	0	-	-4	4	-6	-2	4	-8	-2	0	2	2	8	0	-6	-12	-2	-12	0	10	-16	-8	10	10	-
546A	+	+	-1	+	-1	-	-1	7	3	-3	8	7	8	7	8	-10	4	7	2	4	-1	2	-6	14	-14	
546B	+	-	1	+	3	+	5	1	3	5	4	-5	-8	-1	8	6	0	13	-10	8	-15	6	-2	-2	-2	
546C	+	-	-2	+	-4	-	-2	-4	-4	-2	0	-2	2	4	-12	6	0	-10	4	-8	-6	8	8	-6	2	
546D	+	-	3	-	3	-	-3	-7	9	-9	-4	-7	12	-1	0	-6	12	-1	14	12	-7	-10	-6	-6	-10	
546E	-	+	3	+	1	+	7	1	-7	3	0	-5	4	11	0	-14	4	1	-6	-12	5	-10	-14	-6	6	
546F	-	-	-1	-	5	+	-3	-1	3	9	4	-11	0	-5	-8	-2	4	-15	-2	-12	11	10	-14	6	-14	
546G	-	-	2	-	-4	+	6	-4	0	-6	-8	10	-6	4	4	10	4	-6	-8	0	-10	-8	4	-6	-2	
549A	1	+	0	-2	-4	-2	2	-4	0	6	-6	2	4	-2	-4	-6	12	+	-10	8	10	-6	-4	-2	-2	
549B	-1	+	0	-2	4	-2	-2	-4	0	-6	-6	2	-4	-2	4	6	-12	+	-10	-8	10	-6	4	2	-2	
549C	1	-	3	1	5	1	-4	-4	9	6	0	8	-5	-8	-4	-6	-9	+	-7	8	-11	3	-4	4	-14	
550A	+	-1	+	1	+	-2	3	-1	-6	-9	5	-5	-6	-8	-6	-9	6	5	-8	-9	10	14	6	-15	-8	
550B	+	1	+	-3	-	6	7	5	6	5	-3	-3	2	-4	2	1	-10	7	-8	7	-14	10	6	-15	12	
550C	+	-2	+	0	-	-3	4	-1	-3	5	-3	12	8	5	8	10	8	10	-14	-5	4	-8	9	3	-3	
550D	+	-2	-	-4	+	5	0	-7	3	3	5	-4	12	5	0	6	12	-10	14	3	8	-4	-15	3	-13	
550E	+	3	-	1	+	0	5	-7	8	3	-5	1	-8	-10	0	1	12	5	4	-7	-2	-4	0	-7	-8	
550F	+	1	-	-3	-	-4	-3	-5	-4	5	7	7	-8	6	-8	-9	0	-13	12	-3	6	0	-4	-15	12	
550G	+	-2	-	0	-	2	-6	4	2	-10	-8	-8	-2	0	-2	0	-12	-10	6	0	-6	12	-16	18	12	
550H	-	2	+	4	+	-5	0	-7	-3	3	5	4	12	-5	0	-6	12	-10	-14	3	-8	-4	15	3	13	
550I	-	-1	+	-5	-	-2	-3	-7	6	-3	-7	7	6	-8	-6	3	-6	-1	-8	3	-2	-10	6	9	4	
550J	-	-3	-	-1	+	0	-5	-7	-8	3	-5	-1	-8	10	0	-1	12	5	-4	-7	2	-4	0	-7	8	
550K	-	-1	-	3	-	4	3	-5	4	5	7	-7	-8	-6	8	9	0	-13	-12	-3	-6	0	4	-15	-12	

TABLE 3: HECKE EIGENVALUES 550L-570H

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
550L	-	2	-	0	-	-2	6	4	-2	-10	-8	8	-2	0	2	0	-12	-10	-6	0	6	12	16	18	-12	
550M	-	2	-	0	-	3	-4	-1	3	5	-3	-12	8	-5	-8	-10	8	10	14	-5	-4	-8	-9	3	3	
551A	1	1	-1	-4	1	-1	0	-	-4	-	5	10	-6	-7	7	-3	-10	-8	10	0	-12	-9	-10	2	8	
551B	-1	1	-1	2	-3	-5	2	-	0	-	-7	2	0	-11	-9	1	8	0	-4	0	2	3	4	0	4	
551C	2	-2	-1	-1	-3	-2	-1	-	0	-	2	-4	-6	1	3	4	14	-3	2	0	11	-12	4	0	-8	
551D	-2	-2	-1	-1	1	2	3	-	8	-	-10	-8	-6	5	7	-12	-10	1	-2	-12	-9	12	-4	8	8	
552A	+	+	-2	2	-2	-2	-4	0	+	-10	0	-4	-6	-4	8	-6	4	8	-4	8	6	6	-6	-4	10	
552B	+	+	0	-2	0	2	8	6	-	2	-4	6	10	6	0	12	4	-10	-6	0	2	-6	0	-12	6	
552C	+	+	4	2	0	2	-4	-6	-	10	4	-2	-6	-6	8	8	-4	-2	6	0	-14	-10	16	0	-18	
552D	-	+	2	-4	-4	-2	-2	0	-	-2	0	-10	-6	8	-8	-6	-4	14	8	-8	-6	12	-12	-2	10	
552E	-	-	-2	-4	0	-2	-2	-4	+	-2	-8	2	10	-4	0	6	12	2	-12	-16	10	-4	0	6	-14	
555A	0	-	+	-2	4	5	-2	6	6	3	-6	+	0	-11	9	3	-1	-2	-8	12	6	12	-1	1	2	
555B	0	-	-	2	0	-1	6	2	-6	9	2	-	0	-1	9	3	-3	-10	-4	0	-10	-4	-9	3	-10	
556A	-	0	-1	-1	1	-3	2	-6	-6	-3	5	10	-2	-8	-8	6	-4	12	1	-15	4	15	-9	3	10	-
557A	1	-1	0	2	-3	2	-2	-8	0	-5	-9	4	8	0	9	-2	-4	-6	-4	8	9	-14	18	0	17	+
557B	2	2	0	5	-6	-4	-1	4	0	5	0	10	-2	3	6	8	-11	-6	11	-5	6	-5	-9	0	-7	-
558A	+	+	-1	0	-3	-1	-3	1	-2	-2	+	-2	0	-4	-7	0	6	-9	3	1	-8	1	-5	6	-7	
558B	+	+	3	-4	3	5	3	-7	6	6	-	2	-12	8	-3	12	6	5	11	-3	-4	-13	-3	-6	-7	
558C	+	-	2	0	0	2	6	4	-8	-2	+	10	6	8	8	6	12	-6	-12	-8	10	-8	-8	6	2	
558D	+	-	-1	-2	3	-1	-3	-5	-4	0	-	-2	-2	-6	7	-14	-10	7	-7	3	-6	15	1	-10	13	
558E	-	+	1	0	3	-1	3	1	2	2	+	-2	0	-4	7	0	-6	-9	3	-1	-8	1	5	-6	-7	
558F	-	+	-3	-4	-3	5	-3	-7	-6	-6	-	2	12	8	3	-12	-6	5	11	3	-4	-13	3	6	-7	
558G	-	-	-3	-2	-5	-7	1	7	-4	8	+	-6	2	-10	1	-6	10	1	-3	-3	14	-11	-7	6	-3	
558H	-	-	1	2	-3	3	-1	7	0	-4	-	-10	6	6	5	2	-6	3	-3	-7	-10	-1	-17	-6	5	
560A	+	1	+	-	5	1	3	6	6	-9	0	6	8	-6	-3	-12	-8	-4	4	-8	10	3	12	-16	7	
560B	+	3	-	+	5	-5	-7	2	2	7	-4	-6	-12	2	-1	0	4	4	-8	0	6	3	4	0	13	
560C	-	-1	+	+	3	5	3	-2	6	3	4	2	-12	10	-9	12	0	8	4	0	2	1	-12	-12	-1	
560D	-	0	+	-	-4	-6	2	0	0	6	-8	-10	2	-4	-8	-2	8	-14	12	16	2	8	-8	10	2	
560E	-	-3	+	-	5	-3	-1	-6	-6	-9	4	2	-4	-10	1	4	8	-8	-12	-8	2	-13	4	4	-13	
560F	-	-1	-	+	-3	-1	-3	-2	6	-9	-8	-10	0	-2	3	0	-12	8	-8	0	14	-5	12	12	17	
561A	0	+	-2	-3	-	2	+	2	2	9	8	-12	-3	0	5	11	7	-2	3	-4	-13	8	16	15	16	
561B	0	-	-2	1	-	-6	+	-6	-6	1	-8	-4	-3	8	9	-1	11	-2	11	12	7	0	-16	-5	-16	
561C	-2	-	0	-3	-	-4	+	-2	6	-9	-4	2	-1	6	-9	13	-15	-14	-9	-14	-13	0	2	13	14	
561D	-1	-	2	0	-	-2	-	8	-8	6	8	6	2	8	-4	6	-8	10	4	0	-10	0	12	-14	-14	
562A	+	2	2	4	2	-2	2	-6	-2	-2	-4	-2	10	-4	-6	2	4	-10	2	14	2	16	6	-6	18	-
563A	-1	-1	-4	-5	-4	2	-3	-3	-3	2	-2	-6	-10	-8	-3	2	-3	-1	-3	1	6	-12	-8	-14	-6	-
564A	-	+	-1	-1	3	-2	-6	-6	5	-5	-10	-3	-6	10	-	12	8	-10	-2	-10	-2	3	0	6	17	
564B	-	-	-3	-1	-3	-4	0	2	-9	-3	-4	5	-6	8	+	6	6	2	8	-6	-4	-1	6	-6	-19	
565A	1	-2	+	1	4	4	1	-3	4	4	8	11	-11	-12	6	6	0	-3	-14	13	-1	8	15	-12	10	-
566A	+	0	0	1	-3	-5	4	-4	-1	7	0	-12	3	-4	-2	-2	-11	-1	10	8	-2	4	0	-9	-7	+
566B	-	1	-2	3	0	4	8	7	-4	-6	-6	3	-7	-5	0	5	-6	-2	-5	-12	7	10	-2	-6	7	+
567A	1	+	-1	+	2	-5	-3	-2	-6	5	-6	-3	10	-4	6	6	-6	7	-2	12	-15	14	-18	5	-18	
567B	-1	+	1	+	-2	-5	3	-2	6	-5	-6	-3	-10	-4	-6	-6	6	7	-2	-12	-15	14	18	-5	-18	
568A	+	-1	2	5	2	-1	-2	-3	5	6	11	-2	-6	-11	7	6	6	-2	-14	-	-9	2	-12	-7	10	
570A	+	+	+	2	-6	0	2	+	4	-8	-8	-4	-4	-6	-12	6	-4	2	-8	0	6	8	4	-4	12	
570B	+	+	+	-2	4	-6	4	-	4	6	-6	10	4	12	4	-10	10	2	12	8	-2	10	2	-8	2	
570C	+	+	-	-2	-2	0	-2	-	-8	0	0	4	-8	-6	-8	-10	-8	2	0	8	-2	-8	-16	16	8	
570D	+	-	+	4	0	2	-2	+	0	10	0	2	2	-4	0	-6	8	6	12	0	-14	0	-12	10	2	
570E	+	-	-	-4	-4	-6	-6	+	4	6	-8	2	10	-8	12	2	-4	-2	-12	-16	-14	8	0	-6	14	
570F	+	-	-	2	0	2	0	-	0	-6	2	2	0	8	0	6	-6	2	-4	0	14	2	6	-12	-10	
570G	-	+	+	0	4	2	2	+	4	6	4	-6	10	-4	-12	6	-12	-2	4	8	-6	-4	-12	10	2	
570H	-	+	-	-2	0	6	8	-	-4	2	-2	-2	-12	4	12	10	6	-14	-12	-8	-10	14	2	0	2	

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
570 I	-	+	-	4	0	-6	2	-	8	2	-8	10	6	-8	0	-2	-12	-2	-12	-8	2	-16	-4	6	-10	
570 J	-	-	+	2	-4	6	4	-	0	-10	-2	-2	8	-8	0	-6	-2	2	4	0	-10	-2	-10	-12	-2	
570 K	-	-	+	2	6	-4	-6	-	0	0	8	8	-12	2	0	-6	-12	2	-16	0	-10	8	0	-12	8	
570 L	-	-	-	-2	2	4	-2	+	4	0	-8	8	-8	-6	-12	-6	0	2	8	-8	14	0	4	0	-12	
570 M	-	-	-	4	-4	-2	-2	+	-8	6	4	-10	-2	12	0	6	0	-10	-4	-8	2	-12	-8	6	18	
571 A	0	2	-2	2	5	3	0	0	4	9	5	-7	0	11	0	-12	9	-11	-16	2	-8	10	11	-4	1	-
571 B	-2	-2	-2	-4	-3	-5	-2	-2	-8	5	-7	-7	2	-1	6	2	-3	13	-14	10	2	0	-9	-12	-7	-
572 A	-	1	3	2	-	-	0	2	-3	-6	-1	-7	6	8	12	-6	9	2	-7	-3	8	-4	-12	-15	-13	
573 A	-1	-	2	2	0	2	-6	-2	8	8	-2	2	12	0	0	8	-4	-6	-8	12	-10	16	-4	-12	-6	+
573 B	2	-	2	2	-3	-1	0	-8	2	5	10	-4	-9	9	9	-7	-4	12	-5	9	-10	-17	8	6	-9	+
573 C	-2	-	-2	-2	-1	7	-4	0	-6	-1	-10	12	-3	-7	-13	-13	12	8	-5	3	-14	-9	0	2	-17	-
574 A	+	-1	1	+	0	2	-5	-4	6	-9	3	-8	+	7	-10	-13	-14	-5	4	-15	-6	-3	16	13	17	
574 B	+	2	-2	+	-6	-4	-2	2	0	0	0	10	+	4	8	-4	-8	10	-14	-12	-6	0	4	-14	-10	
574 C	+	2	2	-	-2	4	6	-6	8	-4	-8	10	+	-4	8	-8	-4	-10	-2	12	10	0	0	18	-10	
574 D	+	-2	4	-	4	4	-2	-6	-8	6	4	2	+	0	4	2	14	4	-8	8	-14	-8	6	10	6	
574 E	+	3	-1	-	4	-6	3	4	2	1	9	-8	+	-5	-6	-3	14	-11	-8	3	-14	7	16	5	1	
574 F	+	1	-3	-	0	2	-3	-4	-6	-3	-1	-4	-	-1	-6	9	-6	-1	8	3	-10	-1	-12	3	-1	
574 G	-	-1	-1	+	-6	-4	7	0	-8	1	5	-2	-	-5	-6	-3	-10	-3	14	3	8	7	-2	5	5	
574 H	-	0	-4	-	-2	-6	-6	4	8	-8	0	-2	+	-8	0	0	2	-8	10	-12	10	16	-2	-10	-2	
574 I	-	-3	-1	-	-2	0	-3	-8	-4	-5	-3	10	+	-5	6	-9	-10	13	-2	9	4	-11	-14	-1	7	
574 J	-	-1	1	-	2	4	3	0	4	-5	7	-2	-	-1	-2	-1	10	-13	-2	-3	4	-15	-6	-15	-7	
575 A	1	0	+	-1	-1	-1	0	-5	+	-5	-2	4	-5	9	6	-2	8	-8	-8	-10	3	-3	-3	10	2	
575 B	-2	0	+	-1	2	2	-3	-2	+	7	-5	-11	1	0	0	-11	-13	-8	-5	5	-6	-12	-9	4	14	
575 C	2	2	-	1	0	2	-5	8	+	-5	-5	7	-7	4	-2	-1	3	-6	13	13	8	-14	-3	-14	14	
575 D	-1	0	-	1	-1	1	0	-5	-	-5	-2	-4	-5	-9	-6	2	8	-8	8	-10	-3	-3	3	10	-2	
575 E	-2	-2	-	-1	0	-2	5	8	-	-5	-5	-7	-7	-4	2	1	3	-6	-13	13	-8	-14	3	-14	-14	
576 A	+	+	0	-4	0	-2	0	-8	0	0	-4	10	0	-8	0	0	0	-14	16	0	-10	-4	0	0	14	
576 B	+	-	2	4	-4	2	6	-4	0	2	-4	2	-2	4	8	10	4	-6	4	-16	-6	-4	-12	-10	-14	
576 C	+	-	2	-4	4	2	6	4	0	2	4	2	-2	-4	-8	10	-4	-6	-4	16	-6	4	12	-10	-14	
576 D	+	-	-2	0	4	2	-2	4	8	6	8	-6	6	-4	0	-2	4	2	4	-8	10	-8	-4	6	2	
576 E	-	+	0	4	0	-2	0	8	0	0	4	10	0	8	0	0	0	-14	-16	0	-10	4	0	0	14	
576 F	-	+	4	0	0	6	-8	0	0	-4	0	2	8	0	0	-4	0	10	0	0	6	0	0	-16	-18	
576 G	-	+	-4	0	0	6	8	0	0	4	0	2	-8	0	0	4	0	10	0	0	6	0	0	16	-18	
576 H	-	-	-2	0	0	-6	-2	0	0	-10	0	2	-10	0	0	14	0	10	0	0	-6	0	0	-10	18	
576 I	-	-	-2	0	-4	2	-2	-4	-8	6	-8	-6	6	4	0	-2	-4	2	-4	8	10	8	4	6	2	
578 A	-	2	0	4	-6	2	+	-4	0	0	4	4	-6	8	0	-6	0	4	8	0	-2	-8	0	-6	-14	
579 A	2	+	2	1	-1	6	7	-6	4	-5	0	10	-6	1	-9	1	8	-10	-15	-13	14	-6	10	-5	-19	-
579 B	-1	-	0	0	-6	-6	4	0	0	0	0	-10	0	-4	-6	4	0	-10	4	14	-2	0	-4	4	-2	-
580 A	-	0	+	0	-2	-2	0	-2	-8	-	2	-4	-10	4	12	-6	-12	-10	12	12	12	2	-4	-10	8	
580 B	-	0	-	-2	-4	-6	-4	4	6	+	0	-8	-2	4	-4	-2	8	10	-10	-8	0	8	-6	6	-12	
582 A	+	+	0	-2	4	-4	-2	0	-2	4	-4	-8	-2	-4	0	-6	0	-14	4	-10	-6	8	4	-6	+	
582 B	-	+	0	-2	4	2	4	6	4	-8	8	-2	-8	8	0	6	-12	-2	10	-16	6	-4	4	18	+	
582 C	-	+	-2	-2	0	-4	-4	-4	0	2	-8	4	0	-4	0	10	10	10	-4	0	2	0	-14	2	-	
582 D	-	-	-2	0	4	2	6	0	4	-2	-8	-6	6	-4	-8	-10	4	6	-16	-4	10	8	4	-6	-	
583 A	2	1	3	0	+	4	0	-4	-3	6	-3	7	-2	-2	0	-	5	0	7	3	-2	6	-10	-1	-7	
583 B	1	-1	4	4	-	1	1	-3	5	-3	4	-3	10	-6	-10	+	-10	12	-4	3	-8	-7	-15	-6	-7	
583 C	2	3	-3	2	-	0	6	-8	-5	-4	-5	11	12	-2	-8	+	13	-8	-3	1	-4	-10	2	7	1	
585 A	-1	+	+	2	-4	+	-4	6	0	-4	-10	-2	-6	-8	-8	-4	12	2	-10	0	-6	12	-4	14	-14	
585 B	0	+	+	-1	3	-	3	-4	9	6	2	-1	3	2	6	-9	12	5	-4	-9	14	-7	0	-15	5	
585 C	1	+	-	2	4	+	4	6	0	4	-10	-2	6	-8	8	4	-12	2	-10	0	-6	12	4	-14	-14	
585 D	0	+	-	-1	-3	-	-3	-4	-9	-6	2	-1	-3	2	-6	9	-12	5	-4	9	14	-7	0	15	5	
585 E	-2	-	+	3	1	+	1	-2	3	2	-6	11	5	4	10	-11	-8	13	12	5	10	-3	12	15	17	

TABLE 3: HECKE EIGENVALUES 585F–602C

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
585F	1	-	+	0	-4	-	-2	-4	-8	2	-8	6	6	-4	8	-6	12	-2	-4	0	-6	16	4	-10	18	
585G	-2	-	+	-3	5	-	-5	2	1	-10	-2	-3	9	-4	-10	-9	0	-11	-4	-15	6	-11	-8	11	-9	
585H	1	-	-	-4	-2	+	-2	-6	6	-2	-10	-2	6	10	-4	-2	-6	2	-4	-6	-6	-12	16	-2	-2	
585I	-2	-	-	-1	-5	+	7	-6	-3	-2	2	7	-9	-8	-10	-5	0	5	-4	-9	-6	-3	4	-11	-11	
586A	+	2	3	0	2	-1	1	6	-7	1	-2	-4	0	0	7	-6	9	2	-9	10	7	-5	0	0	-5	-
586B	-	-1	0	-3	-4	-4	-2	3	5	-2	4	-1	-12	-6	1	3	6	-7	12	4	7	-8	-12	-12	13	-
586C	-	-1	-2	-1	0	0	-2	-5	-9	-6	2	-3	10	12	3	-3	-4	3	-10	-6	7	12	16	-14	-7	-
588A	-	+	-2	+	2	-3	8	-1	8	4	3	-1	6	11	6	-12	4	-6	13	-10	-11	-3	2	0	10	
588B	-	+	0	-	-6	-2	0	4	-6	6	-8	2	-12	-4	-12	-6	0	10	8	6	10	-4	12	-12	10	
588C	-	+	-2	-	2	4	-6	-8	-6	-10	-4	6	6	4	-8	2	4	8	-8	-10	-4	4	-12	14	-4	
588D	-	-	2	-	2	3	-8	1	8	4	-3	-1	-6	11	-6	-12	-4	6	13	-10	11	-3	-2	0	-10	
588E	-	-	2	-	2	-4	6	8	-6	-10	4	6	-6	4	8	2	-4	-8	-8	-10	4	4	12	-14	4	
588F	-	-	-4	-	2	6	4	4	2	-2	0	2	0	-4	-12	-6	8	-6	-8	14	2	12	4	0	2	
590A	+	-2	+	5	-3	-1	3	-4	0	0	8	11	-3	-1	6	12	-	2	-4	-3	2	-1	-3	6	2	
590B	+	0	-	4	4	2	-6	4	0	-2	0	2	10	-4	0	6	+	14	4	0	-10	8	-12	-14	-10	
590C	+	0	-	1	-5	-7	1	-2	4	6	-10	-7	-7	7	-4	12	-	-6	12	-13	-10	5	17	0	-12	
590D	-	-2	-	-3	-5	1	3	-8	-4	-8	0	-3	-3	1	6	0	+	-10	12	5	-6	15	-5	-10	-2	
591A	0	+	0	1	2	0	-4	-7	-5	-3	-4	-1	-1	1	11	-6	8	-7	-4	0	0	-8	-1	6	10	+
592A	+	1	-2	-1	-1	-6	-4	8	-6	2	4	+	7	-2	-9	-3	12	4	0	-7	7	0	-3	-12	-8	
592B	+	1	0	3	3	0	2	2	6	-2	4	-	7	-4	-1	9	-8	-4	-12	5	-13	10	1	-2	-12	
592C	-	3	-2	1	5	-2	0	0	-2	6	4	+	-9	-2	9	1	-8	-8	-8	-9	-1	-4	15	4	4	
592D	-	1	-4	3	-5	0	-6	-2	6	-6	-4	-	-9	-4	7	9	4	-8	12	-3	-5	-6	1	2	0	
592E	-	-1	0	1	-3	-4	6	-2	-6	-6	4	-	-9	-8	-3	-3	-12	8	4	15	11	10	-9	6	8	
593A	1	1	-2	-1	-4	6	1	-8	-6	10	-6	-3	8	1	8	9	-12	-8	4	8	7	-10	0	-6	2	+
593B	-1	-2	2	2	-2	-6	2	4	0	2	0	6	10	10	10	-6	0	10	-2	4	-2	8	0	-6	14	-
594A	+	+	-2	1	+	-2	-1	0	-3	-1	-8	1	-11	1	5	-4	3	-2	-12	-8	12	-17	14	2	-5	
594B	+	-	1	4	+	1	5	0	-3	-10	10	4	7	-2	8	5	6	-5	-3	4	-6	-17	5	14	1	
594C	+	-	-3	-4	+	5	-3	8	9	6	2	-4	-9	-10	0	9	6	-1	5	12	2	11	-3	6	-7	
594D	+	-	-2	-1	-	6	-5	-8	-1	-9	0	-3	9	-5	-9	4	-3	-2	4	0	-12	-7	2	2	19	
594E	-	+	2	-1	+	6	5	-8	1	9	0	-3	-9	-5	9	-4	3	-2	4	0	-12	-7	-2	-2	19	
594F	-	-	-1	4	-	1	-5	0	3	10	10	4	-7	-2	-8	-5	-6	-5	-3	-4	-6	-17	-5	-14	1	
594G	-	-	2	1	-	-2	1	0	3	1	-8	1	11	1	-5	4	-3	-2	-12	8	12	-17	-14	-2	-5	
594H	-	-	3	-4	-	5	3	8	-9	-6	2	-4	9	-10	0	-9	-6	-1	5	-12	2	11	3	-6	-7	
595A	-2	2	+	+	2	-1	-	6	9	6	5	-7	-5	-8	9	2	14	13	2	-8	-8	16	1	-6	-16	
595B	2	2	+	-	6	1	+	-6	-5	6	9	-5	-9	0	-1	-6	-6	-7	14	12	-8	0	15	-10	-4	
595C	2	2	-	-	-2	-1	-	2	-1	2	-5	-1	-3	4	1	6	6	-5	-6	16	4	4	-15	14	-4	
598A	+	0	0	0	-2	-	-2	-6	-	-6	8	8	-2	-6	-8	-12	-4	0	10	0	-10	4	-14	14	-14	
598B	+	-3	-3	3	-2	-	1	6	-	-6	-4	-7	4	-9	-5	-12	14	0	-2	-3	-10	16	16	2	-14	
598C	-	-1	3	3	2	+	-1	-4	+	0	8	3	-2	-1	-3	-2	0	-4	-10	3	-12	8	6	-14	-12	
598D	-	-1	-1	-1	-6	+	3	-4	-	0	-8	7	-2	11	-11	-10	0	12	-2	-13	4	8	14	10	4	
600A	+	+	+	0	-4	-6	6	-4	0	-2	-8	2	-6	-12	-8	-6	12	14	-4	8	6	-8	12	10	-2	
600B	+	+	+	-3	2	3	-6	-7	-6	-2	-5	-10	12	-3	10	0	-6	-13	-7	-4	6	-8	6	16	7	
600C	+	+	-	2	2	-2	-6	8	4	8	0	10	2	12	0	-10	-6	2	8	-4	-4	-8	-4	6	-8	
600D	+	-	+	0	4	2	-2	-4	8	6	8	-6	-6	-4	0	2	4	-2	4	8	-10	-8	4	-6	-2	
600E	+	-	-	-5	-6	-3	-2	1	-2	6	3	-6	4	11	-10	-8	-6	3	-1	-12	10	-8	-6	-16	-7	
600F	-	+	+	-4	0	6	2	4	8	-6	0	6	10	4	-8	-10	0	6	4	0	14	16	-12	2	-2	
600G	-	+	+	5	-6	3	2	1	2	6	3	6	4	-11	10	8	-6	3	1	-12	-10	-8	6	-16	7	
600H	-	-	-	-2	2	2	6	8	-4	8	0	-10	2	-12	0	10	-6	2	-8	-4	4	-8	4	6	8	
600I	-	-	-	3	2	-3	6	-7	6	-2	-5	10	12	3	-10	0	-6	-13	7	-4	-6	-8	-6	16	-7	
602A	+	0	-4	+	0	2	6	4	0	2	2	2	-2	-	-6	-6	6	12	0	-8	4	4	18	-16	18	
602B	+	-1	2	+	5	2	0	-3	6	-9	9	9	0	-	12	2	-4	2	15	3	14	-4	-4	-3	-4	
602C	+	3	2	+	-3	2	0	1	6	-1	5	-7	-8	-	-12	-6	12	-6	15	-5	-2	4	-12	-7	12	

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
603A	1	+	-2	4	4	2	0	4	6	-4	8	2	-6	4	-2	-10	6	-2	-	-6	-10	-8	2	-16	6	
603B	-1	+	2	4	-4	2	0	4	-6	4	8	2	6	4	2	10	-6	-2	-	6	-10	-8	-2	16	6	
603C	-1	-	3	-3	0	4	-2	-2	7	8	-1	-3	9	9	0	-1	9	14	+	4	11	-16	-5	0	16	
603D	2	-	0	0	6	4	7	-5	1	-1	-4	3	0	-6	-9	-10	-3	2	+	16	-7	8	4	15	4	
603E	1	-	1	-5	4	-4	-6	-2	3	-4	-7	5	3	7	-8	5	-3	-2	-	12	-13	-8	-1	-4	-12	
603F	-2	-	-2	-2	4	2	-3	7	-9	5	-10	-1	0	-2	1	-10	-9	-2	-	0	-7	-8	-4	-7	0	
605A	1	-3	-	3	-	-4	0	-4	-8	-6	-2	-8	5	-5	-3	4	-2	11	-13	2	8	-10	-4	1	-8	
605B	-1	0	-	0	-	-2	-6	4	4	-6	-8	-2	-2	-4	-12	-2	4	10	-16	8	-14	-8	4	10	10	
605C	-1	-3	-	-3	-	4	0	4	-8	6	-2	-8	-5	5	-3	4	-2	-11	-13	2	-8	10	4	1	-8	
606A	+	-	2	4	4	-2	-6	4	-8	2	0	-2	-10	4	-8	10	4	10	-4	-8	2	0	-4	14	2	+
606B	+	-	0	-3	-2	-6	-1	-5	4	3	-2	-2	-12	13	8	11	-10	-11	-8	6	-2	-14	-4	-8	-7	-
606C	-	+	0	-1	2	2	3	7	8	-7	-2	2	0	1	8	1	6	-5	-12	6	-6	6	-16	0	-7	+
606D	-	+	3	2	2	-4	-6	4	-4	8	7	-4	9	-8	8	-14	-6	-5	3	-6	-12	-3	14	9	-13	+
606E	-	-	-4	-5	-2	-2	3	-5	4	-7	-6	-2	8	-11	4	1	10	-5	8	-2	-10	6	-16	-16	-7	+
606F	-	-	1	-2	2	4	-2	0	4	0	7	-12	-3	4	-12	-6	-10	-3	13	2	4	5	-6	5	3	-
608A	+	0	-1	1	-3	-4	-3	+	8	0	-2	-8	0	-11	7	2	-6	-1	10	-2	5	2	0	6	-12	
608B	-	0	3	5	5	-4	-3	+	0	0	-10	8	0	5	-5	-6	10	-5	10	-10	-11	10	0	-10	-12	
608C	-	3	0	-1	2	-1	3	+	3	3	8	-10	-12	8	-8	-9	-5	10	7	-10	1	-14	6	-4	-6	
608D	-	0	-1	-1	3	-4	-3	-	-8	0	2	-8	0	11	-7	2	6	-1	-10	2	5	-2	0	6	-12	
608E	-	0	3	-5	-5	-4	-3	-	0	0	10	8	0	-5	5	-6	-10	-5	-10	10	-11	-10	0	-10	-12	
608F	-	-3	0	1	-2	-1	3	-	-3	3	-8	-10	-12	-8	8	-9	5	10	-7	10	1	14	-6	-4	-6	
609A	1	+	0	-	0	-6	-2	0	-6	-	-4	6	-10	-10	8	10	-12	-8	12	14	16	-14	4	-6	-8	
609B	-1	+	-2	-	4	-2	2	-4	0	-	-8	-10	-6	12	-8	6	12	-10	-12	-16	2	0	4	-6	-6	
610A	+	0	+	0	2	1	7	-1	6	1	-3	4	9	-1	11	2	0	-	13	0	-4	7	-6	-2	-12	
610B	+	0	-	0	-4	-2	-2	-4	0	-2	0	10	-6	-4	-4	2	-12	-	4	0	2	-8	0	-14	18	
610C	-	2	-	0	-6	6	6	-4	4	-2	-10	2	-2	-12	-2	6	14	-	8	-10	-14	6	6	-6	10	
611A	2	3	-2	2	-3	-	3	-4	-4	4	-5	8	7	-2	+	9	-14	6	3	-8	9	3	-18	-6	4	
612A	-	+	3	2	3	-1	+	-7	3	6	-4	2	9	-1	-12	12	-6	2	-4	0	8	14	-12	-6	-16	
612B	-	+	-3	2	-3	-1	-	-7	-3	-6	-4	2	-9	-1	12	-12	6	2	-4	0	8	14	12	6	-16	
612C	-	-	-1	0	-5	-5	+	1	3	-2	2	-8	5	-9	-6	6	-6	-4	12	12	-2	10	2	-12	16	
612D	-	-	1	4	-3	3	-	1	-3	10	6	-4	-5	-1	2	14	6	8	-12	-12	2	-14	-6	-16	0	
614A	-	0	-2	-3	-3	0	-1	-1	-2	6	-2	-3	5	2	6	-1	-8	-14	-8	3	14	-4	-4	-6	-2	-
614B	-	-2	0	-1	-3	-4	3	-1	-6	-6	-4	11	-3	-10	-12	9	6	14	2	9	-4	-16	0	6	14	-
615A	-1	+	+	0	2	0	0	2	-8	-10	4	2	+	-4	-12	0	0	2	-8	14	-6	10	-12	-18	-16	
615B	0	-	+	0	-1	-4	-3	-6	0	-5	-5	-3	-	-5	1	6	0	13	2	7	-7	-6	14	6	-6	
616A	+	0	0	+	+	-6	0	-2	4	-2	2	2	-8	0	-2	-10	-4	10	4	0	-8	8	-2	-6	2	
616B	+	2	2	-	+	0	4	4	-4	2	-2	-6	4	-4	2	2	-6	4	0	-12	16	-8	-12	10	-2	
616C	+	-2	2	-	+	4	0	-4	4	10	2	10	0	4	-2	2	6	0	-8	12	-12	16	-4	-6	-10	
616D	+	-1	-1	-	-	0	-2	-2	-7	-10	7	-9	-2	-4	8	2	-15	-14	3	3	10	10	0	-11	7	
616E	-	0	-2	-	+	2	-2	-4	-8	-2	-4	6	6	-4	-12	-10	8	10	4	-8	-2	-8	12	10	10	
618A	+	+	-1	-2	6	-1	0	-8	3	-6	-5	6	4	-9	-10	-6	-5	15	-15	0	-16	6	-9	2	1	+
618B	+	+	2	-2	-3	-4	0	1	-3	9	-5	-9	-2	-6	-4	6	-2	-12	-12	6	14	-6	-12	-7	7	+
618C	+	-	0	-4	-3	2	-6	-1	-9	9	-1	5	0	2	-6	-12	-12	8	14	-12	-16	2	12	3	-1	-
618D	+	-	-3	2	-6	-1	0	-4	3	-6	-7	-10	0	5	6	6	3	-1	-13	-12	-16	14	-9	6	17	-
618E	-	+	-2	-2	1	-4	-4	-3	-5	3	-3	-11	6	6	0	6	6	4	8	2	-10	10	0	1	-17	-
618F	-	-	-4	-4	-3	-6	2	3	1	-5	-3	11	-12	6	-10	-4	0	8	-2	0	-4	-14	0	-9	-17	+
618G	-	-	3	-2	-2	3	0	0	-3	-2	-3	2	0	-3	-10	10	9	-5	-5	8	-4	-2	-3	10	-7	-
620A	-	1	+	-4	0	2	-3	-7	0	-6	-	5	-3	-1	6	9	-3	-4	-10	3	-1	-10	-9	12	2	
620B	-	0	-	-2	-4	-4	0	0	-4	2	+	8	6	-8	-6	-8	4	2	-2	0	-4	0	-8	6	-2	
620C	-	-3	-	-2	2	2	-3	-3	-4	-4	+	-7	-3	-5	-6	1	-11	8	4	15	-13	-12	7	-18	10	
621A	1	-	3	4	1	-3	1	2	+	8	1	-10	8	-8	-6	9	6	-6	2	-16	-17	4	0	-1	-4	
621B	-1	-	-3	4	-1	-3	-1	2	-	-8	1	-10	-8	-8	6	-9	-6	-6	2	16	-17	4	0	1	-4	

TABLE 3: HECKE EIGENVALUES 622A–643A

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
622A	-	0	-4	1	-1	-4	-8	8	-8	-3	6	-10	2	1	9	-8	5	-1	-4	6	1	-1	6	15	4	-
623A	1	-1	1	-	-4	2	-3	-5	5	-6	-9	-8	-6	11	6	9	-4	-6	-6	-2	-11	12	12	-	3	-
624A	+	+	0	0	-6	+	2	0	-4	-6	4	-2	0	-4	-10	-10	6	-6	12	-2	6	16	-6	4	14	-
624B	+	+	-4	4	2	+	-6	-4	-4	-6	-8	-10	-4	4	6	6	6	-6	0	-10	-2	0	10	8	-10	-
624C	+	+	2	0	0	-	2	4	0	6	0	-2	6	12	4	6	8	-2	-4	12	-14	0	-8	-18	-6	-
624D	+	-	0	4	2	+	-6	4	-4	10	8	-2	0	4	-2	-2	-10	10	-8	-2	-10	-8	-6	-12	-2	-
624E	+	-	4	0	2	+	2	-8	-4	-6	4	6	-12	-4	6	-2	14	10	4	-2	-2	8	-14	0	-10	-
624F	+	-	-2	-4	0	-	2	-8	-8	-2	-4	-10	2	4	12	6	0	-2	-8	12	10	8	0	-14	2	-
624G	-	+	0	-2	0	-	-6	-2	0	-6	-2	2	-12	4	0	6	-12	2	10	-12	14	-8	-12	0	-10	-
624H	-	-	2	4	-4	-	2	0	0	-10	-4	-2	6	12	0	6	-12	-2	8	0	2	-8	-4	-2	10	-
624I	-	-	2	-4	4	-	2	8	0	6	4	-2	-10	-4	-8	-10	-4	-2	16	8	2	-8	-12	14	10	-
624J	-	-	-4	2	4	-	2	2	0	-6	10	10	8	-4	4	-10	8	-14	-2	-16	-10	16	0	-4	-2	-
626A	+	0	0	0	0	-2	-2	-4	0	6	-4	0	-2	-10	12	-4	-10	-8	2	-16	10	0	-12	10	2	+
626B	+	1	2	5	-1	4	2	0	-1	-10	-8	7	-2	8	1	5	4	-13	-8	-8	14	8	15	-6	-11	-
627A	0	-	4	2	+	1	-3	+	2	-4	-4	2	6	-4	6	1	11	10	-6	-3	0	-11	-13	-9	-6	-
627B	0	-	0	2	-	-1	3	-	6	0	8	2	6	8	-6	9	3	-10	-10	-3	-4	-13	-3	15	-10	-
628A	-	2	4	-1	0	1	-1	0	-3	0	-6	-3	-2	5	-6	6	-3	14	6	0	2	4	0	-7	-12	+
629A	1	0	1	-1	-5	-2	+	3	2	3	-4	+	6	-1	-6	1	-7	1	14	-15	-10	4	-6	16	1	-
629B	2	3	-2	1	-3	4	-	2	-2	-6	-2	+	-3	4	3	-3	0	2	4	-13	9	0	5	16	8	-
629C	0	-3	0	3	-1	0	-	-2	-2	2	-8	-	3	-12	3	-11	-4	4	4	-3	-1	-2	1	6	0	-
629D	-1	0	3	-1	-5	-2	-	1	-6	1	4	-	-6	-11	-10	1	3	-5	-6	1	14	-8	6	0	-13	-
630A	+	+	+	-	0	2	0	2	0	6	8	-4	6	2	-6	6	12	8	2	6	2	-16	0	6	-10	-
630B	+	+	-	+	-4	6	4	6	0	-6	-4	8	10	-2	10	14	-4	-8	6	-2	-10	16	-8	2	2	-
630C	+	-	+	+	4	-2	-2	4	8	2	0	6	6	-4	0	10	-12	14	-12	8	10	16	12	-10	2	-
630D	+	-	+	-	-4	-2	-2	-4	8	-6	-8	-2	-2	-12	8	-6	-4	-2	12	-8	-14	0	-12	-2	10	-
630E	+	-	-	+	-4	-6	-2	0	0	-6	8	-10	-2	4	-8	2	8	-14	-12	16	2	-8	-8	-10	2	-
630F	+	-	-	-	0	2	6	-4	0	6	-4	2	-6	8	12	-6	12	2	8	0	14	-16	-12	-6	14	-
630G	-	+	+	+	4	6	-4	6	0	6	-4	8	-10	-2	-10	-14	4	-8	6	2	-10	16	8	-2	2	-
630H	-	+	-	-	0	2	0	2	0	-6	8	-4	-6	2	6	-6	-12	8	2	-6	2	-16	0	-6	-10	-
630I	-	-	+	-	0	2	6	8	0	-6	-4	-10	6	-4	0	6	12	-10	-4	-12	-10	8	-12	6	-10	-
630J	-	-	-	+	4	-2	6	0	8	-10	-8	2	2	8	-4	-10	-4	-6	0	12	-6	-8	4	-14	2	-
632A	-	1	-1	-5	4	1	-8	2	-6	0	-4	-2	0	8	3	-10	15	-4	0	3	-14	-	6	9	-7	-
633A	-1	+	-3	2	5	-3	-4	7	-6	-10	6	-11	-6	5	3	6	-4	12	-8	-13	-2	-7	0	0	-8	+
635A	0	1	-	-1	-3	-4	0	-4	-3	0	8	-4	-6	-1	6	-6	-12	5	-4	9	14	5	3	6	-10	-
635B	-2	-1	-	1	-3	-2	4	0	7	-8	-4	-6	6	-11	-4	-6	-6	1	4	5	-16	5	-11	-12	2	-
637A	1	0	0	+	-3	+	7	-7	-6	-5	0	8	0	2	7	-3	-7	-7	-3	-5	14	-6	0	0	-14	-
637B	0	2	3	-	0	+	6	7	3	-9	-5	2	6	-1	-3	-9	0	10	14	-6	-11	-1	-3	-15	1	-
637C	1	0	0	-	-3	-	-7	7	-6	-5	0	8	0	2	-7	-3	7	7	-3	-5	-14	-6	0	0	14	-
637D	-2	0	3	-	-6	-	-4	-5	3	-5	3	-4	6	-1	-7	-9	-8	10	-6	-8	13	3	-15	-3	-7	-
639A	-1	-	-2	2	0	-2	0	0	0	2	-10	-6	0	-4	-12	4	-12	10	2	-	-10	4	4	-6	-2	-
640A	+	0	+	2	-6	-2	-6	2	6	-6	4	-6	-2	-4	-10	-2	-10	10	4	16	-6	0	8	6	2	-
640B	+	0	+	-2	6	-2	-6	-2	-6	-6	-4	-6	-2	4	10	-2	10	10	-4	-16	-6	0	-8	6	2	-
640C	+	0	-	2	6	2	-6	-2	6	6	4	6	-2	4	-10	2	10	-10	-4	16	-6	0	-8	6	2	-
640D	+	2	-	0	2	-2	6	6	0	-10	8	-2	-6	-2	12	-10	-6	6	-14	4	-10	-8	10	14	6	-
640E	-	2	+	0	2	2	6	6	0	10	-8	2	-6	-2	-12	10	-6	-6	-14	-4	-10	8	10	14	6	-
640F	-	-2	+	0	-2	2	6	-6	0	10	8	2	-6	2	12	10	6	-6	14	4	-10	-8	-10	14	6	-
640G	-	0	-	-2	-6	2	-6	2	-6	6	-4	6	-2	-4	10	2	-10	-10	4	-16	-6	0	8	6	2	-
640H	-	-2	-	0	-2	-2	6	-6	0	-10	-8	-2	-6	2	-12	-10	6	6	14	-4	-10	8	-10	14	6	-
642A	+	+	2	2	4	-2	2	-4	6	-4	-2	2	6	12	6	4	-4	-2	-4	12	10	4	12	-6	-18	-
642B	+	-	-3	2	0	2	0	2	3	6	2	8	9	-1	-3	6	-3	-10	5	6	2	5	-6	9	2	+
642C	-	+	-1	-2	-4	-6	0	2	-1	-6	10	-4	-7	1	1	-6	-5	10	-5	6	14	9	-2	1	-10	-
643A	-1	-2	-2	-3	-6	-4	-4	-4	-1	3	-3	6	2	0	-6	11	-4	-12	0	-6	-8	-16	9	-3	7	-

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
644A	-	1	-2	+	-2	-1	0	-6	-	1	1	-6	3	0	-3	6	8	-10	-2	5	-7	14	-4	-12	-8	
644B	-	-1	0	-	-2	-3	0	0	+	1	-5	-8	-7	-4	3	-12	4	-6	-12	13	3	4	16	4	10	
645A	1	+	+	0	4	6	-2	0	4	2	8	2	10	-	-4	-14	-4	-2	-4	0	14	8	8	-10	-6	
645B	1	+	-	4	-2	2	0	6	0	10	8	-4	-10	+	0	12	-6	-10	12	4	-8	-16	-12	-10	6	
645C	2	+	-	0	5	1	5	4	3	-8	-9	8	-5	+	8	-13	8	-8	3	-8	4	0	-9	12	5	
645D	-2	+	-	4	1	5	-3	0	3	-8	-1	8	-1	+	0	3	0	8	-9	4	4	8	15	-16	-15	
645E	0	-	-	-2	-5	-5	5	-6	-9	8	-5	8	-7	+	-8	-5	-4	0	9	10	4	16	-9	-6	3	
645F	-2	-	-	-4	-3	5	-7	0	-9	0	7	-8	3	+	-8	-1	-8	0	-9	-12	-4	-8	3	-8	9	
646A	+	0	4	-2	4	6	+	-6	0	6	-4	6	-4	0	14	-4	0	4	10	10	-10	-12	14	-10		
646B	-	2	2	0	-4	2	+	+	8	-8	-2	-8	4	4	8	2	-12	2	12	14	6	-10	-12	6	4	
646C	-	-2	4	4	2	-6	+	+	0	-8	8	-4	10	0	-8	-6	0	-8	8	-12	-6	-4	8	18	2	
646D	-	0	-2	-2	-2	-6	+	-	6	0	0	8	0	-4	-12	2	-4	6	4	-8	-2	-4	0	-10	8	
646E	-	-2	0	2	0	2	-	-	6	6	8	2	0	-4	12	-6	0	-4	-4	12	-10	-16	-12	6	8	
648A	+	+	-1	0	-4	-5	-5	8	-4	3	-4	3	-6	4	-12	-10	8	-5	8	16	-5	4	4	3	2	
648B	+	+	-1	-3	5	-5	-2	-4	-1	-9	-1	-6	3	1	-3	2	11	7	-1	4	-2	1	1	-18	-13	
648C	-	+	1	0	4	-5	5	8	4	-3	-4	3	6	4	12	10	-8	-5	8	-16	-5	4	-4	-3	2	
648D	-	-	1	-3	-5	-5	2	-4	1	9	-1	-6	-3	1	3	-2	-11	7	-1	-4	-2	1	-1	18	-13	
649A	-1	1	-1	1	-	-4	-2	-1	6	-9	2	-4	-3	6	-12	-13	-	8	10	0	-4	-1	-6	0	-2	
650A	+	0	+	0	0	+	-2	-8	4	-2	-4	-6	10	0	-8	-6	8	-2	-4	-12	-10	-8	-12	10	14	
650B	+	2	+	-5	-3	+	-3	-4	-6	9	5	-2	0	-2	9	9	-9	-1	-5	0	-14	-16	15	-6	-8	
650C	+	-3	+	0	-3	+	7	1	4	4	-10	-12	-5	-12	4	-6	-4	4	5	0	11	4	-15	-11	2	
650D	+	1	+	4	1	-	7	-3	0	-4	6	8	-5	4	-12	10	4	8	9	-8	-13	8	-3	-11	10	
650E	+	-2	+	4	-2	-	-2	6	-6	2	-6	2	10	10	12	-2	10	2	12	10	-10	-4	0	-14	-14	
650F	+	3	+	-1	-2	-	3	6	4	2	4	-3	0	5	-13	-12	-10	-8	2	-5	10	-4	0	6	-14	
650G	+	-2	-	-1	3	-	3	-4	-6	-3	-1	2	0	-10	-3	3	-15	-13	-13	0	-10	-4	15	6	-4	
650H	-	-1	+	1	6	+	3	2	0	6	-4	7	0	1	-3	0	-6	8	-14	-3	-2	8	-12	-6	10	
650I	-	2	+	1	3	+	-3	-4	6	-3	-1	-2	0	10	3	-3	-15	-13	13	0	10	-4	-15	6	4	
650J	-	2	+	4	-6	+	6	2	-6	-6	2	-2	-6	-2	12	-6	6	2	4	-6	10	-4	0	-6	-2	
650K	-	-1	-	-4	1	+	-7	-3	0	-4	6	-8	-5	-4	12	-10	4	8	-9	-8	13	8	3	-11	-10	
650L	-	-2	-	5	-3	-	3	-4	6	9	5	2	0	2	-9	-9	-9	-1	5	0	14	-16	-15	-6	8	
650M	-	3	-	0	-3	-	-7	1	-4	4	-10	12	-5	12	-4	6	-4	4	-5	0	-11	4	15	-11	-2	
651A	1	+	-2	-	2	4	8	-4	6	6	+	6	-10	2	-8	10	12	-12	8	4	-4	-6	8	8	14	
651B	1	+	4	-	2	-2	2	8	-6	0	+	6	8	-4	-2	4	-6	-6	-4	4	-10	12	-16	2	-10	
651C	1	-	-2	+	-2	-4	0	-4	2	-2	-	-2	6	-2	-8	2	-4	-4	0	12	4	-2	8	0	-2	
651D	-1	-	-2	-	0	-6	6	-4	-4	-2	+	-10	-6	-8	8	-2	-4	-6	-4	-8	14	4	-4	14	-6	
651E	0	-	-3	-	0	5	0	2	3	3	-	2	9	8	0	-3	-12	-1	5	-6	-7	8	6	-12	-10	
654A	+	-	-1	-2	-3	0	-4	-1	-1	-9	2	6	-2	0	1	-4	-4	-1	4	6	3	-8	-10	-15	1	-
654B	-	+	-1	-2	-5	-4	4	-3	-3	-1	-2	2	6	-4	3	-4	-4	-5	-4	6	11	0	6	5	1	-
655A	-2	-3	+	-3	-4	-5	-2	-6	-6	-6	-2	-8	5	1	-2	9	9	-7	-14	-8	16	-14	14	9	-10	-
656A	+	0	-2	2	0	-4	-2	-4	4	0	-4	-6	+	-12	6	-4	4	10	-12	6	-2	2	4	-6	14	
656B	+	-2	2	2	-2	6	-6	2	0	6	8	10	-	0	6	-2	4	-2	10	2	-2	2	-12	10	-6	
656C	-	2	-2	4	2	4	-2	-6	8	0	8	2	+	12	-4	-4	-8	-14	2	-8	10	-4	-12	-14	6	
657A	-1	-	4	2	4	-2	0	-4	0	-8	6	-2	10	-6	8	12	-4	-14	8	8	+	8	-16	14	-2	
657B	2	-	1	2	4	-2	3	-1	0	10	-6	1	-2	6	-7	-3	-1	-5	-13	-10	+	-1	11	2	-11	
657C	0	-	3	-4	0	-4	-3	-1	-6	6	-10	-7	0	2	3	-9	9	-1	-13	-12	-	11	-15	18	5	
657D	-1	-	-2	2	2	-6	-2	8	-4	-2	-2	-6	-6	-2	-6	-10	6	-14	8	0	-	-4	14	6	-10	
658A	+	-1	-1	+	1	2	0	-2	8	8	-4	-1	9	5	-	1	15	-12	16	-2	13	14	-1	0	-8	
658B	+	2	2	+	-2	2	6	4	8	-4	8	-10	6	2	-	-14	-6	0	-14	16	-2	-16	14	-6	-2	
658C	+	1	3	-	3	2	0	2	0	0	-4	-1	-3	-1	+	9	9	-4	8	-6	-7	2	9	0	-16	
658D	-	-1	-1	+	-5	2	-6	4	-4	-10	2	5	3	-1	-	-5	3	-12	4	4	-5	-4	-1	12	-2	
658E	-	0	-4	-	-2	0	-2	-6	-4	0	-4	-6	-2	10	+	10	-12	2	-2	0	2	16	0	6	2	
658F	-	-3	-1	-	1	-6	-2	0	-4	-6	2	-3	-5	-11	+	-5	9	-4	4	0	11	16	-3	-12	2	

TABLE 3: HECKE EIGENVALUES 659A–676B

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
659A	1	2	-3	-3	0	-1	-2	2	0	0	-3	-7	6	-1	-1	2	-6	1	11	3	7	-6	0	1	-12	+
659B	2	1	2	0	1	5	6	-7	0	6	-6	-1	6	2	-12	-4	-12	-5	16	4	-1	0	6	1	12	-
660A	-	+	+	-2	+	2	8	-2	8	0	0	2	0	6	8	6	-4	10	-12	8	10	-14	4	10	-18	
660B	-	+	+	0	-	-4	-2	2	-8	-4	-4	-2	-12	8	-4	-10	4	6	12	0	-16	6	-2	18	-2	
660C	-	-	+	-4	+	-4	-6	2	0	0	-4	-10	0	-4	12	6	12	-10	-4	0	8	-10	-6	-6	-10	
660D	-	-	+	2	-	2	0	2	0	0	8	2	0	2	0	6	-12	2	-4	0	2	-10	-12	-6	14	
662A	+	-2	1	-2	4	0	-7	5	0	2	-1	-8	-8	-5	0	1	-6	-12	-9	-7	-4	-1	4	6	-4	+
663A	1	+	-4	2	6	+	-	4	0	-6	10	-4	0	12	8	2	-8	-10	12	2	4	-4	-12	-6	8	
663B	-1	+	-2	0	4	-	-	-4	0	-2	-8	-2	2	-4	8	-10	4	14	-4	0	-14	-8	-4	-6	-6	
663C	-1	-	0	-2	-2	+	-	0	-8	-6	6	4	-12	-4	0	-6	0	-2	0	10	-4	12	-4	-10	16	
664A	-	-3	-4	-5	-3	-4	-3	-4	0	5	-5	-3	-2	-4	-2	-6	7	-15	-6	-6	-10	-8	+	-12	16	
664B	-	1	-2	-1	-3	2	-3	-2	-4	-3	7	-7	-2	8	0	6	-5	1	10	6	-16	10	-	-8	0	
664C	-	-1	0	1	-1	-4	-3	4	0	-3	-7	5	-10	-4	-6	-10	5	1	2	-2	14	0	-	0	-12	
665A	-1	-1	+	-	4	0	-1	-	-3	-6	0	-11	5	-7	-8	-3	2	-8	2	1	-11	-12	0	10	-2	
665B	1	0	-	+	-4	-2	2	-	-8	6	-8	-6	6	4	8	-6	-12	-2	4	-12	-6	12	-4	-10	2	
665C	1	-1	-	-	0	-4	-3	+	1	-10	-8	3	1	-11	-4	-5	14	8	-6	-5	15	-12	4	-14	14	
665D	-2	-1	-	-	-3	-1	3	+	4	5	-8	-12	-8	4	-7	4	-10	2	-12	-8	-6	-15	4	10	-7	
665E	-2	3	-	-	-3	3	3	-	-4	1	8	-4	-8	-4	1	-12	6	-6	4	0	10	13	4	-6	5	
666A	+	+	2	-3	5	-3	3	5	3	0	4	-	6	4	-4	-3	14	-14	12	12	13	6	-7	-1	-12	
666B	+	-	0	3	-1	1	3	3	1	4	-6	+	10	12	6	1	0	2	2	0	-3	14	-9	3	-10	
666C	+	-	0	-1	-3	-1	3	-7	-3	0	2	-	6	-4	-6	-9	0	-10	2	-12	5	2	-3	3	2	
666D	-	+	-2	-3	-5	-3	-3	5	-3	0	4	-	-6	4	4	3	-14	-14	12	-12	13	6	7	1	-12	
666E	-	-	-4	-1	1	-3	-3	-5	-5	-4	-10	+	6	4	-2	11	12	10	14	0	-11	-10	9	-11	10	
666F	-	-	-2	0	4	6	-6	8	0	6	4	-	6	-8	-8	-6	4	-2	-12	0	10	-12	4	10	-6	
666G	-	-	4	3	-5	3	-3	-7	-9	0	-2	-	-6	4	10	-3	4	-2	6	12	13	-6	-5	-11	6	
669A	1	+	3	-4	0	-4	-2	-8	6	8	-10	-5	-8	-8	1	2	0	-8	9	0	17	1	9	-4	16	+
670A	+	0	-	1	-5	-2	-6	2	-4	0	0	3	10	-6	-6	-12	14	-15	-	5	-4	14	-7	-15	13	
670B	+	-2	-	-1	3	-4	0	2	0	-6	2	-7	-12	-4	0	-6	-12	11	-	-9	-10	2	-3	-3	5	
670C	-	0	+	-5	-3	6	-6	-2	-4	0	-4	7	-2	2	2	4	6	-13	-	-15	-8	-14	9	-15	3	
670D	-	-2	+	1	-3	-4	4	-2	-8	-10	-10	1	8	4	-4	-6	0	-3	-	-9	14	6	5	13	19	
672A	+	+	0	+	-2	-2	4	-4	-6	-2	0	-6	8	-8	-4	-6	0	-14	4	-2	-2	4	12	0	6	
672B	+	-	-4	-	-2	-2	0	-4	-6	-10	-8	10	-4	-8	-4	10	8	-6	4	14	6	4	-12	4	-2	
672C	-	+	2	+	0	2	2	4	0	6	0	6	-6	8	8	6	-12	10	16	-8	-6	8	-12	-14	-6	
672D	-	+	-4	+	2	-2	0	4	6	-10	8	10	-4	8	4	10	-8	-6	-4	-14	6	-4	12	4	-2	
672E	-	+	-2	-	4	-6	-2	-4	4	-2	-8	-10	-2	-8	0	-10	12	10	8	-12	2	0	-12	6	2	
672F	-	-	-2	+	-4	-6	-2	4	-4	-2	8	-10	-2	8	0	-10	-12	10	-8	12	2	0	12	6	2	
672G	-	-	0	-	2	-2	4	4	6	-2	0	-6	8	8	4	-6	0	-14	-4	2	-2	-4	-12	0	6	
672H	-	-	2	-	0	2	2	-4	0	6	0	6	-6	-8	-8	6	12	10	-16	8	-6	-8	12	-14	-6	
674A	+	1	-2	0	4	-6	-2	-2	5	-9	-3	-4	-5	5	2	9	12	-11	8	-16	-4	-8	-6	0	-18	+
674B	-	0	-2	-4	2	2	-6	-2	-6	6	-6	-6	-10	8	12	14	-6	6	2	-2	-6	-8	14	-14	2	-
674C	-	-3	-2	2	2	-4	0	-8	-3	-9	-3	0	-1	5	12	-7	-12	-3	2	4	0	-2	14	-8	-16	-
675A	0	+	+	1	0	-5	0	-7	0	0	-4	-11	0	-8	0	0	0	-1	-5	0	7	17	0	0	19	
675B	-1	+	+	0	-5	5	-4	-2	3	-10	6	-5	-10	-10	5	2	-5	-11	0	5	-10	12	-12	0	-5	
675C	0	+	-	-4	0	5	0	8	0	0	11	-1	0	-13	0	0	0	14	5	0	17	-13	0	0	14	
675D	-1	+	-	0	5	-5	-4	-2	3	10	6	5	10	10	5	2	5	-11	0	-5	10	12	-12	0	5	
675E	0	-	+	4	0	-5	0	8	0	0	11	1	0	13	0	0	0	14	-5	0	-17	-13	0	0	-14	
675F	1	-	+	0	5	5	4	-2	-3	10	6	-5	10	-10	-5	-2	5	-11	0	-5	-10	12	12	0	-5	
675G	2	-	+	3	-2	5	8	1	-6	2	0	-5	-10	-4	-4	2	-8	7	9	2	5	-3	-6	-12	13	
675H	-2	-	+	3	2	5	-8	1	6	-2	0	-5	10	-4	4	-2	8	7	9	-2	5	-3	6	12	13	
675I	1	-	-	0	-5	-5	4	-2	-3	-10	6	5	-10	10	-5	-2	-5	-11	0	5	10	12	12	0	5	
676A	-	0	-2	2	2	+	6	6	8	2	-10	6	6	4	2	6	10	-2	-10	-10	-2	-4	6	6	-2	
676B	-	-2	3	4	0	+	3	-2	-6	9	-2	7	-3	-4	6	9	0	5	-2	6	1	-4	-12	-6	-14	

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
676C	--	-2	-3	-4	0	+	3	2	-6	9	2	-7	3	-4	-6	9	0	5	2	-6	-1	-4	12	6	14	
676D	-	3	2	1	-5	+	3	-3	-1	-1	-8	3	3	1	4	-6	5	-5	7	-11	14	-4	12	-9	-1	
676E	-	3	-2	-1	5	+	3	3	-1	-1	8	-3	-3	1	-4	-6	-5	-5	-7	11	-14	-4	-12	9	1	
677A	-1	-1	0	1	-3	6	6	-7	-4	8	-8	3	-11	0	-3	-8	0	-14	-7	-8	4	-2	7	12	-7	+
678A	+	+	1	1	-6	-5	-1	6	1	-2	5	-8	-12	2	-8	-2	-5	-5	2	1	8	16	-16	18	-14	+
678B	+	-	-1	-3	2	-5	1	-2	-5	2	-7	-8	0	6	0	-2	9	3	-10	-5	-8	8	-8	-2	10	-
678C	-	+	0	-4	-4	-2	-2	0	-2	0	0	4	6	-8	-6	-6	-4	10	8	-6	2	2	-4	-6	-2	-
678D	-	-	-1	1	-2	7	-3	6	3	2	-3	-4	0	2	-8	-2	-3	-1	-2	-5	4	-4	0	6	-14	-
678E	-	-	2	0	4	-2	-6	-4	-4	10	0	10	-6	4	-4	-2	-4	-10	12	-12	-6	12	12	-6	18	-
678F	-	-	-4	4	4	-2	6	0	-6	-4	0	8	6	8	-2	-14	-12	2	-8	-2	-14	-10	-12	-6	-2	-
680A	-	0	+	0	0	-2	-	-4	-8	2	-8	2	2	-4	0	6	-4	-6	4	-8	2	0	4	-6	18	
680B	-	-1	-	2	4	-1	-	-1	6	-5	3	4	6	10	5	-3	3	5	-6	-9	13	8	-8	-1	-1	
680C	-	2	-	2	-2	2	-	8	-6	-2	6	-2	-6	-8	8	6	0	-10	12	6	-14	2	-8	-10	2	
681A	0	+	0	1	1	0	0	-5	-5	-7	-6	2	10	-1	-12	-3	9	-10	2	11	3	-16	-6	1	-10	+
681B	1	+	2	0	4	-2	6	4	0	6	4	-2	-2	12	0	-2	12	-10	-8	-8	-6	16	-12	2	2	-
681C	-2	+	-4	-3	-5	-2	-6	-5	-3	-9	10	-2	-2	-9	0	-5	15	2	4	13	-9	-8	-6	-1	2	-
681D	0	-	4	1	1	4	-8	-5	3	1	6	-10	6	7	4	-3	9	-10	-2	-5	3	8	-10	9	-2	+
681E	0	-	-2	1	3	-6	-4	-1	-7	-9	0	4	-8	-1	8	-9	3	-2	16	-3	11	16	16	-1	-2	-
682A	-	-2	0	-1	+	-4	-3	2	-3	-6	-	-7	0	-1	-6	-6	3	8	5	12	2	-10	15	0	17	
682B	-	0	-2	-3	-	-4	3	-2	-7	-4	+	7	6	-1	4	-6	9	-6	3	-10	-2	10	-13	6	-7	
684A	-	-	1	-3	-5	-4	3	+	-8	2	4	10	-10	1	1	4	-6	-13	-12	-2	9	8	12	-12	-8	
684B	-	-	-2	0	-2	2	-6	+	-2	-4	-8	-2	8	-8	-2	4	0	2	12	4	6	-16	-6	0	-2	
684C	-	-	3	1	5	-6	5	-	-4	-6	6	-8	8	9	-1	-2	8	11	0	4	-11	-8	4	-10	-10	
685A	1	0	+	3	-6	1	2	-6	-2	-8	-1	8	-6	2	-6	1	2	-5	12	-8	-10	7	0	14	7	+
688A	+	0	-2	2	-1	-1	-7	6	-9	4	-1	-4	-11	+	0	11	-12	0	-7	10	-4	8	3	6	3	
688B	-	2	0	4	3	-1	-3	-2	3	6	-5	8	-3	+	12	-9	12	-10	-11	-6	-10	-8	15	0	-1	
688C	-	2	-4	0	-3	-5	-3	2	1	-6	1	0	5	-	-4	-5	12	2	3	-2	2	8	-15	-4	7	
689A	-1	-2	-2	2	2	+	-2	4	6	6	-8	-6	-10	-12	6	+	-6	10	-8	-12	14	-10	12	10	-14	
690A	+	+	+	-2	6	-2	0	-4	+	-2	-8	-4	2	-8	0	-2	-4	0	0	-8	6	-14	-6	-16	2	
690B	+	+	+	4	-2	0	2	0	-	-4	0	10	6	2	12	6	12	-14	2	-2	6	8	8	-8	0	
690C	+	+	-	-2	2	-2	0	8	+	-10	8	8	-6	12	8	10	4	12	-4	16	-10	10	-10	0	10	
690D	+	+	-	4	2	4	-6	-4	+	8	8	-10	6	6	-4	-14	4	6	14	10	14	-8	-4	0	-8	
690E	+	-	+	0	-4	-6	-6	4	-	-6	-8	6	10	4	-8	-14	0	10	4	8	2	-12	-16	-2	-14	
690F	+	-	-	0	4	-2	2	0	-	6	0	2	2	-12	8	-2	0	6	12	12	-6	4	-4	-6	-10	
690G	-	+	+	0	0	6	2	0	+	6	8	10	-6	-8	8	-6	-4	-6	8	-8	10	-8	-8	-6	18	
690H	-	+	+	-2	-2	-6	-4	0	-	2	0	-8	-6	-4	0	6	0	-8	-4	16	6	14	14	-8	-6	
690I	-	-	+	0	2	0	6	4	-	0	-8	-6	-2	-2	4	-2	0	-2	-2	-10	-10	0	-4	4	16	
690J	-	-	-	0	-2	4	6	-8	+	4	0	-2	-2	2	-12	-6	8	2	-6	10	-2	-8	8	-12	-16	
690K	-	-	-	0	4	-2	-6	4	+	-2	0	-2	10	-4	0	6	-4	-10	-12	-8	10	-8	-4	18	2	
692A	-	-2	2	-2	-2	6	2	2	-4	2	8	-2	10	8	12	2	6	-6	4	-2	10	-2	0	-6	10	+
693A	-1	-	2	+	+	4	-4	0	4	6	10	-6	-4	12	10	6	-2	0	8	12	-8	8	0	6	-10	
693B	0	-	1	+	-	-4	-2	-6	5	-10	1	-5	2	-8	-8	6	-3	-2	-3	-1	10	6	-12	15	-5	
693C	0	-	-3	-	-	-4	6	2	-3	6	5	11	-6	8	0	6	9	-10	5	-9	2	-10	-12	3	-1	
693D	1	-	2	-	-	6	-2	4	0	2	8	6	-10	-4	8	-6	-4	-10	-12	0	2	16	-4	-18	2	
696A	+	+	0	-1	-3	1	-1	0	-6	+	-6	2	-2	4	-7	0	-14	6	-3	-10	2	12	2	-7	4	
696B	+	-	-3	1	-2	4	7	7	0	+	4	-5	3	9	5	-6	-9	-10	4	12	-6	-10	-8	10	16	
696C	+	-	0	-5	-5	1	-3	-4	2	-	2	2	-6	-8	7	12	6	-6	-15	2	-10	-4	-6	3	8	
696D	-	+	1	-3	-2	4	5	5	4	+	8	-3	9	-5	3	10	7	10	8	-4	-14	-14	8	6	-16	
696E	-	+	4	3	1	1	-1	-4	-2	+	-10	6	6	4	-3	4	10	-14	-7	2	-2	16	2	9	-16	
696F	-	+	-2	3	-5	1	-7	2	4	-	-4	-12	-6	-8	3	-14	-2	-8	5	8	4	-2	14	15	14	
696G	-	-	-2	-1	-3	-7	3	-6	4	+	0	-8	6	4	-3	-10	10	0	9	-12	-4	10	6	-3	-10	
699A	2	-	1	1	0	3	3	-4	-6	8	4	-6	7	4	-11	-6	-5	6	-14	2	4	-4	1	14	-8	+

TABLE 3: HECKE EIGENVALUES 700A–707A

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
700A	--1	+	+	3	1	3	2	6	-9	8	10	0	-2	3	0	12	8	-8	0	-14	5	12	12	-17		
700B	-	2	+	+	3	4	0	2	3	9	8	-5	-6	-11	-6	-6	0	-10	-5	15	10	-7	-12	-12	-8	
700C	-	0	+	-	-5	-6	4	-6	3	-3	2	7	-4	-7	-2	-10	-14	4	3	-13	16	1	10	10	-2	
700D	--3	+	-	-5	3	1	6	-6	-9	-4	-2	-4	-10	1	-4	-8	-8	-12	8	-2	13	4	4	13		
700E	-	0	-	+	0	-4	-4	4	-8	2	-8	8	6	-8	-8	0	-4	-6	-8	12	4	-4	0	-10	12	
700F	-	0	-	+	-5	6	-4	-6	-3	-3	2	-7	-4	7	2	10	-14	4	-3	-13	-16	1	-10	10	2	
700G	--3	-	+	3	-1	5	-8	-2	-1	-2	-10	-6	4	-11	-6	-10	0	10	0	10	-7	-12	8	-3		
700H	-	0	-	-	0	4	4	4	8	2	-8	-8	6	8	8	0	-4	-6	8	12	-4	-4	0	-10	-12	
700 I	--2	-	-	3	-4	0	2	-3	9	8	5	-6	11	6	6	0	-10	5	15	-10	-7	12	-12	8		
700 J	-	3	-	-	3	1	-5	-8	2	-1	-2	10	-6	-4	11	6	-10	0	-10	0	-10	-7	12	8	3	
701A	2	2	2	-1	0	-3	7	-1	8	-3	-5	-2	-5	-11	10	12	-4	-8	16	-10	8	0	12	6	3	-
702A	+	+	1	-5	5	+	-6	-6	4	-2	1	-6	-12	4	2	-3	4	-14	-8	2	-13	8	-15	0	-7	
702B	+	+	-2	1	-1	+	0	0	-5	1	-5	-3	-6	1	-4	3	-11	4	-8	8	-16	8	-9	9	-4	
702C	+	+	2	4	2	-	-4	7	0	-9	-10	7	5	-2	-1	14	-8	10	-4	1	-4	-7	0	17	-8	
702D	+	+	2	-5	-1	-	8	4	9	3	-7	7	2	1	8	-7	-11	4	8	-8	8	-4	15	-7	-8	
702E	+	+	-3	-1	-3	-	6	2	0	6	5	2	0	8	-6	9	12	-10	-4	6	11	8	-15	12	17	
702F	+	-	-2	4	2	+	0	-3	4	1	10	-3	9	10	11	-6	-8	-2	4	5	8	-7	12	-3	-16	
702G	+	-	4	1	5	+	6	-6	-5	-5	-5	-9	6	7	-4	9	-5	-2	4	2	14	2	9	-3	14	
702H	+	-	0	-1	-3	-	-6	2	-3	-3	5	-7	6	-1	0	-9	3	-10	-4	-6	2	-10	9	-3	-10	
702 I	-	+	2	1	1	+	0	0	5	-1	-5	-3	6	1	4	-3	11	4	-8	-8	-16	8	9	-9	-4	
702 J	-	+	2	4	-2	+	0	-3	-4	-1	10	-3	-9	10	-11	6	8	-2	4	-5	8	-7	-12	3	-16	
702K	-	+	-2	-5	1	-	-8	4	-9	-3	-7	7	-2	1	-8	7	11	4	8	8	8	-4	-15	7	-8	
702L	-	-	-1	-5	-5	+	6	-6	-4	2	1	-6	12	4	-2	3	-4	-14	-8	-2	-13	8	15	0	-7	
702M	-	-	-4	1	-5	+	-6	-6	5	5	-5	-9	-6	7	4	-9	5	-2	4	-2	14	2	-9	3	14	
702N	-	-	0	-1	3	-	6	2	3	3	5	-7	-6	-1	0	9	-3	-10	-4	6	2	-10	-9	3	-10	
702O	-	-	-2	4	-2	-	4	7	0	9	-10	7	-5	-2	1	-14	8	10	-4	-1	-4	-7	0	-17	-8	
702P	-	-	3	-1	3	-	-6	2	0	-6	5	2	0	8	6	-9	-12	-10	-4	-6	11	8	15	-12	17	
703A	0	3	-2	-1	3	6	2	-	0	6	-4	+	9	4	-13	7	-6	4	-4	-9	3	-12	-3	2	-2	
703B	-2	0	-3	3	-1	2	3	-	0	-4	0	-	-6	-3	-11	-2	-6	5	6	-10	-11	-14	-12	4	-14	
704A	+	1	-1	-2	+	-4	-2	0	-1	0	7	-3	-8	6	8	6	-5	-12	7	-3	4	-10	6	15	-7	
704B	+	-1	3	-4	+	2	-8	-6	5	-4	1	-3	-6	6	-12	6	-3	0	-11	-5	-10	-2	2	-5	13	
704C	+	1	3	4	-	2	-8	6	-5	-4	-1	-3	-6	-6	12	6	3	0	11	5	-10	2	-2	-5	13	
704D	+	-1	3	2	-	4	6	-8	-3	0	5	1	0	10	0	6	-3	4	1	15	-4	2	-6	-9	-7	
704E	+	3	3	-2	-	0	-6	-4	1	8	-7	1	4	-6	-8	-2	1	-4	5	3	16	2	2	15	-7	
704F	-	1	3	-2	+	4	6	8	3	0	-5	1	0	-10	0	6	3	4	-1	-15	-4	-2	6	-9	-7	
704G	--1	-1	4	+	2	0	2	9	-4	5	9	2	6	-4	6	5	0	13	-1	14	-10	-14	-13	-19		
704H	-	3	-1	0	+	6	-4	6	-3	4	9	-7	-2	6	-12	-2	9	-8	-15	3	-6	6	-6	-5	-3	
704 I	--3	3	2	+	0	-6	4	-1	8	7	1	4	6	8	-2	-1	-4	-5	-3	16	-2	-2	15	-7		
704 J	-	1	-1	-4	-	2	0	-2	-9	-4	-5	9	2	-6	4	6	-5	0	-13	1	14	10	14	-13	-19	
704K	--1	-1	2	-	-4	-2	0	1	0	-7	-3	-8	-6	-8	6	5	-12	-7	3	4	10	-6	15	-7		
704L	--3	-1	0	-	6	-4	-6	3	4	-9	-7	-2	-6	12	-2	-9	-8	15	-3	-6	-6	6	-5	-3		
705A	0	+	+	2	2	1	-2	-6	-7	-6	-6	-4	0	7	+	-10	5	7	4	-11	7	-13	-8	-7	10	
705B	-1	+	-	-5	6	3	-3	-1	-5	-7	0	0	-5	-6	-	5	-9	-7	-8	-3	-10	-10	-8	14	12	
705C	0	-	+	2	-6	5	6	2	9	-6	2	-4	0	11	+	6	9	-1	-4	-15	11	11	0	-3	2	
705D	1	-	+	-3	-2	-1	3	-3	-9	-5	-8	4	9	2	-	3	-9	1	8	13	-2	-6	-4	10	-8	
705E	-1	-	+	1	-2	-7	1	-1	-1	7	-8	0	-11	-10	-	-7	9	-15	4	-5	2	-10	12	-6	12	
705F	1	-	-	0	4	2	6	0	0	-2	4	-2	-6	-4	-	-6	0	-2	-4	4	-2	0	-4	-14	10	
706A	+	2	-3	0	-1	-5	-3	4	0	-10	-5	-1	-3	11	6	-6	8	2	2	-5	-3	4	9	-18	9	+
706B	-	0	-1	-4	-1	-3	1	-4	2	0	5	-3	1	-1	10	-6	-10	0	14	-11	-11	-8	5	0	-7	-
706C	-	0	-4	2	-4	0	-2	-4	-4	-6	2	0	10	-4	-8	0	8	-6	-4	-2	10	10	-4	-6	2	-
706D	--2	0	0	-4	-4	-2	4	0	-6	-4	-4	-10	4	0	-4	-6	10	-14	12	6	16	8	10	18	-	
707A	-2	-2	-3	+	-4	-1	-7	-7	-3	-4	-7	10	-8	0	-7	-8	6	10	0	-3	10	-1	10	-12	-2	-

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
708A	-	+	-2	0	-4	6	-2	4	4	10	2	2	10	6	-8	6	+	10	10	0	-2	-4	-4	0	14	
709A	-2	-1	-3	-4	-1	-4	-4	-4	-2	1	-4	-6	4	-12	-3	-4	12	0	7	-8	6	4	6	8	-18	-
710A	+	-1	-	1	-2	-1	-2	-7	9	-2	-5	-10	-6	-3	11	-10	-6	6	-14	-	3	-10	-4	-15	-18	
710B	-	-1	+	-1	-2	-1	-4	-1	-5	4	-9	-8	8	-7	13	-6	-8	10	4	-	-7	6	-16	17	18	
710C	-	-1	-	-3	-6	-3	0	-1	1	0	-3	4	0	1	-9	6	-4	-2	-12	+	9	-10	16	-15	2	
710D	-	-1	-	3	2	-1	8	-5	-1	0	7	-12	12	9	-7	-6	0	2	8	-	-11	10	-16	-15	-2	
711A	1	+	0	-1	-1	-3	-3	-2	1	-3	-4	-2	10	-5	-4	10	0	-4	2	6	-1	+	9	-8	-1	
711B	-1	+	0	-1	1	-3	3	-2	-1	3	-4	-2	-10	-5	4	-10	0	-4	2	-6	-1	+	-9	8	-1	
711C	1	-	3	-1	2	3	6	4	-2	6	-10	-2	10	4	-7	-8	3	-4	8	-15	2	+	6	7	-19	
712A	+	-2	2	-4	0	4	2	2	4	-8	4	8	6	10	8	-2	-2	4	-8	8	-2	8	10	-	14	
713A	1	1	0	-3	-4	2	3	-4	-	-6	-	6	-1	4	6	13	-14	5	-13	-14	0	-8	4	-2	-6	
714A	+	+	2	+	-6	0	+	-2	0	-4	0	8	2	-4	-8	-14	-6	-10	0	-12	14	4	-6	-14	-6	
714B	+	+	1	+	3	-3	-	6	-2	6	4	-11	12	3	12	5	4	-6	9	12	15	-11	7	-13	11	
714C	+	+	1	-	5	-1	+	-6	6	6	4	11	0	-9	4	-7	12	6	13	4	-13	15	13	13	-9	
714D	+	+	-2	-	-2	4	-	-2	-4	0	0	-8	-2	-4	0	-6	-10	-6	0	-8	-6	-12	6	6	14	
714E	-	+	3	+	1	1	+	6	-2	-2	0	5	4	-9	0	11	4	6	-11	-12	-5	-15	1	9	-9	
714F	-	+	-2	+	0	-6	-	0	-8	-6	-8	10	-6	12	0	-10	-8	6	12	0	-6	-8	16	2	2	
714G	-	+	-2	-	4	-2	-	4	8	6	0	-2	10	-4	0	6	-4	6	-12	-8	-6	0	-12	-6	2	
714H	-	+	3	-	-1	3	-	-6	-2	6	0	3	0	11	0	-9	-4	-14	-7	12	-1	-5	3	-1	-13	
714I	-	-	-3	-	3	5	+	2	6	-6	-4	11	-12	-1	12	-9	-12	-10	5	0	-7	-1	-15	9	-19	
715A	0	-2	-	2	+	-	-3	5	-6	-6	-10	-1	-9	-7	9	-12	0	-4	5	0	2	-10	6	0	17	
715B	-2	0	-	0	+	-	-3	-1	-2	0	-2	1	-7	13	-9	4	-14	-10	-1	-10	-4	2	2	-14	-17	
718A	+	0	0	-2	0	3	3	4	9	5	2	6	5	-5	-13	14	9	10	-3	4	1	-7	-3	8	4	-
718B	+	-2	-3	-5	-6	-2	5	-7	4	-8	-11	-5	6	-1	0	0	5	-10	-4	-3	1	-8	7	-14	-4	-
718C	-	0	-3	1	-6	0	-3	-5	6	2	-1	3	-10	1	8	-10	-9	4	12	-5	1	-10	-3	14	10	-
720A	+	+	+	-2	-2	4	-2	-4	-8	-10	-4	0	0	8	-8	6	14	-14	4	-12	6	12	-4	-12	-14	
720B	+	+	-	-2	2	4	2	-4	8	10	-4	0	0	8	8	-6	-14	-14	4	12	6	12	4	12	-14	
720C	+	-	+	0	-4	6	6	4	0	2	8	-2	6	-12	8	-6	12	14	-4	8	-6	8	-12	-10	2	
720D	+	-	+	4	4	-2	-2	-4	4	2	8	6	6	8	4	-6	-4	-2	-8	0	-6	0	-16	6	-14	
720E	+	-	-	-4	0	-6	2	-4	-8	6	0	-6	-10	4	8	-10	0	6	4	0	-14	-16	12	-2	2	
720F	-	+	+	-2	6	-4	6	4	0	6	4	8	0	-8	0	6	6	2	4	-12	-10	4	12	-12	2	
720G	-	+	-	-2	-6	-4	-6	4	0	-6	4	8	0	-8	0	-6	-6	2	4	12	-10	4	-12	12	2	
720H	-	-	+	0	-4	-2	-2	-4	0	2	0	-10	-10	-4	8	10	-4	-2	-12	-8	10	0	12	6	2	
720I	-	-	-	-2	0	2	6	4	6	-6	4	2	-6	10	-6	6	12	2	-2	-12	2	-8	6	6	2	
720J	-	-	-	4	0	2	-6	4	0	6	-8	2	6	4	0	6	0	-10	4	0	2	-8	12	-18	2	
722A	+	1	0	-4	3	2	-6	+	-6	0	2	-10	9	-4	0	6	-9	-4	-7	-6	-1	-4	3	6	17	
722B	+	-3	2	-3	-2	3	-1	+	5	3	6	-6	-12	-10	-8	3	-3	0	-15	0	-11	12	2	-6	-12	
722C	+	1	-4	3	2	1	3	-	-1	5	8	2	8	4	8	1	-15	2	-3	-2	9	10	-6	0	2	
722D	-	3	2	-3	-2	-3	-1	+	5	-3	-6	6	12	-10	-8	-3	3	0	15	0	-11	-12	2	6	12	
722E	-	-1	0	-1	-6	-5	3	-	3	-9	4	-2	0	8	0	3	-9	-10	-5	6	-7	10	-6	12	10	
722F	-	-1	0	-4	3	-2	-6	-	-6	0	-2	10	-9	-4	0	-6	9	-4	7	6	-1	4	3	-6	-17	
723A	-1	+	-2	0	2	2	0	4	-6	-6	-4	6	6	4	-12	-14	-4	-6	12	10	6	-8	-12	12	-2	+
723B	0	-	0	-2	-1	0	-2	-2	-9	-6	2	-10	8	-6	8	-6	10	3	3	4	-6	3	4	-1	13	-
725A	1	0	+	2	-6	-2	2	-2	-2	+	2	-10	2	-8	12	6	-8	-6	-2	-12	6	-10	14	18	-2	
726A	+	+	0	0	+	-6	6	-6	6	-6	4	-2	-6	-6	6	-12	-12	-6	4	-6	12	-12	0	-6	-10	
726B	+	+	-1	4	-	-3	1	8	-8	9	0	3	-3	8	12	11	0	2	4	0	6	4	-16	7	-5	
726C	+	+	2	4	-	6	-2	-4	4	-6	0	6	6	-4	-12	2	12	14	4	-12	6	4	-4	10	-14	
726D	+	-	-1	-4	-	5	-7	0	0	-7	-8	-5	-11	8	4	-5	0	2	12	-16	6	-4	8	-17	-5	
726E	+	-	-4	2	-	-4	2	0	-6	-10	-8	-2	-2	-4	-2	4	0	8	-12	2	6	-10	-4	10	-2	
726F	-	+	0	0	+	6	-6	6	6	6	4	-2	6	6	6	-12	-12	6	4	-6	-12	12	0	-6	-10	
726G	-	+	-1	-4	-	3	-1	-8	-8	-9	0	3	3	-8	12	11	0	-2	4	0	-6	-4	16	7	-5	
726H	-	-	0	-2	-	4	6	4	6	-6	8	-10	-6	-8	-6	0	0	-8	-4	6	-2	-14	12	-6	14	

TABLE 3: HECKE EIGENVALUES 726I–742D

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
726 I	-	-	-1	4	-	-5	7	0	0	7	-8	-5	11	-8	4	-5	0	-2	12	-16	-6	4	-8	-17	-5	
728A	+	-1	0	-	3	+	4	2	1	4	9	3	-5	4	9	-4	10	5	11	-16	11	-5	-4	-2	-7	
728B	+	2	3	-	0	+	-2	5	1	-5	-3	-6	-2	1	3	11	-8	-10	2	14	-7	13	-1	-11	5	
728C	-	0	-1	+	2	-	0	-7	-3	-9	5	-8	-10	5	7	3	0	6	-10	4	-11	-11	11	-3	-15	
728D	-	-2	-1	-	4	+	-6	1	1	3	-7	-10	-10	-7	-9	3	0	6	-6	10	-11	-3	11	-7	17	
730A	+	0	+	-2	2	2	2	0	4	10	10	-6	6	-6	-6	2	6	2	8	16	-	4	6	-6	6	
730B	+	-2	+	-4	0	-4	-3	5	3	-9	-7	-7	9	11	12	6	6	-4	2	-12	-	8	12	9	-16	
730C	+	3	+	1	5	-4	2	0	-2	1	-2	3	9	6	-3	-4	-9	-4	-13	13	-	-17	12	9	-6	
730D	+	-1	-	3	3	0	6	-4	-2	-1	-2	-1	1	6	7	12	9	0	7	5	+	-1	16	1	-14	
730E	+	2	-	0	0	0	3	5	7	5	-5	5	1	-3	4	-6	6	-12	-2	-16	+	8	4	1	16	
730F	+	0	-	2	-6	-2	-6	0	-4	6	-2	-6	6	10	-10	-10	-2	10	-8	0	-	-4	-2	10	-10	
730G	+	-3	-	-1	-3	-2	6	6	-4	-3	4	-3	-3	-8	-13	8	-5	-8	-11	-3	-	11	-2	-11	8	
730H	-	-2	+	4	0	-4	7	1	1	3	9	-1	9	-11	0	6	2	8	-2	-8	+	-4	-4	9	-16	
730 I	-	-1	+	-1	-1	-2	-2	-6	0	-1	-4	-9	-3	12	-5	0	-7	8	7	5	-	-5	-2	-11	12	
730 J	-	-1	-	-3	-3	-6	-6	2	4	5	4	-1	-11	-12	1	0	3	-12	7	5	+	11	10	13	-8	
730K	-	1	-	5	3	-4	-6	-4	6	-9	2	-7	9	2	9	-12	9	-4	-7	-3	-	-1	12	-15	2	
731A	1	1	-1	0	-6	1	+	-2	2	6	2	-3	-4	+	13	1	-3	-7	-9	11	-7	-8	-9	2	10	
732A	-	+	2	2	2	-2	-2	4	6	-2	-2	2	2	6	12	6	10	+	-2	-6	6	-14	0	6	-10	
732B	-	+	1	-5	5	1	-6	-2	-7	4	-8	6	-7	-8	-8	-6	3	-	3	-12	13	9	0	0	-10	
732C	-	-	-2	-2	0	-6	0	0	-4	0	-6	-6	6	-2	0	4	0	+	-2	8	6	10	-12	12	2	
733A	1	-1	0	2	0	6	5	3	4	0	5	1	5	-10	3	-1	7	15	-16	14	-6	1	3	-6	-5	-
734A	-	2	0	0	-2	2	6	6	0	0	8	-6	-10	-10	-8	-6	4	10	-4	-12	14	-8	-4	10	-18	+
735A	1	+	+	-	0	6	-2	8	8	-2	-4	-2	6	4	-8	10	-4	2	4	-12	2	8	4	6	18	
735B	-2	+	+	-	-6	3	4	-1	-4	-8	-1	7	6	1	-2	4	8	14	7	6	-1	-1	-2	12	6	
735C	0	+	-	-	0	1	-6	-5	6	-6	-5	-7	-12	-1	-6	0	6	-2	-7	12	-11	-13	12	-6	10	
735D	0	-	+	+	0	-1	6	5	6	-6	5	-7	12	-1	6	0	-6	2	-7	12	11	-13	-12	6	-10	
735E	-1	-	+	-	-4	2	-2	-4	0	-2	0	-10	-10	4	-8	-10	4	2	12	-8	-10	0	-12	6	-2	
735F	-2	-	-	+	-6	-3	-4	1	-4	-8	1	7	-6	1	2	4	-8	-14	7	6	1	-1	2	-12	-6	
737A	-2	2	-2	-2	-	-2	7	3	-7	-9	-2	-9	4	-6	-1	6	-7	6	-	0	-11	-16	12	15	-16	
738A	+	+	1	-4	4	-5	1	-3	-8	0	7	-4	+	-6	-8	14	-7	-8	-5	-7	1	-14	-15	13	12	
738B	+	-	-1	-2	-2	-1	7	5	6	0	7	-2	+	4	12	6	-5	2	3	3	9	0	-9	-5	-2	
738C	+	-	2	4	4	2	-2	-4	0	6	-8	-2	+	4	-12	6	4	-10	12	12	-6	12	-12	-2	10	
738D	+	-	-1	2	-2	-7	-7	7	2	8	-5	-10	-	-8	-4	2	-9	6	1	-15	1	-8	11	-3	10	
738E	-	+	-1	-4	-4	-5	-1	-3	8	0	7	-4	-	-6	8	-14	7	-8	-5	7	1	-14	15	-13	12	
738F	-	-	-3	-2	-2	1	-5	-1	-6	-8	3	-6	+	-4	12	14	-3	10	-7	3	1	12	-7	15	-10	
738G	-	-	2	2	4	-4	2	-8	-4	8	4	2	-	4	2	-4	-12	-6	16	-6	-2	-14	-4	6	-2	
738H	-	-	2	2	-4	4	2	0	-4	0	4	2	-	-12	2	4	4	10	-8	10	-2	-14	12	-10	-18	
738 I	-	-	2	-4	2	4	2	6	8	0	-8	2	-	-12	-4	4	-8	-14	-2	-8	10	4	-12	14	6	
738 J	-	-	-3	2	6	-1	-3	5	6	0	-1	2	-	8	12	-6	9	-10	-13	-15	-7	-4	-3	-15	2	
739A	2	0	2	2	3	2	1	2	6	-8	-4	3	-9	9	-2	14	-13	-10	-13	6	11	-11	-12	14	16	-
740A	-	3	+	-3	5	2	4	-4	6	6	-4	+	-9	10	-11	-11	-8	-8	-8	3	7	8	-9	-16	12	
740B	-	1	+	-1	-3	-4	0	-4	0	0	2	-	3	2	3	-9	0	2	-4	15	-7	-10	-3	6	-10	
740C	-	-1	-	1	-3	-6	0	0	2	-6	0	+	-9	-10	1	1	0	-12	0	-5	3	16	11	0	8	
741A	1	+	1	3	0	+	6	-	3	6	10	9	-10	-8	8	-1	3	9	2	-3	-8	-11	6	-18	-17	
741B	1	+	-3	1	4	-	2	+	5	10	-6	3	6	8	0	1	7	-7	2	-15	-8	13	-6	18	5	
741C	1	-	3	5	0	+	-6	+	-1	2	-2	5	-2	-4	-8	9	3	-7	-14	5	0	15	-14	-2	-5	
741D	2	-	1	-1	5	+	7	+	-4	2	2	0	-12	-1	3	-2	0	-9	-4	-12	1	4	-4	18	10	
741E	0	-	1	-3	-3	-	-3	+	-8	0	-6	4	-2	-9	3	8	8	-5	14	-8	-9	-14	-4	14	0	
742A	+	0	-1	+	3	4	-3	0	-9	0	-5	0	-2	5	-12	+	7	-10	-8	-12	-14	11	0	-13	1	
742B	+	2	4	+	-4	2	6	2	0	6	-2	-6	-4	4	-8	-	4	8	-8	-12	-12	8	-2	14	-18	
742C	+	0	3	-	3	4	1	0	-1	0	-1	0	6	-11	4	+	3	6	8	4	10	-5	0	-9	5	
742D	+	3	0	-	0	1	1	-3	5	9	-4	-3	6	-2	-2	+	-6	-12	-4	-5	-8	13	9	-6	5	

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
742E	+	2	-4	-	0	-4	-2	6	-8	-6	-8	-6	2	-8	4	-	2	12	-4	-8	-2	8	6	2	-2	
742F	-	3	-2	+	2	1	3	1	1	-9	0	-5	6	-6	0	+	-10	0	-4	-13	-12	1	-11	14	7	
742G	-	-1	-2	-	-6	5	-1	-3	-7	-9	-8	3	-2	10	8	+	-2	0	-4	-13	4	-7	9	-2	-5	
744A	+	+	-1	1	0	-6	0	-3	6	0	+	-8	-11	-8	0	4	-11	-8	12	-7	-4	10	14	10	1	
744B	+	-	-2	0	4	-2	6	4	0	10	+	-2	10	4	-4	2	0	6	-12	-12	2	0	-4	-10	2	
744C	+	-	-3	1	-6	0	-4	-3	0	-2	-	-2	-1	-6	4	-12	7	0	-4	15	2	2	6	-14	-7	
744D	-	+	3	5	4	-2	-8	1	2	-4	+	-4	9	4	0	-12	-11	8	-4	-7	0	2	6	-6	17	
744E	-	+	-3	2	-5	1	1	7	8	-4	+	2	6	10	9	6	10	-7	5	5	-6	5	-15	6	-19	
744F	-	+	1	-3	-2	4	-4	-7	4	-6	-	-6	3	-2	-12	4	7	-8	-4	-1	-2	-14	14	-6	-7	
744G	-	-	-1	-3	-4	-2	0	1	-6	-4	+	4	5	4	-8	4	-15	0	12	-3	8	-6	-2	10	17	
747A	-1	+	-2	0	0	6	-4	2	-4	-4	-4	-2	-8	-2	0	6	-4	2	-14	8	6	2	+	-6	-2	
747B	1	+	2	0	0	6	4	2	4	4	-4	-2	8	-2	0	-6	4	2	-14	-8	6	2	-	6	-2	
747C	1	-	-1	0	3	-6	4	-7	-5	-8	-10	7	2	4	12	-9	1	11	-5	4	12	-4	-	9	-2	
747D	1	-	2	-3	-3	-6	-5	2	4	7	5	-11	2	-8	0	-6	-5	5	-2	-2	0	14	-	0	-8	
747E	-1	-	1	-4	3	2	-4	-1	3	-4	-6	-9	2	4	-8	-7	9	-13	5	0	-12	-12	-	-9	-6	
748A	-	3	3	-2	+	-2	+	2	3	-10	7	-7	4	2	0	-10	-5	0	-9	-7	14	4	18	-9	-17	
749A	-1	1	-2	-	3	-1	0	-3	1	-10	6	-3	-11	-8	0	-3	12	-15	-14	12	4	1	12	3	-16	-
752A	-	0	0	0	-2	-4	-2	2	-4	4	-4	2	6	-6	-	2	-12	2	-2	-8	-14	16	16	-10	-14	
753A	2	+	3	-1	6	-2	5	-4	7	-4	3	-8	3	4	12	-12	0	-10	-7	4	-1	-17	9	-6	-12	-
753B	0	-	3	-1	0	2	-3	8	3	0	-1	2	3	-4	6	6	6	-10	5	-6	11	-1	9	-6	-16	+
753C	0	-	1	-5	6	-6	3	-6	-7	-6	-5	-2	-7	-6	0	10	6	4	5	14	-5	-5	-1	6	-10	-
754A	+	1	3	-1	0	-	3	8	6	+	-4	5	-6	11	-3	0	12	2	-10	9	-16	2	-6	6	-10	
754B	+	1	1	-1	-6	-	-7	-2	-4	-	4	9	12	-1	7	0	-10	-4	-2	-3	6	4	0	-14	6	
754C	+	-2	-2	2	0	-	2	4	-4	-	-8	6	-6	2	-8	-6	-10	-10	-2	-6	6	-14	-6	10	18	
754D	-	1	-3	-3	-4	+	-1	0	2	-	4	-1	-2	3	3	-4	-12	10	-2	-5	0	-10	-6	10	-18	
755A	0	0	+	3	3	-3	0	-4	-9	1	-7	-2	-4	-2	-8	-2	9	-6	-5	-6	5	-10	15	8	2	+
755B	1	1	+	0	-3	-3	-2	-1	3	-3	1	4	10	-10	2	-2	-5	0	-7	0	-7	10	-3	-2	10	+
755C	1	-2	+	2	-4	4	6	4	-6	2	8	6	-2	4	-12	4	12	10	-2	0	0	12	-6	6	-6	-
755D	2	1	-	3	0	6	-4	-1	-3	-3	7	-2	8	8	-8	-10	-4	0	-4	-12	-7	4	12	-4	16	+
755E	2	3	-	-1	-3	-3	-2	5	0	-10	8	4	-6	-4	12	6	-1	6	1	6	10	-8	-15	0	-4	+
755F	2	-3	-	-1	0	6	4	-1	9	5	-1	-2	0	8	0	6	-4	0	4	-12	13	4	-12	-12	-16	+
756A	-	+	1	+	2	0	5	2	2	10	0	5	3	-7	-3	6	1	-6	4	8	10	-3	-13	-6	-14	
756B	-	+	-3	-	0	2	-3	-4	-6	0	-10	-7	-9	5	-3	-6	9	8	8	12	-10	5	-9	-18	8	
756C	-	-	-1	+	-2	0	-5	2	-2	-10	0	5	-3	-7	3	-6	-1	-6	4	-8	10	-3	13	6	-14	
756D	-	-	3	-	0	2	3	-4	6	0	-10	-7	9	5	3	6	-9	8	8	-12	-10	5	9	18	8	
756E	-	-	3	-	3	2	-6	5	-9	6	-1	11	3	-4	-12	0	0	8	-10	3	8	-4	6	3	8	
756F	-	-	-3	-	-3	2	6	5	9	-6	-1	11	-3	-4	12	0	0	8	-10	-3	8	-4	-6	-3	8	
758A	+	-2	-1	-2	6	4	-4	-1	1	0	-4	-3	3	-6	0	-12	2	-3	-3	10	-12	-3	-15	-12	-2	+
758B	-	2	3	-2	2	-4	0	-1	5	0	-8	1	3	-6	8	-12	14	-7	-11	6	4	1	9	4	6	+
759A	-1	+	0	-2	-	2	0	2	-	-10	4	2	-2	2	-8	-4	-12	-6	2	0	-6	2	4	-8	-2	
759B	-1	-	-2	0	-	-2	2	-4	+	-2	0	-10	-6	-12	0	14	-4	6	-4	-8	-6	8	4	-14	2	
760A	+	2	-	4	-4	0	6	+	8	-6	-8	-8	-2	0	12	4	8	-14	-2	-8	-2	4	12	6	0	
760B	+	-2	-	4	4	-4	-2	+	0	2	8	4	6	0	12	-8	0	2	-14	8	6	-4	4	14	-12	
760C	+	3	-	-1	4	1	-7	+	-5	7	-2	-6	6	10	-8	-3	5	-8	11	-12	-9	6	14	-6	-2	
760D	+	-2	-	0	-4	4	-2	-	-4	-6	-8	-4	-2	4	-8	0	-8	2	-14	-8	6	-4	16	-18	-4	
760E	-	0	-	0	-4	-6	-6	+	8	-2	0	2	2	4	-8	-6	-4	-2	8	8	2	-8	4	-14	14	
762A	+	-	0	1	4	-2	6	-7	4	6	8	4	0	-2	-7	0	5	8	10	-7	15	-8	11	-1	-12	+
762B	+	-	3	1	1	-2	3	5	1	-6	-10	-8	9	4	2	9	8	-10	-2	-4	-6	10	8	-10	12	+
762C	+	-	-3	3	-3	-6	-5	1	3	-2	2	-8	-7	-8	10	-9	4	-2	6	0	-14	-10	0	-6	-4	-
762D	-	+	-1	-3	1	-4	-3	3	-9	-6	0	-4	7	-2	6	-3	10	-12	-2	12	6	-8	0	12	-10	-
762E	-	-	-3	-5	-3	0	-3	-1	1	-2	4	-12	-9	6	-2	-1	10	12	-2	-16	6	4	8	-4	10	+
762F	-	-	0	-1	0	2	6	5	0	6	-4	8	-12	-10	-9	0	9	-4	-10	15	-1	-16	-9	-9	8	-

TABLE 3: HECKE EIGENVALUES 762G–782E

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
762G	-	-	-1	1	5	0	-3	-1	3	2	4	-4	7	2	6	5	-10	-8	-2	-12	-10	-4	0	-8	-14	-
763A	-2	0	0	-	-2	3	-5	4	-2	7	-6	8	-10	-1	-3	-6	-9	4	-6	-9	-14	-4	4	-14	-8	-
765A	1	+	+	4	2	2	-	0	0	6	8	8	-2	-2	0	-10	-4	10	2	-14	-8	-8	4	0	-16	-
765B	-1	+	-	4	-2	2	+	0	0	-6	8	8	2	-2	0	10	4	10	2	14	-8	-8	-4	0	-16	-
765C	-1	-	-	-2	-2	2	+	0	-6	6	-10	2	-10	4	-12	10	-8	-14	8	2	-14	-14	-4	-6	2	-
766A	+	2	2	3	-3	1	7	-2	3	6	-8	-5	-6	0	8	11	-11	-13	6	5	3	10	-12	-6	12	-
768A	+	+	0	-4	4	4	-2	-4	-8	-8	-4	-4	6	4	-8	-8	-12	12	12	8	-6	-4	-4	-6	-2	-
768B	+	+	-2	2	0	-4	-2	4	-4	-6	2	-8	-2	4	-12	-6	-4	0	-12	-12	6	10	16	10	-2	-
768C	+	-	0	4	-4	4	-2	4	8	-8	4	-4	6	-4	8	-8	12	12	-12	-8	-6	4	4	-6	-2	-
768D	+	-	2	2	0	4	-2	-4	-4	6	2	8	-2	-4	-12	6	4	0	12	-12	6	10	-16	10	-2	-
768E	-	+	0	4	4	-4	-2	-4	8	8	4	4	6	4	8	8	-12	-12	12	-8	-6	4	-4	-6	-2	-
768F	-	+	2	-2	0	4	-2	4	4	6	-2	8	-2	4	12	6	-4	0	-12	12	6	-10	16	10	-2	-
768G	-	-	0	-4	-4	-4	-2	4	-8	8	-4	4	6	-4	-8	8	12	-12	-12	8	-6	-4	4	-6	-2	-
768H	-	-	-2	-2	0	-4	-2	-4	4	-6	-2	-8	-2	-4	12	-6	4	0	12	12	6	-10	-16	10	-2	-
770A	+	2	+	-	+	2	2	6	6	4	0	8	0	4	-4	-12	0	2	-8	-12	-6	10	-12	14	4	-
770B	+	-2	+	-	+	2	-6	2	-6	0	8	-4	12	-4	12	0	0	2	8	12	2	14	12	6	8	-
770C	+	0	-	+	+	2	6	4	4	-2	8	-10	-6	12	12	6	-12	6	8	-8	14	0	4	-6	-14	-
770D	+	-2	-	+	-	0	0	0	-4	2	-2	-6	8	-12	-6	-6	-10	-4	-8	-4	-4	-16	0	-6	14	-
770E	+	0	-	-	+	-6	-2	-4	-4	6	0	-2	-6	-4	4	-2	12	-2	-8	-8	-10	-8	-12	10	-6	-
770F	-	-2	+	-	+	-4	0	-4	0	-6	-10	2	-12	-4	6	-6	-6	-4	-4	12	-4	8	12	18	-10	-
770G	-	-2	-	-	+	2	6	2	-6	0	8	8	0	-4	0	12	12	-10	-4	-12	14	-10	0	-18	8	-
774A	+	+	3	-1	3	-1	6	-1	0	3	-4	2	0	-	3	0	0	14	-4	6	2	14	-9	18	5	-
774B	+	-	2	4	-4	6	6	-4	4	-6	-8	2	-2	+	-4	6	12	10	12	8	-6	-16	12	-10	2	-
774C	+	-	-3	-1	1	1	-4	1	4	9	2	2	8	+	11	-4	12	0	2	-12	4	14	-3	10	17	-
774D	+	-	1	1	-5	-7	-4	-1	4	5	-10	10	0	-	1	-12	-4	-8	-2	12	4	10	7	-6	-7	-
774E	+	-	-2	-2	4	2	2	-4	-2	-10	-4	-8	-6	-	-2	12	-4	-8	4	0	10	-8	-8	-6	14	-
774F	-	+	-3	-1	-3	-1	-6	-1	0	-3	-4	2	0	-	-3	0	0	14	-4	-6	2	14	9	-18	5	-
774G	-	-	-1	-5	-1	-3	0	-7	4	3	-2	2	-8	+	-7	12	-12	4	6	8	0	-10	3	14	-7	-
774H	-	-	2	2	0	2	-6	4	-6	2	4	4	2	-	-6	4	8	-12	4	0	-14	8	-4	-10	-2	-
774I	-	-	3	-3	5	-3	0	7	4	-1	-6	-6	0	-	3	-12	4	12	10	-8	-16	-14	9	-2	1	-
775A	0	1	+	0	-4	6	-5	-1	-8	-10	+	-1	-3	7	6	-5	11	-12	2	9	9	-10	-9	0	14	-
775B	1	-2	+	-4	4	0	8	4	-2	-6	-	4	-6	6	-8	12	-4	10	-8	0	4	0	-2	14	18	-
775C	2	1	+	2	2	6	7	-5	-4	0	-	7	-3	-9	2	-9	-5	-8	-8	-3	1	0	11	10	-18	-
776A	-	0	-2	2	0	-2	2	-6	-2	-2	-4	-2	10	-12	-8	2	-2	2	-6	10	-10	4	2	-2	-	-
777A	1	+	-2	+	4	2	2	4	0	10	0	-	-6	8	0	14	-8	-14	-12	8	10	4	12	-14	6	-
777B	-2	+	1	+	1	-1	2	4	6	-8	-9	-	6	2	12	11	-5	-2	15	5	-8	10	0	13	9	-
777C	0	+	3	-	-1	-1	-2	4	4	0	3	+	-2	10	10	9	5	-2	3	15	2	-4	-2	7	-3	-
777D	-1	+	-2	-	0	2	-6	4	8	-2	0	-	-10	-8	-8	-2	-12	2	-4	-4	2	4	4	-6	6	-
777E	1	-	-2	+	0	-2	2	-4	-8	-6	-8	-	6	-4	8	6	0	-2	12	-12	2	16	12	10	-14	-
777F	-2	-	1	+	-3	-5	2	-4	-2	0	-5	-	6	2	-4	-9	3	-2	-9	9	8	-14	0	-11	-11	-
777G	0	-	-1	-	-1	-5	-2	-4	-4	8	-1	+	-10	-6	-6	-7	9	-2	-13	7	-14	12	14	11	17	-
780A	-	+	-	-2	-2	+	-2	-2	-4	2	-2	-6	-2	-8	-6	6	6	-2	-14	-6	-6	4	6	14	-14	-
780B	-	+	-	3	1	-	-3	-2	5	-6	10	5	3	4	6	5	-8	1	12	1	-10	-1	0	1	3	-
780C	-	-	+	-2	-6	+	-2	-2	-4	2	-2	2	-6	0	6	-2	-6	14	2	-10	-6	4	2	-14	18	-
780D	-	-	+	-1	3	-	3	2	3	6	2	-7	9	8	-6	3	0	-7	-4	3	-10	-1	0	3	-1	-
781A	0	0	-1	-3	+	7	3	-2	8	4	10	1	-11	-2	2	0	4	-11	8	-	16	6	14	-15	-10	-
781B	0	0	-1	3	-	1	-3	-2	-4	-8	-2	1	-5	-2	2	-12	4	-5	8	-	-8	6	-10	9	2	-
782A	+	-2	2	0	0	-6	+	6	+	-4	4	-2	-2	-10	-8	4	-8	-14	10	12	-14	-8	-6	2	-2	-
782B	-	1	-4	3	6	6	+	-3	+	2	4	-2	4	8	1	1	-8	-2	-11	-6	10	-11	-12	-10	-17	-
782C	-	-2	0	4	-2	2	+	4	+	8	4	12	6	0	0	-14	0	-4	16	-8	-14	16	8	-6	-18	-
782D	-	3	0	-1	-2	2	+	-1	+	-2	4	2	-4	0	5	-9	0	6	-9	2	6	1	-12	-6	7	-
782E	-	0	2	0	0	6	-	4	-	-6	8	-6	2	4	0	-10	4	2	-4	-8	-6	0	-4	10	2	-

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
784A	+	1	-1	+	-3	-6	-5	-1	7	2	5	3	-2	4	-5	-1	-15	-5	9	0	7	-1	-12	7	-2	
784B	+	-3	-1	+	1	2	3	-5	3	-6	1	-5	-10	4	-1	-9	-3	3	-11	-16	7	11	4	-9	6	
784C	+	0	-2	-	4	-2	6	8	0	6	8	-2	-2	4	-8	6	0	6	4	8	-10	-16	8	6	6	
784D	+	-1	1	-	-3	6	5	1	7	2	-5	3	2	4	5	-1	15	5	9	0	-7	-1	12	-7	2	
784E	+	2	4	-	0	0	2	-2	-8	2	4	-6	2	-8	-4	-10	6	-4	12	0	14	8	6	-10	2	
784F	+	3	1	-	1	-2	-3	5	3	-6	-1	-5	10	4	1	-9	3	-3	-11	-16	-7	11	-4	9	-6	
784G	-	-1	3	+	3	2	3	1	-3	-6	7	-1	6	4	9	3	-9	-1	7	0	-1	13	-12	15	-10	
784H	-	0	0	-	-4	0	0	0	-8	2	0	-6	0	12	0	-10	0	0	-4	-16	0	-8	0	0	0	
784I	-	1	-3	-	3	-2	-3	-1	-3	-6	-7	-1	-6	4	-9	3	9	1	7	0	1	13	12	-15	10	
784J	-	-2	0	-	0	4	-6	2	0	-6	-4	2	-6	-8	-12	6	-6	-8	4	0	-2	-8	-6	6	10	
786A	+	+	-1	-1	-1	4	3	0	-7	-8	-2	7	6	2	-3	-9	-13	-10	-7	-15	0	2	6	4	-10	+
786B	+	+	-2	2	-3	3	-5	1	4	9	-5	-8	-12	-6	-8	12	-5	-3	0	-8	-2	-8	-14	-14	12	+
786C	+	+	4	-1	-6	-6	-2	-5	-2	-3	-2	7	-9	12	-8	6	-8	0	3	10	10	-8	1	-11	0	+
786D	+	+	-1	-3	1	-2	1	-4	9	2	10	-7	4	8	5	-5	9	6	-3	5	10	4	6	10	8	-
786E	+	+	2	0	4	-2	-2	8	0	2	-8	2	10	-4	8	10	12	6	0	8	-14	16	12	-14	2	-
786F	+	-	-3	5	3	2	-3	-4	-3	6	2	-1	12	8	9	9	3	-10	-13	-15	2	-4	6	6	-4	+
786G	+	-	1	-5	-3	4	-5	-8	5	0	-10	1	6	-2	-7	1	9	-2	-13	5	-8	-6	6	-8	-6	-
786H	+	-	-2	-2	3	-5	7	-5	-4	-3	-7	-8	-12	-2	8	4	-3	13	8	-16	-2	0	6	-14	12	-
786I	-	+	4	-4	0	6	-2	4	4	-6	-2	-8	6	12	-8	12	-8	-6	-12	4	-14	10	4	10	6	+
786J	-	+	-1	-3	1	-2	-5	-4	3	-10	-2	11	-8	-4	-1	-5	9	-6	15	-1	10	16	-6	-14	8	-
786K	-	+	-4	3	-2	-2	-2	-7	-6	-1	-2	-7	-5	8	-4	-2	-12	12	-15	14	-2	4	15	1	-4	-
786L	-	-	-3	-3	-5	-6	-1	-4	7	10	6	-3	-8	-4	3	9	-5	-14	-7	-5	-6	0	-6	6	12	+
786M	-	-	1	3	-3	4	-7	0	-1	0	2	3	2	-6	3	9	-15	2	-7	7	4	10	-6	0	-2	-
790A	-	-2	-	-2	-4	2	-4	-4	0	-6	-8	8	-2	-10	2	-12	-4	2	4	0	14	+	0	6	14	
791A	1	-2	0	+	4	-2	4	6	-8	10	8	2	10	0	-6	6	6	-6	-8	-8	-4	-4	0	12	10	-
791B	1	-2	-4	+	-4	6	8	-2	-4	-6	-8	2	2	-4	-6	-10	-2	2	12	12	16	8	16	0	10	-
791C	-1	0	2	-	-4	-2	-2	0	0	6	0	-2	-6	-4	-4	-10	-8	-10	4	0	-10	0	-4	6	2	-
792A	+	+	0	-2	+	-6	6	-2	-8	-2	-4	2	10	-6	4	-4	-4	-2	-8	-12	-2	14	4	0	2	
792B	+	-	0	2	+	0	2	8	2	6	0	-2	-2	4	6	8	8	-4	12	10	-6	-10	4	-10	-2	
792C	-	+	0	-2	-	-6	-6	-2	8	2	-4	2	-10	-6	-4	4	4	-2	-8	12	-2	14	-4	0	2	
792D	-	-	-2	0	+	2	-6	0	-4	-2	0	-10	-6	-8	4	6	12	2	4	-12	-14	16	12	-10	-14	
792E	-	-	2	4	-	6	-6	-8	0	6	0	6	10	-8	0	-6	-4	-2	-12	8	2	-4	12	6	2	
792F	-	-	3	-2	-	0	6	4	-1	8	-7	-1	-4	6	8	-2	1	4	-5	-3	16	2	2	-15	-7	
792G	-	-	-4	-2	-	0	6	4	6	-6	0	6	10	-8	-6	12	8	4	-12	-10	2	2	-12	6	14	
793A	1	0	2	-4	4	+	-6	-2	-6	10	-4	-10	2	-2	-2	10	-4	+	-8	12	-6	-2	10	10	-14	
794A	+	-2	-4	-3	0	-4	-8	-6	-6	3	-4	5	0	8	-10	-12	1	-2	10	5	-13	6	-2	0	7	-
794B	-	1	-3	-4	0	-1	0	-1	0	-6	-4	2	0	-1	0	-3	12	2	-4	-3	14	-4	9	0	-7	-
794C	-	-1	-1	0	-4	1	-6	-3	2	6	-2	-8	-2	-11	6	3	4	2	-8	9	-10	0	11	-2	13	-
794D	-	-2	0	-1	0	-4	0	2	-6	-9	-4	-7	0	8	6	0	-9	-10	14	-9	11	-10	-6	12	-1	-
795A	1	+	+	4	-4	-2	-2	4	-8	-10	-8	-2	6	0	-8	+	4	-10	-12	12	6	-8	12	-6	18	
795B	0	+	+	-2	-4	0	5	-4	-2	6	6	8	9	-6	-9	-	14	4	15	9	-5	-6	0	-6	-18	
795C	0	-	+	2	0	-4	3	8	6	-6	2	8	-3	2	9	+	6	8	5	-3	-7	14	-12	-18	2	
795D	1	-	+	0	4	6	6	0	-8	6	4	6	-2	-4	-4	+	-12	-2	-4	16	6	-4	-12	-6	2	
797A	-1	1	0	0	2	-5	-6	7	-5	6	7	-10	-9	2	-7	0	5	-5	-2	-6	-14	5	4	-1	-16	+
798A	+	+	0	+	2	-4	0	+	2	6	-4	-6	-6	0	-4	-10	-4	-2	-6	12	-14	-14	4	-14	-8	
798B	+	-	2	+	0	2	2	+	8	2	4	2	6	-12	-8	10	-4	-10	4	8	10	-4	0	6	-6	
798C	+	-	-2	+	-2	2	-4	-	0	-6	-10	0	-6	-4	6	-6	-12	10	-2	8	-6	16	-12	10	-12	
798D	+	-	-2	-	-4	-2	-2	+	-4	2	4	-6	-10	-4	-8	10	12	-2	-8	-16	10	-8	0	-10	-2	
798E	+	-	0	-	6	-4	0	-	6	6	-4	2	-6	8	12	6	-12	-10	14	-12	2	-10	12	18	8	
798F	+	-	4	-	-2	0	8	-	-6	-2	-8	-10	2	-8	8	-2	12	6	-10	12	-6	-6	-4	2	16	
798G	-	+	-2	+	2	-6	-4	-	-4	-2	-6	-4	6	-4	-6	6	4	-6	-14	8	10	0	8	6	16	
798H	-	-	-4	+	-6	-4	-4	+	2	2	4	2	6	0	-8	-14	-4	-10	10	-4	-14	2	0	6	0	

TABLE 3: HECKE EIGENVALUES 798I-816A

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
798 I	-	-	2	+	0	2	-2	-	4	-2	0	-2	-6	4	0	-10	12	-10	-8	0	-6	-4	4	10	-2	
799A	-1	2	4	-2	0	2	-	4	-4	8	8	-2	-8	-4	+	6	4	6	-4	-6	4	2	0	6	10	
799B	-1	2	0	-2	0	-6	-	-4	4	-4	0	6	4	-12	-	-10	-12	14	4	-6	8	2	16	-10	-6	
800A	+	0	+	0	0	-6	-2	0	0	-10	0	2	10	0	0	-14	0	-10	0	0	6	0	0	10	-18	
800B	+	1	+	-2	-5	0	-5	-5	6	4	-10	10	5	4	-8	10	0	-10	3	0	5	-10	-1	-9	-10	
800C	+	-2	+	-2	4	6	-2	-8	-6	-2	-4	-2	-10	-2	-2	-2	0	2	-6	12	-10	8	-10	-6	-10	
800D	+	0	-	0	0	4	8	0	0	10	0	12	-10	0	0	-4	0	10	0	0	16	0	0	-10	-8	
800E	+	1	-	-2	5	0	5	5	6	4	10	-10	5	4	-8	-10	0	-10	3	0	-5	10	-1	-9	10	
800F	-	-1	+	2	5	0	-5	5	-6	4	10	10	5	-4	8	10	0	-10	-3	0	5	10	1	-9	-10	
800G	-	2	+	2	-4	6	-2	8	6	-2	4	-2	-10	2	2	-2	0	2	6	-12	-10	-8	10	-6	-10	
800H	-	0	-	0	0	-4	-8	0	0	10	0	-12	-10	0	0	4	0	10	0	0	-16	0	0	-10	8	
800 I	-	-1	-	2	-5	0	5	-5	-6	4	-10	-10	5	-4	8	-10	0	-10	-3	0	-5	-10	1	-9	10	
801A	0	-	-4	-2	-2	6	-4	-4	3	-3	8	-8	11	8	2	8	9	-12	3	-10	1	-1	-9	+	7	
801B	-1	-	2	2	4	2	-6	-2	-2	6	6	10	6	2	-12	6	10	-6	12	-4	10	-12	6	+	-18	
801C	0	-	0	2	-6	2	0	-4	-3	3	-4	-4	-3	-4	-6	0	-9	8	-13	6	-7	-1	9	-	-1	
801D	1	-	1	-4	2	2	-3	-5	-7	0	-9	-2	0	-7	12	3	-4	6	12	10	7	-6	-12	-	9	
802A	-	0	4	-2	3	1	4	4	-8	2	-5	-10	7	11	-6	7	-14	-15	10	-9	11	1	-15	-9	10	+
802B	-	-2	-2	0	0	0	6	6	0	6	0	4	2	-4	0	12	6	8	-2	-8	14	-8	0	10	6	+
804A	-	+	4	0	0	2	2	4	-6	-6	0	-6	12	-4	10	12	6	6	+	-2	2	0	-6	10	-18	
804B	-	+	0	0	-2	-4	-3	5	1	-7	-4	7	-8	-2	7	-14	-15	-6	-	0	9	0	-4	5	12	
804C	-	+	-3	3	-2	2	0	-4	-5	-4	5	-11	-5	-5	-8	1	9	-12	-	0	-9	0	-1	14	-6	
804D	-	-	-1	-3	-2	-2	-4	-4	7	-8	3	-3	1	-11	0	11	-3	8	+	8	-9	0	11	-6	-6	
805A	2	-1	+	+	-5	3	-5	0	+	3	6	-4	0	-2	-9	-6	-6	10	4	-8	10	-15	12	-10	7	
805B	-1	0	+	+	2	4	-6	-8	-	10	10	8	-2	0	12	-4	14	-2	-4	8	0	6	-12	10	-2	
805C	-1	0	+	+	-4	-2	6	4	-	-2	4	-10	10	12	-12	14	8	10	-4	8	-6	0	12	-2	-2	
805D	2	3	+	+	-1	7	3	-8	-	-5	-2	-4	-8	6	3	2	2	-14	-4	8	-6	-3	12	-2	7	
806A	+	1	1	-3	0	+	-5	-2	-2	-2	+	11	-6	-1	9	6	4	-14	4	-9	-10	-4	12	-4	-2	
806B	+	-1	-1	1	2	+	7	-4	-6	-2	+	-7	10	1	1	-14	-6	-8	-8	-1	-10	-14	6	-12	4	
806C	-	1	-3	-3	-4	-	3	-4	-4	6	+	-5	-4	-1	5	8	4	-2	2	7	-14	-2	-6	-2	16	
806D	-	-3	1	-3	0	-	3	-8	-4	-10	+	7	-12	11	-11	0	0	-6	-6	-1	-14	10	14	6	-4	
806E	-	1	3	-1	0	-	3	2	6	-6	-	-7	-6	-1	3	6	0	-10	-4	-3	2	8	0	0	-10	
806F	-	-1	1	3	2	-	3	0	-6	10	-	3	2	9	3	-6	-10	-8	8	-3	-6	10	-6	0	-12	
807A	0	-	3	2	3	2	-6	-4	-3	6	2	11	-3	-1	9	3	0	-1	-13	0	2	-4	12	-15	-1	+
808A	-	0	-2	-1	0	4	5	5	6	9	4	-8	0	-1	10	-7	4	-3	-2	6	-4	10	4	8	-11	+
808B	-	2	3	2	-2	-3	3	5	3	-8	-3	-2	-4	-8	5	-2	10	12	-2	-1	4	13	0	-14	-14	+
810A	+	+	-	-1	0	5	-6	5	3	0	8	2	3	-4	9	-9	15	-4	-4	6	14	14	-6	18	-16	
810B	+	+	-	-4	3	-4	3	5	6	6	2	-4	-3	11	0	6	-3	-10	5	6	-7	14	12	6	11	
810C	+	+	-	5	0	-1	6	5	-9	0	-4	-10	-3	8	-3	3	9	8	-4	6	2	2	6	6	-16	
810D	+	-	-	-1	-6	2	0	-4	-9	-3	-4	8	3	8	3	-6	-6	-13	-13	6	-4	-10	9	-9	2	
810E	-	+	+	-1	0	5	6	5	-3	0	8	2	-3	-4	-9	9	-15	-4	-4	-6	14	14	6	-18	-16	
810F	-	+	+	-1	6	2	0	-4	9	3	-4	8	-3	8	-3	6	6	-13	-13	-6	-4	-10	-9	9	2	
810G	-	+	+	5	0	-1	-6	5	9	0	-4	-10	3	8	3	-3	-9	8	-4	-6	2	2	-6	-6	-16	
810H	-	-	+	-4	-3	-4	-3	5	-6	-6	2	-4	3	11	0	-6	3	-10	5	-6	-7	14	-12	-6	11	
811A	0	0	1	0	4	-6	6	-5	-6	-1	-10	-7	9	4	-6	1	3	-10	11	-6	-9	-6	-12	9	2	+
812A	-	3	0	+	2	0	2	1	3	+	0	4	7	-2	-7	-11	-4	-2	5	-11	-3	-4	-6	-3	9	
812B	-	-1	-1	-	1	1	2	-4	-6	+	-5	-4	-4	-11	3	3	2	-2	-4	-10	0	-15	-6	4	4	
813A	2	-	2	-1	6	-4	6	-8	5	-6	1	9	0	-8	3	-6	11	3	-9	-13	-12	-4	4	4	-2	+
813B	0	-	0	-1	0	-4	-6	2	-3	-6	5	-7	6	-10	3	-6	3	-1	-13	15	2	8	-12	12	8	-
814A	+	-2	-3	2	-	2	3	2	-6	6	-10	-	-12	8	3	-12	-12	8	-10	-3	-10	11	3	-12	-16	
814B	-	0	-1	-4	+	4	-7	-4	2	4	-6	-	-2	2	-11	0	4	10	8	3	12	-13	1	-16	-10	
815A	0	-2	-	2	0	-4	6	2	-6	-6	-4	-10	-9	5	-9	9	-12	-1	-4	3	-16	2	15	-12	11	-
816A	+	+	0	-2	0	2	+	-4	-2	0	-6	0	-10	-4	4	-2	4	0	-4	2	-14	-6	12	-2	-2	

TABLE 3: HECKE EIGENVALUES 816B-834C

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
816B	+	+	2	4	-4	6	-	-4	4	-6	4	10	-6	-4	8	6	4	-14	12	12	10	4	-4	-6	-6	
816C	+	+	-3	4	-1	-5	-	7	-1	2	6	8	7	1	6	-2	10	8	12	-12	-14	-10	14	8	12	
816D	+	-	3	0	1	3	+	-1	-7	6	2	-4	9	1	-10	-2	6	-12	4	12	-10	-2	14	4	12	
816E	-	+	0	-2	0	2	+	4	6	0	10	8	6	4	-12	6	12	8	4	-6	2	10	-12	-18	14	
816F	-	+	3	4	3	-1	+	1	-9	6	-2	-4	-3	7	6	-6	-6	8	4	-12	2	10	6	0	-16	
816G	-	+	1	0	-5	-5	-	-1	3	2	-2	-8	-5	9	-6	-6	-6	-4	-12	12	-2	-10	2	12	16	
816H	-	+	-2	0	4	-2	-	-4	0	-10	-8	-2	10	-12	0	6	-12	-10	12	0	10	8	-4	-6	-14	
816I	-	-	-1	-4	-3	3	+	-1	-3	-10	-6	-4	5	1	2	-14	6	8	12	-12	2	14	-6	16	0	
816J	-	-	-4	2	0	-6	+	-4	-6	-4	6	-4	-10	4	-4	-2	-12	-4	12	6	2	-10	12	-2	6	
817A	0	-2	-2	-4	-5	-3	-3	+	3	-4	-3	-8	-5	-	8	-7	-4	2	7	4	-10	-4	3	-16	1	
817B	0	-2	-2	4	3	1	-3	-	-5	-4	1	-8	-1	-	-8	-3	4	-6	11	-4	-2	-12	-13	-8	13	
819A	1	+	0	+	0	+	-2	-4	-6	4	0	-10	-12	-4	10	12	14	-10	0	-8	2	0	6	0	-2	
819B	-1	+	0	+	0	+	2	-4	6	-4	0	-10	12	-4	-10	-12	-14	-10	0	8	2	0	-6	0	-2	
819C	2	-	3	+	6	+	-4	5	-3	5	-3	-4	6	-1	-7	9	-8	-10	-6	8	-13	3	-15	-3	7	
819D	-2	-	-1	+	2	+	0	1	-3	5	9	0	-2	-1	-3	9	0	-2	10	12	15	11	-3	17	3	
819E	0	-	3	-	0	-	6	-7	-3	9	5	2	6	-1	-3	9	0	-10	14	6	11	-1	-3	-15	-1	
819F	2	-	1	-	2	-	4	3	9	1	-5	-8	-6	-9	3	-3	0	10	-2	-12	5	-13	11	-1	1	
822A	+	+	-1	2	-5	2	-3	-1	8	-5	3	0	0	-10	-5	-5	-3	-8	-10	-1	-13	13	2	-12	0	+
822B	+	-	0	4	-4	4	2	4	-2	8	-2	-10	6	4	-2	0	4	-2	4	-6	14	6	0	2	-10	+
822C	+	-	3	2	3	2	-3	-1	0	-9	-1	8	0	-10	-9	-9	-3	8	14	3	11	-7	-6	12	8	+
822D	+	-	-1	-3	2	-4	2	1	-5	-3	-8	-7	6	-4	-3	-2	4	-7	4	8	-3	-2	5	-7	-8	-
822E	-	-	2	0	-4	2	2	4	4	-6	4	-10	-6	-4	12	-14	4	14	4	-4	-6	4	-4	2	10	-
822F	-	-	3	-3	6	-4	2	-7	7	-7	4	-7	6	4	-7	6	-12	-15	4	8	-11	6	1	5	-4	-
825A	0	+	+	1	+	1	6	-7	-6	-6	-7	-2	-6	1	0	6	0	5	-5	-12	-14	-4	6	6	-17	
825B	-1	-	+	-4	-	2	2	0	-8	-6	-8	-6	-2	0	-8	-6	-4	6	4	0	14	-4	-12	-6	-2	
825C	0	-	-	-1	+	-1	-6	-7	6	-6	-7	2	-6	-1	0	-6	0	5	5	-12	14	-4	-6	6	17	
826A	+	2	-3	+	6	-2	4	2	3	6	6	9	10	-4	-8	2	-	-10	4	0	-10	6	9	-1	-1	
826B	+	2	3	-	-2	-2	4	6	3	2	-2	-3	-2	0	0	-2	+	-6	4	4	-6	-6	7	-11	-3	
827A	0	-3	0	0	3	0	-4	5	0	2	-5	-4	4	-2	2	-14	-12	6	8	12	-6	11	-9	-8	6	+
828A	-	+	2	2	4	-2	2	-2	+	4	0	2	0	-2	12	-2	12	-14	2	0	6	6	4	-18	6	
828B	-	+	-2	2	-4	-2	-2	-2	-	-4	0	2	0	-2	-12	2	-12	-14	2	0	6	6	-4	18	6	
828C	-	-	2	-4	-2	-5	-4	-2	+	7	-3	2	9	-8	-9	-2	0	-2	14	3	-3	-6	-8	-12	0	
828D	-	-	0	2	0	-1	6	2	-	3	5	8	-3	8	-9	-6	12	14	8	15	-7	-10	-6	0	-10	
829A	0	-3	-3	4	0	1	3	0	-8	-4	0	-4	2	8	10	-6	-3	5	-12	-8	-8	8	-6	-6	13	+
830A	+	1	-	5	3	-4	3	8	0	-9	-1	-7	-6	2	12	0	-3	-1	14	-12	-4	14	+	-12	2	
830B	-	-1	-	-3	-5	2	-3	-6	-4	9	-3	-3	-2	0	0	-10	13	5	-10	-6	-16	14	+	8	-8	
830C	-	-3	-	-3	3	-4	-1	0	-8	-5	-5	9	-6	-10	-8	0	-11	-13	10	0	16	2	+	16	2	
831A	-1	-	0	-2	-3	3	-2	-2	0	3	-5	-10	-3	1	2	-12	10	-12	0	12	-10	-6	-2	3	-6	-
832A	+	1	-1	-3	2	+	-3	2	-4	-2	-4	-5	-12	7	9	-4	6	4	-10	15	-2	8	-4	2	10	
832B	+	-1	-1	3	-2	+	-3	-2	4	-2	4	-5	-12	-7	-9	-4	-6	4	10	-15	-2	-8	4	2	10	
832C	+	-1	3	-1	-6	+	-3	-2	0	-6	-4	7	0	1	3	0	6	-8	-14	-3	2	8	-12	-6	-10	
832D	+	0	-2	-2	2	-	6	6	8	-2	10	6	-6	-4	-2	-6	10	2	-10	10	2	-4	6	-6	2	
832E	+	-1	1	5	2	-	-3	2	4	6	-4	-11	8	1	9	12	-6	0	-6	7	-2	12	16	-10	-10	
832F	+	3	1	1	2	-	-3	-6	-4	-2	4	-3	0	5	13	-12	10	8	2	-5	-10	-4	0	6	14	
832G	-	1	3	1	6	+	-3	2	0	-6	4	7	0	-1	-3	0	-6	-8	14	3	2	-8	12	-6	-10	
832H	-	0	-2	2	-2	-	6	-6	-8	-2	-10	6	-6	4	2	-6	-10	2	10	-10	2	4	-6	-6	2	
832I	-	1	1	-5	-2	-	-3	-2	-4	6	4	-11	8	-1	-9	12	6	0	6	-7	-2	-12	-16	-10	-10	
832J	-	-3	1	-1	-2	-	-3	6	4	-2	-4	-3	0	-5	-13	-12	-10	8	-2	5	-10	4	0	6	14	
833A	-1	0	2	-	0	2	+	4	4	6	-4	-2	6	4	0	6	12	10	4	-4	6	12	4	-10	-2	
834A	+	-	2	0	0	-2	6	4	0	2	8	-10	2	-4	8	10	-4	-6	-4	8	2	16	0	-6	-14	+
834B	+	-	2	0	3	1	-3	1	6	8	-10	5	2	5	-7	-11	14	0	-4	8	-4	-8	12	12	-2	+
834C	+	-	-3	1	1	-5	-4	4	0	-9	-5	-2	-10	10	-4	6	-14	-14	13	7	-10	5	3	3	6	-

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
834D	-	+	0	4	0	-2	-2	8	-4	8	8	-2	6	4	-10	6	-12	8	-4	2	10	-16	0	10	-18	+
834E	-	+	0	-2	-5	-5	-3	1	8	-8	-2	-1	4	5	3	-9	-4	-4	-2	12	-14	-2	0	8	0	-
834F	-	+	-3	1	1	-5	0	-8	-4	7	-5	-10	-2	-10	12	6	-10	-10	13	-9	10	13	3	-13	6	-
834G	-	-	-4	-2	-3	-1	-7	-5	4	0	2	3	-8	-1	-7	-1	0	12	-2	12	14	-10	-16	0	-12	+
836A	-	-1	1	0	+	-2	-4	-	1	-2	-5	-3	0	2	0	-10	-3	-6	-3	5	0	-4	-4	1	-1	-
836B	-	2	2	0	-	4	-2	-	-4	8	10	-6	12	-4	-12	-14	-6	6	6	10	6	-16	4	2	14	-
840A	+	+	+	+	4	-2	-6	0	0	-6	0	-6	-10	0	-12	-6	-4	2	8	-4	10	0	12	-10	-14	-
840B	+	+	+	-	0	-2	6	-4	4	6	0	6	6	-4	8	14	-4	-2	12	-12	10	8	-4	6	-14	-
840C	+	+	-	+	0	2	2	0	0	6	4	-2	10	4	0	2	4	6	-12	12	-2	0	4	10	-2	-
840D	+	-	+	+	0	6	-2	4	4	6	0	6	-2	-4	8	-2	-12	6	-4	-12	10	-8	-12	14	2	-
840E	-	+	+	-	-4	2	-2	0	-4	-2	-4	-10	6	-4	-8	-6	-12	6	-12	0	6	0	-12	6	-10	-
840F	-	+	-	+	-4	-2	2	-4	0	-10	0	6	-6	-4	-8	6	-4	-10	4	-16	-14	8	-4	10	10	-
840G	-	+	-	-	4	-2	-6	4	8	-2	0	-2	10	4	0	14	12	-2	-4	0	2	-8	-4	-6	-6	-
840H	-	-	+	+	-4	-6	-2	-8	4	6	4	-2	-2	-12	0	2	-4	6	-4	8	6	-16	-4	-18	6	-
840I	-	-	+	-	0	2	2	4	0	2	4	2	-2	8	-4	-2	12	-14	8	-8	-2	8	-4	-18	-2	-
840J	-	-	-	+	4	-2	2	4	0	-2	8	-2	2	4	0	-10	-12	6	12	0	-6	-8	4	2	-14	-
842A	+	-2	2	-3	2	3	-3	-1	-6	6	-1	7	-4	-1	-12	-7	5	-13	-14	-2	-6	-17	-4	6	-15	+
842B	-	-2	-2	1	2	1	-3	-7	-6	2	-5	-11	0	1	-8	3	3	-7	10	14	-2	3	-4	6	-7	-
843A	1	+	0	3	-5	-4	6	-4	3	-8	-9	-10	-3	13	-7	4	6	-12	4	-8	4	-5	3	6	-8	+
845A	1	-2	-	4	-2	+	2	6	-6	2	10	2	6	10	-4	2	-6	2	4	-6	6	-12	16	-2	2	-
846A	+	-	4	-4	0	-2	6	6	4	-4	2	-6	12	-2	+	6	4	2	10	-8	-2	-12	-12	18	14	-
846B	+	-	0	0	-2	-4	2	-2	-4	-4	4	2	-6	6	-	-2	-12	2	2	-8	-14	-16	16	10	-14	-
846C	+	-	-2	0	0	2	-2	0	0	-2	-8	-2	-2	-8	-	2	4	-10	-8	0	10	0	-12	-10	2	-
847A	0	1	3	+	-	4	6	-2	3	6	5	11	-6	-8	0	-6	-9	10	5	9	-2	10	-12	-3	-1	-
847B	0	-3	-1	-	-	4	-2	6	-5	-10	1	-5	2	8	8	-6	3	2	-3	1	-10	-6	-12	-15	-5	-
847C	-1	2	-2	-	-	-4	-4	0	-4	6	10	-6	-4	-12	-10	-6	2	0	8	-12	8	-8	0	-6	-10	-
848A	-	1	-4	0	4	1	5	7	-1	5	4	1	-10	10	6	+	6	4	-4	-15	-8	-1	3	2	17	-
848B	-	-1	0	4	0	5	-3	1	-3	9	4	5	6	10	-6	+	-6	8	4	3	-4	13	-3	18	-7	-
848C	-	2	3	-2	3	-4	3	4	9	6	-5	-10	6	1	0	+	-15	-10	4	-12	8	-11	6	9	-13	-
848D	-	-2	2	0	4	-2	2	-2	2	2	-2	10	2	4	12	+	12	10	2	-6	10	-10	6	-10	14	-
848E	-	3	0	4	0	-3	-3	5	-7	-7	-4	5	6	2	2	+	2	-8	12	-1	-4	1	1	-14	1	-
848F	-	1	-2	2	-2	-7	-3	-5	3	9	8	-3	2	-4	-10	-	2	-10	-4	9	-6	-5	11	-10	-3	-
848G	-	-2	1	2	-5	-4	3	4	3	-6	-7	-6	2	-7	-4	-	-7	2	-16	-12	-12	7	14	17	3	-
849A	-1	+	-4	1	5	-1	-4	4	3	-9	-4	8	-9	12	-6	-6	-3	-13	-6	8	-2	16	16	-5	-7	+
850A	+	-1	+	-2	0	1	-	-1	6	-3	5	-8	6	10	3	3	3	11	-2	9	-11	8	12	15	7	-
850B	+	2	+	4	6	-2	-	-4	0	0	-4	4	6	-8	0	6	0	-4	-8	0	-2	8	0	-6	-14	-
850C	+	1	-	0	-6	3	-	-7	-8	-5	5	8	0	-4	3	9	5	-3	-2	-15	-11	8	4	-1	-9	-
850D	+	1	-	-5	4	3	-	-2	-8	0	-5	-12	-10	-4	-2	-1	0	2	8	5	4	-17	-16	-6	16	-
850E	+	-3	-	-1	-4	3	-	6	0	0	-9	4	6	12	-10	-9	0	-14	-8	-15	-12	3	0	-6	16	-
850F	-	-1	+	5	4	-3	+	-2	8	0	-5	12	-10	4	2	1	0	2	-8	5	-4	-17	16	-6	-16	-
850G	-	2	+	2	-2	6	+	-8	2	6	-2	-6	2	4	-4	10	0	-10	-8	14	-10	-14	4	6	14	-
850H	-	2	+	-2	6	-2	+	8	6	-6	2	-2	-6	4	-12	-6	0	2	-8	-6	-2	-10	-12	6	-2	-
850I	-	3	+	1	-4	-3	+	6	0	0	-9	-4	6	-12	10	9	0	-14	8	-15	12	3	0	-6	-16	-
850J	-	-3	+	-2	-4	3	+	3	6	9	-3	8	-6	-6	13	9	15	7	2	9	3	0	-12	-9	-7	-
850K	-	-1	+	-2	0	-5	-	-1	-6	-9	-1	4	-6	-2	9	9	3	-7	-14	3	-11	8	0	-9	7	-
850L	-	-1	-	0	-6	-3	+	-7	8	-5	5	-8	0	4	-3	-9	5	-3	2	-15	11	8	-4	-1	9	-
851A	-2	1	0	-3	1	4	0	2	+	-8	2	+	3	-6	-13	7	14	-10	-12	-7	-9	-10	11	-4	-16	-
854A	+	1	-2	+	3	0	3	-3	-1	-6	-6	-6	0	-11	-3	4	0	+	1	-1	0	-5	9	3	2	-
854B	+	1	0	-	-3	-4	-3	-7	3	6	-4	2	6	-1	3	-6	6	-	-13	3	2	-1	9	-3	-10	-
854C	-	-1	0	+	-5	0	-3	-1	1	-2	-4	-2	10	-7	-7	6	6	-	-11	-7	-2	-11	-1	13	10	-
854D	-	-1	-2	-	1	-4	-1	-5	-3	2	-2	-2	-12	-1	-5	0	4	+	3	-3	-4	1	7	7	-2	-
855A	-1	-	+	4	-4	2	-2	+	4	2	0	-6	6	8	12	14	-4	14	-4	0	-14	16	0	6	-10	-

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
855B	-1	-	-	-2	2	-4	-2	+	4	-4	0	0	0	-10	-12	2	-4	2	-16	0	-2	-8	12	0	-16	
855C	1	-	-	-2	6	0	6	-	8	-4	0	4	0	-2	8	-2	-12	2	-8	-16	14	8	0	0	-12	
856A	+	1	0	-4	-3	5	-4	5	-9	2	-10	-3	-1	6	8	-3	-2	-1	4	-8	14	-17	0	5	10	+
856B	+	1	-4	2	5	-5	-6	1	-3	-6	-8	3	-5	0	8	-13	-6	-11	10	10	8	13	-12	1	18	+
856C	-	-1	0	0	3	-3	0	-5	1	2	-2	-11	-9	6	0	-11	-14	15	8	-12	-6	9	0	-11	2	-
856D	-	2	-3	0	-6	0	-6	-2	1	8	-2	4	-3	9	-9	-2	-5	0	11	0	-12	15	12	1	-16	-
858A	+	+	2	0	-	+	2	4	8	-6	-4	-6	-2	8	12	14	12	2	12	4	6	-8	-12	10	2	
858B	+	-	0	-4	+	-	0	-4	0	0	-10	2	-6	-10	0	6	0	2	2	12	-10	-10	-12	12	-10	
858C	+	-	-2	4	-	-	0	2	-2	2	-2	12	6	0	0	4	12	-6	-4	4	4	-8	-12	10	2	
858D	+	-	3	-1	-	-	0	2	3	-3	8	2	-9	5	0	-6	-3	-1	11	-6	-1	2	18	0	2	
858E	-	+	2	4	+	+	2	-4	4	-2	0	-2	10	-4	-4	2	-4	10	-4	12	2	-4	-12	2	2	
858F	-	+	-3	1	+	-	-8	-6	-1	1	0	6	-11	-11	-12	2	7	11	9	-2	-11	6	14	0	6	
858G	-	+	-1	-3	-	+	-4	-2	-1	-9	-4	-6	1	11	0	-10	-3	5	3	10	9	10	-6	-8	2	
858H	-	+	-2	0	-	-	6	0	4	6	4	2	10	-4	8	-6	4	-2	-4	-8	14	-8	4	-6	2	
858I	-	+	4	0	-	-	0	0	-8	0	-2	2	-2	2	8	6	-8	10	2	4	-10	-14	4	0	-10	
858J	-	-	-3	5	+	-	0	2	3	-3	8	-10	-3	5	12	-6	-9	-13	5	6	-7	-10	6	0	-10	
858K	-	-	-1	1	-	+	4	6	3	-5	4	10	-7	-5	-8	-2	-3	13	-9	2	-3	10	-14	-8	-14	
858L	-	-	2	4	-	+	-8	-6	-6	-2	-2	4	2	4	4	-8	12	-2	12	-16	0	-8	4	10	10	
858M	-	-	4	-4	-	+	4	-4	8	0	-6	-10	-2	-10	12	-2	-8	-2	-14	-8	2	10	-4	12	6	
861A	-1	+	2	-	0	2	-2	-4	8	-6	4	6	+	12	0	-6	12	14	-4	4	2	8	4	-10	6	
861B	1	-	-1	+	-6	3	-6	6	-9	5	-8	-7	-	2	3	9	-12	6	13	-6	4	-11	2	-4	-9	
861C	-1	-	3	+	-6	-7	-6	-6	5	3	0	1	-	6	-5	-9	-8	-2	9	-10	4	9	-2	16	13	
861D	-1	-	-3	-	-2	5	-2	-2	-3	1	-8	1	+	-10	-9	-11	-4	2	-5	-6	4	11	14	8	1	
862A	+	1	-1	-2	-1	6	-2	1	-3	-9	-2	-10	2	6	0	-3	-9	2	-8	0	4	6	-2	0	3	+
862B	+	-3	-1	-2	3	-2	-2	5	9	-1	2	2	-6	6	-12	-11	-5	2	4	-12	0	10	2	-4	-5	+
862C	-	0	2	4	0	4	-2	-4	0	2	-4	-4	6	6	0	-2	4	-10	-14	12	-6	4	2	2	-2	+
862D	-	1	-3	2	3	2	6	5	3	9	-4	-4	-6	-10	6	3	-9	2	-10	-6	2	-4	12	6	-13	+
862E	-	-1	1	-2	-3	-6	-2	-5	9	5	-8	8	2	-6	-2	-1	-15	2	-2	2	14	0	-16	10	3	-
862F	-	-1	-3	2	5	-6	-6	-5	-3	1	8	-8	-6	-6	-10	-13	9	2	2	10	2	12	0	-14	19	-
864A	+	+	-1	-3	3	0	4	-6	-6	-2	-9	-2	-10	-6	-6	13	12	8	-6	-12	9	0	3	14	-9	
864B	+	+	2	-3	-6	-3	-2	3	-6	-8	0	7	8	12	-6	4	-6	-1	3	-12	-15	-9	12	-10	9	
864C	+	+	-2	1	-2	1	-6	-5	6	-8	8	-5	-8	-4	-10	-4	14	3	-13	-4	9	11	-12	2	1	
864D	+	-	-1	3	-3	0	4	6	6	-2	9	-2	-10	6	6	13	-12	8	6	12	9	0	-3	14	-9	
864E	+	-	2	-1	-2	1	6	5	6	8	-8	-5	8	4	-10	4	14	3	13	-4	9	-11	-12	-2	1	
864F	+	-	2	3	6	-3	-2	-3	6	-8	0	7	8	-12	6	4	6	-1	-3	12	-15	9	-12	-10	9	
864G	-	+	1	3	3	0	-4	6	-6	2	9	-2	10	6	-6	-13	12	8	6	-12	9	0	3	-14	-9	
864H	-	+	2	1	2	1	6	-5	-6	8	8	-5	8	-4	10	4	-14	3	-13	4	9	11	12	-2	1	
864I	-	+	-2	-3	6	-3	2	3	6	8	0	7	-8	12	6	-4	6	-1	3	12	-15	-9	-12	10	9	
864J	-	-	1	-3	-3	0	-4	-6	6	2	-9	-2	10	-6	6	-13	-12	8	-6	12	9	0	-3	-14	-9	
864K	-	-	-2	-1	2	1	-6	5	-6	-8	-8	-5	-8	4	10	-4	-14	3	13	4	9	-11	12	2	1	
864L	-	-	-2	3	-6	-3	2	-3	-6	8	0	7	-8	-12	-6	-4	-6	-1	-3	-12	-15	9	12	10	9	
866A	-	-2	0	-1	0	-1	-3	-4	0	-6	5	-7	-9	-1	3	-9	0	-4	-1	-3	14	-10	15	12	8	-
867A	0	+	-3	4	3	-1	+	-1	-9	-6	-2	4	3	-7	-6	-6	6	-8	-4	-12	-2	10	-6	0	16	
867B	-1	+	0	-4	4	2	+	4	-4	0	4	-8	-8	4	-8	-6	12	-8	12	-12	0	-4	-12	-10	-16	
867C	2	+	-3	2	-5	-1	+	-5	-1	-6	10	-2	-5	1	-2	6	0	10	-12	0	6	-4	6	-10	8	
867D	-1	-	0	4	-4	2	+	4	4	0	-4	8	8	4	-8	-6	12	8	12	12	0	4	-12	-10	16	
867E	2	-	3	-2	5	-1	+	-5	1	6	-10	2	5	1	-2	6	0	-10	-12	0	-6	4	6	-10	-8	
869A	1	1	1	-5	+	5	0	-4	0	-6	-8	-4	0	4	1	-2	11	14	-6	-7	10	+	-6	-3	-7	
869B	-2	1	1	-2	+	2	0	2	-9	0	-5	5	12	-8	4	-2	-13	-10	9	-1	-14	+	12	-9	-7	
869C	-1	-2	2	4	-	2	0	-4	0	0	4	2	0	4	2	2	-2	-4	12	10	-2	+	12	-6	14	
869D	1	-1	1	1	-	1	-8	0	-4	6	-8	0	-8	-12	-1	6	5	-6	2	-9	10	-	6	-3	17	
870A	+	+	-	0	0	-6	-2	-4	4	-	4	-6	-2	-12	0	-2	4	-6	0	0	-10	4	-12	-2	6	

TABLE 3: HECKE EIGENVALUES 870B–886D

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
870B	+	-	+	2	-6	-4	-6	-4	0	-	2	2	0	-4	0	6	-12	-4	8	0	2	-10	18	0	-10	
870C	+	-	-	-4	0	-4	-6	2	-6	+	-4	2	-12	8	0	-6	12	-4	14	0	2	-4	-12	-12	-10	
870D	+	-	-	0	0	-2	6	0	8	-	-4	2	2	4	0	6	-4	14	-12	0	-2	-4	12	-14	-2	
870E	-	+	+	-2	-2	0	-2	-8	0	-	-10	6	-8	-4	0	-2	12	-12	8	0	6	-6	6	8	2	
870F	-	+	-	-4	-4	-4	-2	2	-6	+	0	-2	0	0	0	10	-12	0	-10	16	-10	-16	-4	0	2	
870G	-	+	-	4	0	-2	2	0	-4	-	4	6	-6	4	8	-10	4	-2	-8	0	10	-4	8	-6	2	
870H	-	-	-	2	-2	-4	6	4	0	+	-2	-10	0	-4	-8	10	0	-8	8	8	10	10	-6	8	-18	
870I	-	-	-	-2	2	4	-2	0	4	+	2	-2	-8	4	8	-6	0	-8	-12	-8	-6	-10	14	0	-2	
871A	2	2	2	2	0	+	-5	-5	1	3	-2	11	-12	10	11	14	-3	10	-	-8	-11	-4	12	11	-8	
872A	-	-2	1	0	-3	4	-2	-5	1	9	0	-8	-12	-10	-9	4	-4	5	-4	0	-9	-8	-6	13	5	-
873A	1	-	0	2	4	-2	8	-2	4	0	8	10	12	-8	-8	2	8	-10	2	-8	6	4	-8	-10	+	
873B	1	-	2	-4	-4	6	-2	-8	-4	-6	8	-2	-10	-4	0	10	-8	14	8	4	-6	-8	-8	-10	-	
873C	2	-	-3	-2	0	-4	-6	6	0	-7	7	4	-5	1	10	-10	5	5	-14	-15	7	-5	9	8	-	
873D	-2	-	-1	2	-4	0	-2	-2	8	3	-1	4	-7	-7	-6	-2	7	5	-10	-5	-9	-5	-5	-16	-	
874A	+	3	1	2	-1	-2	4	+	-	-1	0	-4	6	1	3	-4	3	-2	-16	-12	3	-9	11	-3	9	
874B	+	-3	1	2	5	-2	-2	+	-	-1	6	2	0	7	-3	2	9	10	2	12	-9	-15	17	9	-15	
874C	+	-1	-1	-2	5	6	-4	-	-	-9	-8	8	-2	-5	-9	8	-9	2	-4	-16	11	-11	1	-13	-1	
874D	-	1	-3	-2	-5	2	-4	+	-	-3	8	-4	2	5	-13	8	1	-2	8	0	11	-15	-9	3	-1	
874E	-	-1	1	-2	-3	-6	-2	+	-	5	2	-2	-8	-1	13	-6	-5	2	-2	-8	-1	-5	9	-5	3	
874F	-	1	3	2	-3	2	0	-	-	-3	-4	-4	6	-13	3	0	9	2	8	-12	11	-1	9	-3	17	
876A	-	+	1	-4	0	4	3	-7	-6	-6	2	-3	-8	-2	7	-11	1	-5	5	-4	-	1	-15	-18	13	
876B	-	-	-1	-2	-4	-2	1	-7	0	6	-2	-3	-6	2	-7	3	11	7	-3	-2	+	-3	-9	6	-19	
880A	+	0	+	2	+	-4	-4	0	0	-6	0	-2	6	-2	0	-10	-12	-6	12	-16	4	4	-2	6	-2	
880B	+	0	+	2	-	0	0	8	8	10	-8	-10	-2	6	8	14	4	10	-4	0	-8	4	-10	6	-10	
880C	+	0	-	-4	-	6	-6	-4	-4	-2	-8	-10	10	0	-4	-10	4	-2	8	0	-14	16	8	-6	2	
880D	+	-3	-	-1	-	-6	3	5	2	-5	-5	-1	-2	-12	2	-13	-2	1	-16	-15	10	-2	14	9	-16	
880E	-	-1	+	-5	+	2	3	7	6	-3	7	-7	6	-8	-6	-3	6	-1	-8	-3	2	10	6	9	-4	
880F	-	-1	+	1	-	2	-3	1	-6	-9	-5	5	-6	-8	-6	9	-6	5	-8	9	-10	-14	6	-15	8	
880G	-	1	-	-3	+	-6	-7	-5	6	5	3	3	2	-4	2	-1	10	7	-8	-7	14	-10	6	-15	-12	
880H	-	-2	-	0	+	0	-4	4	-6	2	0	-6	-10	-4	-10	2	4	-14	-2	-4	-4	8	-12	6	6	
880I	-	0	-	0	-	2	6	4	-4	6	8	-2	2	-4	12	-2	-4	-10	16	-8	14	-8	4	10	10	
880J	-	2	-	4	-	-4	0	4	6	-6	-8	2	6	-8	-6	-6	12	2	10	12	-16	-8	0	6	14	
882A	+	+	-3	+	3	2	-6	2	6	-9	-7	-10	0	-4	-12	3	3	-4	2	0	2	5	-9	6	-13	
882B	+	+	3	-	3	-2	6	-2	6	-9	7	-10	0	-4	12	3	-3	4	2	0	-2	5	9	-6	13	
882C	+	-	-1	+	-5	0	4	8	4	5	3	-4	0	2	6	9	11	-6	-2	-2	10	3	7	6	7	
882D	+	-	1	-	-5	0	-4	-8	4	5	-3	-4	0	2	-6	9	-11	6	-2	-2	-10	3	-7	-6	-7	
882E	+	-	-2	-	4	-6	2	4	-8	2	0	-10	-6	-4	0	-6	4	-6	4	-8	-10	0	-4	-6	14	
882F	-	+	3	+	-3	2	6	2	-6	9	-7	-10	0	-4	12	-3	-3	-4	2	0	2	5	9	-6	-13	
882G	-	+	-3	-	-3	-2	-6	-2	-6	9	7	-10	0	-4	-12	-3	3	4	2	0	-2	5	-9	6	13	
882H	-	-	-3	+	-3	-4	0	-4	0	-9	-1	8	0	-10	6	3	-3	-10	-10	6	2	-1	9	-6	-1	
882I	-	-	0	-	0	4	6	-2	0	6	4	2	6	8	-12	-6	-6	-8	-4	0	-2	8	-6	-6	10	
882J	-	-	3	-	-3	4	0	4	0	-9	1	8	0	-10	-6	3	3	10	-10	6	-2	-1	-9	6	1	
882K	-	-	4	-	4	-4	0	-4	0	-2	-8	-6	0	4	-8	10	4	4	4	-8	16	-8	-12	8	-8	
882L	-	-	-4	-	4	4	0	4	0	-2	8	-6	0	4	8	10	-4	-4	4	-8	-16	-8	12	-8	8	
885A	2	+	+	2	3	3	1	3	6	8	-4	-1	-2	-7	4	11	-	-8	-9	-12	3	11	-2	3	-18	
885B	1	+	-	0	-4	6	-6	-4	-8	6	-4	-10	-6	0	-8	6	-	6	0	-8	10	0	12	-18	2	
885C	0	-	-	0	-5	-5	-3	-5	2	-2	0	3	6	1	-12	-9	+	2	-9	8	-9	-9	10	-1	10	
885D	-2	-	-	-2	-3	-1	3	-5	-6	0	-8	3	2	-11	8	9	+	-8	3	-8	-1	-5	-6	5	-2	
886A	+	0	-4	3	3	1	3	1	-9	-6	-7	-1	6	-8	-8	-4	-6	-15	2	6	2	14	1	-17	8	+
886B	+	-2	1	2	-5	0	-2	7	0	-10	2	-2	7	4	-1	-9	-4	-10	-16	3	-14	-17	12	-13	12	+
886C	+	2	4	3	-3	-1	3	-1	7	-8	-11	1	-6	-8	2	4	6	-1	4	-8	8	-10	7	-1	10	-
886D	-	0	0	-3	-3	-7	3	-1	1	2	-1	-1	6	8	0	12	-6	9	-14	-14	-14	-2	-9	-9	4	-

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
886E	-	0	-3	0	3	-4	-6	-1	-2	-10	8	2	-9	-4	3	3	0	0	4	7	4	-5	12	-9	-8	-
888A	+	+	-4	-1	-3	-5	7	5	1	-8	6	-	-2	12	-2	-1	-4	14	14	4	-11	10	5	-9	-10	-
888B	+	-	-2	0	-4	-2	-6	0	-4	6	4	-	-6	-8	-8	6	0	14	4	-16	-6	-4	-4	18	2	-
888C	-	+	0	0	0	-6	-4	4	6	-8	-4	-	10	-8	-8	-6	2	-10	-12	-8	-10	8	8	16	6	-
888D	-	-	4	0	0	2	0	0	-6	-4	0	-	2	-12	8	2	6	-10	-4	-8	6	4	8	12	-2	-
890A	+	0	+	2	4	-6	-6	-2	6	0	-4	-10	2	0	4	-10	-10	-4	4	8	-14	0	0	+	-10	-
890B	+	-2	+	-2	4	4	2	-4	-6	6	4	-12	-2	-6	-8	6	0	6	-8	0	-6	-8	-6	+	-10	-
890C	+	2	+	4	0	0	2	6	-4	4	4	-4	-2	6	0	14	-6	-8	-8	-16	14	0	6	-	-2	-
890D	+	1	-	-4	-1	4	0	-6	-4	-2	-8	-2	-4	11	-5	11	-12	-7	0	6	-12	-8	7	-	-8	-
890E	+	-2	-	2	-4	-2	6	-6	2	-2	-2	-2	2	-10	4	2	-6	-10	-12	-12	-6	4	-2	-	-2	-
890F	-	-3	+	0	3	-4	-4	2	-4	2	-8	-10	-12	7	-3	1	0	-5	-4	-14	-4	12	11	-	16	-
890G	-	-1	-	-2	-3	-6	-2	0	4	-10	-8	8	2	9	3	-11	0	7	-2	-8	-16	0	9	+	-2	-
890H	-	0	-	4	-4	6	2	0	4	-2	-4	-2	-6	8	-8	6	-8	-2	4	0	10	8	0	-	2	-
891A	-1	+	-1	-2	+	7	-1	6	-8	-3	-2	-3	-10	0	4	-6	10	-9	2	0	-11	-4	6	-15	14	-
891B	0	+	-3	-4	-	2	-6	2	3	-6	8	2	0	8	3	3	0	8	-13	0	2	2	-18	3	2	-
891C	1	+	1	-2	-	7	1	6	8	3	-2	-3	10	0	-4	6	-10	-9	2	0	-11	-4	-6	15	14	-
891D	1	+	1	4	-	-2	4	6	-4	6	7	3	-2	6	-7	-9	-7	0	11	9	4	8	-12	6	-19	-
891E	-2	+	-2	4	-	4	4	-6	-1	0	1	3	-2	12	-7	3	11	0	-4	15	-8	-10	12	3	17	-
891F	0	-	3	-4	+	2	6	2	-3	6	8	2	0	8	-3	-3	0	8	-13	0	2	2	18	-3	2	-
891G	-1	-	-1	4	+	-2	-4	6	4	-6	7	3	2	6	7	9	7	0	11	-9	4	8	12	-6	-19	-
891H	2	-	2	4	+	4	-4	-6	1	0	1	3	2	12	7	-3	-11	0	-4	-15	-8	-10	-12	-3	17	-
892A	-	3	0	4	1	0	-3	-6	-1	-5	-8	-1	11	-6	8	9	11	8	-7	-12	-5	-4	0	-9	-12	+
892B	-	1	0	-4	3	-4	-3	2	-3	3	-4	-1	3	-10	-12	9	9	8	11	-12	-13	-4	12	15	8	-
892C	-	-1	2	-2	-3	6	-7	0	-9	7	-4	-1	-9	0	4	1	-9	6	-11	0	-5	4	-12	-9	-18	-
894A	+	+	0	0	-2	-1	1	1	-1	-8	6	4	-12	12	-6	0	6	-14	-13	-7	-7	-11	-12	-10	18	+
894B	+	+	-3	0	1	5	-2	1	2	-5	-6	-2	-3	-6	3	-6	-9	10	8	-4	-1	-17	9	-1	-18	+
894C	+	-	3	2	-3	5	0	-1	6	-3	-10	8	-9	2	-9	6	-9	8	-4	6	11	17	-15	-3	-16	+
894D	+	-	1	-2	-3	-5	-4	-5	2	-1	-2	0	3	2	9	2	7	-12	12	-6	3	-5	9	-7	-8	-
894E	-	+	0	-4	-2	1	-3	1	-3	-8	-10	-4	12	4	-6	12	-6	-10	11	-13	17	-5	-4	-18	6	-
894F	-	+	-3	2	-5	1	0	-5	-6	-5	2	-4	-3	-2	-3	-6	9	8	-4	2	-13	1	-1	15	12	-
894G	-	-	-3	-4	-1	-5	2	1	-6	-5	-2	2	-5	-10	11	10	1	-2	-8	0	-1	-7	15	9	6	+
895A	-1	1	+	0	3	-2	0	-2	-2	3	-4	-5	-8	3	-3	-5	-14	9	-12	5	5	-11	4	-11	18	+
895B	-1	3	+	-4	1	2	4	6	-6	3	4	11	0	1	7	3	10	-7	-12	-9	13	-17	-4	5	-14	-
896A	+	0	0	+	2	-4	-2	4	-4	-6	-8	2	-2	10	0	-2	-8	-8	2	0	-14	-4	12	-6	6	-
896B	+	0	0	+	-2	4	-2	-4	-4	6	-8	-2	-2	-10	0	2	8	8	-2	0	-14	-4	-12	-6	6	-
896C	+	0	0	-	2	4	-2	4	4	6	8	-2	-2	10	0	2	-8	8	2	0	-14	4	12	-6	6	-
896D	-	0	0	-	-2	-4	-2	-4	4	-6	8	2	-2	-10	0	-2	8	-8	-2	0	-14	4	-12	-6	6	-
897A	1	+	-4	2	-2	-	2	6	+	2	4	4	-6	0	0	6	0	-2	-14	12	14	8	10	16	4	-
897B	-1	+	2	4	0	-	2	0	+	-2	8	-6	2	4	-8	6	-4	14	8	8	2	16	0	-10	-2	-
897C	-1	+	-2	0	0	-	6	-4	-	6	0	2	-6	-8	-8	2	12	-10	-4	-16	2	-4	0	-6	-10	-
897D	1	-	0	-2	-6	-	-6	2	+	2	4	-8	2	0	0	-2	8	14	-10	-12	6	0	-18	4	0	-
897E	-1	-	2	-4	-4	-	6	-8	+	-10	0	-2	2	8	-8	2	-12	-10	0	-8	10	-12	4	-2	-6	-
897F	-1	-	-4	2	2	-	-6	-2	+	2	0	-8	-10	8	-8	-10	0	-10	-6	4	-2	0	-2	-8	0	-
898A	+	1	-2	1	-4	6	0	-5	1	-6	-6	9	-9	-7	8	-10	-4	8	0	-10	2	2	11	-1	1	+
898B	+	2	-2	0	3	1	2	0	8	10	0	1	7	8	1	6	-3	2	-13	5	-2	-11	-6	-5	17	-
898C	-	2	2	0	0	4	-2	-6	-8	8	0	4	2	2	8	-6	0	-10	-4	-8	10	4	-6	14	2	+
898D	-	-1	-2	3	-6	-2	2	-3	-1	-2	-6	1	-5	-1	-2	0	0	-10	2	-4	16	-2	-3	-17	5	-
899A	1	-2	1	2	0	2	-3	-5	-1	+	+	-3	-10	-6	-9	-10	8	-7	-2	14	-11	4	9	13	2	-
899B	2	1	2	5	-3	2	-3	-2	4	-	+	-6	-2	0	6	-8	4	11	7	-8	1	-8	12	-7	8	-
900A	-	+	+	1	0	7	0	-7	0	0	11	10	0	13	0	0	0	-1	-11	0	10	-4	0	0	19	-
900B	-	+	+	4	0	-2	0	8	0	0	-4	10	0	-8	0	0	0	14	16	0	10	-4	0	0	-14	-
900C	-	+	-	-1	0	-7	0	-7	0	0	11	-10	0	-13	0	0	0	-1	11	0	-10	-4	0	0	-19	-

TABLE 3: HECKE EIGENVALUES 900D–912K

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
900D	-	-	+	1	-6	-5	-6	5	-6	6	-1	-2	0	1	6	-12	6	-13	-11	0	-2	8	-6	0	7	
900E	-	-	+	-2	0	-2	-6	-4	6	-6	-4	-2	-6	10	-6	-6	-12	2	-2	12	-2	8	6	6	-2	
900F	-	-	-	-1	-6	5	6	5	6	6	-1	2	0	-1	-6	12	6	-13	11	0	2	8	6	0	-7	
900G	-	-	-	4	4	0	-4	0	-4	6	4	-8	10	4	4	12	-4	2	-4	0	-8	-12	-4	10	8	
900H	-	-	-	-4	4	0	4	0	4	6	4	8	10	-4	-4	-12	-4	2	4	0	8	-12	4	10	-8	
901A	-1	0	0	2	-6	6	+	4	-8	2	4	2	0	-8	-8	+	4	-8	-12	-8	-4	-4	-4	-14	-2	
901B	-1	2	0	-4	0	2	+	0	2	-2	-6	-10	-4	-12	8	+	-12	-8	8	6	16	14	-16	6	6	
901C	0	1	-3	-4	6	5	-	-4	0	0	8	8	-3	11	-3	+	12	-10	-4	3	11	-1	-6	15	2	
901D	-1	0	3	-1	0	3	-	1	4	-4	4	8	-6	4	10	+	-2	10	9	16	-1	2	11	-5	-8	
901E	2	-3	1	-2	0	1	-	-4	-8	-6	-8	4	-3	7	9	-	12	2	-14	-13	3	3	-4	7	8	
901F	-2	-1	3	2	0	-7	-	0	-8	-10	0	8	7	-9	1	-	-12	-2	2	-15	1	17	-8	-9	0	
902A	+	-2	3	1	+	-6	-7	1	7	-5	-3	7	+	-12	-6	-14	3	6	-12	8	0	4	12	16	-14	
902B	-	-2	3	5	+	2	-3	5	-9	-9	5	-1	+	8	6	-6	3	-10	8	0	-4	-4	0	12	-10	
903A	0	-	0	+	-3	1	1	-6	1	-10	-7	2	-7	-	8	-7	-8	-8	13	10	-6	-8	11	2	-11	
903B	0	-	0	-	-3	5	-3	2	9	6	5	2	-3	-	0	9	0	8	5	-6	2	8	-9	-6	17	
904A	-	0	0	-4	4	6	-2	-4	-6	0	-8	-4	-10	-12	-2	10	-8	10	12	6	2	-10	12	-6	-2	-
905A	1	2	+	-4	0	-6	6	2	-4	-10	-2	10	-6	-6	8	-2	0	2	2	-14	2	16	0	-6	2	+
905B	1	1	-	2	0	6	3	8	-2	-5	9	6	2	-4	-12	-9	-12	0	7	15	0	-6	-10	0	-3	+
906A	+	+	0	-3	1	4	2	-6	4	-2	-8	7	-1	-8	-1	-9	9	-8	-3	-12	-2	-11	-8	10	-7	+
906B	+	+	3	0	-5	-5	2	-6	-5	-5	1	-2	11	-2	8	12	-12	-14	-3	3	-8	-2	-8	-5	2	+
906C	+	-	0	-1	-3	-4	-6	2	0	-6	-4	11	-3	8	-9	9	-3	8	-13	-12	2	-1	12	6	17	-
906D	+	-	-3	2	-3	-1	0	-4	-3	-3	5	-4	-3	-10	0	-6	0	-10	5	9	2	-4	0	-3	-10	-
906E	-	+	1	0	-3	3	6	6	-3	1	5	6	9	-2	-4	0	-4	2	9	5	4	-10	-16	9	2	+
906F	-	+	4	-3	3	0	6	-6	0	10	-4	-9	-3	4	5	-3	11	-4	-3	8	-2	5	-4	-18	-7	+
906G	-	+	-1	-2	-1	-3	0	0	-3	-9	-3	-8	-3	6	8	6	8	2	-1	-15	-14	-8	16	-3	14	-
906H	-	-	-3	-4	-3	1	-6	2	-1	-3	-7	-2	-1	-2	8	-12	4	-2	15	-1	16	-6	-8	15	2	+
906I	-	-	-1	2	3	-5	0	8	7	-1	1	-4	-5	-10	12	2	0	6	-11	3	-2	4	-8	-5	-18	-
909A	0	-	3	0	2	-3	7	-5	5	-6	7	10	-6	4	7	4	10	-2	10	9	-8	7	-2	8	-10	+
909B	2	-	1	-2	6	1	5	7	3	6	-1	-10	2	-12	-11	-4	-4	10	-2	-1	2	11	-8	-14	-10	+
909C	0	-	1	-2	2	1	-3	-5	-1	4	-9	-2	-8	-8	-7	2	14	4	2	-13	8	-9	4	-14	2	-
910A	+	0	+	-	4	+	2	-4	8	-2	4	10	-6	0	8	2	12	-2	-4	12	-2	-8	4	2	14	
910B	+	1	+	-	-3	-	-6	2	-3	-6	-7	5	9	2	-9	0	0	-13	-13	12	-13	5	6	18	5	
910C	+	-2	+	-	0	-	0	2	-6	6	8	-10	-12	-4	0	-12	6	2	-4	-6	-10	-16	0	-12	2	
910D	+	0	-	-	-2	+	-4	-4	-4	-2	-2	-2	0	6	8	-4	-12	-2	8	-12	-2	-8	4	-16	2	
910E	+	1	-	-	-3	-	6	2	9	6	5	-7	9	-10	3	0	12	-1	-13	-12	11	17	-6	-6	-19	
910F	-	0	+	+	-6	-	-8	0	0	6	-2	10	-8	-6	8	-12	-8	6	8	4	2	-8	-12	0	-18	
910G	-	-3	+	+	3	-	-2	-6	-3	-6	-5	-5	7	-6	-7	-12	-8	15	-7	4	-7	1	6	6	15	
910H	-	-1	+	-	-5	+	2	-6	-3	-10	9	-5	1	-10	-9	8	4	5	-3	-8	-5	5	-18	10	5	
910I	-	2	+	-	4	-	0	-2	2	-2	0	2	0	-4	0	4	-6	6	4	-6	-14	-8	-12	0	-18	
910J	-	-2	+	-	0	-	0	2	6	6	-4	2	0	8	0	12	6	-10	-4	6	2	8	12	0	-10	
910K	-	-2	-	+	-4	+	0	-6	-2	6	-8	-6	-8	4	-8	0	-10	-14	-4	6	10	16	0	-16	14	
912A	+	+	-3	3	1	-2	-5	+	4	-6	2	8	-8	-13	-13	-6	-4	-13	-4	8	-3	4	-4	-6	2	
912B	+	+	2	0	0	2	2	-	0	2	4	2	6	4	0	10	4	-2	12	0	-6	4	8	6	-14	
912C	+	-	1	3	5	-2	-1	+	-4	-6	10	0	0	11	-9	10	-4	-5	4	-8	13	-4	4	-6	2	
912D	+	-	4	-4	4	-4	6	+	6	2	-2	4	-6	-4	2	-6	4	-10	-8	0	-2	-14	16	-18	14	
912E	-	+	0	4	0	-4	6	+	6	6	-2	-4	6	4	-6	6	12	14	-8	0	14	10	12	-6	-10	
912F	-	+	1	-3	3	-6	3	-	-4	-10	-2	8	-8	1	-3	-6	0	7	-8	-12	-11	0	-4	10	-2	
912G	-	+	-2	0	0	6	-6	-	-4	2	-8	-10	-2	4	-12	-6	12	-2	4	0	10	0	-16	-2	10	
912H	-	-	0	-4	-4	0	-2	+	2	-6	-6	-8	10	12	-10	2	-4	-10	0	16	-2	-10	16	-2	-10	
912I	-	-	-3	-1	5	-6	-5	+	-4	6	-6	-8	-8	-9	-1	2	8	11	0	4	-11	8	4	10	-10	
912J	-	-	2	0	-2	2	6	-	-2	4	8	-2	-8	8	-2	-4	0	2	-12	4	6	16	-6	0	-2	
912K	-	-	2	0	4	2	-6	-	4	-2	-4	10	10	-4	4	-10	-12	14	12	-8	-6	4	-12	-6	10	

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
912L	-	-	-3	5	-1	2	-1	-	4	-2	6	0	0	1	9	10	8	-1	-8	12	-11	-16	-12	-6	-10	
913A	-1	0	0	-1	+	1	0	2	-2	10	4	7	-2	10	-8	8	12	-8	13	13	11	0	-	6	0	
913B	2	3	3	-4	+	-2	-6	2	7	4	-5	-5	-2	10	-8	14	-3	-2	-5	7	-4	12	-	-15	3	
914A	+	0	0	0	2	0	-6	-4	8	-6	0	-4	6	-6	-8	12	-10	8	-4	0	-6	-16	-10	10	-10	+
914B	+	1	3	4	2	-1	6	-1	0	-4	-3	6	-10	-12	2	-11	-10	10	12	5	17	-4	4	0	-4	-
915A	2	+	-	3	-4	4	8	7	-6	-5	0	11	-2	1	-1	2	-7	+	-12	13	-8	-12	9	-6	-4	
915B	1	+	-	0	0	-2	-2	-4	0	-10	-4	2	-6	-4	8	-14	0	-	-4	4	2	4	4	10	2	
915C	2	-	+	1	4	4	4	-1	-6	3	0	-7	2	-5	13	6	-15	+	-4	-11	-8	0	-5	2	-8	
915D	-1	-	-	-2	-2	-2	-2	-4	0	0	0	-4	-10	10	-8	-6	6	+	-10	-14	-2	12	4	-16	10	
916A	-	0	-2	-2	4	2	-2	8	2	10	2	6	10	-8	-2	10	-6	2	-2	0	-14	14	0	-6	6	+
916B	-	3	1	4	-5	-4	1	5	2	4	-10	-6	-2	-11	10	-2	6	-7	-2	9	-8	8	9	-6	15	+
916C	-	-3	-3	-4	-3	-4	-7	-5	-6	8	2	-6	10	-5	6	6	-14	-11	-2	-5	4	-4	-1	-6	-17	+
916D	-	1	-3	2	-3	2	-3	-1	-6	0	-4	2	0	-1	-6	-6	0	-7	8	3	8	-4	3	6	-1	-
916E	-	-1	1	2	-5	-2	-3	1	-2	0	8	-6	-12	1	-10	10	-4	5	-4	1	-16	4	-11	-18	-9	-
918A	+	+	-1	-2	0	3	+	7	-9	-7	0	-10	5	-1	-2	-2	0	-10	9	-3	-10	-8	-12	10	-6	
918B	+	+	-1	3	-5	-2	+	-8	6	-2	5	10	-10	-6	-12	3	0	-10	-6	12	-5	-8	-7	0	-11	
918C	+	+	2	-2	0	-6	+	-8	3	-10	6	-7	11	2	10	-11	-3	2	-6	0	-4	10	-3	4	0	
918D	+	+	3	-1	3	2	-	-4	6	6	-7	2	6	2	0	-9	0	2	2	-12	11	8	9	12	5	
918E	+	-	0	-4	6	-4	-	2	-3	0	-10	5	-9	-4	-12	-3	3	2	2	12	-16	-10	15	-6	-10	
918F	+	-	-3	2	0	-1	-	-1	-3	3	-4	-10	3	-1	-6	-6	-12	2	-7	-9	2	8	12	-6	2	
918G	-	+	3	2	0	-1	+	-1	3	-3	-4	-10	-3	-1	6	6	12	2	-7	9	2	8	-12	6	2	
918H	-	+	-2	-2	0	-6	-	-8	-3	10	6	-7	-11	2	-10	11	3	2	-6	0	-4	10	3	-4	0	
918I	-	-	0	-4	-6	-4	+	2	3	0	-10	5	9	-4	12	3	-3	2	2	-12	-16	-10	-15	6	-10	
918J	-	-	-3	-1	-3	2	+	-4	-6	-6	-7	2	-6	2	0	9	0	2	2	12	11	8	-9	-12	5	
918K	-	-	1	-2	0	3	-	7	9	7	0	-10	-5	-1	2	2	0	-10	9	3	-10	-8	12	-10	-6	
918L	-	-	1	3	5	-2	-	-8	-6	2	5	10	10	-6	12	-3	0	-10	-6	-12	-5	-8	7	0	-11	
920A	+	0	-	1	-6	-2	-3	-6	-	3	-3	1	9	-8	4	1	1	8	-7	-5	-6	0	-11	4	6	
920B	+	-3	-	-2	0	1	0	0	-	-3	3	-8	3	-2	-11	-14	-8	-4	-4	7	-9	0	4	-2	18	
920C	-	1	+	-2	0	1	-4	-4	-	-3	-1	-8	-5	-6	9	2	0	0	4	3	7	4	8	-14	-14	
920D	-	-1	-	0	2	-5	-4	-2	+	-3	7	-2	-9	-4	-9	-6	0	2	-2	-1	1	-14	0	16	-4	
921A	-2	+	0	0	5	0	1	-1	0	6	-4	3	3	4	-4	-10	6	14	2	7	-4	11	11	15	-5	-
921B	0	-	0	-1	0	-4	-3	5	-6	-6	-10	2	6	2	0	-9	0	-10	-4	9	2	-13	15	15	11	-
922A	-	-2	2	-1	-2	3	6	7	-6	6	-3	1	5	1	8	6	4	8	5	1	-3	8	-16	-9	5	+
923A	0	3	2	0	0	+	0	4	2	1	7	-8	-5	1	5	0	7	4	-13	-	4	-15	2	-6	2	
924A	-	+	-3	+	+	1	-4	3	2	5	0	9	0	10	5	-6	13	6	-1	0	-9	12	-4	-14	-2	
924B	-	+	-1	+	-	-1	6	1	-8	-7	-10	-3	0	-4	7	4	7	-14	-13	-16	13	-8	-14	10	-12	
924C	-	+	-1	-	+	-3	2	-5	4	3	-6	-3	0	-8	-9	-4	-9	-2	-5	0	-9	-4	-2	10	16	
924D	-	+	1	-	-	-1	0	5	2	-1	8	1	0	6	1	-2	9	10	7	0	9	0	0	-6	-2	
924E	-	-	-3	+	+	3	-2	-3	-4	-9	-2	-11	-4	-4	-3	-4	-3	10	11	4	9	-4	10	6	12	
924F	-	-	-1	+	-	1	4	3	6	7	4	1	-4	-2	7	10	-9	-2	-9	-4	-9	16	4	-2	-14	
924G	-	-	3	-	+	-1	0	5	6	-3	-4	-7	-12	2	3	6	3	2	-1	12	-7	-4	0	6	2	
924H	-	-	-3	-	-	-7	-6	-1	0	-3	2	5	-12	8	-3	-12	-3	-10	-13	-12	11	8	6	6	8	
925A	0	1	+	3	-5	-4	4	-8	-4	4	2	+	-5	6	-9	-3	-8	-10	4	5	15	-14	-11	-2	-10	
925B	0	-1	+	1	3	4	-6	2	-6	-6	-4	+	-9	-8	-3	3	12	8	4	-15	-11	-10	-9	6	-8	
925C	-1	2	+	2	0	2	-2	2	8	2	-6	-	10	4	10	6	-6	2	14	0	-2	-6	-18	2	10	
925D	2	-1	+	5	3	2	4	-4	2	2	0	-	7	10	-11	3	0	-4	-16	-15	-11	-12	3	-4	-8	
925E	2	3	+	1	-5	2	0	0	-2	6	-4	-	-9	-2	9	-1	8	-8	-8	9	1	4	15	4	-4	
926A	-	2	0	0	2	0	2	6	-4	6	0	8	-2	-4	0	-12	-12	-2	-16	16	10	8	-6	6	-18	+
927A	1	-	1	-2	2	-5	0	-8	-1	2	5	2	-8	-11	2	-10	11	-5	11	-16	12	6	-1	6	-7	-
928A	+	1	-1	0	-5	1	-6	4	-6	+	9	0	-8	-1	9	-9	14	10	-4	-6	-4	-17	6	0	-4	
928B	+	-1	-1	0	5	1	-6	-4	6	+	-9	0	-8	1	-9	-9	-14	10	4	6	-4	17	-6	0	-4	
930A	+	+	+	0	-4	6	2	-4	-4	2	+	-2	-6	-4	0	2	-4	-6	16	-12	-6	-16	-12	-18	-14	

TABLE 3: HECKE EIGENVALUES 930B–944B

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
930B	+	+	+	-3	3	-2	-4	-3	5	4	-	0	4	1	10	3	6	-2	2	7	5	-1	12	1	-10	
930C	+	+	-	3	3	-2	8	-7	7	-8	+	-4	0	1	6	5	6	2	10	9	1	13	-16	-3	6	
930D	+	+	-	2	-4	-4	-6	0	0	4	-	4	-6	-8	-12	-2	6	10	-10	-14	4	-8	-4	-8	-2	
930E	+	+	-	-4	2	2	0	0	-6	-8	-	-2	6	4	-12	-2	-12	-8	-4	-8	4	4	-16	-2	-2	
930F	+	-	+	1	5	2	-4	1	-5	4	+	12	4	11	-10	9	-10	10	6	15	-13	13	-8	3	-18	
930G	+	-	+	4	-4	2	2	4	4	-2	+	-6	10	-4	8	-6	8	10	-12	0	14	-8	4	-6	-6	
930H	+	-	-	-2	-4	-4	2	-8	-8	4	+	-12	10	8	-4	6	2	10	-6	6	-4	-8	4	0	-18	
930I	+	-	-	-1	-3	2	0	5	9	0	-	8	0	11	-6	-9	6	-10	14	9	-1	-1	-12	-9	14	
930J	+	-	-	4	2	2	0	0	-6	0	-	-2	-10	-4	4	6	-4	0	4	-16	4	4	8	6	14	
930K	-	+	+	-2	0	4	6	0	0	8	+	4	10	8	-4	-14	14	-6	-10	6	-8	8	-12	16	-10	
930L	-	+	+	3	5	-6	-4	5	5	8	+	4	0	-7	6	11	14	-6	10	-9	-3	-7	-12	-9	10	
930M	-	+	+	0	-6	-2	-4	0	2	-8	-	-6	-2	4	4	-6	0	4	-4	-8	-4	-4	0	-2	14	
930N	-	-	+	2	0	-4	6	8	0	0	-	-4	-6	8	-12	-6	-6	2	2	-6	8	8	12	0	-10	
930O	-	-	-	0	-4	6	2	4	-8	6	+	-2	10	-4	0	-10	-12	-2	-4	0	2	0	4	-14	18	
931A	2	2	3	+	4	-6	-7	-	3	0	0	-2	-4	5	4	6	8	-2	8	-2	10	12	-3	-8	4	
931B	0	2	-3	-	3	4	3	+	0	6	4	2	6	-1	3	12	6	1	-4	6	7	8	-12	-12	-8	
931C	2	-2	-3	-	4	6	7	+	3	0	0	-2	4	5	-4	6	-8	2	8	-2	-10	12	3	8	-4	
933A	0	+	0	-1	4	1	-3	4	-9	1	-4	-6	-5	-2	-8	6	9	8	-12	3	-1	-12	2	-2	-16	+
933B	0	-	-2	3	-4	1	-5	-2	-7	3	2	-4	9	-6	-12	-12	3	-14	-12	-3	11	8	6	-12	4	-
934A	+	1	1	-2	0	-4	-2	2	-6	-2	-9	11	1	7	4	2	0	-7	2	-12	2	-9	15	-9	7	+
934B	-	1	3	2	0	-4	6	2	-6	-6	5	-7	9	-1	-12	-6	0	11	-10	12	14	5	15	-9	-1	+
934C	-	3	-1	-2	-2	6	2	2	2	-2	3	-11	9	-7	-2	-12	12	-13	-4	-6	-4	-13	5	7	3	+
935A	0	-2	+	3	-	0	-	0	-2	3	-10	-4	-1	-8	-3	-9	1	2	-3	2	-11	0	6	-7	-12	
935B	0	-2	-	5	+	-4	+	-4	6	-3	2	8	9	-4	-3	-9	9	14	5	6	11	8	6	9	-4	
936A	+	+	-2	2	-4	+	0	-2	-4	0	2	-10	-2	8	0	-12	-12	-6	-6	8	-2	12	4	-14	-10	
936B	+	-	1	5	2	+	3	-2	-4	6	-4	11	-8	-1	-9	12	-6	0	6	-7	-2	12	16	10	-10	
936C	+	-	4	-4	2	+	6	4	-4	6	8	-10	4	-4	6	-6	6	-6	0	-10	-2	0	10	-8	-10	
936D	+	-	-4	0	2	+	-2	8	-4	6	-4	6	12	4	6	2	14	10	-4	-2	-2	-8	-14	0	-10	
936E	+	-	-2	0	0	-	-2	-4	0	-6	0	-2	-6	-12	4	-6	8	-2	4	12	-14	0	-8	18	-6	
936F	-	+	2	2	4	+	0	-2	4	0	2	-10	2	8	0	12	12	-6	-6	-8	-2	12	-4	14	-10	
936G	-	-	0	0	-6	+	-2	0	-4	6	-4	-2	0	4	-10	10	6	-6	-12	-2	6	-16	-6	-4	14	
936H	-	-	0	-4	2	+	6	-4	-4	-10	-8	-2	0	-4	-2	2	-10	10	8	-2	-10	8	-6	12	-2	
936I	-	-	2	4	0	-	-2	8	-8	2	4	-10	-2	-4	12	-6	0	-2	8	12	10	-8	0	14	2	
938A	+	1	-1	-	2	-7	-2	-2	1	-5	9	-9	3	-6	12	-10	-8	-2	-	9	-2	-8	2	2	-18	
938B	+	-2	2	-	-4	2	-2	-8	4	10	0	-6	0	-12	-6	-10	4	-2	-	0	-2	-8	-16	2	0	
938C	-	-1	-1	-	-4	-3	0	4	-9	5	-7	-11	5	-8	6	6	10	-10	+	-5	-6	-2	-6	-12	-2	
938D	-	1	3	-	-6	5	6	2	-3	-9	5	11	3	-10	-12	-6	0	-10	-	-3	2	8	6	6	-10	
939A	0	+	-1	4	-6	1	3	4	-4	-6	0	-2	-6	-12	8	-13	3	-8	-6	2	0	-9	-12	5	-7	+
939B	1	-	0	-4	0	2	-4	-4	-2	-2	0	-2	-12	0	-6	0	6	6	12	12	-2	0	16	0	-14	-
939C	-2	-	-1	-4	0	1	7	4	0	-8	-6	-8	6	-8	4	-13	-9	-12	0	14	-6	-1	8	-7	17	-
940A	-	-2	+	2	0	-5	8	-4	-1	-8	10	2	6	11	+	8	5	-5	16	9	9	1	6	15	-12	
940B	-	3	+	-3	5	5	-2	1	4	-8	-5	2	6	6	+	8	0	5	-4	-16	-11	-4	11	-10	-2	
940C	-	1	+	-1	3	-7	-6	-1	0	0	5	2	-6	-10	-	12	-12	5	8	0	-7	8	-15	6	2	
940D	-	-1	-	1	-1	-5	2	-5	-4	8	-7	-2	-10	2	+	-4	-12	13	-8	-12	11	-12	-1	-2	10	
940E	-	2	-	2	4	1	0	-4	-3	0	-2	10	2	1	-	-12	5	-5	8	9	3	-15	2	-1	0	
942A	+	-	2	2	3	2	4	-8	-1	4	1	-2	3	-3	-4	-8	-6	7	-2	14	-14	2	12	8	8	+
942B	-	+	-2	-1	0	1	5	6	3	0	6	9	-2	5	0	-6	3	8	-6	-12	2	16	12	-13	0	+
942C	-	-	-2	-5	0	-7	-7	6	3	0	-2	-7	6	9	-8	2	-13	0	2	-12	2	8	-4	-1	0	+
942D	-	-	-4	-1	-6	-1	7	-8	-7	-8	-2	7	0	3	-4	4	-9	4	10	-10	4	8	0	17	-10	+
943A	1	0	-2	2	2	6	8	-2	+	2	8	10	-	-8	12	-4	12	-6	-14	-8	-6	10	0	8	-16	
944A	+	1	-1	-1	0	-2	-6	-3	6	-3	4	-2	-5	0	-2	3	+	12	-4	0	-6	-15	14	12	6	
944B	+	1	-1	-1	-4	2	2	-3	-6	5	-4	-6	3	-8	2	11	+	0	8	8	-6	1	-6	-16	-10	

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
962A	-	0	2	-2	6	-	2	0	6	6	0	-	2	-6	-2	10	-4	-10	-14	-6	2	-6	6	-6	6	
964A	-	-2	1	1	-4	4	2	1	3	3	-8	4	9	-1	10	11	2	13	-2	7	8	14	6	2	-17	+
965A	1	0	+	4	2	2	2	6	0	2	4	10	2	-8	-6	2	4	2	-4	-2	2	2	-12	2	14	-
966A	+	+	-2	+	0	4	0	6	+	-6	-10	-6	-2	12	10	-10	-12	-14	-12	-12	14	0	2	-12	12	
966B	+	+	3	-	4	3	0	0	+	1	-2	-5	5	-7	-3	12	-2	-6	-12	10	0	4	4	10	19	
966C	+	+	2	-	-4	-4	-4	-2	-	-6	6	-2	-10	8	10	-6	-12	-10	8	-4	-2	-8	-6	0	-8	
966D	+	+	-4	-	2	2	2	-2	-	-6	0	4	-10	-10	-8	0	0	-4	2	8	-2	-8	-6	6	-2	
966E	+	-	0	+	-2	-6	2	-6	-	-6	0	0	6	-6	8	-4	0	-8	-2	0	-2	8	-2	-2	14	
966F	+	-	0	-	6	2	-6	2	-	-6	8	8	6	2	0	-12	0	8	-10	0	14	8	6	6	-10	
966G	-	+	-2	-	-4	-2	-6	4	+	-2	-8	6	-6	-4	-8	6	4	-10	4	-8	-6	0	-12	2	10	
966H	-	+	1	-	0	-1	4	4	-	-1	10	-5	7	-3	3	0	6	-6	4	-2	-4	12	4	-6	-1	
966I	-	-	-2	+	4	2	6	0	-	-2	4	6	-6	12	-12	6	-4	-10	4	-16	2	8	-16	6	-2	
966J	-	-	3	+	4	-3	-4	0	-	3	-6	-9	9	-3	-7	-4	6	10	4	-6	-8	8	4	-14	-7	
966K	-	-	-3	-	0	5	0	8	+	3	2	-7	9	-1	-3	-12	-6	14	-4	6	-4	-16	-12	6	-1	
968A	+	1	1	-4	+	-4	-4	4	-3	-8	9	-5	12	-8	4	-10	7	8	11	-9	-4	-8	0	-1	1	
968B	+	0	3	4	-	3	3	4	-8	-5	-4	11	7	-12	-8	-1	-4	-2	4	12	10	8	4	3	-13	
968C	-	1	1	4	+	4	4	-4	-3	8	9	-5	-12	8	4	-10	7	-8	11	-9	4	8	0	-1	1	
968D	-	0	3	-4	-	-3	-3	-4	-8	5	-4	11	-7	12	-8	-1	-4	2	4	12	-10	-8	-4	3	-13	
968E	-	-3	-3	2	-	0	6	-4	1	8	-7	-1	-4	-6	-8	2	-1	-4	-5	3	-16	-2	2	15	-7	
969A	1	-	2	2	2	2	+	+	6	0	-8	12	-4	12	-8	6	-4	-6	12	-8	-10	-12	-4	-6	8	
970A	+	-2	+	0	3	-2	-1	-4	-6	5	-2	-8	10	10	9	-9	6	10	3	5	4	2	11	13	-	
970B	-	-2	-	-4	-1	6	3	0	6	3	-2	12	-6	-2	-1	-3	14	-10	11	-1	12	-6	-5	5	-	
972A	-	+	0	-1	0	2	0	8	0	0	11	11	0	5	0	0	0	-13	11	0	17	-4	0	0	14	
972B	-	+	0	5	0	5	0	-7	0	0	11	-1	0	5	0	0	0	14	-16	0	-10	17	0	0	-19	
972C	-	-	0	-1	0	-7	0	-1	0	0	-7	11	0	-13	0	0	0	14	-16	0	-10	-13	0	0	5	
972D	-	-	0	-4	0	5	0	-7	0	0	-7	-10	0	-13	0	0	0	-13	11	0	17	17	0	0	-19	
973A	2	1	4	-	4	-2	-1	-7	-4	-1	-2	-3	2	10	6	-6	12	3	7	-11	-6	-4	-12	0	13	+
973B	0	1	0	-	0	-4	3	-7	6	-9	-4	-7	0	8	6	-6	-12	-1	-13	-3	2	8	6	-6	17	-
974A	+	0	3	-4	3	-5	8	0	6	10	7	6	3	-7	2	-2	5	-8	-2	-7	9	-4	-12	5	7	-
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974F	-	-1	-1	2	-4	-5	0	-6	2	4	5	6	-10	-11	-6	-4	0	0	-10	13	-3	4	-14	13	-2	-
974G	-	-3	1	-2	-4	5	-4	-2	2	0	-7	-6	-2	-1	-2	0	-8	-12	-10	9	-3	4	-2	-3	-2	-
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975I	-2	-	+	-3	-1	-	1	-2	3	-2	-6	-11	-5	-4	10	-11	8	13	-12	-5	-10	-3	12	-15	-17	
975J	0	-	-	-1	-1	+	-1	-4	-3	-8	-4	3	-9	-8	10	-1	4	-11	-4	-1	14	1	6	-15	-15	
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978A	+	+	0	3	-3	2	1	6	2	-4	1	3	10	-11	7	-10	3	14	14	9	14	4	16	3	-7	-
978B	+	+	2	2	4	4	-2	0	0	-2	2	4	-10	12	-6	4	4	-10	4	-6	2	14	0	10	14	-

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978C	+	+	-3	-3	-6	-1	-2	0	5	-7	7	-6	-5	7	4	14	-6	-10	14	-6	2	4	-5	0	-1	-
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978G	-	-	-3	-3	-4	1	-6	-2	-1	-3	7	-6	5	-7	2	-6	-10	8	6	14	-4	8	-1	-10	11	+
978H	-	-	0	-1	3	2	3	2	-6	0	5	11	6	-7	9	-6	-3	-10	-10	-9	2	-4	12	9	-7	-
979A	0	2	3	0	+	-1	6	4	0	7	8	2	5	-11	-11	-9	8	3	8	-8	6	16	-7	-	7	-
979B	1	2	-2	-2	-	-2	-2	-6	2	6	-2	-6	-2	6	-4	-6	14	6	-4	-12	10	-4	6	-	14	-
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980B	-	-3	+	+	-2	-6	2	0	-9	3	2	8	5	1	8	4	-8	7	-3	8	14	4	-1	13	-10	-
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984A	+	+	-1	-2	2	-3	-3	7	6	0	7	10	-	-12	12	10	11	10	-7	1	1	4	15	1	-18	-
984B	+	+	2	4	5	0	-3	-2	0	3	1	-11	-	9	3	-2	8	1	2	1	-11	-14	6	-2	6	-
984C	-	+	-2	0	1	4	-7	2	4	-9	-7	-3	-	1	-9	-2	-8	1	2	5	-11	10	2	-18	2	-
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986A	+	-2	0	5	0	5	-	-1	0	+	-4	5	-3	5	-3	-9	6	11	2	15	-7	8	12	6	-10	-
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987B	-1	+	2	+	0	2	2	-4	-4	6	0	6	6	8	-	-10	-4	6	8	16	14	0	-12	2	18	-
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987E	-1	-	-2	+	-2	4	6	-2	-6	-8	6	-6	-10	-4	-	-6	-12	-14	4	0	12	8	12	-14	-6	-
988A	-	2	4	2	0	+	-3	+	3	-2	-11	-5	-5	11	-6	-14	11	7	-7	12	4	-2	-2	2	17	-
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988C	-	0	-3	3	3	+	-7	-	0	-8	2	-8	2	1	-7	-4	-12	-9	2	4	-13	6	4	-6	4	-
988D	-	-2	0	2	0	-	-3	-	3	6	-1	-7	9	11	6	6	9	-1	11	12	8	-10	-6	6	11	-
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990A	+	+	+	0	+	0	-2	2	-6	-2	0	-6	-2	-2	2	-2	-8	4	-4	2	-10	4	-12	0	-18	-
990B	+	+	-	-4	+	-4	6	2	6	6	8	2	6	-10	6	-6	0	8	-4	6	14	-16	-12	0	14	-
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990E	+	-	+	0	-	-2	-2	-4	0	2	0	-2	-2	-12	-8	-6	12	6	4	0	-6	-16	-4	-10	2	-
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990G	+	-	-	4	-	2	-2	4	4	-6	0	-10	6	-12	4	6	4	10	-12	4	10	4	-4	-10	18	-
990H	-	+	+	-4	-	-4	-6	2	-6	-6	8	2	-6	-10	-6	6	0	8	-4	-6	14	-16	12	0	14	-
990I	-	+	-	0	-	0	2	2	6	2	0	-6	2	-2	-2	2	8	4	-4	-2	-10	4	12	0	-18	-
990J	-	-	+	-4	+	-2	2	-8	0	-2	-8	-10	10	0	0	-14	4	14	-4	-8	10	12	-4	6	-14	-

TABLE 3: HECKE EIGENVALUES 990K–999B

	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79	83	89	97	W_q
990K	-	-	-	0	+	2	2	8	-4	-2	8	-2	-6	8	4	-2	-4	-6	-12	12	2	0	-4	6	-14	
990L	-	-	-	5	+	2	-3	-7	6	3	-7	-7	-6	8	-6	3	6	-1	8	-3	2	-10	6	-9	-4	
994A	+	1	0	+	-5	5	-2	4	-7	-5	-4	1	5	-8	8	-6	-8	13	-8	+	-4	-6	-9	0	-7	
994B	+	-2	2	+	4	0	-4	6	0	-2	0	-2	0	-4	4	6	8	8	12	-	2	4	6	14	-8	
994C	+	2	-2	-	4	0	0	2	8	6	-8	6	12	-4	12	6	-4	0	4	+	-14	4	-14	6	-12	
994D	+	1	0	-	-3	-1	-6	-4	3	-9	8	5	3	-4	-12	6	12	-1	8	-	-4	-10	-9	-12	-1	
994E	-	-2	4	+	2	6	-2	-6	0	2	0	-2	6	-4	8	-12	4	14	-6	+	2	16	6	-14	-2	
994F	-	0	-2	-	-4	-6	6	0	-8	-2	0	6	-10	-4	8	-2	12	10	-12	+	-6	-8	-16	2	14	
994G	-	-2	0	-	-6	2	-6	2	0	-6	8	-10	-6	-4	0	-12	-12	2	2	+	2	8	6	-6	2	
995A	1	-2	-	4	0	-2	0	4	0	2	0	4	6	8	0	6	0	6	2	-4	8	8	-14	-10	0	+
995B	0	1	-	2	-6	-4	3	-4	0	-6	-7	2	-6	8	12	6	-6	-13	5	0	-13	8	-12	3	14	-
996A	-	+	4	4	4	-2	6	-2	-8	-6	-8	-2	-6	2	-12	4	12	2	-10	-8	14	6	+	4	6	
996B	-	-	-1	-2	-3	0	-8	3	-3	6	-4	-5	-8	4	0	7	-9	-9	9	-2	10	8	+	-15	8	
996C	-	-	-3	2	-3	-4	0	-7	-3	-6	8	11	0	-4	-12	-3	-9	-1	-13	6	-10	8	+	3	8	
997A	-2	-1	0	4	0	-5	-4	4	1	0	8	-2	-10	6	2	-11	-11	-8	-5	-9	10	5	-12	9	-6	+
997B	0	-1	-4	-2	-2	-5	-2	-4	-3	2	-4	-10	-2	6	-6	9	-3	4	-9	-13	10	-11	12	-7	-18	-
997C	-2	-1	-2	-4	-2	-1	-6	-4	1	-4	-8	4	-6	-12	8	1	-7	-10	3	11	-10	1	0	1	18	-
999A	1	+	1	-1	-2	-2	-3	0	-1	-3	-4	+	0	-4	0	4	11	-8	-7	6	2	-2	-6	10	-8	
999B	-1	+	-1	-1	2	-2	3	0	1	3	-4	+	0	-4	0	-4	-11	-8	-7	-6	2	-2	6	-10	-8	

TABLE 4

BIRCH–SWINNERTON-DYER DATA

In Table 4 we give the numbers relating to the Birch–Swinnerton-Dyer conjecture, for each “strong Weil” curve E_f . Each curve is identified as before with a single letter X after the conductor, and is¹ the curve NX1 of Table 1.

For each curve we first give the rank r , and then list (to 10 decimal places): the real period $\Omega(f)$, the value of $L^{(r)}(f, 1)/r!$, the regulator R of E_f , and the ratio $L^{(r)}(f, 1)/r!R\Omega$. For curves of rank 0 we obviously have $R = 1$ exactly; also the ratio $L(f, 1)/\Omega(f)$ is known exactly in these cases, so we give it as an exact rational rather than as a decimal. Finally we give the value of

$$S = \frac{L^{(r)}(f, 1)}{r! \Omega(f)} \bigg/ \frac{(\prod c_p) R}{|T|^2},$$

which according to the Birch–Swinnerton-Dyer conjecture should equal the order of the Tate–Shafarevich group $\text{III}(E_f/\mathbb{Q})$. The value S is known exactly in case $r = 0$ and approximately in case $r > 0$; in each case it is a positive integer to within 10 decimal places, and is (to this precision) equal to 1 in all but 4 cases. The exceptions are

$$\begin{aligned} S = 4 & \quad \text{for 571A, 960D, and 960N;} \\ S = 9 & \quad \text{for 681B.} \end{aligned}$$

These are all curves of rank 0, so that the value of S is exact.

We have also computed the corresponding data for all the curves in each isogeny class, but this extra data is not included here for reasons of space. For a summary of the results obtained, see [16]. As well as $S = 1, 4$ and 9 , the values $S = 16, 25$ and 49 were also obtained, all for curves of rank 0.

¹As remarked earlier, for class 990H the “strong” curve is 990H3 and not 990H1, and so the data here is for the curve 990H3.

TABLE 4: BIRCH-SWINNERTON-DYER DATA 11A-66B

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
11 A (B)	0	1.2692093043	0.2538418609	1	1/5	1
14 A (C)	0	1.9813419561	0.3302236593	1	1/6	1
15 A (C)	0	2.8012060847	0.3501507606	1	1/8	1
17 A (C)	0	1.5470797536	0.3867699384	1	1/4	1
19 A (B)	0	1.3597597335	0.4532532445	1	1/3	1
20 A (B)	0	2.8243751420	0.4707291903	1	1/6	1
21 A (B)	0	3.6089232431	0.4511154054	1	1/8	1
24 A (B)	0	4.3130312950	0.5391289119	1	1/8	1
26 A (B)	0	1.5467299538	0.5155766513	1	1/3	1
26 B (D)	0	4.3467574468	0.6209653495	1	1/7	1
27 A (B)	0	1.7666387503	0.5888795834	1	1/3	1
30 A (A)	0	3.3519482592	0.5586580432	1	1/6	1
32 A (B)	0	2.6220575543	0.6555143886	1	1/4	1
33 A (B)	0	2.9893565910	0.7473391477	1	1/4	1
34 A (A)	0	4.4956633263	0.7492772211	1	1/6	1
35 A (B)	0	2.1087337174	0.7029112391	1	1/3	1
36 A (A)	0	4.2065463160	0.7010910527	1	1/6	1
37 A (A)	1	5.9869172925	0.3059997738	0.0511114082	1.0000000000	1.0
37 B (C)	0	2.1770431858	0.7256810619	1	1/3	1
38 A (D)	0	1.8906322299	0.6302107433	1	1/3	1
38 B (A)	0	4.0962281653	0.8192456331	1	1/5	1
39 A (B)	0	3.3067514027	0.8266878507	1	1/4	1
40 A (B)	0	2.9688249468	0.7422062367	1	1/4	1
42 A (A)	0	3.4754474575	0.8688618644	1	1/4	1
43 A (A)	1	5.4686895300	0.3435239746	0.0628165071	1.0000000000	1.0
44 A (A)	0	2.4139388627	0.8046462876	1	1/3	1
45 A (A)	0	1.8431817533	0.9215908766	1	1/2	1
46 A (A)	0	1.3218082226	0.6609041113	1	1/2	1
48 A (B)	0	3.3715007096	0.8428751774	1	1/4	1
49 A (A)	0	1.9333117056	0.9666558528	1	1/2	1
50 A (E)	0	2.1394949443	0.7131649814	1	1/3	1
50 B (A)	0	4.7840561329	0.9568112266	1	1/5	1
51 A (A)	0	2.5801770484	0.8600590161	1	1/3	1
52 A (B)	0	1.6909664173	0.8454832086	1	1/2	1
53 A (A)	1	4.6876410489	0.4358638242	0.0929814846	1.0000000000	1.0
54 A (E)	0	2.1047244760	0.7015748253	1	1/3	1
54 B (A)	0	3.0915655491	1.0305218497	1	1/3	1
55 A (B)	0	4.1146794247	1.0286698562	1	1/4	1
56 A (C)	0	3.4981932568	0.8745483142	1	1/4	1
56 B (A)	0	1.8960382312	0.9480191156	1	1/2	1
57 A (E)	1	5.5555045214	0.4174916397	0.0375745927	2.0000000000	1.0
57 B (B)	0	4.3412090891	1.0853022723	1	1/4	1
57 C (F)	0	1.4662739791	0.5865095917	1	2/5	1
58 A (A)	1	5.4655916989	0.4637041648	0.0424203078	2.0000000000	1.0
58 B (B)	0	2.5830187734	1.0332075094	1	2/5	1
61 A (A)	1	6.1331931484	0.4856736514	0.0791877314	1.0000000000	1.0
62 A (A)	0	4.4349626414	1.1087406604	1	1/4	1
63 A (A)	0	2.2066209287	1.1033104643	1	1/2	1
64 A (B)	0	3.7081493546	0.9270373387	1	1/4	1
65 A (A)	1	5.3828534706	0.5053343423	0.3755140987	0.2500000000	1.0
66 A (A)	0	4.7825487441	0.7970914573	1	1/6	1
66 B (E)	0	4.4087701205	1.1021925301	1	1/4	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
66 C (I)	0	2.3832296312	1.1916148156	1	1/2	1
67 A (A)	0	1.2737700365	1.2737700365	1	1	1
69 A (A)	0	2.4058638676	1.2029319338	1	1/2	1
70 A (A)	0	2.3606086931	1.1803043466	1	1/2	1
72 A (A)	0	1.9465368423	0.9732684211	1	1/2	1
73 A (B)	0	2.3653209345	1.1826604672	1	1/2	1
75 A (A)	0	1.4025399402	1.4025399402	1	1	1
75 B (E)	0	2.5054748897	1.2527374449	1	1/2	1
75 C (C)	0	3.1361746475	0.6272349295	1	1/5	1
76 A (A)	0	1.1104197465	1.1104197465	1	1	1
77 A (F)	1	3.1997813616	0.6273362019	0.0980279793	2.0000000000	1.0
77 B (D)	0	1.5489100656	1.0326067104	1	2/3	1
77 C (A)	0	2.6516148585	1.3258074292	1	1/2	1
78 A (A)	0	1.4504359262	0.7252179631	1	1/2	1
79 A (A)	1	5.9508003526	0.5811802159	0.0976642101	1.0000000000	1.0
80 A (F)	0	4.0378116400	1.0094529100	1	1/4	1
80 B (B)	0	2.2741651990	1.1370825995	1	1/2	1
82 A (A)	1	5.1889950404	0.5830015595	0.2247069249	0.5000000000	1.0
83 A (A)	1	3.3744689001	0.5982673327	0.1772922941	1.0000000000	1.0
84 A (C)	0	2.1903171092	1.0951585546	1	1/2	1
84 B (A)	0	1.9449239505	0.9724619753	1	1/2	1
85 A (A)	0	2.7953844365	1.3976922183	1	1/2	1
88 A (A)	1	4.2525295331	0.6849015939	0.0402643643	4.0000000000	1.0
89 A (C)	1	5.5526265646	0.6224765415	0.1121048812	1.0000000000	1.0
89 B (A)	0	2.8446096585	1.4223048292	1	1/2	1
90 A (M)	0	2.4599353712	0.8199784571	1	1/3	1
90 B (A)	0	3.9710942585	1.3236980862	1	1/3	1
90 C (E)	0	1.3375959946	1.3375959946	1	1	1
91 A (A)	1	3.8972609371	0.5549393665	0.1423921507	1.0000000000	1.0
91 B (B)	1	6.0394915365	0.7108113038	1.0592450864	0.1111111111	1.0
92 A (A)	0	3.4103924343	1.1367974781	1	1/3	1
92 B (C)	1	4.7070877612	0.7033574920	0.0498083973	3.0000000000	1.0
94 A (A)	0	2.7134690132	1.3567345066	1	1/2	1
96 A (E)	0	4.6856803366	1.1714200841	1	1/4	1
96 B (A)	0	4.0043095218	1.0010773805	1	1/4	1
98 A (B)	0	1.0019771958	1.0019771958	1	1	1
99 A (A)	1	4.4984528866	0.6805515591	0.3025713846	0.5000000000	1.0
99 B (H)	0	3.1692296155	0.7923074039	1	1/4	1
99 C (F)	0	2.7285845914	1.3642922957	1	1/2	1
99 D (C)	0	1.6844963330	1.6844963330	1	1	1
100 A (A)	0	2.5261979246	1.2630989623	1	1/2	1
101 A (A)	1	4.5902472119	0.7560295657	0.1647034529	1.0000000000	1.0
102 A (E)	1	4.7278638235	0.6772848980	0.1432538929	1.0000000000	1.0
102 B (G)	0	2.9593558556	1.4796779278	1	1/2	1
102 C (A)	0	2.7860649844	0.9286883281	1	1/3	1
104 A (A)	0	1.1836111473	1.1836111473	1	1	1
105 A (A)	0	5.8634534537	1.4658633634	1	1/4	1
106 A (B)	0	3.7857830586	1.2619276862	1	1/3	1
106 B (A)	1	5.0128820343	0.6909022751	0.0689126804	2.0000000000	1.0
106 C (E)	0	0.5717994122	1.5247984324	1	8/3	1
106 D (D)	0	1.0421614431	1.0421614431	1	1	1
108 A (A)	0	3.3387380236	1.1129126745	1	1/3	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
109 A (A)	0	1.4110259162	1.4110259162	1	1	1
110 A (C)	0	1.3595418147	1.3595418147	1	1	1
110 B (A)	0	4.6222420328	1.5407473443	1	1/3	1
110 C (E)	0	2.8261738995	0.9420579665	1	1/3	1
112 A (K)	1	3.3732121966	0.8093007286	0.2399198987	1.0000000000	1.0
112 B (A)	0	2.2962114575	1.1481057287	1	1/2	1
112 C (E)	0	1.3254912397	1.3254912397	1	1	1
113 A (B)	0	2.0183698932	1.0091849466	1	1/2	1
114 A (A)	0	3.1488707131	1.5744353566	1	1/2	1
114 B (E)	0	1.5271441130	0.7635720565	1	1/2	1
114 C (G)	0	1.1101789855	1.3877237318	1	5/4	1
115 A (A)	0	1.8080388810	1.8080388810	1	1	1
116 A (E)	0	0.2863256792	0.8589770377	1	3	1
116 B (A)	0	3.8294768331	1.2764922777	1	1/3	1
116 C (D)	0	2.6462381980	1.3231190990	1	1/2	1
117 A (A)	1	2.6403251263	0.7461133887	1.1303356262	0.2500000000	1.0
118 A (A)	1	4.1584572190	0.7311083080	0.0879061957	2.0000000000	1.0
118 B (B)	0	3.4884515797	1.3953806319	1	2/5	1
118 C (D)	0	1.6840974200	1.6840974200	1	1	1
118 D (E)	0	1.0990077472	1.0990077472	1	1	1
120 A (E)	0	5.0779771119	1.2694942780	1	1/4	1
120 B (A)	0	2.4898202911	1.2449101455	1	1/2	1
121 A (H)	0	1.0197948618	1.0197948618	1	1	1
121 B (D)	1	4.8024213220	0.8623722967	0.0897851562	2.0000000000	1.0
121 C (F)	0	1.6661569204	1.6661569204	1	1	1
121 D (A)	0	0.8796995193	1.7593990387	1	2	1
122 A (A)	1	2.9660992729	0.7168659467	0.1208432154	2.0000000000	1.0
123 A (A)	1	3.9950826936	0.6715905138	0.8405214175	0.2000000000	1.0
123 B (C)	1	2.9874191593	0.8720710560	0.2919145287	1.0000000000	1.0
124 A (B)	1	4.9333272334	0.8559827493	0.5205306939	0.3333333333	1.0
124 B (A)	0	1.1755295616	1.1755295616	1	1	1
126 A (A)	0	1.5305454481	1.5305454481	1	1	1
126 B (G)	0	0.9104737369	0.9104737369	1	1	1
128 A (C)	1	4.0364616539	0.8725440838	0.4323311646	0.5000000000	1.0
128 B (F)	0	1.9426724598	0.9713362299	1	1/2	1
128 C (A)	0	2.7473537399	1.3736768699	1	1/2	1
128 D (G)	0	2.8542094075	1.4271047037	1	1/2	1
129 A (E)	1	4.4728045672	0.8941955093	0.0999591527	2.0000000000	1.0
129 B (B)	0	2.1455286131	1.6091464598	1	3/4	1
130 A (E)	1	3.7842289994	0.7382173988	1.1704641536	0.1666666667	1.0
130 B (A)	0	3.1294275088	1.5647137544	1	1/2	1
130 C (J)	0	0.8865474519	1.7730949038	1	2	1
131 A (A)	1	4.1716092763	0.9014647353	0.2160951987	1.0000000000	1.0
132 A (A)	0	2.6563291575	1.3281645788	1	1/2	1
132 B (C)	0	2.1888884346	1.0944442173	1	1/2	1
135 A (A)	1	3.7814246602	0.6703486968	0.0295456852	6.0000000000	1.0
135 B (B)	0	0.9833356547	1.9666713094	1	2	1
136 A (A)	1	3.8871054798	0.8998959875	0.2315079928	1.0000000000	1.0
136 B (C)	0	2.8672616758	1.4336308379	1	1/2	1
138 A (E)	1	4.4583602683	0.7907712971	0.1773681913	1.0000000000	1.0
138 B (G)	0	3.0602034222	1.0200678074	1	1/3	1
138 C (A)	0	2.9754615368	1.4877307684	1	1/2	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
139 A (A)	0	1.7396869770	1.7396869770	1	1	1
140 A (A)	0	1.3443015422	1.3443015422	1	1	1
140 B (C)	0	1.5555116321	1.5555116321	1	1	1
141 A (E)	1	2.9765040248	0.7185501725	0.0344867750	7.0000000000	1.0
141 B (G)	0	1.3805576056	0.6902788028	1	1/2	1
141 C (A)	0	3.8499118765	0.9624779691	1	1/4	1
141 D (I)	1	4.7295280333	0.9387887853	0.1984952365	1.0000000000	1.0
141 E (H)	0	2.1075355786	2.1075355786	1	1	1
142 A (F)	1	3.4643325948	1.0535114537	0.0337891426	9.0000000000	1.0
142 B (E)	1	4.4304737755	0.8015334521	0.1809137110	1.0000000000	1.0
142 C (A)	0	1.8851444898	0.9425722449	1	1/2	1
142 D (C)	0	5.2000441497	1.7333480499	1	1/3	1
142 E (G)	0	1.3008551919	1.3008551919	1	1	1
143 A (A)	1	1.9699231646	0.9456964112	0.2400338318	2.0000000000	1.0
144 A (A)	0	2.4286506479	1.2143253239	1	1/2	1
144 B (E)	0	2.4901297792	1.2450648896	1	1/2	1
145 A (A)	1	5.6915790981	0.8316217075	0.5844576299	0.2500000000	1.0
147 A (C)	0	1.4445724908	0.7222862454	1	1/2	1
147 B (I)	0	2.1340337345	2.1340337345	1	1	1
147 C (A)	0	1.9200537453	1.9200537453	1	1	1
148 A (A)	1	3.3849941577	0.9773274900	0.0962411794	3.0000000000	1.0
150 A (A)	0	1.7692592483	1.7692592483	1	1	1
150 B (G)	0	0.7912367898	0.7912367898	1	1	1
150 C (I)	0	1.4990368329	1.4990368329	1	1	1
152 A (A)	1	3.6456468990	0.9539180658	0.0654148696	4.0000000000	1.0
152 B (B)	0	1.3670652145	1.3670652145	1	1	1
153 A (C)	1	5.0866355584	0.7068417367	0.0694802811	2.0000000000	1.0
153 B (A)	1	2.2547575497	1.0181559607	0.1128897385	4.0000000000	1.0
153 C (E)	0	3.1705064379	1.5852532190	1	1/2	1
153 D (D)	0	1.0383987258	2.0767974516	1	2	1
154 A (C)	1	3.5423530689	0.8716922254	0.2460771720	1.0000000000	1.0
154 B (E)	0	1.1263920237	1.6895880356	1	3/2	1
154 C (A)	0	1.2199842863	1.2199842863	1	1	1
155 A (D)	1	2.2165110787	0.6715528888	1.5148872822	0.2000000000	1.0
155 B (A)	0	2.1141393763	1.0570696881	1	1/2	1
155 C (C)	1	5.3453690009	0.9843172157	0.1841439226	1.0000000000	1.0
156 A (E)	1	4.5382403755	1.0035030550	0.1474144120	1.5000000000	1.0
156 B (A)	0	2.7522991873	1.3761495936	1	1/2	1
158 A (E)	1	3.5817087725	1.1172146322	0.0389902803	8.0000000000	1.0
158 B (D)	1	5.3375846274	0.8451399432	0.0791687629	2.0000000000	1.0
158 C (H)	0	1.9304530479	1.5443624383	1	4/5	1
158 D (B)	0	1.6852937385	1.1235291590	1	2/3	1
158 E (F)	0	3.8328305468	1.9164152734	1	1/2	1
160 A (D)	1	5.4517057364	0.9777440810	0.3586929040	0.5000000000	1.0
160 B (A)	0	3.0115524554	1.5057762277	1	1/2	1
161 A (B)	0	3.4513373072	0.8628343268	1	1/4	1
162 A (K)	1	4.3954427629	0.8964795141	0.3059348839	0.6666666667	1.0
162 B (G)	0	5.0519074124	1.6839691375	1	1/3	1
162 C (A)	0	2.8006777426	0.9335592475	1	1/3	1
162 D (E)	0	2.6051702504	1.7367801669	1	2/3	1
163 A (A)	1	5.5180730712	1.0479330218	0.1899092325	1.0000000000	1.0
166 A (A)	1	4.7801672708	0.8600170155	0.0899567909	2.0000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
168 A (B)	0	2.9025247409	1.4512623705	1	1/2	1
168 B (E)	0	2.3215093041	1.1607546520	1	1/2	1
170 A (A)	1	4.0409291005	0.8289341482	0.2051345440	1.0000000000	1.0
170 B (H)	0	1.9117819746	0.6372606582	1	1/3	1
170 C (F)	0	0.7915291521	1.8469013550	1	7/3	1
170 D (D)	0	3.3291061788	1.1097020596	1	1/3	1
170 E (C)	0	1.3360256749	1.3360256749	1	1	1
171 A (D)	0	3.4734453619	0.8683613405	1	1/4	1
171 B (A)	1	2.3827779033	1.0768577238	0.2259668688	2.0000000000	1.0
171 C (I)	0	1.0700870190	2.1401740379	1	2	1
171 D (H)	0	1.1075480554	2.2150961108	1	2	1
172 A (A)	1	4.0096273260	1.0159589215	0.7601396630	0.3333333333	1.0
174 A (I)	0	0.4362942674	1.0180199573	1	7/3	1
174 B (G)	0	1.8598039585	1.8598039585	1	1	1
174 C (F)	0	1.5847007717	1.5847007717	1	1	1
174 D (A)	0	2.2764447598	1.1382223799	1	1/2	1
174 E (E)	0	0.8687758077	0.8687758077	1	1	1
175 A (B)	1	2.6245205399	0.6977340315	0.1329259994	2.0000000000	1.0
175 B (C)	1	2.8291631631	1.0476202635	0.0925733338	4.0000000000	1.0
175 C (F)	0	1.1737212671	2.3474425342	1	2	1
176 A (C)	0	1.6554236532	1.6554236532	1	1	1
176 B (D)	0	1.4588166169	1.4588166169	1	1	1
176 C (A)	1	3.0540697209	1.0693142864	0.1750638303	2.0000000000	1.0
178 A (A)	0	1.4760248421	1.9680331227	1	4/3	1
178 B (C)	0	2.5803848480	1.2901924240	1	1/2	1
179 A (A)	0	2.2601982547	2.2601982547	1	1	1
180 A (A)	0	2.6259797797	1.3129898898	1	1/2	1
182 A (E)	0	0.7188679094	1.7971697734	1	5/2	1
182 B (A)	0	1.9204065876	1.9204065876	1	1	1
182 C (J)	0	1.2089756349	1.2089756349	1	1	1
182 D (D)	0	2.1353664019	2.1353664019	1	1	1
182 E (I)	0	1.3931977005	1.3931977005	1	1	1
184 A (C)	1	4.4163757466	1.0875847398	0.1231309112	2.0000000000	1.0
184 B (B)	1	5.3243221525	1.0975263443	0.1030672368	2.0000000000	1.0
184 C (D)	0	2.5979495920	1.2989747960	1	1/2	1
184 D (A)	0	0.8765533735	1.7531067469	1	2	1
185 A (D)	1	3.5930803975	0.8196298176	0.1140567044	2.0000000000	1.0
185 B (A)	1	4.8983715401	1.0803747090	0.1102789672	2.0000000000	1.0
185 C (B)	1	3.4935106383	1.2447948639	1.4252652907	0.2500000000	1.0
186 A (D)	0	0.7823354594	0.7823354594	1	1	1
186 B (B)	0	1.9528976690	1.9528976690	1	1	1
186 C (A)	0	1.1823572975	1.1823572975	1	1	1
187 A (A)	0	2.1146369273	1.4097579516	1	2/3	1
187 B (C)	0	2.3270976414	2.3270976414	1	1	1
189 A (A)	1	4.0550955533	0.7689943976	0.0632121888	3.0000000000	1.0
189 B (C)	1	5.4604576374	1.1304624983	1.8632435522	0.1111111111	1.0
189 C (F)	0	3.7360320500	1.2453440167	1	1/3	1
189 D (B)	0	2.2649180731	2.2649180731	1	1	1
190 A (D)	1	2.7276120737	1.2342342404	0.0205680115	22.0000000000	1.0
190 B (C)	1	3.4751792755	0.9162065638	0.1318214819	2.0000000000	1.0
190 C (A)	0	0.9862313357	1.9724626715	1	2	1
192 A (Q)	1	3.3132763405	1.1195591687	1.3516037344	0.2500000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
192 B (A)	0	5.6629488337	1.4157372084	1	1/4	1
192 C (K)	0	3.0497736762	1.5248868381	1	1/2	1
192 D (E)	0	2.3840110146	1.1920055073	1	1/2	1
194 A (A)	0	3.7750977496	1.8875488748	1	1/2	1
195 A (A)	0	4.0228485413	1.0057121353	1	1/4	1
195 B (I)	0	2.4793554937	2.4793554937	1	1	1
195 C (K)	0	0.8075282392	2.4225847175	1	3	1
195 D (J)	0	2.1212032775	2.1212032775	1	1	1
196 A (A)	1	4.3609185288	1.1255807768	0.0860354510	3.0000000000	1.0
196 B (C)	0	1.5672749721	1.5672749721	1	1	1
197 A (A)	1	5.6695604223	0.7873204700	0.1388679918	1.0000000000	1.0
198 A (I)	1	2.5334819295	0.9882757884	0.1950429914	2.0000000000	1.0
198 B (E)	0	1.8053975906	1.8053975906	1	1	1
198 C (M)	0	2.7073977438	1.8049318292	1	2/3	1
198 D (A)	0	2.9230911893	0.9743637298	1	1/3	1
198 E (Q)	0	1.0857413665	1.0857413665	1	1	1
200 A (B)	0	0.8086102703	0.8086102703	1	1	1
200 B (C)	1	3.7121152732	1.0884331301	0.2932110266	1.0000000000	1.0
200 C (G)	0	2.6553977578	1.3276988789	1	1/2	1
200 D (E)	0	1.6601084182	1.6601084182	1	1	1
200 E (A)	0	1.8081075317	1.8081075317	1	1	1
201 A	1	3.4396827280	0.7311199568	0.1062772376	2.0000000000	1.0
201 B	1	3.9730794904	1.0237848640	0.0858934794	3.0000000000	1.0
201 C	1	3.1413000178	1.3514120376	0.4302078853	1.0000000000	1.0
202 A	0	1.0396665953	1.0396665953	1	1	1
203 A	0	1.7527085873	0.3505417175	1	1/5	1
203 B	1	2.1960815500	0.9130968311	0.2078922869	2.0000000000	1.0
203 C	0	4.1972127996	2.0986063998	1	1/2	1
204 A	0	0.3761481280	1.1284443840	1	3	1
204 B	0	1.5198335215	1.5198335215	1	1	1
205 A	1	4.8982345731	0.9812396964	1.6026013973	0.1250000000	1.0
205 B	0	2.2297855138	1.1148927569	1	1/2	1
205 C	0	4.1540692640	2.0770346320	1	1/2	1
206 A	0	2.8621154666	1.4310577333	1	1/2	1
207 A	1	2.7562368184	0.9785571203	0.3550337597	1.0000000000	1.0
208 A	1	1.7396717417	1.1571474716	0.1662881916	4.0000000000	1.0
208 B	1	3.7085476616	1.1687186183	0.0787854657	4.0000000000	1.0
208 C	0	2.8023148363	1.4011574181	1	1/2	1
208 D	0	0.9020285967	1.8040571934	1	2	1
209 A	1	2.8242928435	1.2523241408	0.6651173640	0.6666666667	1.0
210 A	0	1.9904248604	1.9904248604	1	1	1
210 B	0	2.3533261493	1.1766630746	1	1/2	1
210 C	0	1.6773824351	1.6773824351	1	1	1
210 D	1	3.5846020263	0.9546630067	0.5326465810	0.5000000000	1.0
210 E	0	1.0259330100	2.0518660200	1	2	1
212 A	1	3.5702124333	1.1728370526	0.1095020791	3.0000000000	1.0
212 B	0	1.1700042516	1.7550063774	1	3/2	1
213 A	0	3.8962324367	1.9481162183	1	1/2	1
214 A	1	3.9012468358	1.3657652583	0.0500120425	7.0000000000	1.0
214 B	1	3.4346780132	0.8945585519	0.2604490285	1.0000000000	1.0
214 C	1	3.2209151387	1.0782012651	0.1673749880	2.0000000000	1.0
214 D	0	3.0714435058	2.0476290039	1	2/3	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
215 A	1	1.3878357096	1.2136934223	0.4372612024	2.0000000000	1.0
216 A	1	3.3065764467	1.2396536502	0.0312421238	12.0000000000	1.0
216 B	0	1.3100970703	1.3100970703	1	1	1
216 C	0	1.3892474299	1.3892474299	1	1	1
216 D	0	0.7284172462	1.4568344924	1	2	1
218 A	1	3.6044066451	1.3776391764	0.5733145475	0.6666666667	1.0
219 A	1	5.6338346986	0.7445712952	0.1321606570	1.0000000000	1.0
219 B	1	3.0575161484	1.2803214992	1.2562368639	0.3333333333	1.0
219 C	1	1.5909301236	1.4044028079	1.7655116175	0.5000000000	1.0
220 A	1	3.1205498611	1.1411181831	0.7313571222	0.5000000000	1.0
220 B	0	3.4764596611	1.7382298305	1	1/2	1
221 A	0	1.9250249682	0.9625124841	1	1/2	1
221 B	0	4.3522001524	2.1761000762	1	1/2	1
222 A	0	2.0529956743	2.0529956743	1	1	1
222 B	0	1.6808078275	1.6808078275	1	1	1
222 C	0	1.7930022995	0.8965011497	1	1/2	1
222 D	0	1.2987881634	1.2987881634	1	1	1
222 E	0	0.6807637973	0.6807637973	1	1	1
224 A	1	2.9056796324	1.1435250754	0.7870964594	0.5000000000	1.0
224 B	0	3.4350375032	1.7175187516	1	1/2	1
225 A	1	4.0529757590	1.2453986751	0.1536400350	2.0000000000	1.0
225 B	0	1.8125458617	1.2083639078	1	2/3	1
225 C	0	0.8242959390	0.8242959390	1	1	1
225 D	0	1.2056681714	2.4113363427	1	2	1
225 E	1	2.6959559895	0.8278123885	0.0255880905	12.0000000000	1.0
226 A	1	3.6884308015	1.3960301214	0.2523259342	1.5000000000	1.0
228 A	0	2.4763879622	1.2381939811	1	1/2	1
228 B	1	2.6951042086	1.2048427141	0.0745081093	6.0000000000	1.0
229 A	1	4.0779758306	1.0708644692	0.2625970613	1.0000000000	1.0
231 A	0	4.6583108957	0.5822888620	1	1/8	1
232 A	1	2.1738325628	1.2149825685	0.2794563365	2.0000000000	1.0
232 B	0	0.7973190120	1.5946380240	1	2	1
233 A	0	2.7842671971	1.3921335986	1	1/2	1
234 A	0	1.0415729062	1.0415729062	1	1	1
234 B	0	0.9776832784	1.9553665569	1	2	1
234 C	1	3.8376981202	1.0636553091	0.2771597129	1.0000000000	1.0
234 D	0	0.4593769991	1.8375079963	1	4	1
234 E	0	2.0087998968	2.0087998968	1	1	1
235 A	1	3.8526221204	0.9646622016	0.0834636923	3.0000000000	1.0
235 B	0	0.6184601660	0.6184601660	1	1	1
235 C	0	2.8253354850	2.8253354850	1	1	1
236 A	1	4.3419651590	1.2389727062	0.0951161253	3.0000000000	1.0
236 B	0	4.9840035894	1.6613345298	1	1/3	1
238 A	1	1.9151889279	1.4310007528	0.1067407377	7.0000000000	1.0
238 B	1	3.3407446902	1.0723237703	0.6419669084	0.5000000000	1.0
238 C	0	3.9826564062	1.9913282031	1	1/2	1
238 D	0	2.3178772183	2.3178772183	1	1	1
238 E	0	1.5219415283	1.5219415283	1	1	1
240 A	0	2.4128899940	1.2064449970	1	1/2	1
240 B	0	1.1583921113	1.1583921113	1	1	1
240 C	1	2.7966571710	1.2467793271	0.8916211397	0.5000000000	1.0
240 D	0	1.5962422221	1.5962422221	1	1	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
242 A	1	2.9307853161	1.4486447784	0.1235713829	4.0000000000	1.0
242 B	0	0.8024848730	0.5349899154	1	2/3	1
243 A	1	2.5479339785	1.2901905904	0.5063673554	1.0000000000	1.0
243 B	0	3.6747566856	1.2249188952	1	1/3	1
244 A	1	3.0228752686	1.3210444110	0.1456719507	3.0000000000	1.0
245 A	1	3.2744322569	0.6333450086	0.0322368866	6.0000000000	1.0
245 B	0	0.5312987757	1.0625975514	1	2	1
245 C	1	0.8334283978	1.2247323722	0.3673778022	4.0000000000	1.0
246 A	0	0.5888594615	1.7665783844	1	3	1
246 B	0	0.4325831931	2.1629159655	1	5	1
246 C	0	0.1839332913	1.1035997477	1	6	1
246 D	1	3.1765117809	1.0083093657	0.3174266098	1.0000000000	1.0
246 E	0	4.1719564061	2.0859782030	1	1/2	1
246 F	0	3.9949562396	1.3316520799	1	1/3	1
246 G	0	0.9386205914	0.9386205914	1	1	1
248 A	1	4.2937936508	1.2011105983	0.1398658967	2.0000000000	1.0
248 B	0	2.1235184936	1.0617592468	1	1/2	1
248 C	1	2.6434410671	1.3306506590	0.2516891100	2.0000000000	1.0
249 A	1	4.4420811152	0.9837596203	0.2214636777	1.0000000000	1.0
249 B	1	3.5061259800	1.5162589107	0.4324599057	1.0000000000	1.0
252 A	0	1.3797968090	1.3797968090	1	1	1
252 B	1	2.2699063939	1.3316363642	0.0977746900	6.0000000000	1.0
254 A	1	3.4367838321	1.4824320446	0.4313428243	1.0000000000	1.0
254 B	0	3.0580721852	1.5290360926	1	1/2	1
254 C	1	3.1765169064	1.1103341928	0.3495445564	1.0000000000	1.0
254 D	0	2.6996100272	2.0247075204	1	3/4	1
256 A	1	5.0378540936	1.2094018572	0.4801257975	0.5000000000	1.0
256 B	1	4.4097575960	1.3421296388	0.6087090320	0.5000000000	1.0
256 C	0	3.1181694995	1.5590847498	1	1/2	1
256 D	0	3.5623007922	1.7811503961	1	1/2	1
258 A	1	2.0851846863	1.0523025616	0.2523283833	2.0000000000	1.0
258 B	0	1.5081807103	0.7540903551	1	1/2	1
258 C	1	3.5120735391	1.1803278628	0.0336077206	10.0000000000	1.0
258 D	0	2.2171216802	1.6628412601	1	3/4	1
258 E	0	0.9267300968	1.8534601936	1	2	1
258 F	0	1.0680841434	2.1361682868	1	2	1
258 G	0	4.4942612748	2.2471306374	1	1/2	1
259 A	0	1.2860376931	1.9290565397	1	3/2	1
260 A	0	3.5288383291	1.7644191645	1	1/2	1
262 A	1	2.4323576003	1.5125639330	0.0565319064	11.0000000000	1.0
262 B	1	5.2733474636	1.1340473415	0.2150526491	1.0000000000	1.0
264 A	0	3.2306740713	1.6153370356	1	1/2	1
264 B	0	2.5539883833	1.2769941916	1	1/2	1
264 C	0	3.0763437704	1.5381718852	1	1/2	1
264 D	0	0.5045516422	1.7659307477	1	7/2	1
265 A	1	3.9017641877	1.0720624179	1.0990540344	0.2500000000	1.0
267 A	0	4.2548896082	1.4182965361	1	1/3	1
267 B	0	1.1529749720	1.1529749720	1	1	1
268 A	0	1.9329143630	1.9329143630	1	1	1
269 A	1	4.1094064150	1.3612506241	0.3312523724	1.0000000000	1.0
270 A	0	2.9614322886	0.9871440962	1	1/3	1
270 B	0	1.1639830028	1.9399716713	1	5/3	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
270 C	0	2.0452697337	2.0452697337	1	1	1
270 D	0	3.2587738278	1.0862579426	1	1/3	1
272 A	1	3.6319660374	1.2387792395	0.3410767685	1.0000000000	1.0
272 B	1	2.7457391181	1.3977853806	0.5090743587	1.0000000000	1.0
272 C	0	3.4730634628	1.7365317314	1	1/2	1
272 D	0	1.8641750575	1.8641750575	1	1	1
273 A	1	2.9063397998	0.7937029426	0.0455156082	6.0000000000	1.0
273 B	0	0.3565960398	2.8527683184	1	8	1
274 A	1	3.6347902471	1.5350419553	0.0603313240	7.0000000000	1.0
274 B	1	2.3854034454	1.1270779102	0.4724894283	1.0000000000	1.0
274 C	1	4.1119537843	1.1574087493	0.5629483258	0.5000000000	1.0
275 A	1	1.8401405799	1.0969494293	2.3844904924	0.2500000000	1.0
275 B	0	2.8380382820	2.8380382820	1	1	1
277 A	1	2.6649848531	1.5289849020	0.5737311791	1.0000000000	1.0
278 A	1	2.9825516172	1.5630333549	0.0655073891	8.0000000000	1.0
278 B	0	1.1816539317	0.7877692878	1	2/3	1
280 A	1	3.4289082752	1.3363376683	0.0974317159	4.0000000000	1.0
280 B	1	1.7288713940	1.1698436166	0.0112775269	60.0000000000	1.0
282 A	0	1.2412727555	1.8619091332	1	3/2	1
282 B	1	3.1689198047	1.6378344218	0.1292107818	4.0000000000	1.0
285 A	1	1.7646479001	1.1896124823	0.2696543559	2.5000000000	1.0
285 B	1	1.5380021359	1.6159662807	2.1013836627	0.5000000000	1.0
285 C	0	1.0845657569	1.6268486353	1	3/2	1
286 A	0	2.3766175169	0.7922058390	1	1/3	1
286 B	1	1.7061664868	1.6606489871	0.0374354527	26.0000000000	1.0
286 C	1	4.0135805233	1.0773574603	0.1342140084	2.0000000000	1.0
286 D	0	0.9232396695	1.8464793390	1	2	1
286 E	0	0.8172352255	2.4517056765	1	3	1
286 F	0	2.5074142997	2.5074142997	1	1	1
288 A	1	2.8175842074	1.4121235928	0.5011823920	1.0000000000	1.0
288 B	1	2.3118891803	1.4337635566	1.2403393457	0.5000000000	1.0
288 C	0	2.7052788037	1.3526394018	1	1/2	1
288 D	0	3.0276912696	1.5138456348	1	1/2	1
288 E	0	1.6267330006	1.6267330006	1	1	1
289 A	1	1.5008878200	1.1388947570	3.0352561777	0.2500000000	1.0
290 A	1	1.6518740857	1.1736436288	1.4209843703	0.5000000000	1.0
291 A	0	0.6260093262	0.6260093262	1	1	1
291 B	0	1.9049982068	0.4762495517	1	1/4	1
291 C	1	4.6468112382	1.0274509269	0.4422176302	0.5000000000	1.0
291 D	0	2.4120051562	2.4120051562	1	1	1
294 A	0	1.8280004280	1.8280004280	1	1	1
294 B	0	2.2120648950	2.2120648950	1	1	1
294 C	0	0.5960449739	2.3841798957	1	4	1
294 D	0	1.3860857439	1.3860857439	1	1	1
294 E	0	0.6693039984	0.6693039984	1	1	1
294 F	0	0.5279473056	1.0558946111	1	2	1
294 G	1	2.3005218481	1.2377532023	0.1345078730	4.0000000000	1.0
296 A	1	3.9715557928	1.3559367025	0.0853529935	4.0000000000	1.0
296 B	1	4.1037703943	1.3782253637	0.1679218415	2.0000000000	1.0
297 A	1	2.5629322349	0.8902546662	0.0578929773	6.0000000000	1.0
297 B	1	3.6511448387	1.1489951990	0.3146944999	1.0000000000	1.0
297 C	1	1.6944590385	1.7349705760	0.3413027475	3.0000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
297D	0	1.3749252643	2.7498505287	1	2	1
298 A	1	3.3567106288	1.6025685289	0.0530469229	9.0000000000	1.0
298 B	1	2.7479579664	1.1527590057	0.4194965934	1.0000000000	1.0
300 A	0	1.2133770573	1.2133770573	1	1	1
300 B	0	0.5426387165	1.6279161494	1	3	1
300 C	0	1.7025229104	1.7025229104	1	1	1
300 D	1	3.8069569608	1.3840260497	0.1211839328	3.0000000000	1.0
302 A	1	2.4415192618	1.6931432719	1.1557989721	0.6000000000	1.0
302 B	0	3.1902035597	1.5951017799	1	1/2	1
302 C	1	3.3905862727	1.5310645332	0.0903126722	5.0000000000	1.0
303 A	1	1.3761905514	1.4855694061	0.0771056743	14.0000000000	1.0
303 B	1	3.8524410972	1.0003648583	0.0649175960	4.0000000000	1.0
304 A	1	1.1779812107	1.5451623821	0.3279259398	4.0000000000	1.0
304 B	0	0.6013138790	1.2026277580	1	2	1
304 C	1	3.3697463259	1.3932747404	0.1033664411	4.0000000000	1.0
304 D	0	1.8637144373	1.8637144373	1	1	1
304 E	0	2.0635461959	2.0635461959	1	1	1
304 F	1	4.3504123451	1.2842245406	0.1475980251	2.0000000000	1.0
306 A	0	0.6894353592	2.0683060777	1	3	1
306 B	1	2.1525639425	1.2110949196	0.5626290098	1.0000000000	1.0
306 C	0	1.1471740369	1.1471740369	1	1	1
306 D	0	2.2491921794	2.2491921794	1	1	1
307 A	0	1.4189146241	1.4189146241	1	1	1
307 B	0	2.3490923578	2.3490923578	1	1	1
307 C	0	2.8002613258	2.8002613258	1	1	1
307 D	0	3.1747546988	3.1747546988	1	1	1
308 A	1	3.1762112278	1.3891063820	0.0728911630	6.0000000000	1.0
309 A	1	3.4344206940	1.2276894797	0.0714932496	5.0000000000	1.0
310 A	0	0.8355225373	2.5065676119	1	3	1
310 B	1	2.5186376495	1.6415121933	0.9776191071	0.6666666667	1.0
312 A	0	1.6605942610	1.6605942610	1	1	1
312 B	1	3.6898752967	1.4061524822	0.3810840121	1.0000000000	1.0
312 C	0	3.5553735765	1.7776867882	1	1/2	1
312 D	0	4.5800008728	1.1450002182	1	1/4	1
312 E	0	1.4416548405	1.4416548405	1	1	1
312 F	1	2.5056872215	1.5607603031	0.2076290408	3.0000000000	1.0
314 A	1	1.8289592463	1.2254919999	0.3350244141	2.0000000000	1.0
315 A	0	1.2730829064	1.2730829064	1	1	1
315 B	1	2.1956280654	1.1391577593	1.0376600456	0.5000000000	1.0
316 A	0	1.3028108436	1.3028108436	1	1	1
316 B	1	3.1195572511	1.2203951480	0.1304026020	3.0000000000	1.0
318 A	0	1.8177609136	1.8177609136	1	1	1
318 B	0	3.9710894510	1.3236964837	1	1/3	1
318 C	1	1.7722504261	1.1170141948	0.3151400554	2.0000000000	1.0
318 D	1	2.4120272485	1.7453725128	0.0328914668	22.0000000000	1.0
318 E	0	1.0987950980	1.0987950980	1	1	1
319 A	0	0.9677788808	1.9355577615	1	2	1
320 A	0	2.8551639918	1.4275819959	1	1/2	1
320 B	1	4.1985525041	1.5161085153	0.7222053381	0.5000000000	1.0
320 C	0	3.9942696310	1.9971348155	1	1/2	1
320 D	0	3.8549380952	1.9274690476	1	1/2	1
320 E	0	2.1294891631	1.0647445816	1	1/2	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
320 F	1	3.2161552676	1.3296773088	0.8268738280	0.5000000000	1.0
322 A	1	2.4232379290	1.2132690207	0.1668936434	3.0000000000	1.0
322 B	0	1.4602213361	1.4602213361	1	1	1
322 C	0	4.9916139661	2.4958069830	1	1/2	1
322 D	1	2.8164413550	1.6590507174	0.2356236837	2.5000000000	1.0
323 A	0	1.8660949120	1.8660949120	1	1	1
324 A	0	4.8202741093	1.6067580364	1	1/3	1
324 B	0	1.6541764509	1.6541764509	1	1	1
324 C	1	3.8631214144	1.4974408733	0.3876245949	1.0000000000	1.0
324 D	0	3.8507505887	1.2835835296	1	1/3	1
325 A	1	2.3706727615	1.5716909753	1.9889176619	0.3333333333	1.0
325 B	1	5.3009854472	1.3630614806	0.2571336017	1.0000000000	1.0
325 C	0	2.4072852546	2.4072852546	1	1	1
325 D	0	1.0054544996	3.0163634989	1	3	1
325 E	0	2.2482646094	0.4496529219	1	1/5	1
326 A	1	1.5955523534	1.2315028128	0.7718347882	1.0000000000	1.0
326 B	1	4.0636363622	1.6801606398	0.0826924700	5.0000000000	1.0
326 C	0	1.3118305721	0.4372768574	1	1/3	1
327 A	1	2.2027451583	1.2543242932	0.1423592158	4.0000000000	1.0
328 A	1	2.6783928660	1.5196480443	1.1347461857	0.5000000000	1.0
328 B	0	4.0075079103	2.0037539552	1	1/2	1
329 A	0	0.7968920264	0.7968920264	1	1	1
330 A	0	0.7813961756	0.7813961756	1	1	1
330 B	0	2.4188689938	2.4188689938	1	1	1
330 C	0	0.9389804493	1.8779608987	1	2	1
330 D	0	0.5190109186	1.8165382150	1	7/2	1
330 E	1	2.2164710033	1.1562800329	0.5216761380	1.0000000000	1.0
331 A	1	5.4083773317	0.9790098108	0.1810172905	1.0000000000	1.0
333 A	1	2.0410609926	1.4792994921	0.7247698611	1.0000000000	1.0
333 B	1	2.0333288908	1.8286710385	1.7986967546	0.5000000000	1.0
333 C	1	2.5034325770	1.2023832503	0.9605876837	0.5000000000	1.0
333 D	0	2.8306206392	2.8306206392	1	1	1
334 A	0	2.2906969318	2.2906969318	1	1	1
335 A	1	4.2194745615	1.4976179639	0.1774649831	2.0000000000	1.0
336 A	0	2.3898781774	1.1949390887	1	1/2	1
336 B	0	5.4362515690	1.3590628923	1	1/4	1
336 C	0	1.1982307057	1.7973460585	1	3/2	1
336 D	0	0.7884933856	1.5769867712	1	2	1
336 E	1	1.9109897808	1.4261701612	0.3731496043	2.0000000000	1.0
336 F	0	3.9315932027	1.9657966013	1	1/2	1
338 A	1	3.7619828722	1.2776773619	0.1698143513	2.0000000000	1.0
338 B	0	1.0433863187	2.0867726374	1	2	1
338 C	0	1.2869571133	2.5739142267	1	2	1
338 D	0	0.5396273391	1.0792546783	1	2	1
338 E	1	1.9456540409	1.7940807919	0.1536827509	6.0000000000	1.0
338 F	1	1.2055736044	0.9530562430	0.1976354325	4.0000000000	1.0
339 A	1	2.5311158528	1.5850840591	0.0695821374	9.0000000000	1.0
339 B	0	2.6056637603	2.6056637603	1	1	1
339 C	1	3.9967355775	1.0125851133	0.0844510137	3.0000000000	1.0
340 A	1	4.7549194562	1.5553431744	0.4361358627	0.7500000000	1.0
342 A	0	0.6943374598	2.0830123794	1	3	1
342 B	0	2.1705272030	2.1705272030	1	1	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
342 C	1	1.7816724980	1.2648109168	0.3549504520	2.0000000000	1.0
342 D	0	2.2375637156	2.2375637156	1	1	1
342 E	1	3.9793483508	1.2390587277	0.3113722696	1.0000000000	1.0
342 F	0	0.9193234687	0.9193234687	1	1	1
342 G	0	1.3602155382	1.3602155382	1	1	1
344 A	1	2.7525869611	1.5494621523	0.2814556223	2.0000000000	1.0
345 A	0	0.4590292151	0.9180584301	1	2	1
345 B	1	4.2977618837	1.5964898077	0.1857350234	2.0000000000	1.0
345 C	0	0.8426900245	2.1067250613	1	5/2	1
345 D	0	2.1209719918	1.0604859959	1	1/2	1
345 E	0	1.2779994655	2.5559989310	1	2	1
345 F	1	2.0934610738	1.0776581097	0.0214488924	24.0000000000	1.0
346 A	0	2.3864267831	2.3864267831	1	1	1
346 B	1	3.2277857995	1.8161375715	0.0803796288	7.0000000000	1.0
347 A	1	3.0596247368	1.0692572441	0.3494733296	1.0000000000	1.0
348 A	1	3.3700624663	1.4721036783	0.1456059735	3.0000000000	1.0
348 B	0	1.8569948187	1.8569948187	1	1	1
348 C	0	1.1001223703	1.1001223703	1	1	1
348 D	1	2.7246446828	1.6385158325	0.0286366013	21.0000000000	1.0
350 A	0	1.0556963012	1.0556963012	1	1	1
350 B	0	1.2015896516	2.4031793031	1	2	1
350 C	1	2.6868361419	1.1698407546	0.2176985668	2.0000000000	1.0
350 D	0	2.6582491803	2.6582491803	1	1	1
350 E	0	0.8585438894	1.7170877788	1	2	1
350 F	1	1.9197624983	1.6277461855	0.0128468084	66.0000000000	1.0
352 A	0	0.9197112746	1.8394225491	1	2	1
352 B	1	2.6902393719	1.6620028340	0.3088949726	2.0000000000	1.0
352 C	1	3.6135445187	1.4943761442	0.2067742817	2.0000000000	1.0
352 D	1	1.7453844567	1.4403302305	0.4126111657	2.0000000000	1.0
352 E	0	1.1446739244	2.2893478489	1	2	1
352 F	1	1.8476278580	1.2616805128	0.1138108438	6.0000000000	1.0
353 A	0	2.7058312639	1.3529156319	1	1/2	1
354 A	0	3.6819566662	1.8409783331	1	1/2	1
354 B	0	1.8675548347	1.2450365565	1	2/3	1
354 C	1	2.5265851358	1.1762556419	0.2327757781	2.0000000000	1.0
354 D	0	1.9702699829	0.9851349914	1	1/2	1
354 E	0	0.3856294396	2.1209619178	1	11/2	1
354 F	1	3.0155788594	1.8179484925	0.0430608748	14.0000000000	1.0
355 A	0	2.2892784269	0.7630928090	1	1/3	1
356 A	1	1.9261487706	1.4738366671	0.2550576033	3.0000000000	1.0
357 A	0	0.5059809459	1.0119618919	1	2	1
357 B	1	1.3908121474	1.4226357062	0.2557203194	4.0000000000	1.0
357 C	0	1.5730848489	3.1461696978	1	2	1
357 D	1	2.5259384793	1.0297863326	0.0291203318	14.0000000000	1.0
358 A	0	1.4369776837	1.4369776837	1	1	1
358 B	0	4.8020823416	1.6006941139	1	1/3	1
359 A	1	5.2576534953	1.6996723486	0.3232758397	1.0000000000	1.0
359 B	1	5.4008701535	1.2244318944	0.2267101152	1.0000000000	1.0
360 A	0	1.3930826876	1.3930826876	1	1	1
360 B	0	1.4341648133	1.4341648133	1	1	1
360 C	0	1.5689797603	1.5689797603	1	1	1
360 D	0	1.6146507705	1.6146507705	1	1	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
360 E	1	2.3312316373	1.5940059578	0.6837612927	1.0000000000	1.0
361 A	1	4.1905500198	1.5194236564	0.1812916740	2.0000000000	1.0
361 B	0	0.9468199298	1.8936398596	1	2	1
362 A	1	5.3424976181	1.2148757892	0.2273984709	1.0000000000	1.0
362 B	1	2.4995552224	1.8658498774	0.1066389652	7.0000000000	1.0
363 A	0	1.6550761880	0.4137690470	1	1/4	1
363 B	0	2.7390143163	2.7390143163	1	1	1
363 C	0	0.7861735761	0.7861735761	1	1	1
364 A	1	1.9913710109	1.5755626644	0.0527463292	15.0000000000	1.0
364 B	1	3.4220544534	1.3982710251	0.1362019068	3.0000000000	1.0
366 A	0	0.6308411291	2.5233645163	1	4	1
366 B	0	2.4635740364	2.4635740364	1	1	1
366 C	0	0.4340530637	1.3021591910	1	3	1
366 D	0	0.2600804041	1.8205628289	1	7	1
366 E	0	1.5897058463	0.7948529232	1	1/2	1
366 F	1	2.7041672516	1.3650563331	0.3785979766	1.3333333333	1.0
366 G	1	2.8119685919	1.8579940284	0.0330372472	20.0000000000	1.0
368 A	1	1.9950448236	1.6292204503	0.8166335067	1.0000000000	1.0
368 B	0	1.8184117667	1.8184117667	1	1	1
368 C	0	1.6505359739	1.6505359739	1	1	1
368 D	1	2.3725142889	1.6792666204	0.7078004243	1.0000000000	1.0
368 E	1	3.0187971397	1.5065697188	0.4990629211	1.0000000000	1.0
368 F	0	2.1965830501	2.1965830501	1	1	1
368 G	1	4.9462587640	1.2657315052	0.2558967425	1.0000000000	1.0
369 A	1	2.4151226244	1.5724499828	0.3255424728	2.0000000000	1.0
369 B	0	0.7651091552	3.0604366208	1	4	1
370 A	1	4.2831069879	1.2956210363	0.6049912085	0.5000000000	1.0
370 B	0	1.5799277405	1.5799277405	1	1	1
370 C	0	1.6518622652	0.5506207551	1	1/3	1
370 D	0	1.6806015939	1.6806015939	1	1	1
371 A	1	0.9763159406	1.8303154738	0.9373581839	2.0000000000	1.0
371 B	0	1.0113136188	3.0339408564	1	3	1
372 A	1	4.2046190818	1.4965237117	0.0593206219	6.0000000000	1.0
372 B	0	3.3409804579	1.6704902290	1	1/2	1
372 C	0	0.3199701361	1.9198208165	1	6	1
372 D	1	3.0344782146	1.6984822680	0.0466439958	12.0000000000	1.0
373 A	1	3.9392451210	1.1341580880	0.2879125450	1.0000000000	1.0
374 A	1	2.1703306562	1.3150843335	0.6059373164	1.0000000000	1.0
377 A	1	5.3853739690	1.9439778472	1.4438944136	0.2500000000	1.0
378 A	0	2.1727069014	2.1727069014	1	1	1
378 B	0	3.2456774250	1.0818924750	1	1/3	1
378 C	0	1.1058292241	2.2116584483	1	2	1
378 D	1	3.3687530822	1.3035890492	0.0644941426	6.0000000000	1.0
378 E	0	1.1994469958	2.3988939916	1	2	1
378 F	1	4.3724105667	1.2660299663	1.3029734425	0.2222222222	1.0
378 G	0	0.4860952950	2.4304764752	1	5	1
378 H	0	0.7591617028	0.7591617028	1	1	1
380 A	1	2.9974122712	1.6365169446	1.0919531893	0.5000000000	1.0
380 B	0	1.3073814431	1.9610721647	1	3/2	1
381 A	1	2.6366565023	1.6669090042	0.1264411199	5.0000000000	1.0
381 B	0	3.3176310732	3.3176310732	1	1	1
384 A	0	3.5791319964	1.7895659982	1	1/2	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
384B	0	2.5308285054	1.2654142527	1	1/2	1
384C	0	3.3922393877	1.6961196939	1	1/2	1
384D	1	4.7973509489	1.5338705503	0.6394656412	0.5000000000	1.0
384E	0	1.3847627016	2.0771440525	1	3/2	1
384F	0	2.9677994642	1.4838997321	1	1/2	1
384G	0	1.9583501933	0.9791750967	1	1/2	1
384H	1	4.1971022527	1.7104633501	0.2716895685	1.5000000000	1.0
385A	1	2.3974795614	1.2551027987	1.0470185598	0.5000000000	1.0
385B	1	3.0431080851	1.0141520788	0.3332619317	1.0000000000	1.0
387A	0	0.6570811277	1.3141622553	1	2	1
387B	1	1.1323623368	1.9788330815	0.8737632016	2.0000000000	1.0
387C	1	4.3310513612	1.2498477542	0.1442891864	2.0000000000	1.0
387D	0	2.7905613828	0.6976403457	1	1/4	1
387E	0	1.5740674722	3.1481349443	1	2	1
389A	2	4.9804251217	0.7593165003	0.1524601779	1.0000000000	1.0
390A	1	4.0725001283	1.1966774321	0.2938434363	1.0000000000	1.0
390B	0	1.9598275838	1.9598275838	1	1	1
390C	0	2.4442145217	2.4442145217	1	1	1
390D	0	0.4686191249	1.4058573748	1	3	1
390E	0	1.8466374169	1.8466374169	1	1	1
390F	0	0.8835342653	0.8835342653	1	1	1
390G	0	1.3107516519	1.3107516519	1	1	1
392A	1	0.8678863534	1.6420053999	1.8919590029	1.0000000000	1.0
392B	0	0.9021374286	1.8042748572	1	2	1
392C	1	3.5611188974	1.5286009201	0.0715412283	6.0000000000	1.0
392D	0	1.2749543703	1.2749543703	1	1	1
392E	0	1.1107639394	2.2215278788	1	2	1
392F	1	5.0380294438	1.3056845741	0.1295828646	2.0000000000	1.0
395A	0	3.2068928320	0.8017232080	1	1/4	1
395B	0	0.9401374803	1.4102062205	1	3/2	1
395C	0	3.1249773129	0.6249954626	1	1/5	1
396A	0	0.4597228734	1.3791686201	1	3	1
396B	1	1.7552831658	1.6410582466	0.3116416919	3.0000000000	1.0
396C	0	1.7632679755	1.7632679755	1	1	1
398A	0	2.7300308931	1.3650154466	1	1/2	1
399A	1	1.3293482923	1.8881722279	2.8407487170	0.5000000000	1.0
399B	1	2.5299798485	1.1285450388	0.8921375713	0.5000000000	1.0
399C	0	2.6662366521	1.3331183261	1	1/2	1
400A	1	1.8057642615	1.6854886296	0.9333935030	1.0000000000	1.0
400B	0	0.8930706751	1.7861413502	1	2	1
400C	1	1.9969667383	1.5552187610	0.0648992099	12.0000000000	1.0
400D	0	4.1418019047	2.0709009524	1	1/2	1
400E	0	2.0340751908	1.0170375954	1	1/2	1
400F	0	1.8522701217	0.9261350608	1	1/2	1
400G	0	1.1310974387	2.2621948774	1	2	1
400H	1	2.5292107621	1.2987503631	0.1283750629	4.0000000000	1.0
402A	1	1.5449271648	1.2440174086	0.4026136108	2.0000000000	1.0
402B	0	2.8944959015	1.4472479508	1	1/2	1
402C	0	4.1774362503	2.0887181252	1	1/2	1
402D	1	2.1211176461	1.4143116144	0.3333883005	2.0000000000	1.0
404A	1	3.3748684624	1.6772917538	0.1656649015	3.0000000000	1.0
404B	0	3.7312884804	1.2437628268	1	1/3	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
405 A	0	4.0159643401	1.3386547800	1	1/3	1
405 B	1	4.4575296500	1.5847069791	1.0665371429	0.3333333333	1.0
405 C	1	4.3122245542	2.0170136488	0.4677431853	1.0000000000	1.0
405 D	1	1.3820485348	1.2688418437	0.3060292534	3.0000000000	1.0
405 E	0	2.9578666342	2.9578666342	1	1	1
405 F	1	3.6601877915	0.9965850007	0.2722770135	1.0000000000	1.0
406 A	1	1.6875693147	1.3611714716	0.8065870004	1.0000000000	1.0
406 B	1	1.7971708524	1.4272791044	0.5956358166	1.3333333333	1.0
406 C	1	3.0327084838	1.9468317637	0.0401215566	16.0000000000	1.0
406 D	0	0.3392055689	1.6960278446	1	5	1
408 A	0	1.8120385466	1.8120385466	1	1	1
408 B	0	3.6474030320	1.8237015160	1	1/2	1
408 C	0	0.7343586417	1.4687172834	1	2	1
408 D	1	2.4678501327	1.7688362275	0.0358375941	20.0000000000	1.0
410 A	1	3.2967108102	1.3868548095	0.4206783335	1.0000000000	1.0
410 B	0	0.7852065211	2.3556195632	1	3	1
410 C	0	2.2878165418	0.7626055139	1	1/3	1
410 D	1	2.6404517311	1.8491499415	0.1750789382	4.0000000000	1.0
414 A	0	0.5627606669	2.2510426675	1	4	1
414 B	0	1.1633580465	2.3267160931	1	2	1
414 C	1	1.3905616429	1.3266055581	0.4770035060	2.0000000000	1.0
414 D	1	2.0997210460	2.0793639154	0.0990304840	10.0000000000	1.0
415 A	0	0.7384429369	2.9537717476	1	4	1
416 A	0	1.9071354957	1.9071354957	1	1	1
416 B	1	3.3952295292	1.5990341270	0.2354824782	2.0000000000	1.0
417 A	0	1.7339868091	1.7339868091	1	1	1
418 A	0	4.8048009610	2.4024004805	1	1/2	1
418 B	1	1.2722415997	1.9863516582	0.0600500257	26.0000000000	1.0
418 C	0	1.5273830792	3.0547661583	1	2	1
420 A	0	0.7540181380	1.1310272070	1	3/2	1
420 B	0	2.6795163333	1.3397581666	1	1/2	1
420 C	0	3.4562422219	1.7281211110	1	1/2	1
420 D	0	3.6614886666	1.8307443333	1	1/2	1
422 A	1	2.1324420574	1.4027291428	0.3289020534	2.0000000000	1.0
423 A	1	2.9332316212	1.6367929761	0.1395042386	4.0000000000	1.0
423 B	0	1.9670814874	1.9670814874	1	1	1
423 C	1	1.0527165607	2.0430649788	1.9407550475	1.0000000000	1.0
423 D	0	1.5929150000	3.1858300000	1	2	1
423 E	0	1.5476958786	3.0953917572	1	2	1
423 F	1	3.1909530365	1.0465381336	0.1639851984	2.0000000000	1.0
423 G	1	4.8510516054	0.9620915002	0.0991631896	2.0000000000	1.0
425 A	1	2.7675003964	2.0769440878	1.5009530553	0.5000000000	1.0
425 B	1	2.2952233022	1.9443756342	0.2823800241	3.0000000000	1.0
425 C	1	5.1322753273	1.4080433297	0.2743507002	1.0000000000	1.0
425 D	1	1.2501339247	1.0183803087	1.6292339382	0.5000000000	1.0
426 A	0	2.6994377236	2.6994377236	1	1	1
426 B	1	1.9951714954	1.2152039603	0.6090724347	1.0000000000	1.0
426 C	0	0.9108770414	1.5181284024	1	5/3	1
427 A	0	2.1028874777	2.1028874777	1	1	1
427 B	1	5.3267648633	2.1617002677	0.4058186016	1.0000000000	1.0
427 C	1	2.0761302548	1.4102148914	0.6792516453	1.0000000000	1.0
428 A	0	0.6740067866	2.0220203597	1	3	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
428 B	1	2.6164175615	1.6383190525	0.2087229343	3.0000000000	1.0
429 A	1	3.2842888785	1.1462496668	0.6980200032	0.5000000000	1.0
429 B	1	2.5841361086	1.3666547477	1.0577265982	0.5000000000	1.0
430 A	1	4.7241011000	1.3745515859	0.2909657429	1.0000000000	1.0
430 B	1	1.7919479982	1.4065602885	0.1569867306	5.0000000000	1.0
430 C	1	2.4806272616	1.8882295645	0.7611903625	1.0000000000	1.0
430 D	1	1.3842895617	1.9231655442	0.0185237308	75.0000000000	1.0
431 A	1	2.4239283338	1.4372881309	0.5929581790	1.0000000000	1.0
431 B	0	1.3899255898	1.3899255898	1	1	1
432 A	0	1.5299540371	1.5299540371	1	1	1
432 B	1	1.9276212967	1.7360954879	0.4503206856	2.0000000000	1.0
432 C	0	0.7563848962	1.5127697924	1	2	1
432 D	1	2.4062471328	1.7185974478	0.0595185973	12.0000000000	1.0
432 E	0	0.8924581010	1.7849162020	1	2	1
432 F	1	1.8227448642	1.6865597223	0.2313214202	4.0000000000	1.0
432 G	0	1.9090528016	1.9090528016	1	1	1
432 H	0	1.2616556796	1.2616556796	1	1	1
433 A	2	4.2147101930	0.9470207809	0.2246941634	1.0000000000	1.0
434 A	1	2.9955292798	1.3951818152	0.9315093827	0.5000000000	1.0
434 B	0	2.8187709466	2.8187709466	1	1	1
434 C	0	3.0097830443	3.0097830443	1	1	1
434 D	1	1.8196638478	1.8789054223	0.2065112658	5.0000000000	1.0
434 E	0	0.3423513497	1.0270540490	1	3	1
435 A	0	4.4008169706	1.4669389902	1	1/3	1
435 B	0	0.8094965277	0.8094965277	1	1	1
435 C	0	4.7719988507	2.3859994254	1	1/2	1
435 D	0	2.0907731352	1.0453865676	1	1/2	1
437 A	1	1.7599083319	1.8940046259	0.2690487612	4.0000000000	1.0
437 B	0	1.8460399927	3.6920799854	1	2	1
438 A	0	0.8715299494	2.6145898483	1	3	1
438 B	0	2.5149090457	2.5149090457	1	1	1
438 C	1	3.3565130070	1.2587504610	0.3750173047	1.0000000000	1.0
438 D	1	1.4530902095	1.5258264643	1.5750843832	0.6666666667	1.0
438 E	0	1.4191115085	1.4191115085	1	1	1
438 F	1	3.0391118606	2.0220564891	0.6653445420	1.0000000000	1.0
438 G	1	3.3567264352	1.4368065918	0.2140190182	2.0000000000	1.0
440 A	1	1.9276140142	1.7340332813	0.8995749504	1.0000000000	1.0
440 B	1	2.1964080283	1.7346746206	0.7897779457	1.0000000000	1.0
440 C	0	2.2356730543	1.6767547907	1	3/4	1
440 D	0	0.8303368442	2.4910105327	1	3	1
441 A	0	0.6045886821	1.2091773641	1	2	1
441 B	1	2.7705733999	1.6517711259	0.8942757802	0.6666666667	1.0
441 C	1	0.7875316161	2.0779091177	1.3192544116	2.0000000000	1.0
441 D	1	2.9531824110	1.2945586181	0.4383605338	1.0000000000	1.0
441 E	0	0.4189900774	0.8379801549	1	2	1
441 F	1	3.2597903746	0.9944972960	0.0762700344	4.0000000000	1.0
442 A	0	2.4942559031	2.4942559031	1	1	1
442 B	1	1.3616952016	2.1459321167	0.1313272428	12.0000000000	1.0
442 C	0	1.7613965761	1.7613965761	1	1	1
442 D	0	2.7807390608	2.7807390608	1	1	1
442 E	0	0.2845834364	3.1304178002	1	11	1
443 A	1	3.9135658341	1.7609825329	0.4499688028	1.0000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
443 B	1	3.6564568242	1.0212452670	0.2792991456	1.0000000000	1.0
443 C	0	1.5792505220	1.5792505220	1	1	1
444 A	0	2.5015053363	1.2507526681	1	1/2	1
444 B	1	3.1314379361	1.8584060832	0.1978224404	3.0000000000	1.0
446 A	1	3.9771573679	1.2749242492	0.1602808402	2.0000000000	1.0
446 B	1	2.0484809659	2.0453212377	0.0713183947	14.0000000000	1.0
446 C	0	1.9305232636	2.8957848955	1	3/2	1
446 D	2	4.8297343529	0.9402826047	0.0973430976	2.0000000000	1.0
448 A	1	2.4735961738	1.7234498942	0.6967385835	1.0000000000	1.0
448 B	1	1.6236666926	1.7358759869	1.0691085768	1.0000000000	1.0
448 C	0	2.1015304995	2.1015304995	1	1	1
448 D	0	2.0546257720	2.0546257720	1	1	1
448 E	0	2.3852212186	2.3852212186	1	1	1
448 F	0	0.9372638440	0.9372638440	1	1	1
448 G	1	2.4289383121	1.4865234745	0.6120054458	1.0000000000	1.0
448 H	0	1.3407014907	1.3407014907	1	1	1
450 A	0	1.1492430291	2.2984860582	1	2	1
450 B	0	2.3058985678	2.3058985678	1	1	1
450 C	1	2.5697855357	1.4111308701	0.1372809959	4.0000000000	1.0
450 D	0	1.0312291894	1.0312291894	1	1	1
450 E	0	1.1001165420	2.2002330841	1	2	1
450 F	1	1.7759273414	1.4100767515	0.1984986546	4.0000000000	1.0
450 G	0	0.5981911141	1.1963822281	1	2	1
451 A	1	2.8355647891	1.7563718021	0.3097040507	2.0000000000	1.0
455 A	1	2.3100747043	2.1253916593	1.8401064307	0.5000000000	1.0
455 B	1	4.2935157508	1.3274338885	2.4733742054	0.1250000000	1.0
456 A	0	3.3984495514	1.6992247757	1	1/2	1
456 B	0	1.3176900889	1.9765351334	1	3/2	1
456 C	1	2.4058503309	1.8547501445	0.0321222210	24.0000000000	1.0
456 D	1	1.4353571813	1.6586130651	0.0962950248	12.0000000000	1.0
458 A	1	4.4546212951	1.0227700195	0.1147987620	2.0000000000	1.0
458 B	1	2.5233506473	2.0857010259	0.0826560125	10.0000000000	1.0
459 A	1	2.3708814092	2.1304937413	0.8986083121	1.0000000000	1.0
459 B	1	2.1438415359	0.9525608247	0.2221621348	2.0000000000	1.0
459 C	0	1.5234809750	1.5234809750	1	1	1
459 D	0	2.8921721983	2.8921721983	1	1	1
459 E	0	1.6822303161	3.3644606323	1	2	1
459 F	0	3.0444866762	1.0148288921	1	1/3	1
459 G	0	0.9552159908	0.9552159908	1	1	1
459 H	1	2.4618961763	1.3287697414	0.1799114241	3.0000000000	1.0
460 A	0	1.3854698570	1.3854698570	1	1	1
460 B	0	1.1935074150	2.3870148301	1	2	1
460 C	1	1.5839825779	1.9113524379	0.6033375823	2.0000000000	1.0
460 D	1	4.0282816130	1.6666314043	0.0689554325	6.0000000000	1.0
462 A	1	1.4946342423	1.2866215632	0.4304135175	2.0000000000	1.0
462 B	0	1.0018600266	1.0018600266	1	1	1
462 C	1	2.4613024381	1.2390297777	1.0068082317	0.5000000000	1.0
462 D	0	0.3229321592	1.2917286370	1	4	1
462 E	1	1.2302485152	2.0363867119	0.0394110599	42.0000000000	1.0
462 F	0	1.3789297070	2.7578594140	1	2	1
462 G	0	1.3136031374	2.6272062748	1	2	1
464 A	1	2.2740483412	1.8709794251	0.4113763527	2.0000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
464 B	1	3.6733647241	1.6722315624	0.2276157812	2.0000000000	1.0
464 C	0	0.9667176524	1.9334353048	1	2	1
464 D	0	1.5984880757	1.5984880757	1	1	1
464 E	0	1.3661402106	0.6830701053	1	1/2	1
464 F	0	2.5655588249	2.5655588249	1	1	1
464 G	0	1.1118048150	2.2236096300	1	2	1
465 A	1	2.9030659041	2.0294354736	1.3981325541	0.5000000000	1.0
465 B	1	2.6840015689	1.4675682335	2.1871346879	0.2500000000	1.0
466 A	0	3.1242578300	1.5621289150	1	1/2	1
466 B	0	3.9592169668	2.6394779779	1	2/3	1
467 A	1	5.1775927316	1.2788013566	0.2469876298	1.0000000000	1.0
468 A	0	0.6621220594	1.9863661783	1	3	1
468 B	0	1.2989007218	1.2989007218	1	1	1
468 C	1	3.2358344502	1.7484928207	0.1801176634	3.0000000000	1.0
468 D	0	1.5799514360	1.5799514360	1	1	1
468 E	0	1.8269518880	1.8269518880	1	1	1
469 A	1	1.6007144150	2.2667465654	1.4160843084	1.0000000000	1.0
469 B	1	5.2760385039	0.9096891283	0.1724189707	1.0000000000	1.0
470 A	1	3.5287956897	1.5542574208	0.2202249092	2.0000000000	1.0
470 B	0	2.0374386103	1.3582924069	1	2/3	1
470 C	1	2.1318477419	1.3213107468	0.0442711443	14.0000000000	1.0
470 D	0	3.9537193611	2.6358129074	1	2/3	1
470 E	1	4.4113118028	2.1138922125	0.1197995238	4.0000000000	1.0
470 F	1	1.6512381529	1.8346862509	0.0264546976	42.0000000000	1.0
471 A	1	3.5535684064	1.1344335682	0.1596189293	2.0000000000	1.0
472 A	1	3.2876310754	1.3378014267	0.2034597855	2.0000000000	1.0
472 B	0	0.5854786616	1.1709573233	1	2	1
472 C	0	2.3105577484	2.3105577484	1	1	1
472 D	0	1.1381594447	2.2763188895	1	2	1
472 E	1	2.7652148704	1.6407951896	0.1483424676	4.0000000000	1.0
473 A	1	2.3797197308	1.1745884799	0.2467913479	2.0000000000	1.0
474 A	1	1.1990226570	1.3453488048	0.5610189253	2.0000000000	1.0
474 B	1	3.0414875397	1.5313917147	0.0503500900	10.0000000000	1.0
475 A	0	1.8243091183	1.8243091183	1	1	1
475 B	1	1.6025344539	2.1875734114	2.7301421271	0.5000000000	1.0
475 C	1	3.5833759752	1.3360934191	0.7457176854	0.5000000000	1.0
477 A	1	1.7789208762	2.1819346902	1.2265496006	1.0000000000	1.0
480 A	1	3.1797233168	1.6466507576	1.0357195225	0.5000000000	1.0
480 B	0	2.6833296776	1.3416648388	1	1/2	1
480 C	0	3.4874928122	1.7437464061	1	1/2	1
480 D	0	1.2325760765	1.8488641147	1	3/2	1
480 E	0	2.0939543480	1.0469771740	1	1/2	1
480 F	1	3.2610572384	1.6836136414	1.0325569399	0.5000000000	1.0
480 G	0	3.8220157083	1.9110078541	1	1/2	1
480 H	0	2.0395949022	2.0395949022	1	1	1
481 A	1	2.6783175658	2.1620581228	3.2289794914	0.2500000000	1.0
482 A	1	0.9101999656	1.1364955128	0.6243108964	2.0000000000	1.0
483 A	0	0.7534884814	3.7674424072	1	5	1
483 B	0	3.4388742008	3.4388742008	1	1	1
484 A	1	0.9208366680	1.9006792135	0.5160196372	4.0000000000	1.0
485 A	0	1.5708346942	0.5236115647	1	1/3	1
485 B	1	4.5625314551	1.7297914316	0.3791297547	1.0000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
486 A	1	2.8478067545	1.4572284936	0.2558510143	2.0000000000	1.0
486 B	1	3.0412502685	1.3830920358	0.4547774480	1.0000000000	1.0
486 C	0	3.7142083977	1.2380694659	1	1/3	1
486 D	0	1.1331617992	2.2663235984	1	2	1
486 E	0	2.6410373364	2.6410373364	1	1	1
486 F	1	2.4821083008	2.2536351937	0.3026506664	3.0000000000	1.0
490 A	1	1.1268749665	1.5859684246	2.1111061188	0.6666666667	1.0
490 B	0	1.4955620274	1.4955620274	1	1	1
490 C	0	0.7299718606	0.7299718606	1	1	1
490 D	1	2.9268743131	1.3457322218	0.2298923831	2.0000000000	1.0
490 E	0	1.5212992086	1.5212992086	1	1	1
490 F	0	1.6419298285	3.2838596569	1	2	1
490 G	1	2.2143485488	1.9926925955	0.0899900152	10.0000000000	1.0
490 H	0	1.1848208922	2.3696417844	1	2	1
490 I	0	1.0309311849	3.0927935547	1	3	1
490 J	0	0.3071649459	3.0716494587	1	10	1
490 K	0	0.7046660037	1.4093320074	1	2	1
492 A	1	3.5983226104	1.6799595493	0.1556243220	3.0000000000	1.0
492 B	1	1.3947922133	1.9348232249	0.0513769139	27.0000000000	1.0
493 A	0	0.4817702232	0.4817702232	1	1	1
493 B	1	2.2243974247	0.9092472733	0.0681268601	6.0000000000	1.0
494 A	1	2.0578845874	1.3430728091	0.3263236474	2.0000000000	1.0
494 B	0	2.7476356194	1.3738178097	1	1/2	1
494 C	0	0.8534817324	1.7069634647	1	2	1
494 D	1	1.3176510859	2.1566805189	0.0209841213	78.0000000000	1.0
495 A	1	1.9875553780	1.3104268430	1.3186317800	0.5000000000	1.0
496 A	1	3.7499429781	1.7858094939	0.4762231064	1.0000000000	1.0
496 B	0	2.3224405828	2.3224405828	1	1	1
496 C	0	2.3037511398	2.3037511398	1	1	1
496 D	0	1.8977267714	1.8977267714	1	1	1
496 E	1	5.2038879824	1.8742408437	0.3601616426	1.0000000000	1.0
496 F	1	1.1195541660	1.7961902528	1.6043799463	1.0000000000	1.0
497 A	1	1.5344348416	2.0670876478	0.2694265787	5.0000000000	1.0
498 A	0	3.2946394322	1.6473197161	1	1/2	1
498 B	1	2.6505109415	1.5703804737	0.0592482170	10.0000000000	1.0
501 A	0	2.6222246447	1.3111123223	1	1/2	1
503 A	1	1.0074334168	2.3519086984	2.3345549783	1.0000000000	1.0
503 B	0	2.9188966226	2.9188966226	1	1	1
503 C	0	0.6270525696	0.6270525696	1	1	1
504 A	1	3.5009394562	1.8034162571	0.2575617602	2.0000000000	1.0
504 B	0	1.7758299335	1.7758299335	1	1	1
504 C	0	1.3257183031	1.3257183031	1	1	1
504 D	0	0.8492127775	1.6984255550	1	2	1
504 E	1	4.3429909590	1.8193359362	0.4189131300	1.0000000000	1.0
504 F	1	3.1386213067	1.8042640647	1.1497175915	0.5000000000	1.0
504 G	0	0.6917988205	1.3835976409	1	2	1
504 H	0	1.9475249698	1.9475249698	1	1	1
505 A	1	5.2536146481	2.2232482832	1.6927379963	0.2500000000	1.0
506 A	1	1.8186526896	1.2018688781	0.6608567348	1.0000000000	1.0
506 B	0	0.8951608921	0.8951608921	1	1	1
506 C	0	3.2096550026	1.0698850009	1	1/3	1
506 D	1	1.8611497781	1.4567809076	0.1565463376	5.0000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
506 E	1	3.5504735863	2.2952032476	0.2154832956	3.0000000000	1.0
506 F	1	2.6021500295	2.0097923473	0.0594121821	13.0000000000	1.0
507 A	1	1.3267718509	2.0594887788	0.2587092344	6.0000000000	1.0
507 B	1	4.7837439392	1.2172344104	0.1272261252	2.0000000000	1.0
507 C	1	0.9171278260	1.1623161893	5.0693748738	0.2500000000	1.0
510 A	0	0.8377889603	0.8377889603	1	1	1
510 B	0	0.4600038552	1.3800115656	1	3	1
510 C	0	1.9361123184	1.9361123184	1	1	1
510 D	1	2.1234819095	2.1812264077	0.3423977694	3.0000000000	1.0
510 E	0	2.0809871839	2.0809871839	1	1	1
510 F	0	2.5858436071	2.5858436071	1	1	1
510 G	0	0.9466241610	2.8398724830	1	3	1
513 A	1	0.9183260617	2.2618541806	2.4630186107	1.0000000000	1.0
513 B	1	4.0784452383	1.3431909599	0.1097796571	3.0000000000	1.0
514 A	1	1.5692657075	2.3372236738	1.4893740829	1.0000000000	1.0
514 B	1	3.7592158470	2.0083555695	0.5342485378	1.0000000000	1.0
516 A	0	1.5157563762	1.5157563762	1	1	1
516 B	1	1.3986133273	1.6697886091	0.1989814455	6.0000000000	1.0
516 C	0	1.2281719842	1.8422579763	1	3/2	1
516 D	0	0.7668761526	2.3006284579	1	3	1
517 A	0	1.5383608886	3.0767217772	1	2	1
517 B	0	1.1898355917	2.3796711833	1	2	1
517 C	1	0.5994133999	2.6598825791	0.3697896671	12.0000000000	1.0
520 A	1	3.9206170977	1.8629119109	0.9503156592	0.5000000000	1.0
520 B	0	2.2507080525	2.2507080525	1	1	1
522 A	1	1.0344899410	1.5670842850	0.7574188124	2.0000000000	1.0
522 B	0	1.4203697590	1.4203697590	1	1	1
522 C	0	1.1900744638	0.7933829759	1	2/3	1
522 D	0	0.6150738882	1.2301477764	1	2	1
522 E	1	2.6723250627	1.4761194092	0.1380931749	4.0000000000	1.0
522 F	1	1.1162693937	1.4521616721	0.6504530539	2.0000000000	1.0
522 G	0	0.2048719257	2.2535911826	1	11	1
522 H	0	3.8963687108	2.5975791405	1	2/3	1
522 I	1	2.8948486012	2.3142517113	0.0799437909	10.0000000000	1.0
522 J	1	1.6391662953	2.3288102501	0.0273217017	52.0000000000	1.0
522 K	0	2.1298310269	2.1298310269	1	1	1
522 L	0	1.2838016184	2.5676032368	1	2	1
522 M	0	0.1265100626	2.7832213780	1	22	1
524 A	1	3.3330958841	1.9866907710	0.5960496908	1.0000000000	1.0
525 A	1	2.6222161011	1.2004647677	1.8312217169	0.2500000000	1.0
525 B	0	1.6139595394	1.6139595394	1	1	1
525 C	1	1.6844284064	2.1137948119	2.5098066546	0.5000000000	1.0
525 D	1	3.7664964200	1.5211195389	0.2692368662	1.5000000000	1.0
528 A	1	2.9816925531	1.7218133021	0.5774617173	1.0000000000	1.0
528 B	0	1.8534676594	0.9267338297	1	1/2	1
528 C	0	1.7314419103	1.7314419103	1	1	1
528 D	0	4.0963917014	2.0481958507	1	1/2	1
528 E	0	3.0402396248	1.5201198124	1	1/2	1
528 F	0	0.9402796053	0.9402796053	1	1	1
528 G	1	1.5635201774	1.7226781594	1.1017946454	1.0000000000	1.0
528 H	1	2.7446333575	1.9892866004	0.2415971269	3.0000000000	1.0
528 I	0	0.7962633741	1.9906584352	1	5/2	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
528 J	0	2.1940597110	2.1940597110	1	1	1
530 A	0	0.9734826979	1.2979769306	1	4/3	1
530 B	1	3.5353430676	1.5252750504	0.8628724405	0.5000000000	1.0
530 C	1	0.6481692597	1.0573504270	0.0815643762	20.0000000000	1.0
530 D	1	2.2585592113	2.2262282010	0.0821404264	12.0000000000	1.0
532 A	0	2.7311532120	1.3655766060	1	1/2	1
534 A	1	3.3954589269	2.2001094030	0.2159854726	3.0000000000	1.0
537 A	0	1.5587177967	1.5587177967	1	1	1
537 B	0	0.8102552899	1.6205105797	1	2	1
537 C	0	1.9799268697	1.3199512465	1	2/3	1
537 D	0	2.7531352269	2.7531352269	1	1	1
537 E	0	2.2873633818	0.9149453527	1	2/5	1
539 A	0	0.2934722056	0.5869444112	1	2	1
539 B	0	1.1393038364	2.2786076728	1	2	1
539 C	1	0.6771987829	2.0049171697	1.4803018112	2.0000000000	1.0
539 D	1	1.1027617077	1.2255668741	0.5556807357	2.0000000000	1.0
540 A	0	4.5731234994	1.5243744998	1	1/3	1
540 B	1	2.9454449340	1.9340458009	0.6566226306	1.0000000000	1.0
540 C	1	1.8649915990	1.8854414951	0.5054825706	2.0000000000	1.0
540 D	1	1.5026305749	1.8616866147	1.2389516397	1.0000000000	1.0
540 E	0	0.8192711081	1.6385422162	1	2	1
540 F	0	1.7249938232	1.7249938232	1	1	1
542 A	0	0.9359576219	3.2758516767	1	7/2	1
542 B	1	3.4021967825	2.2585197861	0.0948345156	7.0000000000	1.0
544 A	1	4.7941960623	1.9135747397	0.7982880611	0.5000000000	1.0
544 B	0	4.8723530805	2.4361765402	1	1/2	1
544 C	0	2.1962622352	1.0981311176	1	1/2	1
544 D	0	3.2086213244	1.6043106622	1	1/2	1
544 E	0	4.8633070873	2.4316535436	1	1/2	1
544 F	0	3.0166594192	1.5083297096	1	1/2	1
545 A	1	3.5241168854	2.3342244963	0.8831430482	0.7500000000	1.0
546 A	0	0.7389168563	0.7389168563	1	1	1
546 B	0	1.4270848853	1.4270848853	1	1	1
546 C	1	1.7445566571	1.6107831676	0.6155463285	1.5000000000	1.0
546 D	0	1.6909566492	1.6909566492	1	1	1
546 E	0	0.1340210485	2.2783578242	1	17	1
546 F	0	0.3816525370	2.6715677591	1	7	1
546 G	0	2.9547312604	2.9547312604	1	1	1
549 A	1	2.9764069065	2.3235893668	1.5613385131	0.5000000000	1.0
549 B	1	1.5545806290	1.3699084867	1.7624154851	0.5000000000	1.0
549 C	0	1.1514737028	2.3029474055	1	2	1
550 A	1	2.0671294787	1.3714836430	0.1658681347	4.0000000000	1.0
550 B	0	0.6080055832	1.2160111664	1	2	1
550 C	0	0.6304349988	0.6304349988	1	1	1
550 D	0	1.4466970381	0.4822323460	1	1/3	1
550 E	0	0.9764911272	1.9529822544	1	2	1
550 F	1	0.4639646208	1.6665638073	1.7960031139	2.0000000000	1.0
550 G	1	3.4565462699	1.2030087826	0.3480378067	1.0000000000	1.0
550 H	0	3.2349129201	3.2349129201	1	1	1
550 I	1	1.2639033911	2.2720084494	0.0214001480	84.0000000000	1.0
550 J	1	2.1835005398	1.9169774850	0.0399062600	22.0000000000	1.0
550 K	0	1.0374564312	2.0749128624	1	2	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
550 L	0	1.5458144854	3.0916289708	1	2	1
550 M	0	0.2819391026	3.1013301282	1	11	1
551 A	1	1.9602738232	2.4701875222	0.6300618549	2.0000000000	1.0
551 B	1	3.9902199840	1.5318935051	0.1919560214	2.0000000000	1.0
551 C	1	0.3338943271	2.6025453385	0.5567512849	14.0000000000	1.0
551 D	1	3.7489876608	0.7883877278	0.1051467488	2.0000000000	1.0
552 A	1	0.8208541864	1.7081772832	2.0809752957	1.0000000000	1.0
552 B	0	1.1877091685	1.1877091685	1	1	1
552 C	0	1.7517459385	1.7517459385	1	1	1
552 D	1	1.2581503271	1.8063895256	2.8715003075	0.5000000000	1.0
552 E	1	2.4465110075	2.0234563853	0.8270783900	1.0000000000	1.0
555 A	0	1.3704794284	1.3704794284	1	1	1
555 B	0	0.3400020988	1.7000104939	1	5	1
556 A	1	4.5857844464	1.9014122128	0.1382106111	3.0000000000	1.0
557 A	1	4.3192896340	2.1645043698	0.5011250815	1.0000000000	1.0
557 B	0	4.1429400648	4.1429400648	1	1	1
558 A	1	3.8035017777	1.5014219878	0.1973736409	2.0000000000	1.0
558 B	0	1.8840962993	1.2560641995	1	2/3	1
558 C	0	1.2927497983	1.2927497983	1	1	1
558 D	1	1.0317785645	1.4837330244	0.3595085892	4.0000000000	1.0
558 E	0	1.2371828219	2.4743656437	1	2	1
558 F	1	1.4723271192	2.3825303332	0.4854621576	3.3333333333	1.0
558 G	1	2.3163764160	2.4016041200	0.0370283410	28.0000000000	1.0
558 H	0	1.2640616931	2.5281233863	1	2	1
560 A	0	1.7924218478	1.7924218478	1	1	1
560 B	0	0.5287383956	2.6436919780	1	5	1
560 C	0	1.1025221381	1.1025221381	1	1	1
560 D	1	1.5673707144	1.9245473164	0.6139413282	2.0000000000	1.0
560 E	1	1.0762429023	1.3838587046	0.1285823769	10.0000000000	1.0
560 F	1	2.6665625486	1.7989783239	0.1124405354	6.0000000000	1.0
561 A	0	0.3054651772	0.6109303543	1	2	1
561 B	1	2.1348844998	1.9186947105	0.0898734667	10.0000000000	1.0
561 C	1	3.7556570477	1.2558191146	0.0835951671	4.0000000000	1.0
561 D	0	5.2004058717	1.3001014679	1	1/4	1
562 A	0	3.9206453206	1.9603226603	1	1/2	1
563 A	2	5.1752090225	1.1345559170	0.2192290035	1.0000000000	1.0
564 A	1	1.2379453765	1.7326473476	1.3996153469	1.0000000000	1.0
564 B	1	3.0040996567	2.0285919557	0.6752745207	1.0000000000	1.0
565 A	0	1.1493097892	1.1493097892	1	1	1
566 A	1	3.8013747648	1.5506939100	0.2039648819	2.0000000000	1.0
566 B	0	2.5982764742	2.5982764742	1	1	1
567 A	1	2.0002970970	2.3242602526	0.5809787596	2.0000000000	1.0
567 B	1	2.0373477589	1.4299998225	0.1169821415	6.0000000000	1.0
568 A	0	1.6374033064	1.6374033064	1	1	1
570 A	1	2.1867251701	1.3629838038	0.6232990878	1.0000000000	1.0
570 B	0	0.7141450332	0.7141450332	1	1	1
570 C	1	2.2776827930	1.4027686580	0.2052917790	3.0000000000	1.0
570 D	0	0.2865166301	1.4325831504	1	5	1
570 E	1	1.7120662755	1.7232305643	1.0065209443	1.0000000000	1.0
570 F	0	1.5439230957	1.5439230957	1	1	1
570 G	0	3.8258404870	1.9129202435	1	1/2	1
570 H	0	2.0418363509	2.0418363509	1	1	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
570 I	0	1.1330304803	2.2660609607	1	2	1
570 J	0	0.3863130601	2.7041914208	1	7	1
570 K	0	0.6858933343	2.7435733371	1	4	1
570 L	0	0.2800031252	2.8000312519	1	10	1
570 M	0	3.0009972891	3.0009972891	1	1	1
571 A	0	0.4323412563	1.7293650251	1	4	4
571 B	2	5.0953146197	0.9031605170	0.1772531403	1.0000000000	1.0
572 A	0	1.1186077536	2.2372155072	1	2	1
573 A	0	2.7728744064	1.3864372032	1	1/2	1
573 B	0	0.7774083687	3.8870418435	1	5	1
573 C	1	3.5896805447	1.1959599002	0.1110553697	3.0000000000	1.0
574 A	1	3.8864860069	1.4097851799	0.3627403205	1.0000000000	1.0
574 B	1	1.8130899302	1.8100407779	0.9983182565	1.0000000000	1.0
574 C	0	1.8417870162	1.8417870162	1	1	1
574 D	0	0.3594839537	1.0784518611	1	3	1
574 E	0	1.8962043734	1.8962043734	1	1	1
574 F	1	2.0605345766	1.6140546542	2.3499552095	0.3333333333	1.0
574 G	1	2.4854639550	2.3048347394	0.0843023414	11.0000000000	1.0
574 H	1	2.7772218686	2.4117764641	0.5789422135	1.5000000000	1.0
574 I	1	0.6002081912	1.9219551260	1.0673824817	3.0000000000	1.0
574 J	0	2.1296506458	2.1296506458	1	1	1
575 A	1	4.5838545934	2.3699937601	0.5170307460	1.0000000000	1.0
575 B	1	0.8085795688	1.1252987905	0.3479245686	4.0000000000	1.0
575 C	0	2.0323726487	4.0647452974	1	2	1
575 D	1	2.0499620940	1.4098509776	0.2292483004	3.0000000000	1.0
575 E	1	4.5445233981	0.8318833192	0.0915259144	2.0000000000	1.0
576 A	1	2.9744774254	1.9430318018	0.6532346775	1.0000000000	1.0
576 B	0	1.9129209871	1.9129209871	1	1	1
576 C	0	3.2695050335	1.6347525168	1	1/2	1
576 D	0	1.3764094010	1.3764094010	1	1	1
576 E	0	1.7173153423	1.7173153423	1	1	1
576 F	0	3.9846657992	1.9923328996	1	1/2	1
576 G	0	2.3005478718	1.1502739359	1	1/2	1
576 H	1	2.1409010281	1.9024600490	1.7772517497	0.5000000000	1.0
576 I	1	1.7607876529	1.9003720950	0.5396369323	2.0000000000	1.0
578 A	0	1.0903585148	3.2710755444	1	3	1
579 A	0	2.9882076413	2.9882076413	1	1	1
579 B	1	4.0476524302	1.5828958511	0.7821303229	0.5000000000	1.0
580 A	1	2.8796164808	1.9398167218	0.4490914524	1.5000000000	1.0
580 B	1	2.0989217355	2.0014236886	0.2118996684	4.5000000000	1.0
582 A	1	2.4268265774	1.3742277699	0.5662653371	1.0000000000	1.0
582 B	0	0.3248421567	1.9490529401	1	6	1
582 C	1	2.6532549743	2.2748039425	0.1714726978	5.0000000000	1.0
582 D	0	2.5584585727	2.5584585727	1	1	1
583 A	0	1.9615186600	3.9230373200	1	2	1
583 B	0	0.5373731951	2.1494927803	1	4	1
583 C	0	0.7126191531	4.2757149188	1	6	1
585 A	1	1.7604418018	1.3938971243	0.7917882448	1.0000000000	1.0
585 B	0	1.6235990929	1.0823993953	1	2/3	1
585 C	0	1.0680848227	2.1361696453	1	2	1
585 D	1	3.6227637614	1.8522954214	0.7669401913	0.6666666667	1.0
585 E	0	0.3703609311	0.7407218623	1	2	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
585 F	1	0.8510814529	2.3595617156	5.5448552134	0.5000000000	1.0
585 G	1	2.7101707231	1.0788350163	0.0995172562	4.0000000000	1.0
585 H	1	2.9358589644	2.4047265041	1.6381757661	0.5000000000	1.0
585 I	1	1.1139515047	1.1749876122	0.0376711581	28.0000000000	1.0
586 A	0	1.9224714698	1.9224714698	1	1	1
586 B	1	1.5864971379	2.3468475653	0.0821813145	18.0000000000	1.0
586 C	1	4.2827446200	2.2831695064	0.1332772386	4.0000000000	1.0
588 A	0	1.0627828254	1.0627828254	1	1	1
588 B	1	0.9032890459	1.8046862614	0.6659685401	3.0000000000	1.0
588 C	1	2.7659029546	1.7240643602	0.4155519044	1.5000000000	1.0
588 D	0	0.4247946359	2.1239731795	1	5	1
588 E	0	1.4059030758	2.1088546137	1	3/2	1
588 F	0	1.4860025529	1.4860025529	1	1	1
590 A	0	1.0745734886	0.7163823257	1	2/3	1
590 B	0	2.6948119120	1.3474059560	1	1/2	1
590 C	1	3.1049556815	1.6137866012	0.2598727271	2.0000000000	1.0
590 D	1	2.1842665685	2.1864656914	0.0278057445	36.0000000000	1.0
591 A	1	4.2440165034	1.6471095314	0.1940507925	2.0000000000	1.0
592 A	1	2.7500416450	2.1008484303	0.7639333150	1.0000000000	1.0
592 B	0	1.9871937616	1.9871937616	1	1	1
592 C	0	2.4513893820	2.4513893820	1	1	1
592 D	1	3.5284992325	2.0579305688	0.2916155614	2.0000000000	1.0
592 E	1	1.7676106702	1.8129978972	1.0256771628	1.0000000000	1.0
593 A	1	4.6326474009	2.5297477726	0.5460695697	1.0000000000	1.0
593 B	0	1.2822317043	0.6411158522	1	1/2	1
594 A	1	3.5551559587	1.5031473990	0.0704679540	6.0000000000	1.0
594 B	0	1.3661755860	1.3661755860	1	1	1
594 C	0	1.9745189169	0.6581729723	1	1/3	1
594 D	1	1.0795971017	1.4819597787	0.1372697070	10.0000000000	1.0
594 E	0	0.3224010209	2.5792081669	1	8	1
594 F	0	2.4423323193	2.4423323193	1	1	1
594 G	0	0.6648978941	2.6595915765	1	4	1
594 H	0	0.5152499152	2.5762495760	1	5	1
595 A	0	0.9778986867	0.9778986867	1	1	1
595 B	0	0.5740270845	4.0181895917	1	7	1
595 C	0	4.2213530436	4.2213530436	1	1	1
598 A	1	2.5667675804	1.5698916376	0.3058110227	2.0000000000	1.0
598 B	1	1.3807484571	0.9723774253	0.1760598428	4.0000000000	1.0
598 C	0	1.2303609822	2.4607219645	1	2	1
598 D	1	0.7246534041	2.3415704004	0.0950381483	34.0000000000	1.0
600 A	1	2.2709404021	1.8182269343	0.8006493401	1.0000000000	1.0
600 B	1	2.2718298778	1.7882616021	0.1967864781	4.0000000000	1.0
600 C	0	1.3641047525	1.3641047525	1	1	1
600 D	0	1.9288462329	1.9288462329	1	1	1
600 E	1	1.6232936555	2.1589881844	0.0158333892	84.0000000000	1.0
600 F	0	1.1134814845	1.1134814845	1	1	1
600 G	0	0.7259589922	1.4519179844	1	2	1
600 H	0	0.6100461910	1.8301385731	1	3	1
600 I	0	1.0159932080	2.0319864160	1	2	1
602 A	0	0.6308611400	0.6308611400	1	1	1
602 B	0	1.0174606853	1.0174606853	1	1	1
602 C	0	2.1063872057	2.1063872057	1	1	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
603 A	0	3.7681340880	1.8840670440	1	1/2	1
603 B	0	2.3992410444	1.1996205222	1	1/2	1
603 C	0	0.2595867981	1.0383471925	1	4	1
603 D	0	1.5982816107	3.1965632214	1	2	1
603 E	1	1.1635621891	2.4252104008	1.0421490246	2.0000000000	1.0
603 F	1	3.4987061444	1.0565412255	0.1509902778	2.0000000000	1.0
605 A	1	0.3648079087	1.8671028100	0.3412029117	15.0000000000	1.0
605 B	1	1.0379669438	1.4632185424	5.6387866729	0.2500000000	1.0
605 C	1	2.6758306612	0.9214319585	0.0688707228	5.0000000000	1.0
606 A	0	1.1774603810	1.7661905715	1	3/2	1
606 B	1	2.6663147092	1.7325801440	0.3249016588	2.0000000000	1.0
606 C	0	0.9957737146	1.9915474292	1	2	1
606 D	0	0.3527718822	2.4694031751	1	7	1
606 E	1	1.9676984559	2.6292883984	0.0247449124	54.0000000000	1.0
606 F	0	2.8467624701	2.8467624701	1	1	1
608 A	1	1.3513123907	1.9841616741	0.7341609860	2.0000000000	1.0
608 B	0	1.0430839617	2.0861679234	1	2	1
608 C	0	2.5878393778	2.5878393778	1	1	1
608 D	1	3.0958399938	1.9643514437	0.3172566166	2.0000000000	1.0
608 E	1	0.5785570780	2.0903172801	0.3612983679	10.0000000000	1.0
608 F	1	2.3045386410	1.4518436127	0.3149965869	2.0000000000	1.0
609 A	1	3.5623621078	2.2343678812	1.2544305231	0.5000000000	1.0
609 B	1	1.3484815516	1.1844590786	1.7567301195	0.5000000000	1.0
610 A	0	0.9772220217	0.9772220217	1	1	1
610 B	1	2.6389744505	1.6222091650	0.4098079754	1.5000000000	1.0
610 C	0	3.2804783161	3.2804783161	1	1	1
611 A	0	4.4507748026	4.4507748026	1	1	1
612 A	0	3.0101774280	2.0067849520	1	2/3	1
612 B	1	0.8947231692	1.9203816974	1.0731708776	2.0000000000	1.0
612 C	1	2.1134557877	1.9847891382	0.0782600212	12.0000000000	1.0
612 D	0	0.9210983643	1.8421967286	1	2	1
614 A	1	3.8235505367	2.5202550314	0.1098566636	6.0000000000	1.0
614 B	1	1.6220180852	2.1964097812	1.0155912261	1.3333333333	1.0
615 A	1	3.1476280915	1.2171094498	0.3866751136	1.0000000000	1.0
615 B	1	1.0766896496	2.0130725659	0.1335490665	14.0000000000	1.0
616 A	1	1.2592832385	2.0157852699	1.6007401736	1.0000000000	1.0
616 B	0	0.4967001346	2.4835006732	1	5	1
616 C	0	1.2228749426	1.2228749426	1	1	1
616 D	1	1.7515268442	1.7957577414	0.0427188652	24.0000000000	1.0
616 E	1	2.1143298453	1.9497931917	1.8443604682	0.5000000000	1.0
618 A	1	3.2025259469	1.3562810365	0.2117517639	2.0000000000	1.0
618 B	1	0.3275850164	1.4815330940	4.5225911437	1.0000000000	1.0
618 C	1	4.3034232575	1.7322241586	1.2075671308	0.3333333333	1.0
618 D	1	0.5996924071	1.6716285286	0.8362429684	3.3333333333	1.0
618 E	1	3.1261472585	2.3263678897	0.1488329050	5.0000000000	1.0
618 F	1	1.5379775186	2.6608279159	0.0224686033	77.0000000000	1.0
618 G	0	0.3846002426	3.0768019409	1	8	1
620 A	1	3.5636105700	2.1477266018	1.8080482362	0.3333333333	1.0
620 B	1	1.9747591044	2.0518524948	0.1385385853	7.5000000004	1.0
620 C	1	2.3002922132	1.4718967298	0.2132912682	3.0000000000	1.0
621 A	0	2.4706496111	2.4706496111	1	1	1
621 B	1	2.4850697914	1.3566125441	0.5459052091	1.0000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
622 A	1	2.0911722409	2.4885586945	0.1700043535	7.0000000000	1.0
623 A	1	1.6365441158	2.2881851242	0.2330301907	6.0000000000	1.0
624 A	1	3.6944854092	1.8414108744	0.9968429540	0.5000000000	1.0
624 B	1	1.7240319882	1.7131634804	1.9873917563	0.5000000000	1.0
624 C	0	2.9977977348	1.4988988674	1	1/2	1
624 D	0	4.1998547381	2.0999273690	1	1/2	1
624 E	0	0.9490741088	2.3726852720	1	5/2	1
624 F	1	1.9064113550	2.1165060726	2.2204085882	0.5000000000	1.0
624 G	1	2.7365561607	1.8217793937	1.3314394346	0.5000000000	1.0
624 H	0	2.2865886337	2.2865886337	1	1	1
624 I	0	0.3978321511	1.9891607556	1	5	1
624 J	0	3.1643734930	1.5821867465	1	1/2	1
626 A	1	4.8236248257	1.5953820026	0.6614867699	0.5000000000	1.0
626 B	0	1.7509727378	1.7509727378	1	1	1
627 A	0	2.0728597236	2.0728597236	1	1	1
627 B	0	0.5438530027	1.6315590080	1	3	1
628 A	0	2.6439246550	2.6439246550	1	1	1
629 A	1	1.7091284174	2.4969954466	1.4609759110	1.0000000000	1.0
629 B	0	2.2266969590	4.4533939180	1	2	1
629 C	1	2.2765689140	1.3037797972	0.1431737679	4.0000000000	1.0
629 D	1	1.3686582300	1.5536888031	0.2270382436	5.0000000000	1.0
630 A	0	3.1915635134	1.0638545045	1	1/3	1
630 B	0	1.1585165872	1.1585165872	1	1	1
630 C	0	0.4975637810	0.9951275620	1	2	1
630 D	1	1.2617923560	1.5738501163	0.6236565425	2.0000000000	1.0
630 E	1	1.8098438078	1.6379676457	0.4525162997	2.0000000000	1.0
630 F	0	1.2655347079	1.2655347079	1	1	1
630 G	0	0.3258351084	2.2808457589	1	7	1
630 H	0	2.5822999144	2.5822999144	1	1	1
630 I	0	0.5836435418	2.3345741671	1	4	1
630 J	0	2.5079578706	2.5079578706	1	1	1
632 A	1	3.2690386003	2.1533299446	0.3293521747	2.0000000000	1.0
633 A	1	1.0939894596	1.1623092920	0.5312250871	2.0000000000	1.0
635 A	1	2.6532748870	2.1046502583	2.3796821076	0.3333333333	1.0
635 B	1	5.1560085835	1.0641729808	0.2063947264	1.0000000000	1.0
637 A	1	3.6103936643	2.4744038023	0.6853556793	1.0000000000	1.0
637 B	0	0.5482023406	2.1928093623	1	4	1
637 C	1	0.2804003378	2.4750227405	8.8267466432	1.0000000000	1.0
637 D	1	1.2618475557	1.2952727037	0.5132445269	2.0000000000	1.0
639 A	1	1.4159216040	1.3874418666	0.9798860775	1.0000000000	1.0
640 A	1	2.5146785012	2.0316664632	3.2316917845	0.2500000000	1.0
640 B	1	5.1408789950	1.9898721132	1.5482738381	0.2500000000	1.0
640 C	0	1.7781462207	1.7781462207	1	1	1
640 D	0	1.1829167593	2.3658335187	1	2	1
640 E	0	4.2937310983	2.1468655491	1	1/2	1
640 F	0	1.6728969242	0.8364484621	1	1/2	1
640 G	1	3.6351503986	2.0751281923	0.5708507117	1.0000000000	1.0
640 H	1	3.0361263762	1.6979739838	0.2796283444	2.0000000000	1.0
642 A	0	2.3935636629	1.1967818315	1	1/2	1
642 B	0	0.8833018943	1.1777358591	1	4/3	1
642 C	1	1.3096864637	2.4034651156	0.0705825162	26.0000000000	1.0
643 A	2	5.0103134331	1.1482617366	0.2291796216	1.0000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
644 A	1	1.3458310330	2.1671360619	0.8051293248	2.0000000000	1.0
644 B	1	1.8677219381	1.8666034248	0.1665668558	6.0000000000	1.0
645 A	0	2.9547633309	1.4773816655	1	1/2	1
645 B	0	3.7246680737	1.8623340369	1	1/2	1
645 C	0	0.7287600670	2.9150402680	1	4	1
645 D	0	0.2060864423	0.8243457694	1	4	1
645 E	1	0.5418428107	2.1123636266	0.0406091718	96.0000000000	1.0
645 F	1	2.0680590222	1.3395707765	0.0539785842	12.0000000000	1.0
646 A	0	1.4274356437	1.4274356437	1	1	1
646 B	0	1.7333045886	3.4666091772	1	2	1
646 C	0	2.3277953903	2.3277953903	1	1	1
646 D	1	2.1316501109	2.5834562462	0.2019918930	6.0000000000	1.0
646 E	0	1.7417182315	1.7417182315	1	1	1
648 A	1	2.8579610545	2.0264323657	0.3545241393	2.0000000000	1.0
648 B	1	3.7739500820	1.9947552563	0.2642794967	2.0000000000	1.0
648 C	0	0.8564595017	1.7129190035	1	2	1
648 D	1	2.5144198042	2.0822444144	0.1380202046	6.0000000000	1.0
649 A	1	3.4660507043	1.6065376871	0.2317533447	2.0000000000	1.0
650 A	1	1.3995225281	1.6156045801	1.1543969802	1.0000000000	1.0
650 B	1	0.6941100694	1.9367249330	1.3951136990	2.0000000000	1.0
650 C	1	4.1558626857	1.0826851427	0.1302599754	2.0000000000	1.0
650 D	0	0.7630894957	1.5261789915	1	2	1
650 E	0	0.3964760735	0.7929521471	1	2	1
650 F	0	1.9439290266	1.9439290266	1	1	1
650 G	1	3.6999544999	1.2442250387	0.5044217593	0.6666666667	1.0
650 H	0	2.0751559918	2.0751559918	1	1	1
650 I	0	1.6546699551	3.3093399101	1	2	1
650 J	0	1.6923586570	3.3847173140	1	2	1
650 K	1	1.7063199854	2.4329160966	0.0339482478	42.0000000000	1.0
650 L	0	0.3104154598	1.8624927590	1	6	1
650 M	0	1.8585582941	3.7171165882	1	2	1
651 A	0	0.2759207512	1.3796037560	1	5	1
651 B	0	4.2374957336	2.1187478668	1	1/2	1
651 C	1	2.0362564170	2.6338964695	1.2934994078	1.0000000000	1.0
651 D	1	1.2854998192	1.5714303035	1.2224274792	1.0000000000	1.0
651 E	0	1.2556913761	1.2556913761	1	1	1
654 A	1	2.2676191686	1.7461630175	0.0481276530	16.0000000000	1.0
654 B	1	1.3654234799	2.4192177359	0.0553678440	32.0000000000	1.0
655 A	2	4.4879258049	0.8881921108	0.0989535199	2.0000000000	1.0
656 A	1	3.3657759082	2.0060295839	0.5960080643	1.0000000000	1.0
656 B	0	2.5555950990	1.2777975495	1	1/2	1
656 C	0	2.2320363680	2.2320363680	1	1	1
657 A	0	1.3920168887	1.3920168887	1	1	1
657 B	0	0.8683578148	3.4734312592	1	4	1
657 C	1	1.4732708732	1.9853109738	0.6737766320	2.0000000000	1.0
657 D	1	3.2248295382	1.3989934744	0.4338193563	1.0000000000	1.0
658 A	0	0.3453570447	0.6907140895	1	2	1
658 B	0	3.6139800621	1.8069900311	1	1/2	1
658 C	0	2.5856452314	1.7237634876	1	2/3	1
658 D	1	1.4893191366	2.4281008908	0.1358619089	12.0000000000	1.0
658 E	1	0.6112087497	2.5417115238	0.1890227227	22.0000000000	1.0
658 F	1	4.3216254957	2.0088664210	0.1162101172	4.0000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
659 A	1	1.5987465961	2.8102618562	1.7577906737	1.0000000000	1.0
659 B	0	3.9580875305	3.9580875305	1	1	1
660 A	0	1.0700232126	1.0700232126	1	1	1
660 B	1	2.9740851833	1.8332170918	0.2054656562	3.0000000000	1.0
660 C	1	2.3631542925	2.2071066960	0.9339663953	1.0000000000	1.0
660 D	0	0.6405269388	1.9215808163	1	3	1
662 A	1	1.0766666723	1.2847762864	0.5966453311	2.0000000000	1.0
663 A	0	1.1861398733	1.1861398733	1	1	1
663 B	1	3.2047641001	1.1920969618	1.4879060356	0.2500000000	1.0
663 C	1	1.9988726759	1.6326154359	0.4083840496	2.0000000000	1.0
664 A	2	3.5536274747	1.3742026761	0.0966760505	4.0000000000	1.0
664 B	1	3.6509416469	2.1908129135	0.3000339536	2.0000000000	1.0
664 C	1	4.5567302944	1.8865952753	0.2070119530	2.0000000000	1.0
665 A	1	1.3794131390	1.2423862408	0.1801325804	5.0000000000	1.0
665 B	1	2.4602431252	2.5513395947	4.1481097027	0.2500000000	1.0
665 C	1	4.8049229586	2.3451929413	0.4880812786	1.0000000000	1.0
665 D	1	1.0294222183	1.0781458036	0.5236655011	2.0000000000	1.0
665 E	0	0.7606298483	1.5212596966	1	2	1
666 A	0	0.6115596244	1.2231192488	1	2	1
666 B	0	0.6266921957	1.2533843913	1	2	1
666 C	1	1.7584042022	1.6232934654	0.2307907168	4.0000000000	1.0
666 D	1	3.7060696315	2.6092064873	0.0704036013	10.0000000000	1.0
666 E	1	1.2684154139	2.5396068527	0.0385036265	52.0000000000	1.0
666 F	0	1.1123286520	2.2246573039	1	2	1
666 G	0	0.0666962770	3.0680287440	1	46	1
669 A	1	2.0309614263	2.3967060910	1.1800844960	1.0000000000	1.0
670 A	1	0.4721151649	1.6886117386	0.3251540503	11.0000000000	1.0
670 B	1	2.8685877138	1.3009484555	1.3605459397	0.3333333333	1.0
670 C	1	4.0108342189	2.6312306772	0.1312061548	5.0000000000	1.0
670 D	1	0.8786739264	2.2406781479	0.1342140984	19.0000000000	1.0
672 A	1	3.0552339984	1.8770484002	0.6143714037	1.0000000000	1.0
672 B	1	1.0195945289	2.1403725794	0.1399492654	15.0000000000	1.0
672 C	0	2.9459785698	1.4729892849	1	1/2	1
672 D	0	0.7784733987	0.7784733987	1	1	1
672 E	1	3.6459594937	1.8104178877	0.9931091615	0.5000000000	1.0
672 F	1	2.4672111687	2.2099943158	1.7914918219	0.5000000000	1.0
672 G	0	1.9947625016	1.9947625016	1	1	1
672 H	0	2.2280058144	2.2280058144	1	1	1
674 A	1	2.7092586874	1.7379082664	0.6414700355	1.0000000000	1.0
674 B	1	4.0104450489	2.6033014713	0.6491303184	1.0000000000	1.0
674 C	1	0.5759572330	1.9868787305	0.1112805927	31.0000000000	1.0
675 A	1	2.3701946024	1.9226596103	0.4055910870	2.0000000000	1.0
675 B	1	3.7197181779	1.4725103035	0.1319553645	3.0000000000	1.0
675 C	0	3.0994098257	1.0331366086	1	1/3	1
675 D	0	0.8904557232	0.8904557232	1	1	1
675 E	0	1.3860982121	1.3860982121	1	1	1
675 F	0	1.9911195281	1.9911195281	1	1	1
675 G	0	1.6911045184	3.3822090368	1	2	1
675 H	0	0.4397610737	0.8795221474	1	2	1
675 I	1	1.6635085406	2.5377208007	0.5085077190	3.0000000000	1.0
676 A	0	0.9379794035	1.4069691052	1	3/2	1
676 B	0	1.4965418840	1.4965418840	1	1	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
676 C	0	0.4150660384	0.4150660384	1	1	1
676 D	0	2.9442542341	2.9442542341	1	1	1
676 E	0	0.8165892007	2.4497676021	1	3	1
677 A	1	2.8368146928	1.2893981935	0.4545232358	1.0000000000	1.0
678 A	1	4.8406123729	1.5171515354	0.1567107030	2.0000000000	1.0
678 B	1	1.5883609701	1.7574538605	0.1844095783	6.0000000000	1.0
678 C	1	1.4205258825	2.4773594939	0.2491390713	7.0000000000	1.0
678 D	0	1.3970457314	2.7940914629	1	2	1
678 E	0	3.1622103101	3.1622103101	1	1	1
678 F	0	1.2858801950	2.5717603900	1	2	1
680 A	1	3.2750305235	2.0532909813	1.2539064699	0.5000000000	1.0
680 B	0	1.4663778907	1.4663778907	1	1	1
680 C	0	0.6189257201	2.4757028802	1	4	1
681 A	1	3.8078769894	1.7148309965	0.2251689066	2.0000000000	1.0
681 B	0	0.8199178694	1.8448152061	1	9/4	9
681 C	2	3.7708461685	1.0262711474	0.1360796890	2.0000000000	1.0
681 D	0	0.5238009112	2.0952036450	1	4	1
681 E	1	2.2367233352	2.0523231219	0.0917557880	10.0000000000	1.0
682 A	1	3.1826703234	2.2812508096	0.7167725770	1.0000000000	1.0
682 B	1	0.6115533121	2.6265637235	0.0753492204	57.0000000000	1.0
684 A	1	2.5117117385	2.1232510164	0.1408900408	6.0000000000	1.0
684 B	1	0.8914338511	2.0174937497	0.3772001230	6.0000000000	1.0
684 C	0	1.0106719472	2.0213438944	1	2	1
685 A	1	5.0493985695	2.5294065402	0.5009322408	1.0000000000	1.0
688 A	1	2.1361539012	2.0384041307	0.9542402959	1.0000000000	1.0
688 B	0	1.2461451514	2.4922903027	1	2	1
688 C	1	1.3631824182	2.3637294196	1.7339788043	1.0000000000	1.0
689 A	1	5.1047223290	1.0333048176	0.8096854253	0.2500000000	1.0
690 A	1	0.7448541864	1.3963979050	0.9373632655	2.0000000000	1.0
690 B	0	0.9529657858	0.9529657858	1	1	1
690 C	0	0.1132251968	0.9058015746	1	8	1
690 D	0	1.2098350005	1.2098350005	1	1	1
690 E	1	0.9634951715	1.8051068282	0.7493994289	2.5000000000	1.0
690 F	0	3.0827918116	1.5413959058	1	1/2	1
690 G	0	0.2784647670	1.9492533690	1	7	1
690 H	1	2.3108328098	2.4704452833	0.1781785678	6.0000000000	1.0
690 I	0	0.9226753489	2.7680260467	1	3	1
690 J	0	0.6031690708	3.0158453540	1	5	1
690 K	0	1.5219655531	3.0439311063	1	2	1
692 A	0	2.1861975957	1.0930987979	1	1/2	1
693 A	0	1.0344382277	1.0344382277	1	1	1
693 B	1	1.7403153566	1.9629786598	0.5639721136	2.0000000000	1.0
693 C	0	0.4482861989	0.8965723977	1	2	1
693 D	0	1.1413329845	2.2826659690	1	2	1
696 A	1	3.8717115384	1.8979933552	0.2451103777	2.0000000000	1.0
696 B	0	1.6317277401	1.6317277401	1	1	1
696 C	1	2.0763110581	2.2838622912	0.1099961531	10.0000000000	1.0
696 D	0	1.2519112526	1.2519112526	1	1	1
696 E	0	0.9609707865	1.9219415730	1	2	1
696 F	1	0.7587994985	1.8510015342	0.4065635999	6.0000000000	1.0
696 G	1	3.6032980856	2.2393010521	0.1035764550	6.0000000000	1.0
699 A	0	1.3274184293	3.9822552879	1	3	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
700 A	0	0.6011899261	1.2023798523	1	2	1
700 B	0	2.2739538827	2.2739538827	1	1	1
700 C	1	4.3423658689	2.1348487950	0.1638775466	3.0000000000	1.0
700 D	1	0.6956459498	1.5021159277	0.0359885161	60.0000000000	1.0
700 E	1	0.7139819035	2.1110581354	0.9855796648	3.0000000000	1.0
700 F	1	1.9419650532	2.1076136836	0.3617664970	3.0000000000	1.0
700 G	1	3.1315375430	1.4807661532	0.0788093246	6.0000000000	1.0
700 H	0	1.5965120710	1.5965120710	1	1	1
700 I	0	1.0169430919	1.0169430919	1	1	1
700 J	0	1.4004661640	2.8009323281	1	2	1
701 A	0	4.5057330241	4.5057330241	1	1	1
702 A	1	1.2959189179	1.6511976366	0.6370759828	2.0000000000	1.0
702 B	1	3.6218545346	1.5825343783	0.4369403473	1.0000000000	1.0
702 C	0	1.5516100045	1.5516100045	1	1	1
702 D	0	1.0950637552	1.0950637552	1	1	1
702 E	0	1.1045333203	0.7363555469	1	2/3	1
702 F	0	1.1196604817	1.1196604817	1	1	1
702 G	0	1.6442531135	1.6442531135	1	1	1
702 H	1	4.4173331953	1.6516890684	3.3651981768	0.1111111111	1.0
702 I	0	2.7852340926	2.7852340926	1	1	1
702 J	0	0.2651042420	2.9161466624	1	11	1
702 K	1	2.9521289967	2.6395534641	0.0425770765	21.0000000000	1.0
702 L	1	1.6844372293	2.6868354521	0.0531697947	30.0000000000	1.0
702 M	1	0.7927472332	2.6131420167	0.0578300306	57.0000000000	1.0
702 N	0	2.4387240231	2.4387240231	1	1	1
702 O	0	2.3848948730	2.3848948730	1	1	1
702 P	0	0.4731564099	2.8389384593	1	6	1
703 A	0	1.0265530673	2.0531061346	1	2	1
703 B	1	1.7712720083	1.1006854340	0.3107048011	2.0000000000	1.0
704 A	1	4.4873325290	2.2759885635	0.5072030095	1.0000000000	1.0
704 B	1	3.8045730058	2.0086088735	0.5279459404	1.0000000000	1.0
704 C	0	2.4683463702	2.4683463702	1	1	1
704 D	0	1.7069125392	1.7069125392	1	1	1
704 E	0	3.0069924701	3.0069924701	1	1	1
704 F	0	2.1595534099	2.1595534099	1	1	1
704 G	0	1.3006681580	1.3006681580	1	1	1
704 H	0	2.6129403750	2.6129403750	1	1	1
704 I	0	1.1705612909	1.1705612909	1	1	1
704 J	1	5.1103236666	2.2490865778	0.4401064834	1.0000000000	1.0
704 K	1	2.0630782447	1.8879135472	0.9150954658	1.0000000000	1.0
704 L	1	1.6188133885	1.4443889285	0.2974172191	3.0000000000	1.0
705 A	1	0.8998713610	1.6952055621	0.9419154979	2.0000000000	1.0
705 B	1	0.7197724152	1.2855968706	0.1190743855	15.0000000000	1.0
705 C	0	2.2422368953	1.4948245969	1	2/3	1
705 D	1	2.3563327671	2.7399409039	1.1627987957	1.0000000000	1.0
705 E	1	3.5400966154	1.6577969123	0.0936582863	5.0000000000	1.0
705 F	0	3.3938962532	2.5454221899	1	3/4	1
706 A	1	2.4288257338	1.9200718399	0.7905350364	1.0000000000	1.0
706 B	1	1.2043004068	2.6991718144	0.0974468626	23.0000000000	1.0
706 C	1	2.9766469907	2.6216013820	1.7614459425	0.5000000000	1.0
706 D	1	2.6150443165	2.3202308347	0.3549050117	2.5000000000	1.0
707 A	2	4.3145192962	0.9738513460	0.1128574563	2.0000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
708 A	0	2.0619546003	1.0309773002	1	1/2	1
709 A	2	4.0733289400	1.0537016538	0.2586831727	1.0000000000	1.0
710 A	1	2.1059977182	1.5321809045	0.1818830205	4.0000000000	1.0
710 B	1	1.7560060271	2.4992935440	0.0418612650	34.0000000000	1.0
710 C	1	3.1589551069	2.5747640635	0.0582191619	14.0000000000	1.0
710 D	0	1.1567437634	2.3134875269	1	2	1
711 A	1	2.5535933872	2.5771593912	0.5046142828	2.0000000000	1.0
711 B	1	1.7397204637	1.4841485905	0.4265480063	2.0000000000	1.0
711 C	0	2.3245918204	2.3245918204	1	1	1
712 A	0	2.0050824245	1.0025412123	1	1/2	1
713 A	1	4.2159507277	2.7924101607	0.6623441167	1.0000000000	1.0
714 A	1	0.8814893885	1.5642040390	1.7745012696	1.0000000000	1.0
714 B	0	0.9622645215	0.9622645215	1	1	1
714 C	0	0.2171317499	1.0856587497	1	5	1
714 D	1	2.6839654336	1.3951533217	0.5198104656	1.0000000000	1.0
714 E	0	0.3477949055	2.4345643386	1	7	1
714 F	1	1.9409723197	2.4625037273	0.8457973022	1.5000000000	1.0
714 G	0	0.6275698389	1.8827095167	1	3	1
714 H	0	2.5311266382	2.5311266382	1	1	1
714 I	0	0.8643306617	2.5929919852	1	3	1
715 A	1	3.5289602069	1.5787090658	1.3420744128	0.3333333333	1.0
715 B	1	0.6975196333	1.2547951053	0.0856637506	21.0000000000	1.0
718 A	0	0.9761894668	0.9761894668	1	1	1
718 B	2	3.6847793558	1.2517811528	0.1698583595	2.0000000000	1.0
718 C	1	2.5755975942	2.6764950054	0.0865978641	12.0000000000	1.0
720 A	1	2.7175526609	2.0753361001	0.3818391691	2.0000000000	1.0
720 B	0	0.8280154410	1.6560308820	1	2	1
720 C	0	2.9317714525	1.4658857262	1	1/2	1
720 D	0	3.4281037645	1.7140518822	1	1/2	1
720 E	1	1.4374984153	2.1527060376	1.4975362857	1.0000000000	1.0
720 F	0	0.7101221744	1.4202443487	1	2	1
720 G	1	1.1463561696	2.1771249053	0.3165282837	6.0000000000	1.0
720 H	1	1.6172770870	2.1029661686	0.3250782110	4.0000000000	1.0
720 I	0	3.2613074970	1.6306537485	1	1/2	1
720 J	0	0.9676241149	1.9352482298	1	2	1
722 A	1	0.5289503653	1.8191706374	10.317625755	0.3333333333	1.0
722 B	1	1.5826710943	1.1653365368	0.3681549947	2.0000000000	1.0
722 C	0	0.5404948479	1.0809896957	1	2	1
722 D	0	0.6600950502	3.9605703013	1	6	1
722 E	1	0.2759017297	2.5648119356	0.7746755637	12.0000000000	1.0
722 F	1	3.2014095799	2.5300549410	0.2634313498	3.0000000000	1.0
723 A	1	3.4502808494	1.2092337952	0.7009480376	0.5000000000	1.0
723 B	1	1.7498147728	2.1513753427	1.2294874727	1.0000000000	1.0
725 A	1	2.5453515525	2.6227061923	2.0607811048	0.5000000000	1.0
726 A	1	2.0448157422	1.4899240842	0.7286348855	1.0000000000	1.0
726 B	0	0.2420836944	0.9683347776	1	4	1
726 C	0	1.3230677871	1.3230677871	1	1	1
726 D	1	3.5176356020	1.7815333279	0.0633072015	8.0000000000	1.0
726 E	1	0.5670099361	1.7173298337	0.3028747336	10.0000000000	1.0
726 F	0	0.6987232202	2.0961696607	1	3	1
726 G	1	1.0589008371	2.4891357710	0.0391779803	60.0000000000	1.0
726 H	0	0.9428381419	2.8285144257	1	3	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
726 I	0	0.3723785106	2.9790280849	1	8	1
728 A	0	1.3003620819	1.3003620819	1	1	1
728 B	0	0.6814404408	2.7257617631	1	4	1
728 C	1	0.8260062433	2.0890611620	0.4215184367	6.0000000000	1.0
728 D	1	2.2030843903	1.6802562364	0.1271139260	6.0000000000	1.0
730 A	0	0.8825509340	0.8825509340	1	1	1
730 B	0	0.9049726844	0.3016575615	1	1/3	1
730 C	0	2.0285045409	2.0285045409	1	1	1
730 D	0	1.1089286592	1.1089286592	1	1	1
730 E	0	1.7926088874	1.7926088874	1	1	1
730 F	1	2.5571125379	1.7537639412	0.3429188030	2.0000000000	1.0
730 G	1	3.3565977387	1.1443062568	0.3409125388	1.0000000000	1.0
730 H	0	1.7236786394	1.7236786394	1	1	1
730 I	1	3.5781506698	2.5255155328	0.1008308388	7.0000000000	1.0
730 J	1	1.1012771596	2.6051012065	0.0375480527	63.0000000000	1.0
730 K	0	3.2768778391	3.2768778391	1	1	1
731 A	1	2.8737517466	2.8068624879	0.9767240651	1.0000000000	1.0
732 A	0	3.2997896421	1.6498948210	1	1/2	1
732 B	1	2.7343523122	1.9306520942	0.1176788183	6.0000000000	1.0
732 C	1	2.6755394130	2.2682272678	0.1883921057	4.5000000000	1.0
733 A	0	1.6195086138	1.6195086138	1	1	1
734 A	0	1.3368249040	3.3420622601	1	5/2	1
735 A	0	1.4373759725	1.4373759725	1	1	1
735 B	0	0.2066678812	0.4133357625	1	2	1
735 C	1	2.5974756723	1.7905882921	0.3446785491	2.0000000000	1.0
735 D	0	0.7149316291	1.4298632582	1	2	1
735 E	1	1.2066457006	1.6631875266	2.7567123073	0.5000000000	1.0
735 F	1	1.2882643756	1.4518853833	0.0134167723	84.0000000000	1.0
737 A	1	0.8056517670	1.4537684161	0.1503718765	12.0000000000	1.0
738 A	1	1.3681269310	1.6904101465	0.6177826444	2.0000000000	1.0
738 B	0	0.4498240118	0.8996480237	1	2	1
738 C	0	1.5977768394	1.5977768394	1	1	1
738 D	1	1.6152729786	1.6730284056	0.2589389577	4.0000000000	1.0
738 E	1	2.0318963871	2.7547337823	0.1355745204	10.0000000000	1.0
738 F	1	1.7902683597	2.6751400081	0.0339606273	44.0000000000	1.0
738 G	0	0.9655401711	2.8966205133	1	3	1
738 H	0	0.4071435598	2.8500049188	1	7	1
738 I	0	2.5773335958	2.5773335958	1	1	1
738 J	0	1.1131778064	2.2263556127	1	2	1
739 A	0	3.6774435209	3.6774435209	1	1	1
740 A	0	0.4224994326	2.5349965954	1	6	1
740 B	1	1.3678317161	2.3309157286	0.8520476975	2.0000000000	1.0
740 C	1	2.0445936737	2.0073801537	0.0818165886	12.0000000000	1.0
741 A	0	1.8564415480	1.8564415480	1	1	1
741 B	0	0.4119061199	1.2357183596	1	3	1
741 C	0	0.2794796202	3.0742758221	1	11	1
741 D	0	0.1985518805	3.9710376102	1	20	1
741 E	1	2.6257324397	2.2064827886	0.1050412999	8.0000000000	1.0
742 A	1	4.0625082193	1.6214759738	0.1995658699	2.0000000000	1.0
742 B	0	2.0545031092	2.0545031092	1	1	1
742 C	0	0.7444499769	1.4888999537	1	2	1
742 D	0	0.2649897578	2.1199180628	1	8	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
742 E	1	0.5772565826	1.9175643082	0.3321857846	10.000000000	1.0
742 F	0	0.8960291889	3.5841167558	1	4	1
742 G	1	2.1897835762	2.5093311207	0.0572963271	20.000000000	1.0
744 A	1	2.4148251947	1.9176814067	0.1985321143	4.000000000	1.0
744 B	0	1.1678014455	1.7517021682	1	3/2	1
744 C	1	1.9190973642	2.2681277304	0.0738670095	16.000000000	1.0
744 D	0	0.4891399315	1.9565597260	1	4	1
744 E	0	0.9794154660	0.9794154660	1	1	1
744 F	1	1.9869542434	1.9646208882	0.0823966671	12.000000000	1.0
744 G	1	3.0362477634	2.3164305927	0.0635771181	12.000000000	1.0
747 A	1	1.7571979728	1.4144734403	1.6099192716	0.500000000	1.0
747 B	0	4.5044550320	2.2522275160	1	1/2	1
747 C	1	0.5060273272	2.5853819361	2.5545872695	2.000000000	1.0
747 D	1	2.2599386817	2.7052674428	1.1970534708	1.000000000	1.0
747 E	1	1.8087956133	1.5131984023	0.4182889408	2.000000000	1.0
748 A	0	0.5058218300	3.0349309803	1	6	1
749 A	1	1.6335946125	1.6287624673	0.4985210084	2.000000000	1.0
752 A	1	2.1590990894	2.1778815538	1.0086992137	1.000000000	1.0
753 A	0	1.6329306908	3.2658613816	1	2	1
753 B	0	2.8471750728	1.8981167152	1	2/3	1
753 C	1	2.6391949788	2.1958740551	0.2080060466	4.000000000	1.0
754 A	0	2.4638722502	1.6425815001	1	2/3	1
754 B	1	1.0757562718	1.9362290724	0.4499692735	4.000000000	1.0
754 C	1	3.0286050243	1.2261926627	0.8097408892	0.500000000	1.0
754 D	1	1.3796837741	2.9335184328	0.1181236232	18.000000000	1.0
755 A	1	2.7886799378	1.9583126075	0.7022364169	1.000000000	1.0
755 B	1	3.5020797733	2.8444156107	0.8122075438	1.000000000	1.0
755 C	0	2.1787770859	1.0893885430	1	1/2	1
755 D	0	4.1290425737	4.1290425737	1	1	1
755 E	0	5.0574520557	5.0574520557	1	1	1
755 F	0	0.1545463751	2.0091028768	1	13	1
756 A	0	1.7063838931	1.7063838931	1	1	1
756 B	1	2.1611676117	2.0476009778	0.9474512605	1.000000000	1.0
756 C	1	1.8177688025	2.1245818910	0.3895951799	3.000000000	1.0
756 D	0	2.0527415348	2.0527415348	1	1	1
756 E	0	2.0818495119	2.0818495119	1	1	1
756 F	0	1.2639024702	1.2639024702	1	1	1
758 A	1	2.0017018367	1.2248470767	0.3059514295	2.000000000	1.0
758 B	0	0.9266980553	3.7067922210	1	4	1
759 A	1	0.8052905555	1.2941612047	1.6070736157	1.000000000	1.0
759 B	1	1.0932448414	1.6306147760	1.4915366753	1.000000000	1.0
760 A	0	2.6248441565	2.6248441565	1	1	1
760 B	0	0.1879117839	1.3153824873	1	7	1
760 C	0	1.4515241647	2.9030483294	1	2	1
760 D	1	2.7369997770	1.7874773125	0.2176930286	3.000000000	1.0
760 E	1	2.6012959156	2.2403249976	1.7224683929	0.500000000	1.0
762 A	0	1.5128246531	1.5128246531	1	1	1
762 B	0	0.4542920018	1.8171680074	1	4	1
762 C	1	2.7594600422	1.8102970129	0.1640082648	4.000000000	1.0
762 D	1	3.6331443577	2.5474201408	0.0701161278	10.000000000	1.0
762 E	1	1.7475576222	2.9286927531	0.0253920855	66.000000000	1.0
762 F	0	0.9694108898	2.9082326695	1	3	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
762G	0	0.4859865783	2.9159194697	1	6	1
763A	1	3.2220719485	1.2429317687	0.1285851453	3.0000000000	1.0
765A	0	2.0616348322	2.0616348322	1	1	1
765B	0	1.1759321797	1.1759321797	1	1	1
765C	1	3.0783495607	1.5493056096	0.5032909938	1.0000000000	1.0
766A	0	2.0523827032	2.0523827032	1	1	1
768A	1	4.3656747146	1.9178228339	0.4392958613	1.0000000000	1.0
768B	1	3.3463601247	1.8962429969	0.5666583769	1.0000000000	1.0
768C	0	2.1726561662	2.1726561662	1	1	1
768D	0	4.7324678729	2.3662339365	1	1/2	1
768E	0	1.5362999083	1.5362999083	1	1	1
768F	0	2.9074554322	1.4537277161	1	1/2	1
768G	1	3.0869981951	2.3829930020	0.3859725292	2.0000000000	1.0
768H	1	2.0558814521	2.3075439492	1.1224109964	1.0000000000	1.0
770A	0	1.6305047714	1.6305047714	1	1	1
770B	0	1.6894669604	0.5631556535	1	1/3	1
770C	0	0.2245966433	1.1229832167	1	5	1
770D	1	1.3934589960	1.3476401147	0.2417796502	4.0000000000	1.0
770E	1	0.7940452690	1.7753382914	2.2358149600	1.0000000000	1.0
770F	1	1.1288970583	2.3511646469	0.5206773792	4.0000000000	1.0
770G	0	1.8601556792	1.8601556792	1	1	1
774A	0	1.0875097482	1.4500129977	1	4/3	1
774B	0	1.5541356320	1.5541356320	1	1	1
774C	0	0.0950910435	0.7607283478	1	8	1
774D	1	0.4431002071	1.7881748996	1.0088998329	4.0000000000	1.0
774E	1	2.4922340340	1.5855794875	0.3181040516	2.0000000000	1.0
774F	1	2.4512293565	2.7370287348	0.4187228636	2.6666666667	1.0
774G	1	1.7182765282	2.7922038752	0.0677084429	24.0000000000	1.0
774H	0	0.4166411653	2.9164881572	1	7	1
774I	0	0.7011646252	2.8046585008	1	4	1
775A	1	2.3905216902	2.2177911110	0.4638717817	2.0000000000	1.0
775B	0	0.9454718718	0.9454718718	1	1	1
775C	0	0.9912538890	3.9650155558	1	4	1
776A	1	3.7983234120	2.1229515512	0.5589180596	1.0000000000	1.0
777A	0	5.0397845548	1.2599461387	1	1/4	1
777B	0	0.3038812313	0.6077624625	1	2	1
777C	0	0.6617332035	1.3234664070	1	2	1
777D	1	2.8360196315	1.2399116897	1.7488055103	0.2500000000	1.0
777E	1	0.7933334438	2.8167539819	2.8404237877	1.2500000000	1.0
777F	1	3.7264393331	1.4653365029	0.1966134924	2.0000000000	1.0
777G	1	1.5621913918	2.1958152307	0.0702799683	20.0000000000	1.0
780A	1	2.9965748479	2.0100038393	0.2235890354	3.0000000000	1.0
780B	0	0.3205650494	1.6028252468	1	5	1
780C	1	1.7695272041	2.3706082461	0.1116403787	12.0000000000	1.0
780D	0	1.8390125650	1.8390125650	1	1	1
781A	0	0.8627373069	0.8627373069	1	1	1
781B	1	2.8846769626	1.9767169681	0.2284157514	3.0000000000	1.0
782A	1	2.5499874086	1.4230642485	1.1161343336	0.5000000000	1.0
782B	0	0.8576115978	2.5728347933	1	3	1
782C	0	0.2690438605	1.8833070233	1	7	1
782D	0	3.8984248290	3.8984248290	1	1	1
782E	0	1.1016692094	2.7541730235	1	5/2	1

TABLE 4: BIRCH-SWINNERTON-DYER DATA 784A-798H

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
784 A	1	2.3868312845	2.3969872725	1.0042550088	1.0000000000	1.0
784 B	1	1.9041961437	1.4763178646	0.2584323871	3.0000000000	1.0
784 C	0	1.3221927708	1.3221927708	1	1	1
784 D	0	1.3459764274	1.3459764274	1	1	1
784 E	0	0.7166350909	2.8665403635	1	4	1
784 F	0	2.9388051489	2.9388051489	1	1	1
784 G	0	1.6482722736	1.6482722736	1	1	1
784 H	1	2.5575309899	2.2075638093	0.4315810479	2.0000000000	1.0
784 I	1	4.1466198122	2.2889418179	0.5520018525	1.0000000000	1.0
784 J	1	1.1233153024	1.7509775464	0.3896896852	4.0000000000	1.0
786 A	1	5.0581564923	1.4518567044	0.2870327770	1.0000000000	1.0
786 B	1	3.0058818246	1.4457554465	0.2404877388	2.0000000000	1.0
786 C	1	0.3058839233	1.7237614871	2.8176725807	2.0000000000	1.0
786 D	0	0.6243670999	0.6243670999	1	1	1
786 E	0	2.2980284444	1.1490142222	1	1/2	1
786 F	0	4.1760155624	1.3920051875	1	1/3	1
786 G	1	1.5009743618	1.9166428447	1.2769324337	1.0000000000	1.0
786 H	1	2.4343976800	1.8060854772	0.0529930284	14.0000000000	1.0
786 I	0	1.6451388054	2.4677082081	1	3/2	1
786 J	1	1.5930966388	2.5762182307	0.0770054092	21.0000000000	1.0
786 K	1	2.1879905064	2.4916678812	0.1897988035	6.0000000000	1.0
786 L	1	2.1896861019	2.9938328080	0.0390640833	35.0000000000	1.0
786 M	0	1.0935574545	3.2806723636	1	3	1
790 A	1	2.7861735231	2.4402283805	0.2189587583	4.0000000000	1.0
791 A	0	2.2117095243	1.1058547621	1	1/2	1
791 B	0	0.9636216384	0.4818108192	1	1/2	1
791 C	1	2.3695463432	1.6526782132	2.7898643434	0.2500000000	1.0
792 A	1	1.4206345211	2.1984987673	1.5475470536	1.0000000000	1.0
792 B	0	1.7214809982	1.7214809982	1	1	1
792 C	1	3.2592957589	2.2026699195	0.3379058058	2.0000000000	1.0
792 D	1	2.3650528515	2.1181494566	0.8956034345	1.0000000000	1.0
792 E	0	1.0701000521	2.1402001041	1	2	1
792 F	0	0.9557592918	1.9115185836	1	2	1
792 G	0	0.9996484530	0.9996484530	1	1	1
793 A	1	2.3803932304	2.7443839416	4.6116480361	0.2500000000	1.0
794 A	2	4.3426433734	1.2945419121	0.1490499911	2.0000000000	1.0
794 B	1	4.3358043302	2.9800260413	2.0619191834	0.3333333333	1.0
794 C	1	2.0929914585	2.6163624910	0.2500117696	5.0000000000	1.0
794 D	1	0.9328062669	2.4030580639	1.2880799311	2.0000000000	1.0
795 A	1	3.5382943332	2.4330443998	1.3752639948	0.5000000000	1.0
795 B	0	0.6187438638	0.6187438638	1	1	1
795 C	0	0.9180185975	1.5300309959	1	5/3	1
795 D	0	1.5531710671	2.3297566006	1	3/2	1
797 A	1	3.2937076036	1.7511687923	0.5316709930	1.0000000000	1.0
798 A	1	3.3055085609	1.5173058616	0.4590234252	1.0000000000	1.0
798 B	0	1.6218747329	1.6218747329	1	1	1
798 C	1	2.8121213632	1.8270977124	0.1299444424	5.0000000000	1.0
798 D	1	1.3827775712	1.8586019292	0.1344107663	10.0000000000	1.0
798 E	0	1.5364303383	1.5364303383	1	1	1
798 F	0	1.9304569995	1.9304569995	1	1	1
798 G	1	1.1169891799	2.5627859695	0.1529579704	15.0000000000	1.0
798 H	1	1.2821112156	2.9833490048	0.0554024630	42.0000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
798 I	0	1.6166094707	3.2332189414	1	2	1
799 A	0	3.5646854576	1.7823427288	1	1/2	1
799 B	1	2.2398831435	1.9428746696	0.5782666759	1.5000000000	1.0
800 A	1	2.3452395729	2.2273703795	1.8994821725	0.5000000000	1.0
800 B	1	3.8271616725	2.4376901920	0.3184723302	2.0000000000	1.0
800 C	1	1.3468072016	1.7318693925	1.2859074338	1.0000000000	1.0
800 D	0	1.5683562272	1.5683562272	1	1	1
800 E	0	0.6216204919	1.8648614756	1	3	1
800 F	0	1.3899856760	1.3899856760	1	1	1
800 G	0	2.4380769240	2.4380769240	1	1	1
800 H	1	3.5069511370	2.2287681477	0.6355287144	1.0000000000	1.0
800 I	1	1.7115587321	2.0166727413	0.1963777913	6.0000000000	1.0
801 A	0	0.2874283226	0.5748566451	1	2	1
801 B	0	2.5228982039	1.2614491020	1	1/2	1
801 C	1	0.9932060231	2.0400693564	0.5135060876	4.0000000000	1.0
801 D	1	1.3275315217	2.7050732537	2.0376715803	1.0000000000	1.0
802 A	0	2.9586860920	2.9586860920	1	1	1
802 B	0	2.7580954859	1.3790477430	1	1/2	1
804 A	0	1.2181961170	1.8272941755	1	3/2	1
804 B	1	0.3920429442	2.0038536354	0.8518852612	6.0000000000	1.0
804 C	1	3.3871101427	1.8789758549	0.1849143543	3.0000000000	1.0
804 D	1	1.2304488171	2.3844031579	0.0922777168	21.0000000000	1.0
805 A	1	0.3121746385	3.2924658901	5.2734358970	2.0000000000	1.0
805 B	0	1.3368165187	0.6684082593	1	1/2	1
805 C	0	1.1515660853	0.5757830427	1	1/2	1
805 D	0	2.4472776600	4.8945553199	1	2	1
806 A	1	2.4438911360	1.9545918895	0.3998934037	2.0000000000	1.0
806 B	1	1.1144535359	1.4834831912	0.6655652943	2.0000000000	1.0
806 C	1	3.0117139374	3.0094615134	0.0999252112	10.0000000000	1.0
806 D	1	0.7414216038	2.1907922082	0.0447705073	66.0000000000	1.0
806 E	0	0.5520548375	3.3123290252	1	6	1
806 F	0	1.1935183996	2.3870367992	1	2	1
807 A	0	3.1886904948	2.1257936632	1	2/3	1
808 A	0	1.2470006143	1.2470006143	1	1	1
808 B	0	1.4157114782	2.8314229564	1	2	1
810 A	0	3.6661316994	1.2220438998	1	1/3	1
810 B	0	1.5814478510	1.0542985673	1	2/3	1
810 C	0	0.5080077512	1.5240232535	1	3	1
810 D	1	2.4563334791	1.7806753746	0.3624661288	2.0000000000	1.0
810 E	0	2.3454927808	2.3454927808	1	1	1
810 F	0	1.7955568582	2.3940758110	1	4/3	1
810 G	0	1.6014313405	2.6690522342	1	5/3	1
810 H	1	2.0683411935	2.8535912313	0.2299420135	6.0000000000	1.0
811 A	1	1.9340514664	2.0684791373	1.0695057361	1.0000000000	1.0
812 A	0	0.9323743908	2.7971231723	1	3	1
812 B	1	1.9151319254	1.9569919894	0.0851547946	12.0000000000	1.0
813 A	0	4.2392844940	4.2392844940	1	1	1
813 B	1	2.3320513620	2.2554368624	0.9671471646	1.0000000000	1.0
814 A	1	2.2458764704	1.1895559609	1.5889867184	0.3333333333	1.0
814 B	1	4.0087531499	2.8487110366	0.1421245425	5.0000000000	1.0
815 A	1	1.2781104061	1.6300946134	1.9130913171	0.6666666667	1.0
816 A	1	3.2521275353	1.9819661024	0.3047183853	2.0000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
816 B	0	1.7752905638	1.7752905638	1	1	1
816 C	0	1.1027496746	1.1027496746	1	1	1
816 D	0	0.8104871336	2.4314614009	1	3	1
816 E	0	0.5970685354	1.1941370708	1	2	1
816 F	0	1.9526773174	1.9526773174	1	1	1
816 G	1	3.6606128038	2.0695270453	0.2826749449	2.0000000000	1.0
816 H	1	0.9934818585	1.9010184804	0.9567454424	2.0000000000	1.0
816 I	1	1.5953891658	2.3777937997	0.0677461892	22.0000000000	1.0
816 J	1	1.9478575653	2.3068323672	0.5921460604	2.0000000000	1.0
817 A	2	3.0564514640	1.5664703894	0.2562563823	2.0000000000	1.0
817 B	1	1.0755524992	1.4907042069	0.1385989255	10.0000000000	1.0
819 A	1	1.9019631700	2.7129683860	2.8528085388	0.5000000000	1.0
819 B	1	4.3270924952	1.5410481142	0.7122787950	0.5000000000	1.0
819 C	0	1.9275039799	3.8550079598	1	2	1
819 D	0	0.2993838632	0.5987677264	1	2	1
819 E	0	0.8373929073	1.6747858145	1	2	1
819 F	0	0.6025246471	3.6151478828	1	6	1
822 A	1	1.5796960506	1.5173956623	0.4802808938	2.0000000000	1.0
822 B	0	0.4081775441	1.6327101766	1	4	1
822 C	0	1.4591472676	1.9455296901	1	4/3	1
822 D	1	1.6107354638	1.8697090304	0.1160779701	10.0000000000	1.0
822 E	0	2.1838655335	3.2757983002	1	3/2	1
822 F	0	1.6654990728	3.3309981457	1	2	1
825 A	1	3.3747765340	1.8040304300	0.2672814647	2.0000000000	1.0
825 B	1	1.3368809093	1.7134165818	0.8544349113	1.5000000000	1.0
825 C	1	1.5092459478	2.2662910335	0.7508024245	2.0000000000	1.0
826 A	0	1.3649054369	1.3649054369	1	1	1
826 B	0	0.6982483960	2.0947451879	1	3	1
827 A	1	5.0436182817	1.3222292482	0.2621588658	1.0000000000	1.0
828 A	0	4.1001947929	2.0500973964	1	1/2	1
828 B	1	1.2458224988	2.1792708496	1.1661751450	1.5000000000	1.0
828 C	1	1.2681978153	2.3181073582	1.8278752181	1.0000000000	1.0
828 D	0	1.7429033412	1.7429033412	1	1	1
829 A	1	3.4016683815	1.2057007713	0.3544439481	1.0000000000	1.0
830 A	0	1.3300813679	1.7734418239	1	4/3	1
830 B	1	0.8272891182	2.7304793867	0.0257852663	128.0000000000	1.0
830 C	1	2.0301023316	2.2133179782	0.0545124732	20.0000000000	1.0
831 A	1	2.0541939187	1.7484352839	0.0851153958	10.0000000000	1.0
832 A	1	2.4007898238	2.4019028404	0.2501159011	4.0000000000	1.0
832 B	1	1.3485484416	2.0036945644	0.3714539468	4.0000000000	1.0
832 C	1	1.6405548585	2.1713281808	0.3308832023	4.0000000000	1.0
832 D	0	2.3913876409	1.1956938204	1	1/2	1
832 E	0	0.8369394685	1.6738789370	1	2	1
832 F	0	1.5368108334	3.0736216668	1	2	1
832 G	0	1.2301336856	2.4602673712	1	2	1
832 H	1	3.9630716475	2.1688159289	1.0945126012	0.5000000000	1.0
832 I	1	2.6223391999	2.4833924059	0.2367535449	4.0000000000	1.0
832 J	1	0.6378305375	1.5951492806	0.6252245646	4.0000000000	1.0
833 A	0	2.0755836776	1.0377918388	1	1/2	1
834 A	0	0.4778202483	1.6723708692	1	7/2	1
834 B	0	0.8593990205	1.7187980411	1	2	1
834 C	1	2.8261629299	1.8333472976	0.0810881814	8.0000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
834D	0	4.7123016890	2.3561508445	1	1/2	1
834E	1	2.4708181999	2.7025662830	0.5468970325	2.0000000000	1.0
834F	1	1.7964026865	2.5729930149	0.0511536797	28.0000000000	1.0
834G	1	1.8880547711	3.0211249880	0.8000628568	2.0000000000	1.0
836A	1	1.2193960008	2.0763611931	0.8513892090	2.0000000000	1.0
836B	0	1.7821670818	2.6732506227	1	3/2	1
840A	1	1.1320487150	1.9611172165	3.4647223049	0.5000000000	1.0
840B	0	1.2212883443	1.2212883443	1	1	1
840C	0	2.7757566445	1.3878783223	1	1/2	1
840D	0	0.3691044282	1.8455221409	1	5	1
840E	1	2.2308313793	1.9866164430	0.8905273887	1.0000000000	1.0
840F	1	3.7748216876	2.0729414081	2.1965979638	0.2500000000	1.0
840G	0	3.0559620739	1.5279810369	1	1/2	1
840H	1	1.6475613042	2.4547357754	0.4966402503	3.0000000000	1.0
840I	0	1.9451617258	1.9451617258	1	1	1
840J	0	2.1289344074	2.1289344074	1	1	1
842A	1	2.7237772439	1.4033951345	0.5152385855	1.0000000000	1.0
842B	1	2.4004300386	2.3619845303	0.0756910698	13.0000000000	1.0
843A	1	2.6718170788	2.5244694756	0.9448511635	1.0000000000	1.0
845A	0	1.4929349382	1.4929349382	1	1	1
846A	0	0.6897576384	1.3795152768	1	2	1
846B	1	2.4931128810	1.7694946429	0.3548765594	2.0000000000	1.0
846C	1	0.8110632878	1.6551206746	1.0203400275	2.0000000000	1.0
847A	0	0.2341098321	1.8728786567	1	8	1
847B	1	0.9088500537	1.2780958682	0.1757847545	8.0000000000	1.0
847C	1	0.5402177457	1.9187378840	1.1839287023	3.0000000000	1.0
848A	0	0.7199604717	1.4399209433	1	2	1
848B	0	0.7428842336	1.4857684673	1	2	1
848C	0	1.3433600285	2.6867200570	1	2	1
848D	0	2.5003648541	1.2501824271	1	1/2	1
848E	0	1.5405906701	3.0811813403	1	2	1
848F	1	1.5703419300	2.4421024255	1.5551405582	1.0000000000	1.0
848G	1	2.3425055207	1.8785649168	0.2004867118	4.0000000000	1.0
849A	1	2.0010953407	1.1472977579	0.2866674402	2.0000000000	1.0
850A	0	0.3539825981	0.7079651961	1	2	1
850B	0	2.0105217603	2.0105217603	1	1	1
850C	1	1.2971425667	1.9776460914	0.7623086861	2.0000000000	1.0
850D	1	0.2669383439	1.8987366857	0.5080725646	14.0000000000	1.0
850E	1	2.2753196437	1.1214812335	0.0821482553	6.0000000000	1.0
850F	0	0.5968922828	2.3875691312	1	4	1
850G	0	1.8071584322	3.6143168644	1	2	1
850H	0	0.8549748907	3.4198995626	1	4	1
850I	0	1.0175538788	4.0702155151	1	4	1
850J	0	0.5974888457	1.1949776915	1	2	1
850K	1	1.4888215440	2.7115121406	0.1517706040	12.0000000000	1.0
850L	1	2.9004989556	2.7415419874	0.0675140487	14.0000000000	1.0
851A	1	3.8742025615	1.4039603600	0.1811934634	2.0000000000	1.0
854A	1	2.1225419878	1.8547964065	0.4369280837	2.0000000000	1.0
854B	1	2.7912216525	2.0035028254	3.2300418371	0.2222222222	1.0
854C	1	2.9994943944	2.7291045757	0.1137318585	8.0000000000	1.0
854D	1	1.2693965422	2.6429141568	0.0495720017	42.0000000000	1.0
855A	0	0.9424032711	0.9424032711	1	1	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
855 B	1	1.4708261859	1.6027786295	0.3632377152	3.0000000000	1.0
855 C	0	1.0322612259	2.0645224517	1	2	1
856 A	1	4.4255910442	2.4629930198	0.2782671281	2.0000000000	1.0
856 B	1	1.5228754070	2.3348898990	0.7666056882	2.0000000000	1.0
856 C	1	3.7638167988	2.0348872808	0.1351611535	4.0000000000	1.0
856 D	1	0.5234970512	2.6183951496	5.0017381060	1.0000000000	1.0
858 A	0	1.1358111272	1.1358111272	1	1	1
858 B	1	0.8964019921	1.9329491676	2.1563418918	1.0000000000	1.0
858 C	0	1.4531272382	1.4531272382	1	1	1
858 D	0	0.8847016726	1.7694033452	1	2	1
858 E	0	1.7801749731	2.6702624597	1	3/2	1
858 F	1	0.6144950596	2.5969082820	0.0384189535	110.00000000	1.0
858 G	1	2.8332583986	2.6549896366	0.0520599972	18.0000000000	1.0
858 H	0	1.8422546597	1.8422546597	1	1	1
858 I	0	0.3445543329	2.7564346634	1	8	1
858 J	0	1.4366901998	2.8733803997	1	2	1
858 K	0	0.2109237587	2.9529326224	1	14	1
858 L	0	0.5152748186	3.6069237304	1	7	1
858 M	0	1.7091732041	3.4183464083	1	2	1
861 A	0	1.8859710003	0.9429855001	1	1/2	1
861 B	1	0.4004425393	2.9817201838	0.4380036777	17.0000000000	1.0
861 C	1	0.5198123568	1.9612976333	0.1078025070	35.0000000000	1.0
861 D	1	3.0961653380	1.6446967712	0.1062408878	5.0000000000	1.0
862 A	1	2.3772088260	1.8963864630	0.3988682951	2.0000000000	1.0
862 B	1	3.7125048067	1.0545502107	0.1420267805	2.0000000000	1.0
862 C	0	2.0389535297	3.0584302946	1	3/2	1
862 D	0	1.9962060868	2.6616081157	1	4/3	1
862 E	1	1.6300467988	2.7877433020	2.1377785779	0.8000000000	1.0
862 F	1	2.7368168553	2.6102377901	0.1192186913	8.0000000000	1.0
864 A	1	3.4479342806	2.1886718012	0.3173888513	2.0000000000	1.0
864 B	1	3.2770629245	2.3652195151	0.3608749007	2.0000000000	1.0
864 C	1	1.5867673970	2.1953723773	0.2305917029	6.0000000000	1.0
864 D	0	1.6436559999	1.6436559999	1	1	1
864 E	0	0.9161205838	1.8322411676	1	2	1
864 F	0	1.0740205877	2.1480411753	1	2	1
864 G	0	1.9906657850	1.9906657850	1	1	1
864 H	0	1.0026005839	2.0052011678	1	2	1
864 I	0	0.6200860754	1.2401721508	1	2	1
864 J	1	0.9489652340	2.3079806750	0.4053504091	6.0000000000	1.0
864 K	1	1.7365551510	2.1621840308	0.6225497732	2.0000000000	1.0
864 L	1	1.8920131616	2.2253261072	0.1960280679	6.0000000000	1.0
866 A	1	2.2101758802	2.4744969079	0.8396945680	1.3333333333	1.0
867 A	1	0.6257848531	1.6987250823	0.6786378233	4.0000000000	1.0
867 B	1	2.7849176519	1.2964049615	0.4655092622	1.0000000000	1.0
867 C	1	0.5370146722	3.3192111933	3.0904287770	2.0000000000	1.0
867 D	0	0.6754417434	1.3508834868	1	2	1
867 E	0	2.2141682160	4.4283364320	1	2	1
869 A	1	3.8207898925	3.0401844953	0.3978476416	2.0000000000	1.0
869 B	1	1.9650475386	1.4849690043	0.3778455674	2.0000000000	1.0
869 C	0	1.7377151949	0.8688575975	1	1/2	1
869 D	1	1.9846117144	2.5742151775	0.2161812610	6.0000000000	1.0
870 A	1	2.3986779297	1.6315300055	0.2267262847	3.0000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
870 B	1	1.0644927048	1.9885607215	1.8680829963	1.0000000000	1.0
870 C	1	2.0189872282	2.0033437922	0.9922518400	1.0000000000	1.0
870 D	0	1.5632095609	1.5632095609	1	1	1
870 E	1	2.8225873469	2.6829769303	0.3168460471	3.0000000000	1.0
870 F	1	1.2569671923	2.7740230622	0.0630547895	35.000000000	1.0
870 G	0	2.5341570608	2.5341570608	1	1	1
870 H	0	3.2900075814	3.2900075814	1	1	1
870 I	0	0.6243667517	3.1218337583	1	5	1
871 A	0	2.5120439338	5.0240878676	1	2	1
872 A	1	2.6621279676	1.8625114998	0.3498162978	2.0000000000	1.0
873 A	0	2.1363788789	2.1363788789	1	1	1
873 B	1	0.7702196484	2.8470655353	3.6964332722	1.0000000000	1.0
873 C	1	0.1777801853	3.5817480649	5.0367650056	4.0000000000	1.0
873 D	1	2.6074609607	1.2665575443	0.1214359067	4.0000000000	1.0
874 A	0	2.3626280616	2.3626280616	1	1	1
874 B	0	0.7640162400	0.7640162400	1	1	1
874 C	1	4.3943660458	1.4639454722	0.3331414491	1.0000000000	1.0
874 D	1	2.5757683247	3.1207398502	0.2423152595	5.0000000000	1.0
874 E	1	1.2290033293	2.8015532327	2.2795326635	1.0000000000	1.0
874 F	0	0.5062034509	3.5434241560	1	7	1
876 A	1	1.1285506284	2.0503245410	1.8167767484	1.0000000000	1.0
876 B	1	2.3538774689	2.4725343055	0.0700272731	15.000000000	1.0
880 A	1	3.1380735450	2.2742262829	1.4494410346	0.5000000000	1.0
880 B	0	3.0983312416	1.5491656208	1	1/2	1
880 C	1	0.5665638294	2.2927808355	2.6978788227	1.5000000000	1.0
880 D	1	2.4941606033	1.6190601765	0.0540950255	12.000000000	1.0
880 E	0	0.3888578280	0.7777156559	1	2	1
880 F	1	1.0869015844	2.0045110340	0.4610608409	4.0000000000	1.0
880 G	1	1.0122129857	2.5733079776	0.1271129700	20.000000000	1.0
880 H	1	3.4266195477	1.8690062115	1.0908746568	0.5000000000	1.0
880 I	0	1.7212734487	1.7212734487	1	1	1
880 J	0	1.8425381207	2.7638071810	1	3/2	1
882 A	1	1.4743501869	1.6179112815	1.6460586799	0.6666666667	1.0
882 B	0	0.7682933291	1.5365866583	1	2	1
882 C	0	0.4827121345	0.9654242689	1	2	1
882 D	1	2.7923167773	1.8519608668	0.1658086290	4.0000000000	1.0
882 E	1	0.7584048118	1.6782922276	0.5532310059	4.0000000000	1.0
882 F	0	0.5029660481	3.0177962886	1	6	1
882 G	1	2.2521071108	2.8881331620	0.0712452092	18.000000000	1.0
882 H	1	1.0223787452	2.8863902281	0.0470535054	60.000000000	1.0
882 I	0	1.2970927845	2.5941855689	1	2	1
882 J	0	0.3024687067	3.0246870671	1	10	1
882 K	0	0.8064528302	3.2258113209	1	4	1
882 L	0	0.5020150241	2.0080600965	1	4	1
885 A	0	2.8904036931	2.8904036931	1	1	1
885 B	1	1.5393627999	2.5812771705	3.3536956599	0.5000000000	1.0
885 C	1	3.2915493122	2.3961058957	0.2426522860	3.0000000000	1.0
885 D	1	2.0244640686	1.5083772548	0.7450748463	1.0000000000	1.0
886 A	1	4.7336407225	1.5970001613	0.1686862454	2.0000000000	1.0
886 B	1	2.6526645889	1.4452458189	0.5448279534	1.0000000000	1.0
886 C	0	1.1782593182	2.3565186364	1	2	1
886 D	1	0.5609213187	3.0178292399	0.1415823508	38.000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
886 E	1	3.4991428175	2.8836435055	0.1648199948	5.0000000000	1.0
888 A	0	0.6337216869	0.6337216869	1	1	1
888 B	1	2.0800736558	2.4590584170	1.1821977603	1.0000000000	1.0
888 C	1	1.2894745254	2.0698576775	1.6051947027	1.0000000000	1.0
888 D	0	2.6073010855	2.6073010855	1	1	1
890 A	1	3.4473615420	1.7674639432	0.5127004875	1.0000000000	1.0
890 B	1	2.9585607938	1.2590202825	0.4255516010	1.0000000000	1.0
890 C	0	1.8431917777	1.8431917777	1	1	1
890 D	1	4.4512040152	1.9993135464	0.4491624153	1.0000000000	1.0
890 E	1	0.8221408346	1.4484961589	0.8809294576	2.0000000000	1.0
890 F	1	1.8577167478	2.1818938580	0.0903463831	13.0000000000	1.0
890 G	1	1.7698498471	2.8194525619	1.5930461934	1.0000000000	1.0
890 H	0	2.8880852693	2.8880852693	1	1	1
891 A	1	2.3510028056	1.4926709205	0.3174540917	2.0000000000	1.0
891 B	0	1.6877732673	0.5625910891	1	1/3	1
891 C	0	1.0225312693	2.0450625386	1	2	1
891 D	0	2.3923630414	2.3923630414	1	1	1
891 E	0	0.7912355109	0.7912355109	1	1	1
891 F	0	4.1487915620	1.3829305207	1	1/3	1
891 G	0	0.9633519031	0.9633519031	1	1	1
891 H	0	4.0505896063	4.0505896063	1	1	1
892 A	0	3.1558649438	3.1558649438	1	1	1
892 B	1	3.3588715764	2.5062487345	2.2384738542	0.3333333333	1.0
892 C	1	2.6003499001	2.1454643693	0.2750225228	3.0000000000	1.0
894 A	1	1.1906736040	1.5789258426	0.6630389039	2.0000000000	1.0
894 B	1	1.7191484425	1.3981098223	0.8132571846	1.0000000000	1.0
894 C	0	1.1568928656	1.9281547759	1	5/3	1
894 D	1	2.5631776678	2.0563466857	0.2674215306	3.0000000000	1.0
894 E	1	0.4843074326	2.7362803060	0.1228235424	46.0000000000	1.0
894 F	1	3.9572511003	2.6445527611	0.1336560503	5.0000000000	1.0
894 G	1	1.4668493690	3.1291749777	0.0277047098	77.0000000000	1.0
895 A	1	4.9464687269	1.7538892145	0.3545740024	1.0000000000	1.0
895 B	0	1.5004138844	1.5004138844	1	1	1
896 A	1	2.6873620768	2.2979189861	1.7101670117	0.5000000000	1.0
896 B	1	3.8005038960	2.2943272655	1.2073805623	0.5000000000	1.0
896 C	0	3.3322500870	1.6661250435	1	1/2	1
896 D	1	4.7125132663	2.3323381619	0.9898489532	0.5000000000	1.0
897 A	0	1.0302057840	1.0302057840	1	1	1
897 B	0	2.2513865211	1.1256932605	1	1/2	1
897 C	1	2.2862457182	1.2516489192	2.1898764585	0.2500000000	1.0
897 D	1	0.1325510626	3.0756516640	0.7734507752	30.0000000000	1.0
897 E	1	0.7081406416	1.8699514540	1.0562599259	2.5000000000	1.0
897 F	1	1.6831372042	1.6264676608	0.9663310019	1.0000000000	1.0
898 A	1	3.3421121540	1.9128614257	0.5723510575	1.0000000000	1.0
898 B	0	1.5337032793	1.5337032793	1	1	1
898 C	0	2.5896817775	3.8845226663	1	3/2	1
898 D	1	3.5510202364	2.7167655369	0.2550220646	3.0000000000	1.0
899 A	1	3.9981065199	2.3407393199	0.5854619701	1.0000000000	1.0
899 B	0	4.6049071056	4.6049071056	1	1	1
900 A	0	0.8731871573	1.7463743146	1	2	1
900 B	0	1.8812247026	1.8812247026	1	1	1
900 C	1	1.9525058408	2.3172745410	0.5934103992	2.0000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
900D	1	1.7136269959	2.3474457037	0.1141558086	12.0000000000	1.0
900E	1	1.1743738590	2.2986343680	0.3262212668	6.0000000000	1.0
900F	0	0.7663572902	1.5327145804	1	2	1
900G	0	0.6478758022	1.9436274067	1	3	1
900H	0	1.4486943348	1.4486943348	1	1	1
901A	1	1.5965166331	1.6122697072	2.0197343064	0.5000000000	1.0
901B	1	1.1813876044	1.9820576840	3.3554739809	0.5000000000	1.0
901C	0	2.7871636011	0.9290545337	1	1/3	1
901D	0	0.3712575011	1.1137725033	1	3	1
901E	1	0.8647721717	2.9384268771	0.2265280169	15.0000000000	1.0
901F	1	3.3303214003	1.3138606461	0.3945146693	1.0000000000	1.0
902A	1	0.7602202342	1.5657216063	1.0297815921	2.0000000000	1.0
902B	0	3.7125501906	2.4750334604	1	2/3	1
903A	1	2.3072778935	2.3301434116	0.2524775427	4.0000000000	1.0
903B	0	0.4081046798	1.6324187193	1	4	1
904A	1	3.3170381291	2.2509942877	1.3572314819	0.5000000000	1.0
905A	1	4.9675540093	3.2826026256	2.6432345734	0.2500000000	1.0
905B	0	0.5380139112	2.6900695561	1	5	1
906A	1	0.3410288839	1.5305099601	2.2439594301	2.0000000000	1.0
906B	1	2.3722659293	1.7692736909	0.7458159176	1.0000000000	1.0
906C	1	1.4178407129	2.0227288128	0.7133131368	2.0000000000	1.0
906D	1	2.6185194041	1.8951624465	2.1712603430	0.3333333333	1.0
906E	0	0.4571735518	2.2858677589	1	5	1
906F	0	1.3091802355	2.6183604710	1	2	1
906G	1	2.2339918210	2.7197368860	0.4058112270	3.0000000000	1.0
906H	1	1.8388370165	3.1439048924	0.0310859019	55.0000000000	1.0
906I	0	3.0548337452	3.0548337452	1	1	1
909A	0	0.8230901504	1.6461803008	1	2	1
909B	0	1.7762665291	3.5525330582	1	2	1
909C	1	3.1448957012	2.1205886995	0.3371476992	2.0000000000	1.0
910A	0	1.0509058342	1.0509058342	1	1	1
910B	1	2.1311572863	1.9847671663	1.3969643482	0.6666666667	1.0
910C	1	2.6028837106	1.3176683943	1.5187021867	0.3333333333	1.0
910D	1	2.0695112831	1.8764854850	0.3022429338	3.0000000000	1.0
910E	0	0.2537944386	1.5227666315	1	6	1
910F	1	0.5215759252	3.0174721366	0.1051872356	55.0000000000	1.0
910G	1	3.4576823605	2.1767073443	0.0629527851	10.0000000000	1.0
910H	1	1.0529878541	2.7611188149	0.0257076017	102.0000000000	1.0
910I	0	3.5045156420	3.5045156420	1	1	1
910J	0	1.5932468473	1.5932468473	1	1	1
910K	1	1.1077876976	2.5781642664	0.0664945425	35.0000000000	1.0
912A	1	0.8595022649	1.9507400215	1.1348079587	2.0000000000	1.0
912B	0	3.2036916115	1.6018458057	1	1/2	1
912C	0	1.2142468377	2.4284936753	1	2	1
912D	0	2.3936541672	2.3936541672	1	1	1
912E	0	1.5429736445	1.5429736445	1	1	1
912F	1	0.9267225427	2.0970333294	1.1314245812	2.0000000000	1.0
912G	1	3.0080919221	1.9566840158	0.6504734784	1.0000000000	1.0
912H	1	1.8797316974	2.5332700532	0.2695352807	5.0000000000	1.0
912I	1	1.7505351624	2.4090814409	0.3440492789	4.0000000000	1.0
912J	0	1.5440087217	2.3160130825	1	3/2	1
912K	0	0.7961574782	2.3884724345	1	3	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
912 L	0	0.9591647519	1.9183295038	1	2	1
913 A	0	0.7261078737	0.7261078737	1	1	1
913 B	0	2.7848137078	5.5696274157	1	2	1
914 A	1	1.8616267621	1.8025054924	1.9364842933	0.5000000000	1.0
914 B	0	2.0072142904	2.0072142904	1	1	1
915 A	0	0.4655714661	3.2590002627	1	7	1
915 B	1	4.3738322547	2.6130796804	1.1948696375	0.5000000000	1.0
915 C	0	1.3320394729	3.9961184186	1	3	1
915 D	1	1.4688651520	1.8627134122	0.2818068856	4.5000000000	1.0
916 A	0	0.8084778532	1.2127167798	1	3/2	1
916 B	0	1.0750718596	3.2252155789	1	3	1
916 C	2	3.9021255757	1.5975481506	0.1364681992	3.0000000000	1.0
916 D	1	4.2153960915	2.4420882822	1.7379778051	0.3333333333	1.0
916 E	1	3.1904104520	2.1723205570	0.2269635407	3.0000000000	1.0
918 A	1	1.3010831327	1.7144756707	0.6588647672	2.0000000000	1.0
918 B	1	1.4955878600	1.8094793603	0.2016463903	6.0000000000	1.0
918 C	1	0.4499146685	1.9056874752	4.2356642461	1.0000000000	1.0
918 D	0	0.7550553757	1.5101107513	1	2	1
918 E	1	2.3462922188	1.7463400878	0.7442977792	1.0000000000	1.0
918 F	1	1.7651732561	1.6690879842	1.4183491437	0.6666666667	1.0
918 G	0	2.3989561499	3.1986081999	1	4/3	1
918 H	1	2.2570175348	2.9626885215	0.0397774668	33.0000000000	1.0
918 I	1	1.7369042247	3.0516986095	1.7569757538	1.0000000000	1.0
918 J	1	0.8491152910	2.9088946006	0.3425794626	10.0000000000	1.0
918 K	0	0.3288423901	2.6307391207	1	8	1
918 L	0	1.5039717658	3.0079435315	1	2	1
920 A	1	0.5880378493	2.4148869343	0.0684447704	60.0000000000	1.0
920 B	1	2.4127978013	1.6168440738	0.0558426430	12.0000000000	1.0
920 C	1	2.8080443019	2.5036279661	0.2228978336	4.0000000000	1.0
920 D	1	1.4333203773	2.1460465556	0.1871569146	8.0000000000	1.0
921 A	0	0.3209950628	0.6419901256	1	2	1
921 B	1	3.0340231314	2.3484673295	1.1610659648	0.6666666667	1.0
922 A	0	1.9239190433	1.9239190433	1	1	1
923 A	0	2.7221586012	2.7221586012	1	1	1
924 A	0	0.2719911704	0.8159735113	1	3	1
924 B	1	1.2992812640	2.0082329065	0.1030432728	15.0000000000	1.0
924 C	1	1.8340344156	2.0474653986	0.3721241327	3.0000000000	1.0
924 D	0	0.5078494253	1.5235482760	1	3	1
924 E	1	2.9265647575	2.4104695147	0.0549101014	15.0000000000	1.0
924 F	0	0.3714094030	1.8570470150	1	5	1
924 G	0	2.5196994926	2.5196994926	1	1	1
924 H	1	0.7490531114	2.4534577552	0.0727869394	45.0000000000	1.0
925 A	1	2.1906183485	2.4066435608	0.5493069029	2.0000000000	1.0
925 B	1	2.9208099321	1.8478028504	0.3163168596	2.0000000000	1.0
925 C	0	1.5623454535	1.5623454535	1	1	1
925 D	0	1.6068744035	3.2137488070	1	2	1
925 E	0	2.6774308083	5.3548616166	1	2	1
926 A	0	2.3944486185	3.5916729278	1	3/2	1
927 A	1	0.8448400033	2.8850974564	1.7074815616	2.0000000000	1.0
928 A	1	1.4888902545	2.5431935125	0.4270283698	4.0000000000	1.0
928 B	1	2.6895458659	2.0226680670	0.1880120444	4.0000000000	1.0
930 A	1	1.4909281319	1.5301618707	2.0526299530	0.5000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
930 B	0	0.6265047032	0.6265047032	1	1	1
930 C	0	1.2362640714	1.2362640714	1	1	1
930 D	1	0.5179840083	1.7003818844	0.4689559791	7.0000000000	1.0
930 E	1	2.7841974936	1.5851902223	0.2846763252	2.0000000000	1.0
930 F	0	0.1346848356	1.4815331915	1	11	1
930 G	0	3.0709626286	1.5354813143	1	1/2	1
930 H	1	0.7521873500	2.0821067538	0.1845379571	15.0000000000	1.0
930 I	0	1.4978178696	1.4978178696	1	1	1
930 J	0	0.9353812860	1.8707625720	1	2	1
930 K	0	0.9415552095	1.8831104190	1	2	1
930 L	0	0.7411152604	2.2233457812	1	3	1
930 M	1	1.5250007887	2.7780796862	0.3036151092	6.0000000000	1.0
930 N	0	0.5072789872	3.0436739229	1	6	1
930 O	0	0.8109446204	3.2437784816	1	4	1
931 A	0	1.7334096346	5.2002289037	1	3	1
931 B	0	1.5598943009	1.5598943009	1	1	1
931 C	0	1.8757939810	1.8757939810	1	1	1
933 A	1	3.5910691959	1.8296030548	0.5094869954	1.0000000000	1.0
933 B	1	1.0684901563	2.3006452064	0.1957430894	11.0000000000	1.0
934 A	1	4.3580455357	2.0713329133	0.4752894150	1.0000000000	1.0
934 B	0	2.1730807250	3.6218012083	1	5/3	1
934 C	0	1.2985913057	3.8957739172	1	3	1
935 A	1	1.8682654972	1.5708761577	0.4204103111	2.0000000000	1.0
935 B	0	1.4462050325	0.9641366883	1	2/3	1
936 A	1	2.2570695652	2.2673570478	1.0045578935	1.0000000000	1.0
936 B	0	2.1411309907	2.1411309907	1	1	1
936 C	0	0.9953703325	1.9907406649	1	2	1
936 D	0	0.5479481922	1.0958963844	1	2	1
936 E	1	1.7307793292	2.2335648552	0.6452483045	2.0000000000	1.0
936 F	0	1.0573903107	2.1147806214	1	2	1
936 G	1	2.1330121455	2.3589685594	0.5529665090	2.0000000000	1.0
936 H	1	2.4247872636	2.2877842319	0.4717494739	2.0000000000	1.0
936 I	0	1.1006671090	2.2013342180	1	2	1
938 A	1	3.6024689923	2.0114613919	0.2791781687	2.0000000000	1.0
938 B	1	0.9200292966	1.5169169330	0.3297540499	5.0000000000	1.0
938 C	1	1.7860118886	2.7904680859	0.0651000726	24.0000000000	1.0
938 D	0	1.7647855022	3.5295710044	1	2	1
939 A	1	0.4879438291	1.8494900042	3.7903748214	1.0000000000	1.0
939 B	1	3.1462027883	3.0969548932	1.9686937566	0.5000000000	1.0
939 C	1	2.5507838076	1.3745564274	0.1077752198	5.0000000000	1.0
940 A	0	0.3036041233	0.9108123700	1	3	1
940 B	0	2.6935919591	2.6935919591	1	1	1
940 C	1	1.0823244188	2.5466599781	2.3529543766	1.0000000000	1.0
940 D	1	3.4473780398	2.1758274460	0.2103847640	3.0000000000	1.0
940 E	0	0.9196309029	2.7588927086	1	3	1
942 A	0	1.8983844340	1.8983844340	1	1	1
942 B	0	0.1107867230	1.7725875672	1	16	1
942 C	1	0.7173734051	3.2430106243	0.0282542066	160.00000000	1.0
942 D	1	2.8179267391	3.1731945642	0.0469197578	24.0000000000	1.0
943 A	0	3.5161617758	1.7580808879	1	1/2	1
944 A	1	2.0241263360	2.5461928917	0.6289609612	2.0000000000	1.0
944 B	1	3.5140200888	2.5384601650	0.3611903320	2.0000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
944 C	1	1.7116935681	1.9540124811	0.2853916901	4.0000000000	1.0
944 D	0	2.6857753111	2.6857753111	1	1	1
944 E	2	3.4396916312	1.6216175829	0.1178606803	4.0000000000	1.0
944 F	0	2.0369179593	2.0369179593	1	1	1
944 G	0	0.7123740916	1.4247481832	1	2	1
944 H	1	0.5325676265	2.6349557864	1.2369113588	4.0000000000	1.0
944 I	1	1.3545068719	2.4534358947	0.4528282480	4.0000000000	1.0
944 J	1	1.3648615266	2.3102708075	1.6926778010	1.0000000000	1.0
944 K	1	2.9562562217	1.7643718252	0.1492066057	4.0000000000	1.0
946 A	0	2.6183705859	1.3091852930	1	1/2	1
946 B	1	1.7794979329	1.9736736156	1.6636773602	0.6666666667	1.0
946 C	0	0.3661751353	3.6617513529	1	10	1
948 A	0	1.0595880585	1.5893820878	1	3/2	1
948 B	0	1.6952296957	1.6952296957	1	1	1
948 C	0	2.0612524126	2.0612524126	1	1	1
950 A	1	1.8318889258	2.0016845509	0.5463444106	2.0000000000	1.0
950 B	0	0.4410560617	0.8821121233	1	2	1
950 C	0	1.2198252026	2.4396504053	1	2	1
950 D	0	1.5541474188	3.1082948376	1	2	1
950 E	1	2.5365493120	2.8728001790	0.1887603870	6.0000000000	1.0
954 A	1	0.7259892615	1.9067839158	1.3132314877	2.0000000000	1.0
954 B	0	1.1953320239	1.1953320239	1	1	1
954 C	0	0.6676539457	1.3353078914	1	2	1
954 D	1	1.6378943272	1.8540254793	0.2829891783	4.0000000000	1.0
954 E	1	0.8578088245	1.7557498068	1.0233922505	2.0000000000	1.0
954 F	1	1.5511785481	1.6924692457	0.5455430156	2.0000000000	1.0
954 G	0	3.1990540814	3.1990540814	1	1	1
954 H	1	2.8839207271	2.9846183138	0.0739226361	14.0000000000	1.0
954 I	1	2.7048923859	3.0682460481	0.1134332022	10.0000000000	1.0
954 J	1	0.6041744008	2.9240611399	0.0711729909	68.0000000000	1.0
954 K	0	0.4941064717	2.9646388300	1	6	1
954 L	0	1.3885180426	2.7770360851	1	2	1
954 M	0	0.8313387442	3.3253549770	1	4	1
955 A	0	1.4863405386	0.7431702693	1	1/2	1
956 A	0	1.9246145311	1.9246145311	1	1	1
957 A	0	2.1597414573	1.0798707286	1	1/2	1
960 A	1	2.7025732251	2.0503200250	1.5173095078	0.5000000000	1.0
960 B	1	3.5906720505	2.0578659516	0.5731144261	1.0000000000	1.0
960 C	0	1.7605688118	1.7605688118	1	1	1
960 D	0	0.8715629020	1.7431258040	1	2	4
960 E	0	1.1850926721	1.1850926721	1	1	1
960 F	0	1.8974006112	1.8974006112	1	1	1
960 G	0	1.9807518180	1.9807518180	1	1	1
960 H	1	2.9612986380	2.6461902538	0.5957274330	1.5000000000	1.0
960 I	0	1.1287136997	1.1287136997	1	1	1
960 J	0	1.4422113862	1.4422113862	1	1	1
960 K	1	4.9320596338	2.1856460521	1.7726031024	0.2500000000	1.0
960 L	1	1.7061708770	2.5949647962	1.5209290178	1.0000000000	1.0
960 M	1	2.3059156871	2.5176290280	1.0918131318	1.0000000000	1.0
960 N	0	2.2484039196	2.2484039196	1	1	4
960 O	0	0.8191069172	2.4573207515	1	3	1
960 P	0	1.9775352502	1.9775352502	1	1	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
962 A	0	2.7999393875	2.7999393875	1	1	1
964 A	0	1.1198415821	1.1198415821	1	1	1
965 A	0	4.0716220074	2.0358110037	1	1/2	1
966 A	1	0.8853869237	1.4614322919	0.8253071357	2.0000000000	1.0
966 B	0	0.2819418097	1.4097090486	1	5	1
966 C	1	0.5664715755	1.7653369522	1.5581867021	2.0000000000	1.0
966 D	1	1.7441234008	1.3761745479	0.3945175402	2.0000000000	1.0
966 E	1	1.9082662943	2.0669379979	0.2707874163	4.0000000000	1.0
966 F	0	0.4082792341	1.6331169366	1	4	1
966 G	1	1.1385943933	2.7656988559	0.6072616535	4.0000000000	1.0
966 H	0	0.4792307183	2.3961535917	1	5	1
966 I	0	0.4643098624	2.7858591746	1	6	1
966 J	0	0.3992807787	3.5935270082	1	9	1
966 K	0	2.7119467172	2.7119467172	1	1	1
968 A	1	1.6556466339	2.6417815152	0.1994523968	8.0000000000	1.0
968 B	0	0.3816352236	2.2898113418	1	6	1
968 C	0	0.6021890261	2.4087561044	1	4	1
968 D	1	3.0267578391	2.5010482433	0.4131563171	2.0000000000	1.0
968 E	1	0.4991290115	1.4384836253	0.3602484508	8.0000000000	1.0
969 A	0	2.9869009292	2.9869009292	1	1	1
970 A	0	0.5310141996	0.5310141996	1	1	1
970 B	0	1.5883427578	1.5883427578	1	1	1
972 A	0	1.6050978266	1.6050978266	1	1	1
972 B	0	2.0222965389	2.0222965389	1	1	1
972 C	1	2.9166563143	2.3761866009	0.8146954406	1.0000000000	1.0
972 D	1	2.3149516507	2.3144639469	0.9997893244	1.0000000000	1.0
973 A	0	4.8789961045	4.8789961045	1	1	1
973 B	1	4.1607442392	2.4129023423	5.2192876639	0.1111111111	1.0
974 A	0	1.2046434315	1.2046434315	1	1	1
974 B	0	0.2422921289	0.2422921289	1	1	1
974 C	0	2.1052734500	1.0526367250	1	1/2	1
974 D	0	2.2001897331	2.2001897331	1	1	1
974 E	1	4.1853626672	2.8758677503	0.2290417007	3.0000000000	1.0
974 F	1	3.1291701195	2.8497837722	0.1011906126	9.0000000000	1.0
974 G	1	2.4046637022	2.3325867716	0.3233420637	3.0000000000	1.0
974 H	1	1.4607794760	2.1312789050	0.0972667419	15.0000000000	1.0
975 A	1	1.7990725603	2.6002925141	1.4453516615	1.0000000000	1.0
975 B	1	1.1088014849	1.1952547420	0.2694925012	4.0000000000	1.0
975 C	0	0.8069461169	1.6138922337	1	2	1
975 D	0	0.3611376073	0.7222752146	1	2	1
975 E	0	0.4098572926	0.8197145851	1	2	1
975 F	1	1.1032705873	1.8832551063	0.8534874073	2.0000000000	1.0
975 G	0	1.4788241842	1.4788241842	1	1	1
975 H	1	0.9164687672	3.1593865372	0.5745579573	6.0000000000	1.0
975 I	1	0.9486309445	1.4720753967	0.0184736827	84.0000000000	1.0
975 J	1	2.4669880308	2.3914244965	0.0969370125	10.0000000000	1.0
975 K	1	1.8043863715	1.8568392911	0.0571705373	18.0000000000	1.0
976 A	0	1.4719422687	2.9438845375	1	2	1
976 B	0	0.9972054784	1.9944109568	1	2	1
976 C	1	1.8569859777	2.2379967092	1.2051769567	1.0000000000	1.0
978 A	0	1.0617344731	1.0617344731	1	1	1
978 B	0	2.7212716532	1.3606358266	1	1/2	1

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
978 C	0	0.1241533014	0.2483066027	1	2	1
978 D	0	0.7283045343	0.7283045343	1	1	1
978 E	1	4.1996664677	1.9268524917	0.2294054190	2.0000000000	1.0
978 F	1	2.5393784966	2.8120225559	0.0503348387	22.0000000000	1.0
978 G	1	1.6105329585	3.2537064683	0.0360761951	56.0000000000	1.0
978 H	0	3.1205326016	3.1205326016	1	1	1
979 A	0	2.3873792165	2.3873792165	1	1	1
979 B	1	0.4339769970	3.3551350704	2.5770452456	3.0000000000	1.0
980 A	0	1.9162261352	1.9162261352	1	1	1
980 B	0	0.5248079970	0.5248079970	1	1	1
980 C	1	1.7233973087	2.5976248754	0.7536349460	2.0000000000	1.0
980 D	1	1.0078659084	2.0479221272	0.1693282567	12.0000000000	1.0
980 E	0	0.6921975851	1.3843951703	1	2	1
980 F	0	0.1605835588	1.4452520290	1	9	1
980 G	0	1.7191073020	2.5786609530	1	3/2	1
980 H	0	3.1333036925	3.1333036925	1	1	1
980 I	0	0.4067815814	0.8135631628	1	2	1
981 A	1	1.5965483661	2.9392815103	0.9205112644	2.0000000000	1.0
981 B	1	3.4310796281	1.4755066121	0.2150207474	2.0000000000	1.0
982 A	1	3.2102819765	1.9893672757	0.3098430746	2.0000000000	1.0
984 A	0	1.0445702608	1.0445702608	1	1	1
984 B	0	0.4869816017	1.9479264069	1	4	1
984 C	1	1.2706245285	1.9979012463	0.2620628939	6.0000000000	1.0
984 D	1	2.2574282587	2.6460535196	0.1953589968	6.0000000000	1.0
985 A	0	0.7767573734	2.3302721201	1	3	1
985 B	1	3.1840958685	1.0139872562	0.0796134364	4.0000000000	1.0
986 A	0	1.4557074439	0.9704716293	1	2/3	1
986 B	1	0.2363375503	1.6316897708	1.7260162095	4.0000000000	1.0
986 C	1	1.9736513369	2.1668844307	1.0979063983	1.0000000000	1.0
986 D	1	2.2888169878	3.2671750193	0.1784314253	8.0000000000	1.0
986 E	1	0.4067622872	2.6151822165	0.0918466316	70.0000000000	1.0
986 F	1	2.6487806372	3.0414018081	0.1435283921	8.0000000000	1.0
987 A	0	2.1704014659	2.1704014659	1	1	1
987 B	0	1.7012497719	0.8506248859	1	1/2	1
987 C	0	2.9294495045	2.9294495045	1	1	1
987 D	0	0.6856950713	4.7998654990	1	7	1
987 E	1	0.6097264187	1.7241936701	0.1885210172	15.0000000000	1.0
988 A	0	1.0584031179	3.1752093536	1	3	1
988 B	1	0.2075922181	2.4883343573	0.3073498884	39.0000000000	1.0
988 C	1	1.9393544817	2.2502125023	0.1933815713	6.0000000000	1.0
988 D	0	1.0820810042	1.0820810042	1	1	1
989 A	0	4.0825015085	4.0825015085	1	1	1
990 A	1	3.6418359797	1.7927380911	0.2461310862	2.0000000000	1.0
990 B	0	1.4891045559	0.9927363706	1	2/3	1
990 C	0	0.5287307010	1.0574614021	1	2	1
990 D	0	1.1688028795	1.1688028795	1	1	1
990 E	1	0.9415811183	1.7973283849	1.9088407255	1.0000000000	1.0
990 F	0	1.2550458447	1.2550458447	1	1	1
990 G	0	0.1953396646	1.5627173170	1	8	1
990 H	1	0.8123963142	3.0762047385	0.6310968930	6.0000000000	1.0
990 I	0	1.4030386758	2.8060773515	1	2	1
990 J	1	1.7426021642	3.0722751578	0.4407596899	4.0000000000	1.0

Curve	r	Ω	$L^{(r)}(1)/r!$	R	$L^{(r)}(1)/r!\Omega R$	S
990K	0	1.3965264783	2.7930529567	1	2	1
990L	0	0.4490143433	3.1431004032	1	7	1
994A	1	3.3666422532	2.0603062005	0.3059882883	2.0000000000	1.0
994B	0	0.8538300680	0.8538300680	1	1	1
994C	0	0.5412944368	1.6238833103	1	3	1
994D	1	1.0902271710	2.1101578430	2.9032818559	0.6666666667	1.0
994E	0	4.6778604868	2.3389302434	1	1/2	1
994F	1	2.4540446205	3.0834509733	0.6282385715	2.0000000000	1.0
994G	1	0.9475612683	2.6207661260	1.8438674899	1.5000000000	1.0
995A	0	3.0420956800	1.5210478400	1	1/2	1
995B	1	3.7214583261	2.5126961150	2.0255737629	0.3333333333	1.0
996A	0	1.4835654563	2.2253481845	1	3/2	1
996B	1	0.5549429367	2.5867487904	0.1195202023	39.0000000000	1.0
996C	1	2.5646016409	2.5309557847	0.9868806696	1.0000000000	1.0
997A	1	4.9334464641	1.2116129751	0.2455915928	1.0000000000	1.0
997B	2	3.1532626948	1.8005737283	0.5710192593	1.0000000000	1.0
997C	2	4.8868977083	1.2202850721	0.2497054665	1.0000000000	1.0
999A	1	0.8267057148	2.9754511153	3.5991660178	1.0000000000	1.0
999B	1	4.9408397836	1.5561970558	0.3149661037	1.0000000000	1.0

TABLE 5

PARAMETRIZATION DEGREES

This table shows, for each newform f , the degree of the modular parametrization

$$\varphi: X_0(N) \rightarrow E_f.$$

The corresponding curve in each case is the “strong Weil” curve in its isogeny class, which is the first curve in its class¹. In each case the prime factorization of the degree is also shown.

¹except class 990H as before, where it is the curve 990H3.

curve	degree	curve	degree	curve	degree	curve	degree
11A1(B)	1	75B1(E)	$6 = 2 \cdot 3$	116C1(D)	$15 = 3 \cdot 5$	150A1(A)	$8 = 2^3$
14A1(C)	1	75C1(C)	$6 = 2 \cdot 3$	117A1(A)	$8 = 2^3$	150B1(G)	$40 = 2^3 \cdot 5$
15A1(C)	1	76A1(A)	$6 = 2 \cdot 3$	118A1(A)	$4 = 2^2$	150C1(I)	$48 = 2^4 \cdot 3$
17A1(C)	1	77A1(F)	$4 = 2^2$	118B1(B)	$12 = 2^2 \cdot 3$	152A1(A)	$8 = 2^3$
19A1(B)	1	77B1(D)	$20 = 2^2 \cdot 5$	118C1(D)	$6 = 2 \cdot 3$	152B1(B)	$8 = 2^3$
20A1(B)	1	77C1(A)	$6 = 2 \cdot 3$	118D1(E)	$38 = 2 \cdot 19$	153A1(C)	$8 = 2^3$
21A1(B)	1	78A1(A)	$40 = 2^3 \cdot 5$	120A1(E)	$8 = 2^3$	153B1(A)	$16 = 2^4$
24A1(B)	1	79A1(A)	$2 = 2$	120B1(A)	$8 = 2^3$	153C1(E)	$8 = 2^3$
26A1(B)	$2 = 2$	80A1(F)	$4 = 2^2$	121A1(H)	$6 = 2 \cdot 3$	153D1(D)	$24 = 2^3 \cdot 3$
26B1(D)	$2 = 2$	80B1(B)	$4 = 2^2$	121B1(D)	$4 = 2^2$	154A1(C)	$24 = 2^3 \cdot 3$
27A1(B)	1	82A1(A)	$4 = 2^2$	121C1(F)	$6 = 2 \cdot 3$	154B1(E)	$24 = 2^3 \cdot 3$
30A1(A)	$2 = 2$	83A1(A)	$2 = 2$	121D1(A)	$24 = 2^3 \cdot 3$	154C1(A)	$16 = 2^4$
32A1(B)	1	84A1(C)	$6 = 2 \cdot 3$	122A1(A)	$8 = 2^3$	155A1(D)	$20 = 2^2 \cdot 5$
33A1(B)	$3 = 3$	84B1(A)	$6 = 2 \cdot 3$	123A1(A)	$20 = 2^2 \cdot 5$	155B1(A)	$8 = 2^3$
34A1(A)	$2 = 2$	85A1(A)	$4 = 2^2$	123B1(C)	$4 = 2^2$	155C1(C)	$4 = 2^2$
35A1(B)	$2 = 2$	88A1(A)	$8 = 2^3$	124A1(B)	$6 = 2 \cdot 3$	156A1(E)	$12 = 2^2 \cdot 3$
36A1(A)	1	89A1(C)	$2 = 2$	124B1(A)	$6 = 2 \cdot 3$	156B1(A)	$12 = 2^2 \cdot 3$
37A1(A)	$2 = 2$	89B1(A)	$5 = 5$	126A1(A)	$8 = 2^3$	158A1(E)	$32 = 2^5$
37B1(C)	$2 = 2$	90A1(M)	$8 = 2^3$	126B1(G)	$32 = 2^5$	158B1(D)	$8 = 2^3$
38A1(D)	$6 = 2 \cdot 3$	90B1(A)	$8 = 2^3$	128A1(C)	$4 = 2^2$	158C1(H)	$48 = 2^4 \cdot 3$
38B1(A)	$2 = 2$	90C1(E)	$16 = 2^4$	128B1(F)	$8 = 2^3$	158D1(B)	$40 = 2^3 \cdot 5$
39A1(B)	$2 = 2$	91A1(A)	$4 = 2^2$	128C1(A)	$4 = 2^2$	158E1(F)	$6 = 2 \cdot 3$
40A1(B)	$2 = 2$	91B1(B)	$4 = 2^2$	128D1(G)	$8 = 2^3$	160A1(D)	$8 = 2^3$
42A1(A)	$4 = 2^2$	92A1(A)	$2 = 2$	129A1(E)	$8 = 2^3$	160B1(A)	$8 = 2^3$
43A1(A)	$2 = 2$	92B1(C)	$6 = 2 \cdot 3$	129B1(B)	$15 = 3 \cdot 5$	161A1(B)	$10 = 2 \cdot 5$
44A1(A)	$2 = 2$	94A1(A)	$2 = 2$	130A1(E)	$24 = 2^3 \cdot 3$	162A1(K)	$12 = 2^2 \cdot 3$
45A1(A)	$2 = 2$	96A1(E)	$4 = 2^2$	130B1(A)	$8 = 2^3$	162B1(G)	$6 = 2 \cdot 3$
46A1(A)	$5 = 5$	96B1(A)	$4 = 2^2$	130C1(J)	$80 = 2^4 \cdot 5$	162C1(A)	$6 = 2 \cdot 3$
48A1(B)	$2 = 2$	98A1(B)	$16 = 2^4$	131A1(A)	$2 = 2$	162D1(E)	$12 = 2^2 \cdot 3$
49A1(A)	1	99A1(A)	$4 = 2^2$	132A1(A)	$6 = 2 \cdot 3$	163A1(A)	$6 = 2 \cdot 3$
50A1(E)	$2 = 2$	99B1(H)	$12 = 2^2 \cdot 3$	132B1(C)	$30 = 2 \cdot 3 \cdot 5$	166A1(A)	$8 = 2^3$
50B1(A)	$2 = 2$	99C1(F)	$12 = 2^2 \cdot 3$	135A1(A)	$12 = 2^2 \cdot 3$	168A1(B)	$8 = 2^3$
51A1(A)	$2 = 2$	99D1(C)	$6 = 2 \cdot 3$	135B1(B)	$36 = 2^2 \cdot 3^2$	168B1(E)	$24 = 2^3 \cdot 3$
52A1(B)	$3 = 3$	100A1(A)	$12 = 2^2 \cdot 3$	136A1(A)	$8 = 2^3$	170A1(A)	$16 = 2^4$
53A1(A)	$2 = 2$	101A1(A)	$2 = 2$	136B1(C)	$8 = 2^3$	170B1(H)	$160 = 2^5 \cdot 5$
54A1(E)	$6 = 2 \cdot 3$	102A1(E)	$8 = 2^3$	138A1(E)	$8 = 2^3$	170C1(F)	$84 = 2^2 \cdot 3 \cdot 7$
54B1(A)	$2 = 2$	102B1(G)	$16 = 2^4$	138B1(G)	$16 = 2^4$	170D1(D)	$12 = 2^2 \cdot 3$
55A1(B)	$2 = 2$	102C1(A)	$24 = 2^3 \cdot 3$	138C1(A)	$8 = 2^3$	170E1(C)	$20 = 2^2 \cdot 5$
56A1(C)	$2 = 2$	104A1(A)	$8 = 2^3$	139A1(A)	$6 = 2 \cdot 3$	171A1(D)	$12 = 2^2 \cdot 3$
56B1(A)	$4 = 2^2$	105A1(A)	$4 = 2^2$	140A1(A)	$12 = 2^2 \cdot 3$	171B1(A)	$8 = 2^3$
57A1(E)	$4 = 2^2$	106A1(B)	$6 = 2 \cdot 3$	140B1(C)	$60 = 2^2 \cdot 3 \cdot 5$	171C1(I)	$96 = 2^5 \cdot 3$
57B1(B)	$3 = 3$	106B1(A)	$8 = 2^3$	141A1(E)	$28 = 2^2 \cdot 7$	171D1(H)	$32 = 2^5$
57C1(F)	$12 = 2^2 \cdot 3$	106C1(E)	$48 = 2^4 \cdot 3$	141B1(G)	$12 = 2^2 \cdot 3$	172A1(A)	$12 = 2^2 \cdot 3$
58A1(A)	$4 = 2^2$	106D1(D)	$10 = 2 \cdot 5$	141C1(A)	$6 = 2 \cdot 3$	174A1(I)	$1540 = 2^2 \cdot 5 \cdot 7 \cdot 11$
58B1(B)	$4 = 2^2$	108A1(A)	$6 = 2 \cdot 3$	141D1(I)	$4 = 2^2$	174B1(G)	$28 = 2^2 \cdot 7$
61A1(A)	$2 = 2$	109A1(A)	$4 = 2^2$	141E1(H)	$12 = 2^2 \cdot 3$	174C1(F)	$12 = 2^2 \cdot 3$
62A1(A)	$2 = 2$	110A1(C)	$20 = 2^2 \cdot 5$	142A1(F)	$36 = 2^2 \cdot 3^2$	174D1(A)	$10 = 2 \cdot 5$
63A1(A)	$4 = 2^2$	110B1(A)	$4 = 2^2$	142B1(E)	$4 = 2^2$	174E1(E)	$52 = 2^2 \cdot 13$
64A1(B)	$2 = 2$	110C1(E)	$28 = 2^2 \cdot 7$	142C1(A)	$9 = 3^2$	175A1(B)	$8 = 2^3$
65A1(A)	$2 = 2$	112A1(K)	$8 = 2^3$	142D1(C)	$4 = 2^2$	175B1(C)	$16 = 2^4$
66A1(A)	$4 = 2^2$	112B1(A)	$4 = 2^2$	142E1(G)	$324 = 2^2 \cdot 3^4$	175C1(F)	$40 = 2^3 \cdot 5$
66B1(E)	$4 = 2^2$	112C1(E)	$8 = 2^3$	143A1(A)	$4 = 2^2$	176A1(C)	$16 = 2^4$
66C1(I)	$20 = 2^2 \cdot 5$	113A1(B)	$6 = 2 \cdot 3$	144A1(A)	$4 = 2^2$	176B1(D)	$8 = 2^3$
67A1(A)	$5 = 5$	114A1(A)	$12 = 2^2 \cdot 3$	144B1(E)	$8 = 2^3$	176C1(A)	$8 = 2^3$
69A1(A)	$2 = 2$	114B1(E)	$20 = 2^2 \cdot 5$	145A1(A)	$4 = 2^2$	178A1(A)	$32 = 2^5$
70A1(A)	$4 = 2^2$	114C1(G)	$60 = 2^2 \cdot 3 \cdot 5$	147A1(C)	$24 = 2^3 \cdot 3$	178B1(C)	$28 = 2^2 \cdot 7$
72A1(A)	$4 = 2^2$	115A1(A)	$10 = 2 \cdot 5$	147B1(I)	$42 = 2 \cdot 3 \cdot 7$	179A1(A)	$9 = 3^2$
73A1(B)	$3 = 3$	116A1(E)	$120 = 2^3 \cdot 3 \cdot 5$	147C1(A)	$6 = 2 \cdot 3$	180A1(A)	$12 = 2^2 \cdot 3$
75A1(A)	$6 = 2 \cdot 3$	116B1(A)	$8 = 2^3$	148A1(A)	$12 = 2^2 \cdot 3$	182A1(E)	$180 = 2^2 \cdot 3^2 \cdot 5$

curve	degree	curve	degree	curve	degree	curve	degree
182B1(A)	$12 = 2^2 \cdot 3$	208A1	$16 = 2^4$	236B1	$14 = 2 \cdot 7$	269A1	$6 = 2 \cdot 3$
182C1(J)	$308 = 2^2 \cdot 7 \cdot 11$	208B1	$16 = 2^4$	238A1	$112 = 2^4 \cdot 7$	270A1	$36 = 2^2 \cdot 3^2$
182D1(D)	$36 = 2^2 \cdot 3^2$	208C1	$12 = 2^2 \cdot 3$	238B1	$8 = 2^3$	270B1	$60 = 2^2 \cdot 3 \cdot 5$
182E1(I)	$140 = 2^2 \cdot 5 \cdot 7$	208D1	$48 = 2^4 \cdot 3$	238C1	$16 = 2^4$	270C1	$12 = 2^2 \cdot 3$
184A1(C)	$8 = 2^3$	209A1	$24 = 2^3 \cdot 3$	238D1	$16 = 2^4$	270D1	$60 = 2^2 \cdot 3 \cdot 5$
184B1(B)	$8 = 2^3$	210A1	$48 = 2^4 \cdot 3$	238E1	$80 = 2^4 \cdot 5$	272A1	$16 = 2^4$
184C1(D)	$12 = 2^2 \cdot 3$	210B1	$96 = 2^5 \cdot 3$	240A1	$16 = 2^4$	272B1	$16 = 2^4$
184D1(A)	$24 = 2^3 \cdot 3$	210C1	$32 = 2^5$	240B1	$48 = 2^4 \cdot 3$	272C1	$16 = 2^4$
185A1(D)	$48 = 2^4 \cdot 3$	210D1	$16 = 2^4$	240C1	$16 = 2^4$	272D1	$48 = 2^4 \cdot 3$
185B1(A)	$8 = 2^3$	210E1	$128 = 2^7$	240D1	$16 = 2^4$	273A1	$48 = 2^4 \cdot 3$
185C1(B)	$6 = 2 \cdot 3$	212A1	$12 = 2^2 \cdot 3$	242A1	$16 = 2^4$	273B1	$672 = 2^5 \cdot 3 \cdot 7$
186A1(D)	$44 = 2^2 \cdot 11$	212B1	$21 = 3 \cdot 7$	242B1	$176 = 2^4 \cdot 11$	274A1	$28 = 2^2 \cdot 7$
186B1(B)	$20 = 2^2 \cdot 5$	213A1	$6 = 2 \cdot 3$	243A1	$6 = 2 \cdot 3$	274B1	$132 = 2^2 \cdot 3 \cdot 11$
186C1(A)	$28 = 2^2 \cdot 7$	214A1	$28 = 2^2 \cdot 7$	243B1	$9 = 3^2$	274C1	$12 = 2^2 \cdot 3$
187A1(A)	$16 = 2^4$	214B1	$12 = 2^2 \cdot 3$	244A1	$12 = 2^2 \cdot 3$	275A1	$24 = 2^3 \cdot 3$
187B1(C)	$30 = 2 \cdot 3 \cdot 5$	214C1	$60 = 2^2 \cdot 3 \cdot 5$	245A1	$48 = 2^4 \cdot 3$	275B1	$28 = 2^2 \cdot 7$
189A1(A)	$12 = 2^2 \cdot 3$	214D1	$12 = 2^2 \cdot 3$	245B1	$336 = 2^4 \cdot 3 \cdot 7$	277A1	$10 = 2 \cdot 5$
189B1(C)	$12 = 2^2 \cdot 3$	215A1	$8 = 2^3$	245C1	$32 = 2^5$	278A1	$32 = 2^5$
189C1(F)	$12 = 2^2 \cdot 3$	216A1	$24 = 2^3 \cdot 3$	246A1	$84 = 2^2 \cdot 3 \cdot 7$	278B1	$272 = 2^4 \cdot 17$
189D1(B)	$36 = 2^2 \cdot 3^2$	216B1	$24 = 2^3 \cdot 3$	246B1	$300 = 2^2 \cdot 3 \cdot 5^2$	280A1	$16 = 2^4$
190A1(D)	$88 = 2^3 \cdot 11$	216C1	$72 = 2^3 \cdot 3^2$	246C1	$1680 = 2^4 \cdot 3 \cdot 5 \cdot 7$	280B1	$240 = 2^4 \cdot 3 \cdot 5$
190B1(C)	$8 = 2^3$	216D1	$72 = 2^3 \cdot 3^2$	246D1	$48 = 2^4 \cdot 3$	282A1	$48 = 2^4 \cdot 3$
190C1(A)	$24 = 2^3 \cdot 3$	218A1	$24 = 2^3 \cdot 3$	246E1	$24 = 2^3 \cdot 3$	282B1	$64 = 2^6$
192A1(Q)	$8 = 2^3$	219A1	$12 = 2^2 \cdot 3$	246F1	$20 = 2^2 \cdot 5$	285A1	$40 = 2^3 \cdot 5$
192B1(A)	$8 = 2^3$	219B1	$12 = 2^2 \cdot 3$	246G1	$44 = 2^2 \cdot 11$	285B1	$24 = 2^3 \cdot 3$
192C1(K)	$8 = 2^3$	219C1	$60 = 2^2 \cdot 3 \cdot 5$	248A1	$8 = 2^3$	285C1	$72 = 2^3 \cdot 3^2$
192D1(E)	$8 = 2^3$	220A1	$36 = 2^2 \cdot 3^2$	248B1	$16 = 2^4$	286A1	$60 = 2^2 \cdot 3 \cdot 5$
194A1(A)	$14 = 2 \cdot 7$	220B1	$12 = 2^2 \cdot 3$	248C1	$8 = 2^3$	286B1	$104 = 2^3 \cdot 13$
195A1(A)	$24 = 2^3 \cdot 3$	221A1	$120 = 2^3 \cdot 3 \cdot 5$	249A1	$24 = 2^3 \cdot 3$	286C1	$24 = 2^3 \cdot 3$
195B1(I)	$12 = 2^2 \cdot 3$	221B1	$24 = 2^3 \cdot 3$	249B1	$8 = 2^3$	286D1	$120 = 2^3 \cdot 3 \cdot 5$
195C1(K)	$84 = 2^2 \cdot 3 \cdot 7$	222A1	$12 = 2^2 \cdot 3$	252A1	$48 = 2^4 \cdot 3$	286E1	$60 = 2^2 \cdot 3 \cdot 5$
195D1(J)	$84 = 2^2 \cdot 3 \cdot 7$	222B1	$44 = 2^2 \cdot 11$	252B1	$48 = 2^4 \cdot 3$	286F1	$12 = 2^2 \cdot 3$
196A1(A)	$6 = 2 \cdot 3$	222C1	$36 = 2^2 \cdot 3^2$	254A1	$36 = 2^2 \cdot 3^2$	288A1	$16 = 2^4$
196B1(C)	$42 = 2 \cdot 3 \cdot 7$	222D1	$52 = 2^2 \cdot 13$	254B1	$16 = 2^4$	288B1	$32 = 2^5$
197A1(A)	$10 = 2 \cdot 5$	222E1	$2484 = 2^2 \cdot 3^3 \cdot 23$	254C1	$12 = 2^2 \cdot 3$	288C1	$32 = 2^5$
198A1(I)	$32 = 2^5$	224A1	$8 = 2^3$	254D1	$24 = 2^3 \cdot 3$	288D1	$16 = 2^4$
198B1(E)	$32 = 2^5$	224B1	$8 = 2^3$	256A1	$8 = 2^3$	288E1	$48 = 2^4 \cdot 3$
198C1(M)	$32 = 2^5$	225A1	$8 = 2^3$	256B1	$8 = 2^3$	289A1	$72 = 2^3 \cdot 3^2$
198D1(A)	$32 = 2^5$	225B1	$40 = 2^3 \cdot 5$	256C1	$8 = 2^3$	290A1	$48 = 2^4 \cdot 3$
198E1(Q)	$160 = 2^5 \cdot 5$	225C1	$48 = 2^4 \cdot 3$	256D1	$8 = 2^3$	291A1	$1012 = 2^2 \cdot 11 \cdot 23$
200A1(B)	$120 = 2^3 \cdot 3 \cdot 5$	225D1	$48 = 2^4 \cdot 3$	258A1	$24 = 2^3 \cdot 3$	291B1	$120 = 2^3 \cdot 3 \cdot 5$
200B1(C)	$8 = 2^3$	225E1	$48 = 2^4 \cdot 3$	258B1	$196 = 2^2 \cdot 7^2$	291C1	$12 = 2^2 \cdot 3$
200C1(G)	$24 = 2^3 \cdot 3$	226A1	$24 = 2^3 \cdot 3$	258C1	$40 = 2^3 \cdot 5$	291D1	$12 = 2^2 \cdot 3$
200D1(E)	$40 = 2^3 \cdot 5$	228A1	$18 = 2 \cdot 3^2$	258D1	$60 = 2^2 \cdot 3 \cdot 5$	294A1	$84 = 2^2 \cdot 3 \cdot 7$
200E1(A)	$24 = 2^3 \cdot 3$	228B1	$24 = 2^3 \cdot 3$	258E1	$760 = 2^3 \cdot 5 \cdot 19$	294B1	$12 = 2^2 \cdot 3$
201A1	$12 = 2^2 \cdot 3$	229A1	$8 = 2^3$	258F1	$168 = 2^3 \cdot 3 \cdot 7$	294C1	$192 = 2^6 \cdot 3$
201B1	$12 = 2^2 \cdot 3$	231A1	$20 = 2^2 \cdot 5$	258G1	$12 = 2^2 \cdot 3$	294D1	$60 = 2^2 \cdot 3 \cdot 5$
201C1	$60 = 2^2 \cdot 3 \cdot 5$	232A1	$16 = 2^4$	259A1	$36 = 2^2 \cdot 3^2$	294E1	$420 = 2^2 \cdot 3 \cdot 5 \cdot 7$
202A1	$34 = 2 \cdot 17$	232B1	$16 = 2^4$	260A1	$48 = 2^4 \cdot 3$	294F1	$448 = 2^6 \cdot 7$
203A1	$48 = 2^4 \cdot 3$	233A1	$27 = 3^3$	262A1	$44 = 2^2 \cdot 11$	294G1	$64 = 2^6$
203B1	$8 = 2^3$	234A1	$28 = 2^2 \cdot 7$	262B1	$12 = 2^2 \cdot 3$	296A1	$16 = 2^4$
203C1	$12 = 2^2 \cdot 3$	234B1	$48 = 2^4 \cdot 3$	264A1	$16 = 2^4$	296B1	$16 = 2^4$
204A1	$132 = 2^2 \cdot 3 \cdot 11$	234C1	$16 = 2^4$	264B1	$16 = 2^4$	297A1	$72 = 2^3 \cdot 3^2$
204B1	$12 = 2^2 \cdot 3$	234D1	$320 = 2^6 \cdot 5$	264C1	$24 = 2^3 \cdot 3$	297B1	$12 = 2^2 \cdot 3$
205A1	$12 = 2^2 \cdot 3$	234E1	$20 = 2^2 \cdot 5$	264D1	$336 = 2^4 \cdot 3 \cdot 7$	297C1	$36 = 2^2 \cdot 3^2$
205B1	$16 = 2^4$	235A1	$12 = 2^2 \cdot 3$	265A1	$30 = 2 \cdot 3 \cdot 5$	297D1	$24 = 2^3 \cdot 3$
205C1	$8 = 2^3$	235B1	$108 = 2^2 \cdot 3^3$	267A1	$10 = 2 \cdot 5$	298A1	$36 = 2^2 \cdot 3^2$
206A1	$15 = 3 \cdot 5$	235C1	$18 = 2 \cdot 3^2$	267B1	$238 = 2 \cdot 7 \cdot 17$	298B1	$20 = 2^2 \cdot 5$
207A1	$16 = 2^4$	236A1	$6 = 2 \cdot 3$	268A1	$18 = 2 \cdot 3^2$	300A1	$36 = 2^2 \cdot 3^2$

curve	degree	curve	degree	curve	degree	curve	degree
300B1	$180 = 2^2 \cdot 3^2 \cdot 5$	325B1	$12 = 2^2 \cdot 3$	348D1	$84 = 2^2 \cdot 3 \cdot 7$	369B1	$160 = 2^5 \cdot 5$
300C1	$120 = 2^3 \cdot 3 \cdot 5$	325C1	$48 = 2^4 \cdot 3$	350A1	$96 = 2^5 \cdot 3$	370A1	$16 = 2^4$
300D1	$24 = 2^3 \cdot 3$	325D1	$84 = 2^2 \cdot 3 \cdot 7$	350B1	$120 = 2^3 \cdot 3 \cdot 5$	370B1	$44 = 2^2 \cdot 11$
302A1	$120 = 2^3 \cdot 3 \cdot 5$	325E1	$84 = 2^2 \cdot 3 \cdot 7$	350C1	$24 = 2^3 \cdot 3$	370C1	$108 = 2^2 \cdot 3^3$
302B1	$27 = 3^3$	326A1	$36 = 2^2 \cdot 3^2$	350D1	$48 = 2^4 \cdot 3$	370D1	$96 = 2^5 \cdot 3$
302C1	$40 = 2^3 \cdot 5$	326B1	$20 = 2^2 \cdot 5$	350E1	$1320 = 2^3 \cdot 3 \cdot 5 \cdot 11$	371A1	$32 = 2^5$
303A1	$112 = 2^4 \cdot 7$	326C1	$204 = 2^2 \cdot 3 \cdot 17$	350F1	$264 = 2^3 \cdot 3 \cdot 11$	371B1	$60 = 2^2 \cdot 3 \cdot 5$
303B1	$32 = 2^5$	327A1	$16 = 2^4$	352A1	$32 = 2^5$	372A1	$24 = 2^3 \cdot 3$
304A1	$48 = 2^4 \cdot 3$	328A1	$16 = 2^4$	352B1	$32 = 2^5$	372B1	$30 = 2 \cdot 3 \cdot 5$
304B1	$48 = 2^4 \cdot 3$	328B1	$24 = 2^3 \cdot 3$	352C1	$32 = 2^5$	372C1	$360 = 2^3 \cdot 3^2 \cdot 5$
304C1	$16 = 2^4$	329A1	$180 = 2^2 \cdot 3^2 \cdot 5$	352D1	$32 = 2^5$	372D1	$48 = 2^4 \cdot 3$
304D1	$16 = 2^4$	330A1	$160 = 2^5 \cdot 5$	352E1	$96 = 2^5 \cdot 3$	373A1	$22 = 2 \cdot 11$
304E1	$24 = 2^3 \cdot 3$	330B1	$32 = 2^5$	352F1	$96 = 2^5 \cdot 3$	374A1	$40 = 2^3 \cdot 5$
304F1	$24 = 2^3 \cdot 3$	330C1	$192 = 2^6 \cdot 3$	353A1	$24 = 2^3 \cdot 3$	377A1	$14 = 2 \cdot 7$
306A1	$192 = 2^6 \cdot 3$	330D1	$2240 = 2^6 \cdot 5 \cdot 7$	354A1	$16 = 2^4$	378A1	$36 = 2^2 \cdot 3^2$
306B1	$48 = 2^4 \cdot 3$	330E1	$64 = 2^6$	354B1	$40 = 2^3 \cdot 5$	378B1	$36 = 2^2 \cdot 3^2$
306C1	$128 = 2^7$	331A1	$12 = 2^2 \cdot 3$	354C1	$120 = 2^3 \cdot 3 \cdot 5$	378C1	$72 = 2^3 \cdot 3^2$
306D1	$64 = 2^6$	333A1	$20 = 2^2 \cdot 5$	354D1	$48 = 2^4 \cdot 3$	378D1	$24 = 2^3 \cdot 3$
307A1	$13 = 13$	333B1	$48 = 2^4 \cdot 3$	354E1	$1584 = 2^4 \cdot 3^2 \cdot 11$	378E1	$72 = 2^3 \cdot 3^2$
307B1	$10 = 2 \cdot 5$	333C1	$16 = 2^4$	354F1	$56 = 2^3 \cdot 7$	378F1	$72 = 2^3 \cdot 3^2$
307C1	$11 = 11$	333D1	$28 = 2^2 \cdot 7$	355A1	$16 = 2^4$	378G1	$1260 = 2^2 \cdot 3^2 \cdot 5 \cdot 7$
307D1	$15 = 3 \cdot 5$	334A1	$8 = 2^3$	356A1	$12 = 2^2 \cdot 3$	378H1	$420 = 2^2 \cdot 3 \cdot 5 \cdot 7$
308A1	$24 = 2^3 \cdot 3$	335A1	$8 = 2^3$	357A1	$544 = 2^5 \cdot 17$	380A1	$24 = 2^3 \cdot 3$
309A1	$20 = 2^2 \cdot 5$	336A1	$24 = 2^3 \cdot 3$	357B1	$32 = 2^5$	380B1	$240 = 2^4 \cdot 3 \cdot 5$
310A1	$48 = 2^4 \cdot 3$	336B1	$16 = 2^4$	357C1	$48 = 2^4 \cdot 3$	381A1	$20 = 2^2 \cdot 5$
310B1	$96 = 2^5 \cdot 3$	336C1	$48 = 2^4 \cdot 3$	357D1	$112 = 2^4 \cdot 7$	381B1	$44 = 2^2 \cdot 11$
312A1	$16 = 2^4$	336D1	$96 = 2^5 \cdot 3$	358A1	$102 = 2 \cdot 3 \cdot 17$	384A1	$16 = 2^4$
312B1	$16 = 2^4$	336E1	$32 = 2^5$	358B1	$22 = 2 \cdot 11$	384B1	$16 = 2^4$
312C1	$16 = 2^4$	336F1	$24 = 2^3 \cdot 3$	359A1	$16 = 2^4$	384C1	$16 = 2^4$
312D1	$32 = 2^5$	338A1	$12 = 2^2 \cdot 3$	359B1	$8 = 2^3$	384D1	$16 = 2^4$
312E1	$240 = 2^4 \cdot 3 \cdot 5$	338B1	$156 = 2^2 \cdot 3 \cdot 13$	360A1	$64 = 2^6$	384E1	$48 = 2^4 \cdot 3$
312F1	$48 = 2^4 \cdot 3$	338C1	$112 = 2^4 \cdot 7$	360B1	$32 = 2^5$	384F1	$48 = 2^4 \cdot 3$
314A1	$20 = 2^2 \cdot 5$	338D1	$312 = 2^3 \cdot 3 \cdot 13$	360C1	$96 = 2^5 \cdot 3$	384G1	$48 = 2^4 \cdot 3$
315A1	$20 = 2^2 \cdot 5$	338E1	$24 = 2^3 \cdot 3$	360D1	$64 = 2^6$	384H1	$48 = 2^4 \cdot 3$
315B1	$32 = 2^5$	338F1	$336 = 2^4 \cdot 3 \cdot 7$	360E1	$32 = 2^5$	385A1	$64 = 2^6$
316A1	$36 = 2^2 \cdot 3^2$	339A1	$72 = 2^3 \cdot 3^2$	361A1	$20 = 2^2 \cdot 5$	385B1	$32 = 2^5$
316B1	$36 = 2^2 \cdot 3^2$	339B1	$126 = 2 \cdot 3^2 \cdot 7$	361B1	$120 = 2^3 \cdot 3 \cdot 5$	387A1	$64 = 2^6$
318A1	$20 = 2^2 \cdot 5$	339C1	$24 = 2^3 \cdot 3$	362A1	$20 = 2^2 \cdot 5$	387B1	$48 = 2^4 \cdot 3$
318B1	$60 = 2^2 \cdot 3 \cdot 5$	340A1	$24 = 2^3 \cdot 3$	362B1	$28 = 2^2 \cdot 7$	387C1	$16 = 2^4$
318C1	$24 = 2^3 \cdot 3$	342A1	$60 = 2^2 \cdot 3 \cdot 5$	363A1	$180 = 2^2 \cdot 3^2 \cdot 5$	387D1	$60 = 2^2 \cdot 3 \cdot 5$
318D1	$88 = 2^3 \cdot 11$	342B1	$160 = 2^5 \cdot 5$	363B1	$36 = 2^2 \cdot 3^2$	387E1	$48 = 2^4 \cdot 3$
318E1	$204 = 2^2 \cdot 3 \cdot 17$	342C1	$96 = 2^5 \cdot 3$	363C1	$396 = 2^2 \cdot 3^2 \cdot 11$	389A1	$40 = 2^3 \cdot 5$
319A1	$92 = 2^2 \cdot 23$	342D1	$48 = 2^4 \cdot 3$	364A1	$120 = 2^3 \cdot 3 \cdot 5$	390A1	$32 = 2^5$
320A1	$16 = 2^4$	342E1	$16 = 2^4$	364B1	$24 = 2^3 \cdot 3$	390B1	$64 = 2^6$
320B1	$16 = 2^4$	342F1	$480 = 2^5 \cdot 3 \cdot 5$	366A1	$48 = 2^4 \cdot 3$	390C1	$48 = 2^4 \cdot 3$
320C1	$16 = 2^4$	342G1	$60 = 2^2 \cdot 3 \cdot 5$	366B1	$60 = 2^2 \cdot 3 \cdot 5$	390D1	$720 = 2^4 \cdot 3^2 \cdot 5$
320D1	$16 = 2^4$	344A1	$16 = 2^4$	366C1	$228 = 2^2 \cdot 3 \cdot 19$	390E1	$48 = 2^4 \cdot 3$
320E1	$16 = 2^4$	345A1	$80 = 2^4 \cdot 5$	366D1	$364 = 2^2 \cdot 7 \cdot 13$	390F1	$80 = 2^4 \cdot 5$
320F1	$16 = 2^4$	345B1	$16 = 2^4$	366E1	$52 = 2^2 \cdot 13$	390G1	$320 = 2^6 \cdot 5$
322A1	$48 = 2^4 \cdot 3$	345C1	$300 = 2^2 \cdot 3 \cdot 5^2$	366F1	$48 = 2^4 \cdot 3$	392A1	$96 = 2^5 \cdot 3$
322B1	$112 = 2^4 \cdot 7$	345D1	$40 = 2^3 \cdot 5$	366G1	$80 = 2^4 \cdot 5$	392B1	$168 = 2^3 \cdot 3 \cdot 7$
322C1	$24 = 2^3 \cdot 3$	345E1	$96 = 2^5 \cdot 3$	368A1	$24 = 2^3 \cdot 3$	392C1	$24 = 2^3 \cdot 3$
322D1	$40 = 2^3 \cdot 5$	345F1	$192 = 2^6 \cdot 3$	368B1	$120 = 2^3 \cdot 3 \cdot 5$	392D1	$192 = 2^6 \cdot 3$
323A1	$140 = 2^2 \cdot 5 \cdot 7$	346A1	$28 = 2^2 \cdot 7$	368C1	$16 = 2^4$	392E1	$168 = 2^3 \cdot 3 \cdot 7$
324A1	$18 = 2 \cdot 3^2$	346B1	$28 = 2^2 \cdot 7$	368D1	$16 = 2^4$	392F1	$24 = 2^3 \cdot 3$
324B1	$36 = 2^2 \cdot 3^2$	347A1	$14 = 2 \cdot 7$	368E1	$8 = 2^3$	395A1	$36 = 2^2 \cdot 3^2$
324C1	$18 = 2 \cdot 3^2$	348A1	$12 = 2^2 \cdot 3$	368F1	$24 = 2^3 \cdot 3$	395B1	$72 = 2^3 \cdot 3^2$
324D1	$36 = 2^2 \cdot 3^2$	348B1	$12 = 2^2 \cdot 3$	368G1	$48 = 2^4 \cdot 3$	395C1	$68 = 2^2 \cdot 17$
325A1	$60 = 2^2 \cdot 3 \cdot 5$	348C1	$180 = 2^2 \cdot 3^2 \cdot 5$	369A1	$32 = 2^5$	396A1	$240 = 2^4 \cdot 3 \cdot 5$

curve	degree	curve	degree	curve	degree	curve	degree
396B1	$48 = 2^4 \cdot 3$	423F1	$96 = 2^5 \cdot 3$	442A1	$96 = 2^5 \cdot 3$	464C1	$96 = 2^5 \cdot 3$
396C1	$60 = 2^2 \cdot 3 \cdot 5$	423G1	$32 = 2^5$	442B1	$192 = 2^6 \cdot 3$	464D1	$32 = 2^5$
398A1	$55 = 5 \cdot 11$	425A1	$32 = 2^5$	442C1	$64 = 2^6$	464E1	$60 = 2^2 \cdot 3 \cdot 5$
399A1	$120 = 2^3 \cdot 3 \cdot 5$	425B1	$60 = 2^2 \cdot 3 \cdot 5$	442D1	$48 = 2^4 \cdot 3$	464F1	$480 = 2^5 \cdot 3 \cdot 5$
399B1	$24 = 2^3 \cdot 3$	425C1	$12 = 2^2 \cdot 3$	442E1	$10560 = 2^6 \cdot 3 \cdot 5 \cdot 11$	464G1	$96 = 2^5 \cdot 3$
399C1	$168 = 2^3 \cdot 3 \cdot 7$	425D1	$96 = 2^5 \cdot 3$	443A1	$12 = 2^2 \cdot 3$	465A1	$48 = 2^4 \cdot 3$
400A1	$48 = 2^4 \cdot 3$	426A1	$80 = 2^4 \cdot 5$	443B1	$14 = 2 \cdot 7$	465B1	$32 = 2^5$
400B1	$48 = 2^4 \cdot 3$	426B1	$120 = 2^3 \cdot 3 \cdot 5$	443C1	$62 = 2 \cdot 31$	466A1	$20 = 2^2 \cdot 5$
400C1	$48 = 2^4 \cdot 3$	426C1	$1440 = 2^5 \cdot 3^2 \cdot 5$	444A1	$24 = 2^3 \cdot 3$	466B1	$32 = 2^5$
400D1	$16 = 2^4$	427A1	$22 = 2 \cdot 11$	444B1	$48 = 2^4 \cdot 3$	467A1	$34 = 2 \cdot 17$
400E1	$48 = 2^4 \cdot 3$	427B1	$28 = 2^2 \cdot 7$	446A1	$24 = 2^3 \cdot 3$	468A1	$192 = 2^6 \cdot 3$
400F1	$80 = 2^4 \cdot 5$	427C1	$36 = 2^2 \cdot 3^2$	446B1	$56 = 2^3 \cdot 7$	468B1	$576 = 2^6 \cdot 3^2$
400G1	$240 = 2^4 \cdot 3 \cdot 5$	428A1	$54 = 2 \cdot 3^3$	446C1	$36 = 2^2 \cdot 3^2$	468C1	$48 = 2^4 \cdot 3$
400H1	$48 = 2^4 \cdot 3$	428B1	$18 = 2 \cdot 3^2$	446D1	$88 = 2^3 \cdot 11$	468D1	$96 = 2^5 \cdot 3$
402A1	$32 = 2^5$	429A1	$16 = 2^4$	448A1	$32 = 2^5$	468E1	$96 = 2^5 \cdot 3$
402B1	$32 = 2^5$	429B1	$64 = 2^6$	448B1	$32 = 2^5$	469A1	$80 = 2^4 \cdot 5$
402C1	$48 = 2^4 \cdot 3$	430A1	$24 = 2^3 \cdot 3$	448C1	$64 = 2^6$	469B1	$32 = 2^5$
402D1	$144 = 2^4 \cdot 3^2$	430B1	$40 = 2^3 \cdot 5$	448D1	$32 = 2^5$	470A1	$64 = 2^6$
404A1	$12 = 2^2 \cdot 3$	430C1	$72 = 2^3 \cdot 3^2$	448E1	$64 = 2^6$	470B1	$320 = 2^6 \cdot 5$
404B1	$68 = 2^2 \cdot 17$	430D1	$600 = 2^3 \cdot 3 \cdot 5^2$	448F1	$64 = 2^6$	470C1	$112 = 2^4 \cdot 7$
405A1	$24 = 2^3 \cdot 3$	431A1	$10 = 2 \cdot 5$	448G1	$32 = 2^5$	470D1	$80 = 2^4 \cdot 5$
405B1	$24 = 2^3 \cdot 3$	431B1	$40 = 2^3 \cdot 5$	448H1	$64 = 2^6$	470E1	$32 = 2^5$
405C1	$12 = 2^2 \cdot 3$	432A1	$24 = 2^3 \cdot 3$	450A1	$320 = 2^6 \cdot 5$	470F1	$336 = 2^4 \cdot 3 \cdot 7$
405D1	$36 = 2^2 \cdot 3^2$	432B1	$24 = 2^3 \cdot 3$	450B1	$60 = 2^2 \cdot 3 \cdot 5$	471A1	$16 = 2^4$
405E1	$72 = 2^3 \cdot 3^2$	432C1	$144 = 2^4 \cdot 3^2$	450C1	$64 = 2^6$	472A1	$40 = 2^3 \cdot 5$
405F1	$24 = 2^3 \cdot 3$	432D1	$48 = 2^4 \cdot 3$	450D1	$60 = 2^2 \cdot 3 \cdot 5$	472B1	$48 = 2^4 \cdot 3$
406A1	$120 = 2^3 \cdot 3 \cdot 5$	432E1	$48 = 2^4 \cdot 3$	450E1	$192 = 2^6 \cdot 3$	472C1	$24 = 2^3 \cdot 3$
406B1	$96 = 2^5 \cdot 3$	432F1	$48 = 2^4 \cdot 3$	450F1	$192 = 2^6 \cdot 3$	472D1	$112 = 2^4 \cdot 7$
406C1	$64 = 2^6$	432G1	$144 = 2^4 \cdot 3^2$	450G1	$384 = 2^7 \cdot 3$	472E1	$16 = 2^4$
406D1	$640 = 2^7 \cdot 5$	432H1	$48 = 2^4 \cdot 3$	451A1	$36 = 2^2 \cdot 3^2$	473A1	$204 = 2^2 \cdot 3 \cdot 17$
408A1	$32 = 2^5$	433A1	$28 = 2^2 \cdot 7$	455A1	$48 = 2^4 \cdot 3$	474A1	$168 = 2^3 \cdot 3 \cdot 7$
408B1	$64 = 2^6$	434A1	$24 = 2^3 \cdot 3$	455B1	$32 = 2^5$	474B1	$40 = 2^3 \cdot 5$
408C1	$240 = 2^4 \cdot 3 \cdot 5$	434B1	$48 = 2^4 \cdot 3$	456A1	$80 = 2^4 \cdot 5$	475A1	$36 = 2^2 \cdot 3^2$
408D1	$80 = 2^4 \cdot 5$	434C1	$80 = 2^4 \cdot 5$	456B1	$48 = 2^4 \cdot 3$	475B1	$80 = 2^4 \cdot 5$
410A1	$48 = 2^4 \cdot 3$	434D1	$80 = 2^4 \cdot 5$	456C1	$96 = 2^5 \cdot 3$	475C1	$16 = 2^4$
410B1	$288 = 2^5 \cdot 3^2$	434E1	$480 = 2^5 \cdot 3 \cdot 5$	456D1	$96 = 2^5 \cdot 3$	477A1	$28 = 2^2 \cdot 7$
410C1	$128 = 2^7$	435A1	$20 = 2^2 \cdot 5$	458A1	$48 = 2^4 \cdot 3$	480A1	$32 = 2^5$
410D1	$64 = 2^6$	435B1	$140 = 2^2 \cdot 5 \cdot 7$	458B1	$40 = 2^3 \cdot 5$	480B1	$32 = 2^5$
414A1	$128 = 2^7$	435C1	$48 = 2^4 \cdot 3$	459A1	$12 = 2^2 \cdot 3$	480C1	$32 = 2^5$
414B1	$64 = 2^6$	435D1	$80 = 2^4 \cdot 5$	459B1	$60 = 2^2 \cdot 3 \cdot 5$	480D1	$192 = 2^6 \cdot 3$
414C1	$64 = 2^6$	437A1	$80 = 2^4 \cdot 5$	459C1	$18 = 2 \cdot 3^2$	480E1	$192 = 2^6 \cdot 3$
414D1	$160 = 2^5 \cdot 5$	437B1	$40 = 2^3 \cdot 5$	459D1	$198 = 2 \cdot 3^2 \cdot 11$	480F1	$64 = 2^6$
415A1	$112 = 2^4 \cdot 7$	438A1	$288 = 2^5 \cdot 3^2$	459E1	$180 = 2^2 \cdot 3^2 \cdot 5$	480G1	$32 = 2^5$
416A1	$16 = 2^4$	438B1	$32 = 2^5$	459F1	$54 = 2 \cdot 3^3$	480H1	$64 = 2^6$
416B1	$16 = 2^4$	438C1	$32 = 2^5$	459G1	$66 = 2 \cdot 3 \cdot 11$	481A1	$114 = 2 \cdot 3 \cdot 19$
417A1	$72 = 2^3 \cdot 3^2$	438D1	$576 = 2^6 \cdot 3^2$	459H1	$36 = 2^2 \cdot 3^2$	482A1	$112 = 2^4 \cdot 7$
418A1	$20 = 2^2 \cdot 5$	438E1	$112 = 2^4 \cdot 7$	460A1	$36 = 2^2 \cdot 3^2$	483A1	$300 = 2^2 \cdot 3 \cdot 5^2$
418B1	$104 = 2^3 \cdot 13$	438F1	$64 = 2^6$	460B1	$360 = 2^3 \cdot 3^2 \cdot 5$	483B1	$20 = 2^2 \cdot 5$
418C1	$56 = 2^3 \cdot 7$	438G1	$64 = 2^6$	460C1	$144 = 2^4 \cdot 3^2$	484A1	$240 = 2^4 \cdot 3 \cdot 5$
420A1	$840 = 2^3 \cdot 3 \cdot 5 \cdot 7$	440A1	$48 = 2^4 \cdot 3$	460D1	$24 = 2^3 \cdot 3$	485A1	$140 = 2^2 \cdot 5 \cdot 7$
420B1	$120 = 2^3 \cdot 3 \cdot 5$	440B1	$16 = 2^4$	462A1	$64 = 2^6$	485B1	$12 = 2^2 \cdot 3$
420C1	$72 = 2^3 \cdot 3^2$	440C1	$288 = 2^5 \cdot 3^2$	462B1	$480 = 2^5 \cdot 3 \cdot 5$	486A1	$36 = 2^2 \cdot 3^2$
420D1	$24 = 2^3 \cdot 3$	440D1	$144 = 2^4 \cdot 3^2$	462C1	$32 = 2^5$	486B1	$36 = 2^2 \cdot 3^2$
422A1	$24 = 2^3 \cdot 3$	441A1	$168 = 2^3 \cdot 3 \cdot 7$	462D1	$4160 = 2^6 \cdot 5 \cdot 13$	486C1	$108 = 2^2 \cdot 3^3$
423A1	$32 = 2^5$	441B1	$24 = 2^3 \cdot 3$	462E1	$672 = 2^5 \cdot 3 \cdot 7$	486D1	$36 = 2^2 \cdot 3^2$
423B1	$96 = 2^5 \cdot 3$	441C1	$192 = 2^6 \cdot 3$	462F1	$192 = 2^6 \cdot 3$	486E1	$36 = 2^2 \cdot 3^2$
423C1	$48 = 2^4 \cdot 3$	441D1	$32 = 2^5$	462G1	$96 = 2^5 \cdot 3$	486F1	$108 = 2^2 \cdot 3^3$
423D1	$96 = 2^5 \cdot 3$	441E1	$336 = 2^4 \cdot 3 \cdot 7$	464A1	$32 = 2^5$	490A1	$168 = 2^3 \cdot 3 \cdot 7$
423E1	$224 = 2^5 \cdot 7$	441F1	$48 = 2^4 \cdot 3$	464B1	$32 = 2^5$	490B1	$84 = 2^2 \cdot 3 \cdot 7$

curve	degree	curve	degree	curve	degree	curve	degree
490C1	$588 = 2^2 \cdot 3 \cdot 7^2$	514B1	$32 = 2^5$	542A1	$98 = 2 \cdot 7^2$	560F1	$48 = 2^4 \cdot 3$
490D1	$24 = 2^3 \cdot 3$	516A1	$48 = 2^4 \cdot 3$	542B1	$56 = 2^3 \cdot 7$	561A1	$320 = 2^6 \cdot 5$
490E1	$60 = 2^2 \cdot 3 \cdot 5$	516B1	$48 = 2^4 \cdot 3$	544A1	$16 = 2^4$	561B1	$160 = 2^5 \cdot 5$
490F1	$840 = 2^3 \cdot 3 \cdot 5 \cdot 7$	516C1	$42 = 2 \cdot 3 \cdot 7$	544B1	$32 = 2^5$	561C1	$64 = 2^6$
490G1	$160 = 2^5 \cdot 5$	516D1	$288 = 2^5 \cdot 3^2$	544C1	$32 = 2^5$	561D1	$32 = 2^5$
490H1	$192 = 2^6 \cdot 3$	517A1	$120 = 2^3 \cdot 3 \cdot 5$	544D1	$16 = 2^4$	562A1	$40 = 2^3 \cdot 5$
490I1	$420 = 2^2 \cdot 3 \cdot 5 \cdot 7$	517B1	$72 = 2^3 \cdot 3^2$	544E1	$48 = 2^4 \cdot 3$	563A1	$52 = 2^2 \cdot 13$
490J1	$1120 = 2^5 \cdot 5 \cdot 7$	517C1	$432 = 2^4 \cdot 3^3$	544F1	$48 = 2^4 \cdot 3$	564A1	$120 = 2^3 \cdot 3 \cdot 5$
490K1	$120 = 2^3 \cdot 3 \cdot 5$	520A1	$32 = 2^5$	545A1	$102 = 2 \cdot 3 \cdot 17$	564B1	$72 = 2^3 \cdot 3^2$
492A1	$24 = 2^3 \cdot 3$	520B1	$32 = 2^5$	546A1	$120 = 2^3 \cdot 3 \cdot 5$	565A1	$48 = 2^4 \cdot 3$
492B1	$216 = 2^3 \cdot 3^3$	522A1	$240 = 2^4 \cdot 3 \cdot 5$	546B1	$40 = 2^3 \cdot 5$	566A1	$24 = 2^3 \cdot 3$
493A1	$1872 = 2^4 \cdot 3^2 \cdot 13$	522B1	$440 = 2^3 \cdot 5 \cdot 11$	546C1	$96 = 2^5 \cdot 3$	566B1	$26 = 2 \cdot 13$
493B1	$120 = 2^3 \cdot 3 \cdot 5$	522C1	$80 = 2^4 \cdot 5$	546D1	$216 = 2^3 \cdot 3^3$	567A1	$24 = 2^3 \cdot 3$
494A1	$40 = 2^3 \cdot 5$	522D1	$224 = 2^5 \cdot 7$	546E1	$4760 = 2^3 \cdot 5 \cdot 7 \cdot 17$	567B1	$72 = 2^3 \cdot 3^2$
494B1	$34 = 2 \cdot 17$	522E1	$96 = 2^5 \cdot 3$	546F1	$1176 = 2^3 \cdot 3 \cdot 7^2$	568A1	$80 = 2^4 \cdot 5$
494C1	$152 = 2^3 \cdot 19$	522F1	$120 = 2^3 \cdot 3 \cdot 5$	546G1	$48 = 2^4 \cdot 3$	570A1	$192 = 2^6 \cdot 3$
494D1	$312 = 2^3 \cdot 3 \cdot 13$	522G1	$1320 = 2^3 \cdot 3 \cdot 5 \cdot 11$	549A1	$16 = 2^4$	570B1	$336 = 2^4 \cdot 3 \cdot 7$
495A1	$32 = 2^5$	522H1	$80 = 2^4 \cdot 5$	549B1	$48 = 2^4 \cdot 3$	570C1	$96 = 2^5 \cdot 3$
496A1	$16 = 2^4$	522I1	$80 = 2^4 \cdot 5$	549C1	$48 = 2^4 \cdot 3$	570D1	$2240 = 2^6 \cdot 5 \cdot 7$
496B1	$16 = 2^4$	522J1	$416 = 2^5 \cdot 13$	550A1	$96 = 2^5 \cdot 3$	570E1	$128 = 2^7$
496C1	$32 = 2^5$	522K1	$80 = 2^4 \cdot 5$	550B1	$480 = 2^5 \cdot 3 \cdot 5$	570F1	$144 = 2^4 \cdot 3^2$
496D1	$24 = 2^3 \cdot 3$	522L1	$56 = 2^3 \cdot 7$	550C1	$132 = 2^2 \cdot 3 \cdot 11$	570G1	$64 = 2^6$
496E1	$24 = 2^3 \cdot 3$	522M1	$12320 = 2^5 \cdot 5 \cdot 7 \cdot 11$	550D1	$180 = 2^2 \cdot 3^2 \cdot 5$	570H1	$48 = 2^4 \cdot 3$
496F1	$48 = 2^4 \cdot 3$	524A1	$60 = 2^2 \cdot 3 \cdot 5$	550E1	$880 = 2^4 \cdot 5 \cdot 11$	570I1	$960 = 2^6 \cdot 3 \cdot 5$
497A1	$40 = 2^3 \cdot 5$	525A1	$96 = 2^5 \cdot 3$	550F1	$240 = 2^4 \cdot 3 \cdot 5$	570J1	$560 = 2^4 \cdot 5 \cdot 7$
498A1	$28 = 2^2 \cdot 7$	525B1	$64 = 2^6$	550G1	$48 = 2^4 \cdot 3$	570K1	$2880 = 2^6 \cdot 3^2 \cdot 5$
498B1	$80 = 2^4 \cdot 5$	525C1	$240 = 2^4 \cdot 3 \cdot 5$	550H1	$36 = 2^2 \cdot 3^2$	570L1	$2400 = 2^5 \cdot 3 \cdot 5^2$
501A1	$46 = 2 \cdot 23$	525D1	$48 = 2^4 \cdot 3$	550I1	$672 = 2^5 \cdot 3 \cdot 7$	570M1	$96 = 2^5 \cdot 3$
503A1	$22 = 2 \cdot 11$	528A1	$32 = 2^5$	550J1	$176 = 2^4 \cdot 11$	571A1	$120 = 2^3 \cdot 3 \cdot 5$
503B1	$38 = 2 \cdot 19$	528B1	$48 = 2^4 \cdot 3$	550K1	$48 = 2^4 \cdot 3$	571B1	$48 = 2^4 \cdot 3$
503C1	$100 = 2^2 \cdot 5^2$	528C1	$672 = 2^5 \cdot 3 \cdot 7$	550L1	$240 = 2^4 \cdot 3 \cdot 5$	572A1	$120 = 2^3 \cdot 3 \cdot 5$
504A1	$32 = 2^5$	528D1	$32 = 2^5$	550M1	$660 = 2^2 \cdot 3 \cdot 5 \cdot 11$	573A1	$24 = 2^3 \cdot 3$
504B1	$96 = 2^5 \cdot 3$	528E1	$24 = 2^3 \cdot 3$	551A1	$24 = 2^3 \cdot 3$	573B1	$300 = 2^2 \cdot 3 \cdot 5^2$
504C1	$64 = 2^6$	528F1	$480 = 2^5 \cdot 3 \cdot 5$	551B1	$24 = 2^3 \cdot 3$	573C1	$60 = 2^2 \cdot 3 \cdot 5$
504D1	$96 = 2^5 \cdot 3$	528G1	$96 = 2^5 \cdot 3$	551C1	$672 = 2^5 \cdot 3 \cdot 7$	574A1	$24 = 2^3 \cdot 3$
504E1	$32 = 2^5$	528H1	$96 = 2^5 \cdot 3$	551D1	$96 = 2^5 \cdot 3$	574B1	$480 = 2^5 \cdot 3 \cdot 5$
504F1	$64 = 2^6$	528I1	$120 = 2^3 \cdot 3 \cdot 5$	552A1	$96 = 2^5 \cdot 3$	574C1	$224 = 2^5 \cdot 7$
504G1	$192 = 2^6 \cdot 3$	528J1	$96 = 2^5 \cdot 3$	552B1	$448 = 2^6 \cdot 7$	574D1	$8160 = 2^5 \cdot 3 \cdot 5 \cdot 17$
504H1	$96 = 2^5 \cdot 3$	530A1	$112 = 2^4 \cdot 7$	552C1	$64 = 2^6$	574E1	$120 = 2^3 \cdot 3 \cdot 5$
505A1	$16 = 2^4$	530B1	$24 = 2^3 \cdot 3$	552D1	$96 = 2^5 \cdot 3$	574F1	$120 = 2^3 \cdot 3 \cdot 5$
506A1	$56 = 2^3 \cdot 7$	530C1	$1200 = 2^4 \cdot 3 \cdot 5^2$	552E1	$64 = 2^6$	574G1	$88 = 2^3 \cdot 11$
506B1	$2856 = 2^3 \cdot 3 \cdot 7 \cdot 17$	530D1	$48 = 2^4 \cdot 3$	555A1	$60 = 2^2 \cdot 3 \cdot 5$	574H1	$72 = 2^3 \cdot 3^2$
506C1	$88 = 2^3 \cdot 11$	532A1	$30 = 2 \cdot 3 \cdot 5$	555B1	$420 = 2^2 \cdot 3 \cdot 5 \cdot 7$	574I1	$3528 = 2^3 \cdot 3^2 \cdot 7^2$
506D1	$200 = 2^3 \cdot 5^2$	534A1	$48 = 2^4 \cdot 3$	556A1	$18 = 2 \cdot 3^2$	574J1	$120 = 2^3 \cdot 3 \cdot 5$
506E1	$24 = 2^3 \cdot 3$	537A1	$208 = 2^4 \cdot 13$	557A1	$14 = 2 \cdot 7$	575A1	$12 = 2^2 \cdot 3$
506F1	$104 = 2^3 \cdot 13$	537B1	$32 = 2^5$	557B1	$182 = 2 \cdot 7 \cdot 13$	575B1	$240 = 2^4 \cdot 3 \cdot 5$
507A1	$312 = 2^3 \cdot 3 \cdot 13$	537C1	$64 = 2^6$	558A1	$16 = 2^4$	575C1	$280 = 2^3 \cdot 5 \cdot 7$
507B1	$24 = 2^3 \cdot 3$	537D1	$24 = 2^3 \cdot 3$	558B1	$240 = 2^4 \cdot 3 \cdot 5$	575D1	$60 = 2^2 \cdot 3 \cdot 5$
507C1	$168 = 2^3 \cdot 3 \cdot 7$	537E1	$192 = 2^6 \cdot 3$	558C1	$64 = 2^6$	575E1	$56 = 2^3 \cdot 7$
510A1	$1008 = 2^4 \cdot 3^2 \cdot 7$	539A1	$320 = 2^6 \cdot 5$	558D1	$160 = 2^5 \cdot 5$	576A1	$32 = 2^5$
510B1	$336 = 2^4 \cdot 3 \cdot 7$	539B1	$192 = 2^6 \cdot 3$	558E1	$48 = 2^4 \cdot 3$	576B1	$64 = 2^6$
510C1	$80 = 2^4 \cdot 5$	539C1	$288 = 2^5 \cdot 3^2$	558F1	$240 = 2^4 \cdot 3 \cdot 5$	576C1	$64 = 2^6$
510D1	$192 = 2^6 \cdot 3$	539D1	$72 = 2^3 \cdot 3^2$	558G1	$224 = 2^5 \cdot 7$	576D1	$64 = 2^6$
510E1	$128 = 2^7$	540A1	$36 = 2^2 \cdot 3^2$	558H1	$352 = 2^5 \cdot 11$	576E1	$32 = 2^5$
510F1	$32 = 2^5$	540B1	$36 = 2^2 \cdot 3^2$	560A1	$32 = 2^5$	576F1	$32 = 2^5$
510G1	$144 = 2^4 \cdot 3^2$	540C1	$216 = 2^3 \cdot 3^3$	560B1	$480 = 2^5 \cdot 3 \cdot 5$	576G1	$96 = 2^5 \cdot 3$
513A1	$72 = 2^3 \cdot 3^2$	540D1	$108 = 2^2 \cdot 3^3$	560C1	$48 = 2^4 \cdot 3$	576H1	$32 = 2^5$
513B1	$24 = 2^3 \cdot 3$	540E1	$72 = 2^3 \cdot 3^2$	560D1	$96 = 2^5 \cdot 3$	576I1	$64 = 2^6$
514A1	$128 = 2^7$	540F1	$36 = 2^2 \cdot 3^2$	560E1	$240 = 2^4 \cdot 3 \cdot 5$	578A1	$576 = 2^6 \cdot 3^2$

curve	degree	curve	degree	curve	degree	curve	degree
579A1	$80 = 2^4 \cdot 5$	600D1	$64 = 2^6$	621A1	$144 = 2^4 \cdot 3^2$	645D1	$1152 = 2^7 \cdot 3^2$
579B1	$24 = 2^3 \cdot 3$	600E1	$336 = 2^4 \cdot 3 \cdot 7$	621B1	$48 = 2^4 \cdot 3$	645E1	$1536 = 2^9 \cdot 3$
580A1	$24 = 2^3 \cdot 3$	600F1	$192 = 2^6 \cdot 3$	622A1	$84 = 2^2 \cdot 3 \cdot 7$	645F1	$192 = 2^6 \cdot 3$
580B1	$72 = 2^3 \cdot 3^2$	600G1	$1680 = 2^4 \cdot 3 \cdot 5 \cdot 7$	623A1	$96 = 2^5 \cdot 3$	646A1	$192 = 2^6 \cdot 3$
582A1	$48 = 2^4 \cdot 3$	600H1	$480 = 2^5 \cdot 3 \cdot 5$	624A1	$32 = 2^5$	646B1	$192 = 2^6 \cdot 3$
582B1	$2016 = 2^5 \cdot 3^2 \cdot 7$	600I1	$240 = 2^4 \cdot 3 \cdot 5$	624B1	$96 = 2^5 \cdot 3$	646C1	$448 = 2^6 \cdot 7$
582C1	$80 = 2^4 \cdot 5$	602A1	$480 = 2^5 \cdot 3 \cdot 5$	624C1	$32 = 2^5$	646D1	$192 = 2^6 \cdot 3$
582D1	$64 = 2^6$	602B1	$1360 = 2^4 \cdot 5 \cdot 17$	624D1	$32 = 2^5$	646E1	$192 = 2^6 \cdot 3$
583A1	$48 = 2^4 \cdot 3$	602C1	$48 = 2^4 \cdot 3$	624E1	$480 = 2^5 \cdot 3 \cdot 5$	648A1	$48 = 2^4 \cdot 3$
583B1	$384 = 2^7 \cdot 3$	603A1	$28 = 2^2 \cdot 7$	624F1	$64 = 2^6$	648B1	$24 = 2^3 \cdot 3$
583C1	$1248 = 2^5 \cdot 3 \cdot 13$	603B1	$84 = 2^2 \cdot 3 \cdot 7$	624G1	$48 = 2^4 \cdot 3$	648C1	$144 = 2^4 \cdot 3^2$
585A1	$192 = 2^6 \cdot 3$	603C1	$480 = 2^5 \cdot 3 \cdot 5$	624H1	$64 = 2^6$	648D1	$72 = 2^3 \cdot 3^2$
585B1	$48 = 2^4 \cdot 3$	603D1	$96 = 2^5 \cdot 3$	624I1	$960 = 2^6 \cdot 3 \cdot 5$	649A1	$24 = 2^3 \cdot 3$
585C1	$64 = 2^6$	603E1	$96 = 2^5 \cdot 3$	624J1	$48 = 2^4 \cdot 3$	650A1	$192 = 2^6 \cdot 3$
585D1	$48 = 2^4 \cdot 3$	603F1	$120 = 2^3 \cdot 3 \cdot 5$	626A1	$20 = 2^2 \cdot 5$	650B1	$360 = 2^3 \cdot 3^2 \cdot 5$
585E1	$672 = 2^5 \cdot 3 \cdot 7$	605A1	$1320 = 2^3 \cdot 3 \cdot 5 \cdot 11$	626B1	$532 = 2^2 \cdot 7 \cdot 19$	650C1	$120 = 2^3 \cdot 3 \cdot 5$
585F1	$192 = 2^6 \cdot 3$	605B1	$120 = 2^3 \cdot 3 \cdot 5$	627A1	$28 = 2^2 \cdot 7$	650D1	$840 = 2^3 \cdot 3 \cdot 5 \cdot 7$
585G1	$96 = 2^5 \cdot 3$	605C1	$120 = 2^3 \cdot 3 \cdot 5$	627B1	$180 = 2^2 \cdot 3^2 \cdot 5$	650E1	$1920 = 2^7 \cdot 3 \cdot 5$
585H1	$48 = 2^4 \cdot 3$	606A1	$198 = 2 \cdot 3^2 \cdot 11$	628A1	$48 = 2^4 \cdot 3$	650F1	$280 = 2^3 \cdot 5 \cdot 7$
585I1	$672 = 2^5 \cdot 3 \cdot 7$	606B1	$48 = 2^4 \cdot 3$	629A1	$36 = 2^2 \cdot 3^2$	650G1	$72 = 2^3 \cdot 3^2$
586A1	$30 = 2 \cdot 3 \cdot 5$	606C1	$48 = 2^4 \cdot 3$	629B1	$360 = 2^3 \cdot 3^2 \cdot 5$	650H1	$72 = 2^3 \cdot 3^2$
586B1	$144 = 2^4 \cdot 3^2$	606D1	$1428 = 2^2 \cdot 3 \cdot 7 \cdot 17$	629C1	$112 = 2^4 \cdot 7$	650I1	$360 = 2^3 \cdot 3^2 \cdot 5$
586C1	$32 = 2^5$	606E1	$432 = 2^4 \cdot 3^3$	629D1	$220 = 2^2 \cdot 5 \cdot 11$	650J1	$576 = 2^6 \cdot 3^2$
588A1	$180 = 2^2 \cdot 3^2 \cdot 5$	606F1	$100 = 2^2 \cdot 5^2$	630A1	$96 = 2^5 \cdot 3$	650K1	$168 = 2^3 \cdot 3 \cdot 7$
588B1	$288 = 2^5 \cdot 3^2$	608A1	$32 = 2^5$	630B1	$1120 = 2^5 \cdot 5 \cdot 7$	650L1	$1800 = 2^3 \cdot 3^2 \cdot 5^2$
588C1	$48 = 2^4 \cdot 3$	608B1	$480 = 2^5 \cdot 3 \cdot 5$	630C1	$1024 = 2^{10}$	650M1	$600 = 2^3 \cdot 3 \cdot 5^2$
588D1	$1260 = 2^2 \cdot 3^2 \cdot 5 \cdot 7$	608C1	$48 = 2^4 \cdot 3$	630D1	$256 = 2^8$	651A1	$960 = 2^6 \cdot 3 \cdot 5$
588E1	$336 = 2^4 \cdot 3 \cdot 7$	608D1	$32 = 2^5$	630E1	$128 = 2^7$	651B1	$48 = 2^4 \cdot 3$
588F1	$288 = 2^5 \cdot 3^2$	608E1	$480 = 2^5 \cdot 3 \cdot 5$	630F1	$384 = 2^7 \cdot 3$	651C1	$32 = 2^5$
590A1	$352 = 2^5 \cdot 11$	608F1	$48 = 2^4 \cdot 3$	630G1	$3360 = 2^5 \cdot 3 \cdot 5 \cdot 7$	651D1	$128 = 2^7$
590B1	$60 = 2^2 \cdot 3 \cdot 5$	609A1	$32 = 2^5$	630H1	$96 = 2^5 \cdot 3$	651E1	$96 = 2^5 \cdot 3$
590C1	$48 = 2^4 \cdot 3$	609B1	$384 = 2^7 \cdot 3$	630I1	$768 = 2^8 \cdot 3$	654A1	$128 = 2^7$
590D1	$288 = 2^5 \cdot 3^2$	610A1	$60 = 2^2 \cdot 3 \cdot 5$	630J1	$128 = 2^7$	654B1	$256 = 2^8$
591A1	$16 = 2^4$	610B1	$96 = 2^5 \cdot 3$	632A1	$64 = 2^6$	655A1	$144 = 2^4 \cdot 3^2$
592A1	$32 = 2^5$	610C1	$64 = 2^6$	633A1	$128 = 2^7$	656A1	$32 = 2^5$
592B1	$32 = 2^5$	611A1	$58 = 2 \cdot 29$	635A1	$24 = 2^3 \cdot 3$	656B1	$48 = 2^4 \cdot 3$
592C1	$80 = 2^4 \cdot 5$	612A1	$144 = 2^4 \cdot 3^2$	635B1	$40 = 2^3 \cdot 5$	656C1	$96 = 2^5 \cdot 3$
592D1	$48 = 2^4 \cdot 3$	612B1	$144 = 2^4 \cdot 3^2$	637A1	$60 = 2^2 \cdot 3 \cdot 5$	657A1	$480 = 2^5 \cdot 3 \cdot 5$
592E1	$48 = 2^4 \cdot 3$	612C1	$96 = 2^5 \cdot 3$	637B1	$192 = 2^6 \cdot 3$	657B1	$96 = 2^5 \cdot 3$
593A1	$22 = 2 \cdot 11$	612D1	$1056 = 2^5 \cdot 3 \cdot 11$	637C1	$420 = 2^2 \cdot 3 \cdot 5 \cdot 7$	657C1	$96 = 2^5 \cdot 3$
593B1	$58 = 2 \cdot 29$	614A1	$48 = 2^4 \cdot 3$	637D1	$192 = 2^6 \cdot 3$	657D1	$48 = 2^4 \cdot 3$
594A1	$48 = 2^4 \cdot 3$	614B1	$96 = 2^5 \cdot 3$	639A1	$48 = 2^4 \cdot 3$	658A1	$5040 = 2^4 \cdot 3^2 \cdot 5 \cdot 7$
594B1	$48 = 2^4 \cdot 3$	615A1	$32 = 2^5$	640A1	$32 = 2^5$	658B1	$72 = 2^3 \cdot 3^2$
594C1	$720 = 2^4 \cdot 3^2 \cdot 5$	615B1	$112 = 2^4 \cdot 7$	640B1	$32 = 2^5$	658C1	$48 = 2^4 \cdot 3$
594D1	$480 = 2^5 \cdot 3 \cdot 5$	616A1	$96 = 2^5 \cdot 3$	640C1	$32 = 2^5$	658D1	$96 = 2^5 \cdot 3$
594E1	$1440 = 2^5 \cdot 3^2 \cdot 5$	616B1	$960 = 2^6 \cdot 3 \cdot 5$	640D1	$64 = 2^6$	658E1	$1056 = 2^5 \cdot 3 \cdot 11$
594F1	$144 = 2^4 \cdot 3^2$	616C1	$64 = 2^6$	640E1	$64 = 2^6$	658F1	$96 = 2^5 \cdot 3$
594G1	$144 = 2^4 \cdot 3^2$	616D1	$96 = 2^5 \cdot 3$	640F1	$64 = 2^6$	659A1	$50 = 2 \cdot 5^2$
594H1	$240 = 2^4 \cdot 3 \cdot 5$	616E1	$32 = 2^5$	640G1	$32 = 2^5$	659B1	$141 = 3 \cdot 47$
595A1	$1716 = 2^2 \cdot 3 \cdot 11 \cdot 13$	618A1	$48 = 2^4 \cdot 3$	640H1	$64 = 2^6$	660A1	$96 = 2^5 \cdot 3$
595B1	$980 = 2^2 \cdot 5 \cdot 7^2$	618B1	$456 = 2^3 \cdot 3 \cdot 19$	642A1	$120 = 2^3 \cdot 3 \cdot 5$	660B1	$48 = 2^4 \cdot 3$
595C1	$28 = 2^2 \cdot 7$	618C1	$72 = 2^3 \cdot 3^2$	642B1	$240 = 2^4 \cdot 3 \cdot 5$	660C1	$144 = 2^4 \cdot 3^2$
598A1	$96 = 2^5 \cdot 3$	618D1	$720 = 2^4 \cdot 3^2 \cdot 5$	642C1	$208 = 2^4 \cdot 13$	660D1	$288 = 2^5 \cdot 3^2$
598B1	$480 = 2^5 \cdot 3 \cdot 5$	618E1	$40 = 2^3 \cdot 5$	643A1	$32 = 2^5$	662A1	$160 = 2^5 \cdot 5$
598C1	$48 = 2^4 \cdot 3$	618F1	$616 = 2^3 \cdot 7 \cdot 11$	644A1	$48 = 2^4 \cdot 3$	663A1	$288 = 2^5 \cdot 3^2$
598D1	$272 = 2^4 \cdot 17$	618G1	$224 = 2^5 \cdot 7$	644B1	$24 = 2^3 \cdot 3$	663B1	$128 = 2^7$
600A1	$192 = 2^6 \cdot 3$	620A1	$72 = 2^3 \cdot 3^2$	645A1	$44 = 2^2 \cdot 11$	663C1	$64 = 2^6$
600B1	$48 = 2^4 \cdot 3$	620B1	$180 = 2^2 \cdot 3^2 \cdot 5$	645B1	$72 = 2^3 \cdot 3^2$	664A1	$160 = 2^5 \cdot 5$
600C1	$96 = 2^5 \cdot 3$	620C1	$72 = 2^3 \cdot 3^2$	645C1	$2688 = 2^7 \cdot 3 \cdot 7$	664B1	$16 = 2^4$

curve	degree	curve	degree	curve	degree	curve	degree
664C1	$16=2^4$	682B1	$456=2^3 \cdot 3 \cdot 19$	702P1	$1080=2^3 \cdot 3^3 \cdot 5$	720I1	$48=2^4 \cdot 3$
665A1	$120=2^3 \cdot 3 \cdot 5$	684A1	$144=2^4 \cdot 3^2$	703A1	$720=2^4 \cdot 3^2 \cdot 5$	720J1	$384=2^7 \cdot 3$
665B1	$32=2^5$	684B1	$144=2^4 \cdot 3^2$	703B1	$56=2^3 \cdot 7$	722A1	$684=2^2 \cdot 3^2 \cdot 19$
665C1	$24=2^3 \cdot 3$	684C1	$192=2^6 \cdot 3$	704A1	$16=2^4$	722B1	$120=2^3 \cdot 3 \cdot 5$
665D1	$600=2^3 \cdot 3 \cdot 5^2$	685A1	$28=2^2 \cdot 7$	704B1	$32=2^5$	722C1	$720=2^4 \cdot 3^2 \cdot 5$
665E1	$232=2^3 \cdot 29$	688A1	$32=2^5$	704C1	$32=2^5$	722D1	$2280=2^3 \cdot 3 \cdot 5 \cdot 19$
666A1	$240=2^4 \cdot 3 \cdot 5$	688B1	$48=2^4 \cdot 3$	704D1	$64=2^6$	722E1	$720=2^4 \cdot 3^2 \cdot 5$
666B1	$352=2^5 \cdot 11$	688C1	$80=2^4 \cdot 5$	704E1	$128=2^7$	722F1	$36=2^2 \cdot 3^2$
666C1	$96=2^5 \cdot 3$	689A1	$40=2^3 \cdot 5$	704F1	$64=2^6$	723A1	$28=2^2 \cdot 7$
666D1	$80=2^4 \cdot 5$	690A1	$448=2^6 \cdot 7$	704G1	$32=2^5$	723B1	$20=2^2 \cdot 5$
666E1	$416=2^5 \cdot 13$	690B1	$336=2^4 \cdot 3 \cdot 7$	704H1	$96=2^5 \cdot 3$	725A1	$96=2^5 \cdot 3$
666F1	$288=2^5 \cdot 3^2$	690C1	$17280=2^7 \cdot 3^3 \cdot 5$	704I1	$128=2^7$	726A1	$144=2^4 \cdot 3^2$
666G1	$19872=2^5 \cdot 3^3 \cdot 23$	690D1	$144=2^4 \cdot 3^2$	704J1	$32=2^5$	726B1	$5280=2^5 \cdot 3 \cdot 5 \cdot 11$
669A1	$40=2^3 \cdot 5$	690E1	$480=2^5 \cdot 3 \cdot 5$	704K1	$16=2^4$	726C1	$480=2^5 \cdot 3 \cdot 5$
670A1	$528=2^4 \cdot 3 \cdot 11$	690F1	$64=2^6$	704L1	$96=2^5 \cdot 3$	726D1	$96=2^5 \cdot 3$
670B1	$48=2^4 \cdot 3$	690G1	$1792=2^8 \cdot 7$	705A1	$840=2^3 \cdot 3 \cdot 5 \cdot 7$	726E1	$2400=2^5 \cdot 3 \cdot 5^2$
670C1	$80=2^4 \cdot 5$	690H1	$96=2^5 \cdot 3$	705B1	$1440=2^5 \cdot 3^2 \cdot 5$	726F1	$1584=2^4 \cdot 3^2 \cdot 11$
670D1	$304=2^4 \cdot 19$	690I1	$240=2^4 \cdot 3 \cdot 5$	705C1	$88=2^3 \cdot 11$	726G1	$480=2^5 \cdot 3 \cdot 5$
672A1	$32=2^5$	690J1	$240=2^4 \cdot 3 \cdot 5$	705D1	$48=2^4 \cdot 3$	726H1	$480=2^5 \cdot 3 \cdot 5$
672B1	$480=2^5 \cdot 3 \cdot 5$	690K1	$384=2^7 \cdot 3$	705E1	$80=2^4 \cdot 5$	726I1	$1056=2^5 \cdot 3 \cdot 11$
672C1	$64=2^6$	692A1	$123=3 \cdot 41$	705F1	$108=2^2 \cdot 3^3$	728A1	$48=2^4 \cdot 3$
672D1	$480=2^5 \cdot 3 \cdot 5$	693A1	$144=2^4 \cdot 3^2$	706A1	$36=2^2 \cdot 3^2$	728B1	$672=2^5 \cdot 3 \cdot 7$
672E1	$64=2^6$	693B1	$56=2^3 \cdot 7$	706B1	$276=2^2 \cdot 3 \cdot 23$	728C1	$96=2^5 \cdot 3$
672F1	$64=2^6$	693C1	$200=2^3 \cdot 5^2$	706C1	$48=2^4 \cdot 3$	728D1	$96=2^5 \cdot 3$
672G1	$32=2^5$	693D1	$160=2^5 \cdot 5$	706D1	$80=2^4 \cdot 5$	730A1	$384=2^7 \cdot 3$
672H1	$64=2^6$	696A1	$48=2^4 \cdot 3$	707A1	$104=2^3 \cdot 13$	730B1	$420=2^2 \cdot 3 \cdot 5 \cdot 7$
674A1	$36=2^2 \cdot 3^2$	696B1	$80=2^4 \cdot 5$	708A1	$90=2 \cdot 3^2 \cdot 5$	730C1	$840=2^3 \cdot 3 \cdot 5 \cdot 7$
674B1	$48=2^4 \cdot 3$	696C1	$80=2^4 \cdot 5$	709A1	$44=2^2 \cdot 11$	730D1	$648=2^3 \cdot 3^4$
674C1	$1860=2^2 \cdot 3 \cdot 5 \cdot 31$	696D1	$720=2^4 \cdot 3^2 \cdot 5$	710A1	$96=2^5 \cdot 3$	730E1	$36=2^2 \cdot 3^2$
675A1	$48=2^4 \cdot 3$	696E1	$144=2^4 \cdot 3^2$	710B1	$272=2^4 \cdot 17$	730F1	$288=2^5 \cdot 3^2$
675B1	$36=2^2 \cdot 3^2$	696F1	$336=2^4 \cdot 3 \cdot 7$	710C1	$112=2^4 \cdot 7$	730G1	$72=2^3 \cdot 3^2$
675C1	$42=2 \cdot 3 \cdot 7$	696G1	$48=2^4 \cdot 3$	710D1	$720=2^4 \cdot 3^2 \cdot 5$	730H1	$140=2^2 \cdot 5 \cdot 7$
675D1	$540=2^2 \cdot 3^3 \cdot 5$	699A1	$54=2 \cdot 3^3$	711A1	$16=2^4$	730I1	$56=2^3 \cdot 7$
675E1	$210=2 \cdot 3 \cdot 5 \cdot 7$	700A1	$288=2^5 \cdot 3^2$	711B1	$48=2^4 \cdot 3$	730J1	$504=2^3 \cdot 3^2 \cdot 7$
675F1	$108=2^2 \cdot 3^3$	700B1	$36=2^2 \cdot 3^2$	711C1	$60=2^2 \cdot 3 \cdot 5$	730K1	$120=2^3 \cdot 3 \cdot 5$
675G1	$288=2^5 \cdot 3^2$	700C1	$36=2^2 \cdot 3^2$	712A1	$80=2^4 \cdot 5$	731A1	$120=2^3 \cdot 3 \cdot 5$
675H1	$864=2^5 \cdot 3^3$	700D1	$1440=2^5 \cdot 3^2 \cdot 5$	713A1	$20=2^2 \cdot 5$	732A1	$48=2^4 \cdot 3$
675I1	$180=2^2 \cdot 3^2 \cdot 5$	700E1	$360=2^3 \cdot 3^2 \cdot 5$	714A1	$1344=2^6 \cdot 3 \cdot 7$	732B1	$192=2^6 \cdot 3$
676A1	$252=2^2 \cdot 3^2 \cdot 7$	700F1	$180=2^2 \cdot 3^2 \cdot 5$	714B1	$120=2^3 \cdot 3 \cdot 5$	732C1	$72=2^3 \cdot 3^2$
676B1	$60=2^2 \cdot 3 \cdot 5$	700G1	$144=2^4 \cdot 3^2$	714C1	$2040=2^3 \cdot 3 \cdot 5 \cdot 17$	733A1	$48=2^4 \cdot 3$
676C1	$780=2^2 \cdot 3 \cdot 5 \cdot 13$	700H1	$72=2^3 \cdot 3^2$	714D1	$96=2^5 \cdot 3$	734A1	$90=2 \cdot 3^2 \cdot 5$
676D1	$180=2^2 \cdot 3^2 \cdot 5$	700I1	$180=2^2 \cdot 3^2 \cdot 5$	714E1	$840=2^3 \cdot 3 \cdot 5 \cdot 7$	735A1	$192=2^6 \cdot 3$
676E1	$2340=2^2 \cdot 3^2 \cdot 5 \cdot 13$	700J1	$720=2^4 \cdot 3^2 \cdot 5$	714F1	$192=2^6 \cdot 3$	735B1	$4704=2^5 \cdot 3 \cdot 7^2$
677A1	$22=2 \cdot 11$	701A1	$42=2 \cdot 3 \cdot 7$	714G1	$3840=2^8 \cdot 3 \cdot 5$	735C1	$48=2^4 \cdot 3$
678A1	$56=2^3 \cdot 7$	702A1	$120=2^3 \cdot 3 \cdot 5$	714H1	$40=2^3 \cdot 5$	735D1	$336=2^4 \cdot 3 \cdot 7$
678B1	$72=2^3 \cdot 3^2$	702B1	$24=2^3 \cdot 3$	714I1	$1080=2^3 \cdot 3^3 \cdot 5$	735E1	$96=2^5 \cdot 3$
678C1	$224=2^5 \cdot 7$	702C1	$180=2^2 \cdot 3^2 \cdot 5$	715A1	$48=2^4 \cdot 3$	735F1	$672=2^5 \cdot 3 \cdot 7$
678D1	$504=2^3 \cdot 3^2 \cdot 7$	702D1	$504=2^3 \cdot 3^2 \cdot 7$	715B1	$336=2^4 \cdot 3 \cdot 7$	737A1	$672=2^5 \cdot 3 \cdot 7$
678E1	$160=2^5 \cdot 5$	702E1	$1080=2^3 \cdot 3^3 \cdot 5$	718A1	$120=2^3 \cdot 3 \cdot 5$	738A1	$240=2^4 \cdot 3 \cdot 5$
678F1	$288=2^5 \cdot 3^2$	702F1	$660=2^2 \cdot 3 \cdot 5 \cdot 11$	718B1	$112=2^4 \cdot 7$	738B1	$2400=2^5 \cdot 3 \cdot 5^2$
680A1	$64=2^6$	702G1	$456=2^3 \cdot 3 \cdot 19$	718C1	$240=2^4 \cdot 3 \cdot 5$	738C1	$192=2^6 \cdot 3$
680B1	$48=2^4 \cdot 3$	702H1	$72=2^3 \cdot 3^2$	720A1	$64=2^6$	738D1	$672=2^5 \cdot 3 \cdot 7$
680C1	$384=2^7 \cdot 3$	702I1	$72=2^3 \cdot 3^2$	720B1	$192=2^6 \cdot 3$	738E1	$80=2^4 \cdot 5$
681A1	$32=2^5$	702J1	$1980=2^2 \cdot 3^2 \cdot 5 \cdot 11$	720C1	$128=2^7$	738F1	$352=2^5 \cdot 11$
681B1	$375=3 \cdot 5^3$	702K1	$168=2^3 \cdot 3 \cdot 7$	720D1	$64=2^6$	738G1	$384=2^7 \cdot 3$
681C1	$96=2^5 \cdot 3$	702L1	$360=2^3 \cdot 3^2 \cdot 5$	720E1	$128=2^7$	738H1	$13440=2^7 \cdot 3 \cdot 5 \cdot 7$
681D1	$192=2^6 \cdot 3$	702M1	$1368=2^3 \cdot 3^2 \cdot 19$	720F1	$192=2^6 \cdot 3$	738I1	$96=2^5 \cdot 3$
681E1	$160=2^5 \cdot 5$	702N1	$72=2^3 \cdot 3^2$	720G1	$192=2^6 \cdot 3$	738J1	$160=2^5 \cdot 5$
682A1	$72=2^3 \cdot 3^2$	702O1	$60=2^2 \cdot 3 \cdot 5$	720H1	$128=2^7$	739A1	$33=3 \cdot 11$

curve	degree	curve	degree	curve	degree	curve	degree
740A1	$12480 = 2^6 \cdot 3 \cdot 5 \cdot 13$	762B1	$3360 = 2^5 \cdot 3 \cdot 5 \cdot 7$	784C1	$192 = 2^6 \cdot 3$	800G1	$192 = 2^6 \cdot 3$
740B1	$144 = 2^4 \cdot 3^2$	762C1	$96 = 2^5 \cdot 3$	784D1	$336 = 2^4 \cdot 3 \cdot 7$	800H1	$32 = 2^5$
740C1	$96 = 2^5 \cdot 3$	762D1	$80 = 2^4 \cdot 5$	784E1	$384 = 2^7 \cdot 3$	800I1	$240 = 2^4 \cdot 3 \cdot 5$
741A1	$32 = 2^5$	762E1	$528 = 2^4 \cdot 3 \cdot 11$	784F1	$48 = 2^4 \cdot 3$	801A1	$1904 = 2^4 \cdot 7 \cdot 17$
741B1	$1680 = 2^4 \cdot 3 \cdot 5 \cdot 7$	762F1	$180 = 2^2 \cdot 3^2 \cdot 5$	784G1	$168 = 2^3 \cdot 3 \cdot 7$	801B1	$60 = 2^2 \cdot 3 \cdot 5$
741C1	$1760 = 2^5 \cdot 5 \cdot 11$	762G1	$3696 = 2^4 \cdot 3 \cdot 7 \cdot 11$	784H1	$64 = 2^6$	801C1	$80 = 2^4 \cdot 5$
741D1	$15680 = 2^6 \cdot 5 \cdot 7^2$	763A1	$60 = 2^2 \cdot 3 \cdot 5$	784I1	$24 = 2^3 \cdot 3$	801D1	$60 = 2^2 \cdot 3 \cdot 5$
741E1	$64 = 2^6$	765A1	$192 = 2^6 \cdot 3$	784J1	$384 = 2^7 \cdot 3$	802A1	$52 = 2^2 \cdot 13$
742A1	$40 = 2^3 \cdot 5$	765B1	$64 = 2^6$	786A1	$40 = 2^3 \cdot 5$	802B1	$42 = 2 \cdot 3 \cdot 7$
742B1	$336 = 2^4 \cdot 3 \cdot 7$	765C1	$96 = 2^5 \cdot 3$	786B1	$216 = 2^3 \cdot 3^3$	804A1	$210 = 2 \cdot 3 \cdot 5 \cdot 7$
742C1	$600 = 2^3 \cdot 3 \cdot 5^2$	766A1	$132 = 2^2 \cdot 3 \cdot 11$	786C1	$12960 = 2^5 \cdot 3^4 \cdot 5$	804B1	$360 = 2^3 \cdot 3^2 \cdot 5$
742D1	$3840 = 2^8 \cdot 3 \cdot 5$	768A1	$64 = 2^6$	786D1	$504 = 2^3 \cdot 3^2 \cdot 7$	804C1	$72 = 2^3 \cdot 3^2$
742E1	$5600 = 2^5 \cdot 5^2 \cdot 7$	768B1	$32 = 2^5$	786E1	$144 = 2^4 \cdot 3^2$	804D1	$168 = 2^3 \cdot 3 \cdot 7$
742F1	$1120 = 2^5 \cdot 5 \cdot 7$	768C1	$64 = 2^6$	786F1	$152 = 2^3 \cdot 19$	805A1	$5720 = 2^3 \cdot 5 \cdot 11 \cdot 13$
742G1	$160 = 2^5 \cdot 5$	768D1	$32 = 2^5$	786G1	$264 = 2^3 \cdot 3 \cdot 11$	805B1	$144 = 2^4 \cdot 3^2$
744A1	$32 = 2^5$	768E1	$64 = 2^6$	786H1	$168 = 2^3 \cdot 3 \cdot 7$	805C1	$360 = 2^3 \cdot 3^2 \cdot 5$
744B1	$120 = 2^3 \cdot 3 \cdot 5$	768F1	$32 = 2^5$	786I1	$288 = 2^5 \cdot 3^2$	805D1	$360 = 2^3 \cdot 3^2 \cdot 5$
744C1	$128 = 2^7$	768G1	$64 = 2^6$	786J1	$504 = 2^3 \cdot 3^2 \cdot 7$	806A1	$80 = 2^4 \cdot 5$
744D1	$1440 = 2^5 \cdot 3^2 \cdot 5$	768H1	$32 = 2^5$	786K1	$144 = 2^4 \cdot 3^2$	806B1	$176 = 2^4 \cdot 11$
744E1	$144 = 2^4 \cdot 3^2$	770A1	$64 = 2^6$	786L1	$280 = 2^3 \cdot 5 \cdot 7$	806C1	$160 = 2^5 \cdot 5$
744F1	$192 = 2^6 \cdot 3$	770B1	$576 = 2^6 \cdot 3^2$	786M1	$840 = 2^3 \cdot 3 \cdot 5 \cdot 7$	806D1	$1056 = 2^5 \cdot 3 \cdot 11$
744G1	$96 = 2^5 \cdot 3$	770C1	$2560 = 2^9 \cdot 5$	790A1	$128 = 2^7$	806E1	$1008 = 2^4 \cdot 3^2 \cdot 7$
747A1	$84 = 2^2 \cdot 3 \cdot 7$	770D1	$256 = 2^8$	791A1	$144 = 2^4 \cdot 3^2$	806F1	$1040 = 2^4 \cdot 5 \cdot 13$
747B1	$28 = 2^2 \cdot 7$	770E1	$256 = 2^8$	791B1	$176 = 2^4 \cdot 11$	807A1	$96 = 2^5 \cdot 3$
747C1	$192 = 2^6 \cdot 3$	770F1	$576 = 2^6 \cdot 3^2$	791C1	$48 = 2^4 \cdot 3$	808A1	$56 = 2^3 \cdot 7$
747D1	$60 = 2^2 \cdot 3 \cdot 5$	770G1	$192 = 2^6 \cdot 3$	792A1	$192 = 2^6 \cdot 3$	808B1	$112 = 2^4 \cdot 7$
747E1	$64 = 2^6$	774A1	$192 = 2^6 \cdot 3$	792B1	$128 = 2^7$	810A1	$72 = 2^3 \cdot 3^2$
748A1	$384 = 2^7 \cdot 3$	774B1	$480 = 2^5 \cdot 3 \cdot 5$	792C1	$64 = 2^6$	810B1	$216 = 2^3 \cdot 3^3$
749A1	$32 = 2^5$	774C1	$6080 = 2^6 \cdot 5 \cdot 19$	792D1	$128 = 2^7$	810C1	$1080 = 2^3 \cdot 3^3 \cdot 5$
752A1	$48 = 2^4 \cdot 3$	774D1	$1344 = 2^6 \cdot 3 \cdot 7$	792E1	$192 = 2^6 \cdot 3$	810D1	$144 = 2^4 \cdot 3^2$
753A1	$80 = 2^4 \cdot 5$	774E1	$96 = 2^5 \cdot 3$	792F1	$112 = 2^4 \cdot 7$	810E1	$72 = 2^3 \cdot 3^2$
753B1	$56 = 2^3 \cdot 7$	774F1	$192 = 2^6 \cdot 3$	792G1	$2688 = 2^7 \cdot 3 \cdot 7$	810F1	$144 = 2^4 \cdot 3^2$
753C1	$80 = 2^4 \cdot 5$	774G1	$192 = 2^6 \cdot 3$	793A1	$48 = 2^4 \cdot 3$	810G1	$1080 = 2^3 \cdot 3^3 \cdot 5$
754A1	$240 = 2^4 \cdot 3 \cdot 5$	774H1	$1568 = 2^5 \cdot 7^2$	794A1	$88 = 2^3 \cdot 11$	810H1	$216 = 2^3 \cdot 3^3$
754B1	$1248 = 2^5 \cdot 3 \cdot 13$	774I1	$320 = 2^6 \cdot 5$	794B1	$72 = 2^3 \cdot 3^2$	811A1	$24 = 2^3 \cdot 3$
754C1	$48 = 2^4 \cdot 3$	775A1	$96 = 2^5 \cdot 3$	794C1	$40 = 2^3 \cdot 5$	812A1	$144 = 2^4 \cdot 3^2$
754D1	$144 = 2^4 \cdot 3^2$	775B1	$192 = 2^6 \cdot 3$	794D1	$216 = 2^3 \cdot 3^3$	812B1	$96 = 2^5 \cdot 3$
755A1	$20 = 2^2 \cdot 5$	775C1	$480 = 2^5 \cdot 3 \cdot 5$	795A1	$80 = 2^4 \cdot 5$	813A1	$68 = 2^2 \cdot 17$
755B1	$16 = 2^4$	776A1	$40 = 2^3 \cdot 5$	795B1	$180 = 2^2 \cdot 3^2 \cdot 5$	813B1	$108 = 2^2 \cdot 3^3$
755C1	$44 = 2^2 \cdot 11$	777A1	$48 = 2^4 \cdot 3$	795C1	$660 = 2^2 \cdot 3 \cdot 5 \cdot 11$	814A1	$144 = 2^4 \cdot 3^2$
755D1	$40 = 2^3 \cdot 5$	777B1	$21840 = 2^4 \cdot 3 \cdot 5 \cdot 7 \cdot 13$	795D1	$120 = 2^3 \cdot 3 \cdot 5$	814B1	$80 = 2^4 \cdot 5$
755E1	$96 = 2^5 \cdot 3$	777C1	$96 = 2^5 \cdot 3$	797A1	$24 = 2^3 \cdot 3$	815A1	$120 = 2^3 \cdot 3 \cdot 5$
755F1	$5304 = 2^3 \cdot 3 \cdot 13 \cdot 17$	777D1	$80 = 2^4 \cdot 5$	798A1	$64 = 2^6$	816A1	$64 = 2^6$
756A1	$216 = 2^3 \cdot 3^3$	777E1	$240 = 2^4 \cdot 3 \cdot 5$	798B1	$192 = 2^6 \cdot 3$	816B1	$128 = 2^7$
756B1	$72 = 2^3 \cdot 3^2$	777F1	$48 = 2^4 \cdot 3$	798C1	$160 = 2^5 \cdot 5$	816C1	$160 = 2^5 \cdot 5$
756C1	$72 = 2^3 \cdot 3^2$	777G1	$160 = 2^5 \cdot 5$	798D1	$320 = 2^6 \cdot 5$	816D1	$480 = 2^5 \cdot 3 \cdot 5$
756D1	$216 = 2^3 \cdot 3^3$	780A1	$96 = 2^5 \cdot 3$	798E1	$1152 = 2^7 \cdot 3^2$	816E1	$576 = 2^6 \cdot 3^2$
756E1	$72 = 2^3 \cdot 3^2$	780B1	$1560 = 2^3 \cdot 3 \cdot 5 \cdot 13$	798F1	$256 = 2^8$	816F1	$144 = 2^4 \cdot 3^2$
756F1	$216 = 2^3 \cdot 3^3$	780C1	$192 = 2^6 \cdot 3$	798G1	$480 = 2^5 \cdot 3 \cdot 5$	816G1	$48 = 2^4 \cdot 3$
758A1	$80 = 2^4 \cdot 5$	780D1	$72 = 2^3 \cdot 3^2$	798H1	$1344 = 2^6 \cdot 3 \cdot 7$	816H1	$384 = 2^7 \cdot 3$
758B1	$88 = 2^3 \cdot 11$	781A1	$612 = 2^2 \cdot 3^2 \cdot 17$	798I1	$64 = 2^6$	816I1	$528 = 2^4 \cdot 3 \cdot 11$
759A1	$160 = 2^5 \cdot 5$	781B1	$108 = 2^2 \cdot 3^3$	799A1	$88 = 2^3 \cdot 11$	816J1	$192 = 2^6 \cdot 3$
759B1	$128 = 2^7$	782A1	$72 = 2^3 \cdot 3^2$	799B1	$168 = 2^3 \cdot 3 \cdot 7$	817A1	$56 = 2^3 \cdot 7$
760A1	$64 = 2^6$	782B1	$168 = 2^3 \cdot 3 \cdot 7$	800A1	$64 = 2^6$	817B1	$1160 = 2^3 \cdot 5 \cdot 29$
760B1	$2240 = 2^6 \cdot 5 \cdot 7$	782C1	$3360 = 2^5 \cdot 3 \cdot 5 \cdot 7$	800B1	$48 = 2^4 \cdot 3$	819A1	$96 = 2^5 \cdot 3$
760C1	$480 = 2^5 \cdot 3 \cdot 5$	782D1	$40 = 2^3 \cdot 5$	800C1	$192 = 2^6 \cdot 3$	819B1	$32 = 2^5$
760D1	$96 = 2^5 \cdot 3$	782E1	$400 = 2^4 \cdot 5^2$	800D1	$160 = 2^5 \cdot 5$	819C1	$128 = 2^7$
760E1	$64 = 2^6$	784A1	$48 = 2^4 \cdot 3$	800E1	$240 = 2^4 \cdot 3 \cdot 5$	819D1	$5376 = 2^8 \cdot 3 \cdot 7$
762A1	$60 = 2^2 \cdot 3 \cdot 5$	784B1	$336 = 2^4 \cdot 3 \cdot 7$	800F1	$48 = 2^4 \cdot 3$	819E1	$96 = 2^5 \cdot 3$

curve	degree	curve	degree	curve	degree	curve	degree
819F1	$384 = 2^7 \cdot 3$	847A1	$800 = 2^5 \cdot 5^2$	864B1	$96 = 2^5 \cdot 3$	882F1	$1008 = 2^4 \cdot 3^2 \cdot 7$
822A1	$64 = 2^6$	847B1	$480 = 2^5 \cdot 3 \cdot 5$	864C1	$96 = 2^5 \cdot 3$	882G1	$144 = 2^4 \cdot 3^2$
822B1	$1440 = 2^5 \cdot 3^2 \cdot 5$	847C1	$720 = 2^4 \cdot 3^2 \cdot 5$	864D1	$48 = 2^4 \cdot 3$	882H1	$480 = 2^5 \cdot 3 \cdot 5$
822C1	$640 = 2^7 \cdot 5$	848A1	$192 = 2^6 \cdot 3$	864E1	$288 = 2^5 \cdot 3^2$	882I1	$384 = 2^7 \cdot 3$
822D1	$120 = 2^3 \cdot 3 \cdot 5$	848B1	$1152 = 2^7 \cdot 3^2$	864F1	$96 = 2^5 \cdot 3$	882J1	$3360 = 2^5 \cdot 3 \cdot 5 \cdot 7$
822E1	$168 = 2^3 \cdot 3 \cdot 7$	848C1	$144 = 2^4 \cdot 3^2$	864G1	$144 = 2^4 \cdot 3^2$	882K1	$512 = 2^9$
822F1	$88 = 2^3 \cdot 11$	848D1	$84 = 2^2 \cdot 3 \cdot 7$	864H1	$288 = 2^5 \cdot 3^2$	882L1	$3584 = 2^9 \cdot 7$
825A1	$72 = 2^3 \cdot 3^2$	848E1	$128 = 2^7$	864I1	$288 = 2^5 \cdot 3^2$	885A1	$224 = 2^5 \cdot 7$
825B1	$192 = 2^6 \cdot 3$	848F1	$48 = 2^4 \cdot 3$	864J1	$144 = 2^4 \cdot 3^2$	885B1	$128 = 2^7$
825C1	$360 = 2^3 \cdot 3^2 \cdot 5$	848G1	$240 = 2^4 \cdot 3 \cdot 5$	864K1	$96 = 2^5 \cdot 3$	885C1	$48 = 2^4 \cdot 3$
826A1	$220 = 2^2 \cdot 5 \cdot 11$	849A1	$96 = 2^5 \cdot 3$	864L1	$288 = 2^5 \cdot 3^2$	885D1	$400 = 2^4 \cdot 5^2$
826B1	$180 = 2^2 \cdot 3^2 \cdot 5$	850A1	$2016 = 2^5 \cdot 3^2 \cdot 7$	866A1	$96 = 2^5 \cdot 3$	886A1	$88 = 2^3 \cdot 11$
827A1	$44 = 2^2 \cdot 11$	850B1	$288 = 2^5 \cdot 3^2$	867A1	$576 = 2^6 \cdot 3^2$	886B1	$396 = 2^2 \cdot 3^2 \cdot 11$
828A1	$48 = 2^4 \cdot 3$	850C1	$560 = 2^4 \cdot 5 \cdot 7$	867B1	$96 = 2^5 \cdot 3$	886C1	$720 = 2^4 \cdot 3^2 \cdot 5$
828B1	$144 = 2^4 \cdot 3^2$	850D1	$6720 = 2^6 \cdot 3 \cdot 5 \cdot 7$	867C1	$1632 = 2^5 \cdot 3 \cdot 17$	886D1	$5016 = 2^3 \cdot 3 \cdot 11 \cdot 19$
828C1	$84 = 2^2 \cdot 3 \cdot 7$	850E1	$192 = 2^6 \cdot 3$	867D1	$1632 = 2^5 \cdot 3 \cdot 17$	886E1	$60 = 2^2 \cdot 3 \cdot 5$
828D1	$60 = 2^2 \cdot 3 \cdot 5$	850F1	$1344 = 2^6 \cdot 3 \cdot 7$	867E1	$96 = 2^5 \cdot 3$	888A1	$400 = 2^4 \cdot 5^2$
829A1	$66 = 2 \cdot 3 \cdot 11$	850G1	$384 = 2^7 \cdot 3$	869A1	$120 = 2^3 \cdot 3 \cdot 5$	888B1	$128 = 2^7$
830A1	$256 = 2^8$	850H1	$3840 = 2^8 \cdot 3 \cdot 5$	869B1	$84 = 2^2 \cdot 3 \cdot 7$	888C1	$96 = 2^5 \cdot 3$
830B1	$2048 = 2^{11}$	850I1	$960 = 2^6 \cdot 3 \cdot 5$	869C1	$44 = 2^2 \cdot 11$	888D1	$64 = 2^6$
830C1	$320 = 2^6 \cdot 5$	850J1	$480 = 2^5 \cdot 3 \cdot 5$	869D1	$216 = 2^3 \cdot 3^3$	890A1	$64 = 2^6$
831A1	$120 = 2^3 \cdot 3 \cdot 5$	850K1	$288 = 2^5 \cdot 3^2$	870A1	$192 = 2^6 \cdot 3$	890B1	$96 = 2^5 \cdot 3$
832A1	$64 = 2^6$	850L1	$112 = 2^4 \cdot 7$	870B1	$1440 = 2^5 \cdot 3^2 \cdot 5$	890C1	$448 = 2^6 \cdot 7$
832B1	$64 = 2^6$	851A1	$88 = 2^3 \cdot 11$	870C1	$288 = 2^5 \cdot 3^2$	890D1	$72 = 2^3 \cdot 3^2$
832C1	$128 = 2^7$	854A1	$240 = 2^4 \cdot 3 \cdot 5$	870D1	$256 = 2^8$	890E1	$576 = 2^6 \cdot 3^2$
832D1	$48 = 2^4 \cdot 3$	854B1	$288 = 2^5 \cdot 3^2$	870E1	$96 = 2^5 \cdot 3$	890F1	$312 = 2^3 \cdot 3 \cdot 13$
832E1	$128 = 2^7$	854C1	$64 = 2^6$	870F1	$1120 = 2^5 \cdot 5 \cdot 7$	890G1	$200 = 2^3 \cdot 5^2$
832F1	$384 = 2^7 \cdot 3$	854D1	$336 = 2^4 \cdot 3 \cdot 7$	870G1	$256 = 2^8$	890H1	$160 = 2^5 \cdot 5$
832G1	$128 = 2^7$	855A1	$576 = 2^6 \cdot 3^2$	870H1	$64 = 2^6$	891A1	$72 = 2^3 \cdot 3^2$
832H1	$48 = 2^4 \cdot 3$	855B1	$192 = 2^6 \cdot 3$	870I1	$1600 = 2^6 \cdot 5^2$	891B1	$78 = 2 \cdot 3 \cdot 13$
832I1	$128 = 2^7$	855C1	$320 = 2^6 \cdot 5$	871A1	$224 = 2^5 \cdot 7$	891C1	$216 = 2^3 \cdot 3^3$
832J1	$384 = 2^7 \cdot 3$	856A1	$40 = 2^3 \cdot 5$	872A1	$64 = 2^6$	891D1	$216 = 2^3 \cdot 3^3$
833A1	$72 = 2^3 \cdot 3^2$	856B1	$112 = 2^4 \cdot 7$	873A1	$96 = 2^5 \cdot 3$	891E1	$270 = 2 \cdot 3^3 \cdot 5$
834A1	$2352 = 2^4 \cdot 3 \cdot 7^2$	856C1	$48 = 2^4 \cdot 3$	873B1	$480 = 2^5 \cdot 3 \cdot 5$	891F1	$78 = 2 \cdot 3 \cdot 13$
834B1	$96 = 2^5 \cdot 3$	856D1	$240 = 2^4 \cdot 3 \cdot 5$	873C1	$8096 = 2^5 \cdot 11 \cdot 23$	891G1	$72 = 2^3 \cdot 3^2$
834C1	$96 = 2^5 \cdot 3$	858A1	$192 = 2^6 \cdot 3$	873D1	$96 = 2^5 \cdot 3$	891H1	$90 = 2 \cdot 3^2 \cdot 5$
834D1	$64 = 2^6$	858B1	$576 = 2^6 \cdot 3^2$	874A1	$144 = 2^4 \cdot 3^2$	892A1	$312 = 2^3 \cdot 3 \cdot 13$
834E1	$48 = 2^4 \cdot 3$	858C1	$80 = 2^4 \cdot 5$	874B1	$3600 = 2^4 \cdot 3^2 \cdot 5^2$	892B1	$120 = 2^3 \cdot 3 \cdot 5$
834F1	$672 = 2^5 \cdot 3 \cdot 7$	858D1	$3120 = 2^4 \cdot 3 \cdot 5 \cdot 13$	874C1	$96 = 2^5 \cdot 3$	892C1	$72 = 2^3 \cdot 3^2$
834G1	$400 = 2^4 \cdot 5^2$	858E1	$576 = 2^6 \cdot 3^2$	874D1	$80 = 2^4 \cdot 5$	894A1	$1248 = 2^5 \cdot 3 \cdot 13$
836A1	$48 = 2^4 \cdot 3$	858F1	$2640 = 2^4 \cdot 3 \cdot 5 \cdot 11$	874E1	$400 = 2^4 \cdot 5^2$	894B1	$120 = 2^3 \cdot 3 \cdot 5$
836B1	$66 = 2 \cdot 3 \cdot 11$	858G1	$144 = 2^4 \cdot 3^2$	874F1	$1344 = 2^6 \cdot 3 \cdot 7$	894C1	$760 = 2^3 \cdot 5 \cdot 19$
840A1	$192 = 2^6 \cdot 3$	858H1	$288 = 2^5 \cdot 3^2$	876A1	$1848 = 2^3 \cdot 3 \cdot 7 \cdot 11$	894D1	$72 = 2^3 \cdot 3^2$
840B1	$128 = 2^7$	858I1	$1152 = 2^7 \cdot 3^2$	876B1	$120 = 2^3 \cdot 3 \cdot 5$	894E1	$1104 = 2^4 \cdot 3 \cdot 23$
840C1	$64 = 2^6$	858J1	$240 = 2^4 \cdot 3 \cdot 5$	880A1	$32 = 2^5$	894F1	$120 = 2^3 \cdot 3 \cdot 5$
840D1	$1920 = 2^7 \cdot 3 \cdot 5$	858K1	$35280 = 2^4 \cdot 3^2 \cdot 5 \cdot 7^2$	880B1	$96 = 2^5 \cdot 3$	894G1	$616 = 2^3 \cdot 7 \cdot 11$
840E1	$64 = 2^6$	858L1	$1008 = 2^4 \cdot 3^2 \cdot 7$	880C1	$576 = 2^6 \cdot 3^2$	895A1	$24 = 2^3 \cdot 3$
840F1	$128 = 2^7$	858M1	$160 = 2^5 \cdot 5$	880D1	$288 = 2^5 \cdot 3^2$	895B1	$648 = 2^3 \cdot 3^4$
840G1	$256 = 2^8$	861A1	$56 = 2^3 \cdot 7$	880E1	$672 = 2^5 \cdot 3 \cdot 7$	896A1	$32 = 2^5$
840H1	$192 = 2^6 \cdot 3$	861B1	$1632 = 2^5 \cdot 3 \cdot 17$	880F1	$96 = 2^5 \cdot 3$	896B1	$32 = 2^5$
840I1	$64 = 2^6$	861C1	$2240 = 2^6 \cdot 5 \cdot 7$	880G1	$480 = 2^5 \cdot 3 \cdot 5$	896C1	$32 = 2^5$
840J1	$128 = 2^7$	861D1	$80 = 2^4 \cdot 5$	880H1	$48 = 2^4 \cdot 3$	896D1	$32 = 2^5$
842A1	$60 = 2^2 \cdot 3 \cdot 5$	862A1	$32 = 2^5$	880I1	$64 = 2^6$	897A1	$768 = 2^8 \cdot 3$
842B1	$156 = 2^2 \cdot 3 \cdot 13$	862B1	$192 = 2^6 \cdot 3$	880J1	$144 = 2^4 \cdot 3^2$	897B1	$160 = 2^5 \cdot 5$
843A1	$60 = 2^2 \cdot 3 \cdot 5$	862C1	$54 = 2 \cdot 3^3$	882A1	$1008 = 2^4 \cdot 3^2 \cdot 7$	897C1	$48 = 2^4 \cdot 3$
845A1	$336 = 2^4 \cdot 3 \cdot 7$	862D1	$144 = 2^4 \cdot 3^2$	882B1	$144 = 2^4 \cdot 3^2$	897D1	$12480 = 2^6 \cdot 3 \cdot 5 \cdot 13$
846A1	$512 = 2^9$	862E1	$640 = 2^7 \cdot 5$	882C1	$672 = 2^5 \cdot 3 \cdot 7$	897E1	$3360 = 2^5 \cdot 3 \cdot 5 \cdot 7$
846B1	$64 = 2^6$	862F1	$128 = 2^7$	882D1	$96 = 2^5 \cdot 3$	897F1	$96 = 2^5 \cdot 3$
846C1	$384 = 2^7 \cdot 3$	864A1	$48 = 2^4 \cdot 3$	882E1	$1536 = 2^9 \cdot 3$	898A1	$168 = 2^3 \cdot 3 \cdot 7$

curve	degree	curve	degree	curve	degree
898B1	420 = $2^2 \cdot 3 \cdot 5 \cdot 7$	912K1	1440 = $2^5 \cdot 3^2 \cdot 5$	930H1	960 = $2^6 \cdot 3 \cdot 5$
898C1	78 = $2 \cdot 3 \cdot 13$	912L1	160 = $2^5 \cdot 5$	930I1	120 = $2^3 \cdot 3 \cdot 5$
898D1	48 = $2^4 \cdot 3$	913A1	120 = $2^3 \cdot 3 \cdot 5$	930J1	2080 = $2^5 \cdot 5 \cdot 13$
899A1	28 = $2^2 \cdot 7$	913B1	300 = $2^2 \cdot 3 \cdot 5^2$	930K1	160 = $2^5 \cdot 5$
899B1	58 = $2 \cdot 29$	914A1	140 = $2^2 \cdot 5 \cdot 7$	930L1	4680 = $2^3 \cdot 3^2 \cdot 5 \cdot 13$
900A1	720 = $2^4 \cdot 3^2 \cdot 5$	914B1	52 = $2^2 \cdot 13$	930M1	192 = $2^6 \cdot 3$
900B1	144 = $2^4 \cdot 3^2$	915A1	1680 = $2^4 \cdot 3 \cdot 5 \cdot 7$	930N1	1440 = $2^5 \cdot 3^2 \cdot 5$
900C1	144 = $2^4 \cdot 3^2$	915B1	64 = 2^6	930O1	512 = 2^9
900D1	288 = $2^5 \cdot 3^2$	915C1	144 = $2^4 \cdot 3^2$	931A1	546 = $2 \cdot 3 \cdot 7 \cdot 13$
900E1	288 = $2^5 \cdot 3^2$	915D1	144 = $2^4 \cdot 3^2$	931B1	126 = $2 \cdot 3^2 \cdot 7$
900F1	1440 = $2^5 \cdot 3^2 \cdot 5$	916A1	153 = $3^2 \cdot 17$	931C1	78 = $2 \cdot 3 \cdot 13$
900G1	960 = $2^6 \cdot 3 \cdot 5$	916B1	8604 = $2^2 \cdot 3^2 \cdot 239$	933A1	28 = $2^2 \cdot 7$
900H1	192 = $2^6 \cdot 3$	916C1	132 = $2^2 \cdot 3 \cdot 11$	933B1	308 = $2^2 \cdot 7 \cdot 11$
901A1	168 = $2^3 \cdot 3 \cdot 7$	916D1	60 = $2^2 \cdot 3 \cdot 5$	934A1	32 = 2^5
901B1	1480 = $2^3 \cdot 5 \cdot 37$	916E1	36 = $2^2 \cdot 3^2$	934B1	280 = $2^3 \cdot 5 \cdot 7$
901C1	120 = $2^3 \cdot 3 \cdot 5$	918A1	1440 = $2^5 \cdot 3^2 \cdot 5$	934C1	264 = $2^3 \cdot 3 \cdot 11$
901D1	840 = $2^3 \cdot 3 \cdot 5 \cdot 7$	918B1	120 = $2^3 \cdot 3 \cdot 5$	935A1	40 = $2^3 \cdot 5$
901E1	2040 = $2^3 \cdot 3 \cdot 5 \cdot 17$	918C1	792 = $2^3 \cdot 3^2 \cdot 11$	935B1	1464 = $2^3 \cdot 3 \cdot 61$
901F1	72 = $2^3 \cdot 3^2$	918D1	360 = $2^3 \cdot 3^2 \cdot 5$	936A1	64 = 2^6
902A1	2520 = $2^3 \cdot 3^2 \cdot 5 \cdot 7$	918E1	72 = $2^3 \cdot 3^2$	936B1	240 = $2^4 \cdot 3 \cdot 5$
902B1	248 = $2^3 \cdot 31$	918F1	144 = $2^4 \cdot 3^2$	936C1	384 = $2^7 \cdot 3$
903A1	48 = $2^4 \cdot 3$	918G1	144 = $2^4 \cdot 3^2$	936D1	1920 = $2^7 \cdot 3 \cdot 5$
903B1	912 = $2^4 \cdot 3 \cdot 19$	918H1	264 = $2^3 \cdot 3 \cdot 11$	936E1	128 = 2^7
904A1	96 = $2^5 \cdot 3$	918I1	216 = $2^3 \cdot 3^3$	936F1	192 = $2^6 \cdot 3$
905A1	80 = $2^4 \cdot 5$	918J1	1080 = $2^3 \cdot 3^3 \cdot 5$	936G1	128 = 2^7
905B1	200 = $2^3 \cdot 5^2$	918K1	480 = $2^5 \cdot 3 \cdot 5$	936H1	128 = 2^7
906A1	2184 = $2^3 \cdot 3 \cdot 7 \cdot 13$	918L1	360 = $2^3 \cdot 3^2 \cdot 5$	936I1	256 = 2^8
906B1	120 = $2^3 \cdot 3 \cdot 5$	920A1	720 = $2^4 \cdot 3^2 \cdot 5$	938A1	48 = $2^4 \cdot 3$
906C1	216 = $2^3 \cdot 3^3$	920B1	288 = $2^5 \cdot 3^2$	938B1	1200 = $2^4 \cdot 3 \cdot 5^2$
906D1	360 = $2^3 \cdot 3^2 \cdot 5$	920C1	32 = 2^5	938C1	192 = $2^6 \cdot 3$
906E1	24360 = $2^3 \cdot 3 \cdot 5 \cdot 7 \cdot 29$	920D1	64 = 2^6	938D1	528 = $2^4 \cdot 3 \cdot 11$
906F1	120 = $2^3 \cdot 3 \cdot 5$	921A1	456 = $2^3 \cdot 3 \cdot 19$	939A1	748 = $2^2 \cdot 11 \cdot 17$
906G1	72 = $2^3 \cdot 3^2$	921B1	72 = $2^3 \cdot 3^2$	939B1	40 = $2^3 \cdot 5$
906H1	440 = $2^3 \cdot 5 \cdot 11$	922A1	46 = $2 \cdot 23$	939C1	140 = $2^2 \cdot 5 \cdot 7$
906I1	56 = $2^3 \cdot 7$	923A1	90 = $2 \cdot 3^2 \cdot 5$	940A1	4620 = $2^2 \cdot 3 \cdot 5 \cdot 7 \cdot 11$
909A1	896 = $2^7 \cdot 7$	924A1	4200 = $2^3 \cdot 3 \cdot 5^2 \cdot 7$	940B1	360 = $2^3 \cdot 3^2 \cdot 5$
909B1	256 = 2^8	924B1	360 = $2^3 \cdot 3^2 \cdot 5$	940C1	1080 = $2^3 \cdot 3^3 \cdot 5$
909C1	48 = $2^4 \cdot 3$	924C1	72 = $2^3 \cdot 3^2$	940D1	72 = $2^3 \cdot 3^2$
910A1	960 = $2^6 \cdot 3 \cdot 5$	924D1	312 = $2^3 \cdot 3 \cdot 13$	940E1	84 = $2^2 \cdot 3 \cdot 7$
910B1	144 = $2^4 \cdot 3^2$	924E1	120 = $2^3 \cdot 3 \cdot 5$	942A1	108 = $2^2 \cdot 3^3$
910C1	288 = $2^5 \cdot 3^2$	924F1	360 = $2^3 \cdot 3^2 \cdot 5$	942B1	4608 = $2^9 \cdot 3^2$
910D1	96 = $2^5 \cdot 3$	924G1	72 = $2^3 \cdot 3^2$	942C1	2560 = $2^9 \cdot 5$
910E1	21168 = $2^4 \cdot 3^3 \cdot 7^2$	924H1	3240 = $2^3 \cdot 3^4 \cdot 5$	942D1	384 = $2^7 \cdot 3$
910F1	5280 = $2^5 \cdot 3 \cdot 5 \cdot 11$	925A1	192 = $2^6 \cdot 3$	943A1	72 = $2^3 \cdot 3^2$
910G1	240 = $2^4 \cdot 3 \cdot 5$	925B1	96 = $2^5 \cdot 3$	944A1	32 = 2^5
910H1	816 = $2^4 \cdot 3 \cdot 17$	925C1	144 = $2^4 \cdot 3^2$	944B1	96 = $2^5 \cdot 3$
910I1	64 = 2^6	925D1	1152 = $2^7 \cdot 3^2$	944C1	48 = $2^4 \cdot 3$
910J1	576 = $2^6 \cdot 3^2$	925E1	256 = 2^8	944D1	80 = $2^4 \cdot 5$
910K1	1120 = $2^5 \cdot 5 \cdot 7$	926A1	66 = $2 \cdot 3 \cdot 11$	944E1	224 = $2^5 \cdot 7$
912A1	192 = $2^6 \cdot 3$	927A1	160 = $2^5 \cdot 5$	944F1	24 = $2^3 \cdot 3$
912B1	96 = $2^5 \cdot 3$	928A1	64 = 2^6	944G1	912 = $2^4 \cdot 3 \cdot 19$
912C1	192 = $2^6 \cdot 3$	928B1	64 = 2^6	944H1	288 = $2^5 \cdot 3^2$
912D1	160 = $2^5 \cdot 5$	930A1	288 = $2^5 \cdot 3^2$	944I1	96 = $2^5 \cdot 3$
912E1	288 = $2^5 \cdot 3^2$	930B1	360 = $2^3 \cdot 3^2 \cdot 5$	944J1	56 = $2^3 \cdot 7$
912F1	480 = $2^5 \cdot 3 \cdot 5$	930C1	440 = $2^3 \cdot 5 \cdot 11$	944K1	144 = $2^4 \cdot 3^2$
912G1	96 = $2^5 \cdot 3$	930D1	2688 = $2^7 \cdot 3 \cdot 7$	946A1	48 = $2^4 \cdot 3$
912H1	480 = $2^5 \cdot 3 \cdot 5$	930E1	96 = $2^5 \cdot 3$	946B1	120 = $2^3 \cdot 3 \cdot 5$
912I1	96 = $2^5 \cdot 3$	930F1	91080 = $2^3 \cdot 3^2 \cdot 5 \cdot 11 \cdot 23$	946C1	520 = $2^3 \cdot 5 \cdot 13$
912J1	72 = $2^3 \cdot 3^2$	930G1	288 = $2^5 \cdot 3^2$	948A1	126 = $2 \cdot 3^2 \cdot 7$

curve	degree	curve	degree	curve	degree
948B1	864 = 2 ⁵ · 3 ³	970A1	4004 = 2 ² · 7 · 11 · 13	986B1	864 = 2 ⁵ · 3 ³
948C1	96 = 2 ⁵ · 3	970B1	76 = 2 ² · 19	986C1	432 = 2 ⁴ · 3 ³
950A1	160 = 2 ⁵ · 5	972A1	54 = 2 · 3 ³	986D1	96 = 2 ⁵ · 3
950B1	576 = 2 ⁶ · 3 ²	972B1	54 = 2 · 3 ³	986E1	1680 = 2 ⁴ · 3 · 5 · 7
950C1	2112 = 2 ⁶ · 3 · 11	972C1	54 = 2 · 3 ³	986F1	64 = 2 ⁶
950D1	192 = 2 ⁶ · 3	972D1	108 = 2 ² · 3 ³	987A1	112 = 2 ⁴ · 7
950E1	288 = 2 ⁵ · 3 ²	973A1	136 = 2 ³ · 17	987B1	80 = 2 ⁴ · 5
954A1	336 = 2 ⁴ · 3 · 7	973B1	88 = 2 ³ · 11	987C1	288 = 2 ⁵ · 3 ²
954B1	168 = 2 ³ · 3 · 7	974A1	180 = 2 ² · 3 ² · 5	987D1	3360 = 2 ⁵ · 3 · 5 · 7
954C1	704 = 2 ⁶ · 11	974B1	840 = 2 ³ · 3 · 5 · 7	987E1	1200 = 2 ⁴ · 3 · 5 ²
954D1	160 = 2 ⁵ · 5	974C1	189 = 3 ³ · 7	988A1	420 = 2 ² · 3 · 5 · 7
954E1	1440 = 2 ⁵ · 3 ² · 5	974D1	600 = 2 ³ · 3 · 5 ²	988B1	13260 = 2 ² · 3 · 5 · 13 · 17
954F1	144 = 2 ⁴ · 3 ²	974E1	48 = 2 ⁴ · 3	988C1	120 = 2 ³ · 3 · 5
954G1	56 = 2 ³ · 7	974F1	144 = 2 ⁴ · 3 ²	988D1	108 = 2 ² · 3 ³
954H1	112 = 2 ⁴ · 7	974G1	96 = 2 ⁵ · 3	989A1	224 = 2 ⁵ · 7
954I1	240 = 2 ⁴ · 3 · 5	974H1	480 = 2 ⁵ · 3 · 5	990A1	64 = 2 ⁶
954J1	1632 = 2 ⁵ · 3 · 17	975A1	576 = 2 ⁶ · 3 ²	990B1	1728 = 2 ⁶ · 3 ³
954K1	480 = 2 ⁵ · 3 · 5	975B1	288 = 2 ⁵ · 3 ²	990C1	1536 = 2 ⁹ · 3
954L1	192 = 2 ⁶ · 3	975C1	720 = 2 ⁴ · 3 ² · 5	990D1	600 = 2 ³ · 3 · 5 ²
954M1	240 = 2 ⁴ · 3 · 5	975D1	2016 = 2 ⁵ · 3 ² · 7	990E1	256 = 2 ⁸
955A1	440 = 2 ³ · 5 · 11	975E1	720 = 2 ⁴ · 3 ² · 5	990F1	120 = 2 ³ · 3 · 5
956A1	42 = 2 · 3 · 7	975F1	400 = 2 ⁴ · 5 ²	990G1	17920 = 2 ⁹ · 5 · 7
957A1	280 = 2 ³ · 5 · 7	975G1	128 = 2 ⁷	990H3	1728 = 2 ⁶ · 3 ³
960A1	64 = 2 ⁶	975H1	144 = 2 ⁴ · 3 ²	990I1	192 = 2 ⁶ · 3
960B1	128 = 2 ⁷	975I1	2016 = 2 ⁵ · 3 ² · 7	990J1	512 = 2 ⁹
960C1	128 = 2 ⁷	975J1	80 = 2 ⁴ · 5	990K1	1280 = 2 ⁸ · 5
960D1	384 = 2 ⁷ · 3	975K1	144 = 2 ⁴ · 3 ²	990L1	840 = 2 ³ · 3 · 5 · 7
960E1	384 = 2 ⁷ · 3	976A1	192 = 2 ⁶ · 3	994A1	64 = 2 ⁶
960F1	64 = 2 ⁶	976B1	128 = 2 ⁷	994B1	480 = 2 ⁵ · 3 · 5
960G1	128 = 2 ⁷	976C1	48 = 2 ⁴ · 3	994C1	576 = 2 ⁶ · 3 ²
960H1	384 = 2 ⁷ · 3	978A1	2280 = 2 ³ · 3 · 5 · 19	994D1	384 = 2 ⁷ · 3
960I1	128 = 2 ⁷	978B1	60 = 2 ² · 3 · 5	994E1	128 = 2 ⁷
960J1	128 = 2 ⁷	978C1	40560 = 2 ⁴ · 3 · 5 · 13 ²	994F1	128 = 2 ⁷
960K1	64 = 2 ⁶	978D1	1560 = 2 ³ · 3 · 5 · 13	994G1	864 = 2 ⁵ · 3 ³
960L1	128 = 2 ⁷	978E1	80 = 2 ⁴ · 5	995A1	44 = 2 ² · 11
960M1	128 = 2 ⁷	978F1	176 = 2 ⁴ · 11	995B1	72 = 2 ³ · 3 ²
960N1	64 = 2 ⁶	978G1	448 = 2 ⁶ · 7	996A1	270 = 2 · 3 ³ · 5
960O1	384 = 2 ⁷ · 3	978H1	72 = 2 ³ · 3 ²	996B1	624 = 2 ⁴ · 3 · 13
960P1	128 = 2 ⁷	979A1	40 = 2 ³ · 5	996C1	144 = 2 ⁴ · 3 ²
962A1	64 = 2 ⁶	979B1	1104 = 2 ⁴ · 3 · 23	997A1	80 = 2 ⁴ · 5
964A1	72 = 2 ³ · 3 ²	980A1	432 = 2 ⁴ · 3 ³	997B1	48 = 2 ⁴ · 3
965A1	84 = 2 ² · 3 · 7	980B1	1008 = 2 ⁴ · 3 ² · 7	997C1	96 = 2 ⁵ · 3
966A1	960 = 2 ⁶ · 3 · 5	980C1	96 = 2 ⁵ · 3	999A1	72 = 2 ³ · 3 ²
966B1	1560 = 2 ³ · 3 · 5 · 13	980D1	576 = 2 ⁶ · 3 ²	999B1	24 = 2 ³ · 3
966C1	4224 = 2 ⁷ · 3 · 11	980E1	672 = 2 ⁵ · 3 · 7		
966D1	192 = 2 ⁶ · 3	980F1	3024 = 2 ⁴ · 3 ³ · 7		
966E1	192 = 2 ⁶ · 3	980G1	180 = 2 ² · 3 ² · 5		
966F1	4800 = 2 ⁶ · 3 · 5 ²	980H1	144 = 2 ⁴ · 3 ²		
966G1	512 = 2 ⁹	980I1	2880 = 2 ⁶ · 3 ² · 5		
966H1	360 = 2 ³ · 3 ² · 5	981A1	128 = 2 ⁷		
966I1	960 = 2 ⁶ · 3 · 5	981B1	128 = 2 ⁷		
966J1	3960 = 2 ³ · 3 ² · 5 · 11	982A1	112 = 2 ⁴ · 7		
966K1	120 = 2 ³ · 3 · 5	984A1	432 = 2 ⁴ · 3 ³		
968A1	96 = 2 ⁵ · 3	984B1	288 = 2 ⁵ · 3 ²		
968B1	1056 = 2 ⁵ · 3 · 11	984C1	480 = 2 ⁵ · 3 · 5		
968C1	1056 = 2 ⁵ · 3 · 11	984D1	96 = 2 ⁵ · 3		
968D1	96 = 2 ⁵ · 3	985A1	96 = 2 ⁵ · 3		
968E1	960 = 2 ⁶ · 3 · 5	985B1	144 = 2 ⁴ · 3 ²		
969A1	80 = 2 ⁴ · 5	986A1	176 = 2 ⁴ · 11		

BIBLIOGRAPHY

1. A. O. L. Atkin and J. Lehner, *Hecke operators on $\Gamma_0(m)$* , Math. Ann. **185** (1970), 134–160.
2. B. J. Birch and W. Kuyk (eds.), *Modular Functions of One Variable IV*, Lecture Notes in Mathematics 476, Springer-Verlag, 1975.
3. B. J. Birch and H. P. F. Swinnerton-Dyer, *Notes on elliptic curves. I.*, J. Reine Angew. Math. **212** (1963), 7–25.
4. A. Brumer and K. Kramer, *The rank of elliptic curves*, Duke Math. J. **44** (1977), 715–743.
5. J. P. Buhler and B. H. Gross, *Arithmetic on curves with complex multiplication. II*, Invent. Math. **79** (1985), 11–29.
6. J. P. Buhler, B. H. Gross and D. B. Zagier, *On the conjecture of Birch and Swinnerton Dyer for an elliptic curve of rank 3*, Math. Comp. **44** (1985), 473–481.
7. H. Carayol, *Sur les représentations l -adiques attachées aux formes modulaires de Hilbert*, C. R. Acad. Sc. Paris **296** (1983), 629.
8. J. W. S. Cassels, *Elliptic curves*, LMS Student Texts, Cambridge University Press, 1991.
9. H. Cohen, *A Course in Computational Algebraic Number Theory*, Graduate Texts in Mathematics 138, Springer-Verlag, 1993.
10. I. Connell, *Notes on elliptic curves*, (Unpublished lecture notes, McGill University), 1991.
11. D. Cox, *The arithmetic-geometric mean of Gauss*, l'Enseignement Math. **30** (1984), 270–330.
12. J. E. Cremona, *Hyperbolic tessellations, modular symbols, and elliptic curves over complex quadratic fields*, Compositio Math. **51** (1984), 275–323.
13. J. E. Cremona, *Hyperbolic tessellations, modular symbols, and elliptic curves over complex quadratic fields (Addendum and Errata)*, Compositio Math. **63** (1987), 271–272.
14. J. E. Cremona, *Modular symbols for $\Gamma_1(N)$ and elliptic curves with everywhere good reduction*, Math. Proc. Camb. Phil. Soc. **111** (1992), 199–218.
15. J. E. Cremona and E. Whitley, *Periods of cusp forms and elliptic curves over imaginary quadratic fields*, Math. Comp. **62** (1994), 407–429.
16. J. E. Cremona, *The analytic order of III for modular elliptic curves*, J. de Théorie des Nombres de Bordeaux **5** (1993), 179–184.
17. J. E. Cremona, *Computing the degree of the modular parametrization of a modular elliptic curve*, Math. Comp. **64** (1995), 1235–1250.
18. J. E. Cremona, *Computing periods of cusp forms and modular elliptic curves*, Experimental Mathematics (1996) (to appear).
19. J. E. Cremona and P. Serf, *Computing the rank of elliptic curves over real quadratic number fields of class number 1*, University of Exeter Department of Mathematics Preprint **M96/14** (1996).

20. J. E. Cremona, *Classical invariants and 2-descent on elliptic curves*, University of Exeter Department of Mathematics Preprint **M96/16** (1996).
21. B. Edixhoven, *On the Manin constants of modular elliptic curves*, Arithmetic Algebraic Geometry, Progress in Mathematics **89**, Birkhauser, 1991, pp. 25–39.
22. G. Faltings, *Endlichkeitssätze für abelsche Varietäten über Zahlkörpern*, Invent. Math. **73** (1983), 349–366.
23. S. Fermigier, *Construction of high rank elliptic curves over $\mathbb{Q}(T)$ and \mathbb{Q} with nontrivial 2-torsion*, C. R. Acad. Sc. Paris **322** (1996), 949–952.
24. L. Figueiredo, *Serre's conjecture over imaginary quadratic fields*, PhD thesis, University of Cambridge, 1995.
25. B. H. Gross and D. Zagier, *Points de Heegner et dérivées de fonctions L* , C. R. Acad. Sc. Paris **297** (1983), 85–87.
26. B. H. Gross and D. Zagier, *Heegner points and derivatives of L -series*, Invent. Math. **84** (1986), 225–320.
27. D. Husemöller, *Elliptic curves*, Springer-Verlag, 1987.
28. A. W. Knap, *Elliptic Curves*, Mathematical Notes **40**, Princeton University Press, 1992.
29. V. I. Kolyvagin, *Finiteness of $E(\mathbb{Q})$ and $\text{III}_{E/\mathbb{Q}}$ for a subclass of Weil curves*, Math. USSR Izvest. **32** (1989), 523–542.
30. A. Kraus, *Quelques remarques à propos des invariants c_4 , c_6 et Δ d'une courbe elliptique*, Acta Arith. **54** (1989), 75–80.
31. S. Lang, *Elliptic functions*, Addison-Wesley, 1973.
32. S. Lang, *Introduction to modular forms*, Springer-Verlag, 1976.
33. S. Lang, *Elliptic curves: diophantine analysis*, Springer-Verlag, 1976.
34. M. Laska, *An algorithm for finding a minimal Weierstrass equation for an elliptic curve*, Math. Comp. **38** (1982), 257–260.
35. M. Laska, *Elliptic curves over number fields with prescribed reduction type*, Aspects of Mathematics Vol. E4, Vieweg, 1983.
36. E. Lutz, *Sur l'équation $y^2 = x^3 - Ax - B$ dans les corps p -adic*, J. Reine Angew. Math. **177** (1937), 237–247.
37. Ju. I. Manin, *Parabolic points and zeta-functions of modular curves*, Math. USSR-Izv. **6** (1972), 19–64.
38. B. Mazur, *Courbes elliptiques et symboles modulaires*, Séminaire Bourbaki No. 414 (1972), 277–294.
39. B. Mazur, *Modular curves and the Eisenstein ideal*, IHES Publ. Math. **47** (1977), 33–186.
40. B. Mazur, *Rational isogenies of prime degree*, Invent. Math. **44** (1978), 129–162.
41. B. Mazur and H. P. F. Swinnerton-Dyer, *Arithmetic of Weil curves*, Invent. Math. **25** (1974), 1–61.
42. L. Merel, *Opérateurs de Hecke pour $\Gamma_0(N)$ et fractions continues*, Annales de l'Institut Fourier de l'Université de Grenoble **41 no. 3** (1991), 519–537.
43. J.-F. Mestre, *La méthode des graphes. Exemples et applications.*, Proceedings of the International Conference on Class Numbers and Units of Algebraic Number Fields (Y. Yamamoto and H. Yokoi, eds.), Katata, 1986, pp. 217–242.
44. M. R. Murty and V. K. Murty, *Mean values of derivatives of modular L -series*, Ann. Math. **133** (1991), 447–475.

45. K. Nagao, *An example of an elliptic curve over \mathbb{Q} with rank ≥ 20* , Proceedings of the Japan Academy Series A **69** (1993), 291–293.
46. T. Nagell, *Solution de quelque problèmes dans la théorie arithmétique des cubiques planes du premier genre*, Wid. Akad. Skrifter Oslo I (1935).
47. A. Néron, *Modèles minimaux des variétés abéliennes sur les corps locaux et globaux*, IHES Publ. Math. **21** (1964), 361–482.
48. R. G. E. Pinch, *Elliptic curves over number fields*, DPhil thesis, University of Oxford, 1982.
49. R. G. E. Pinch, *Elliptic curves with everywhere good reduction*, (preprint).
50. M. Pohst and H. Zassenhaus, *Algorithmic Algebraic Number Theory*, Encyclopedia of mathematics and its applications, Cambridge University Press, 1989.
51. R. Schoof, *Elliptic curves over finite fields and the computation of square roots mod p* , Math. Comp. **44** (1985), 483–494.
52. P. Serf, *The rank of elliptic curves over real quadratic number fields of class number 1*, PhD thesis, Universität des Saarlandes, Saarbrücken, 1995.
53. J.-P. Serre, *Propriétés galoisiennes des points d'ordre fini des courbes elliptiques*, Invent. Math. **15** (1972), 259–331.
54. J.-P. Serre, *Résumé de cours*, Annuaire du Collège de France, 1984–85.
55. G. Shimura, *Introduction to the arithmetic theory of automorphic functions*, Math. Soc. Japan No. 11, 1971.
56. S. Siksek, *Descent on curves of genus one*, PhD thesis, University of Exeter, 1995.
57. S. Siksek, *Infinite Descent on Elliptic Curves*, Rocky Mountain Journal of Mathematics **25 no. 4** (1995), 1501–1538.
58. J. H. Silverman, *The Arithmetic of Elliptic Curves*, GTM 106, Springer-Verlag, 1986.
59. J. H. Silverman, *Computing heights on elliptic curves*, Math. Comp. **51** (1988), 339–358.
60. J. H. Silverman, *The difference between the Weil height and the canonical height on elliptic curves*, Math. Comp. **55** (1990), 723–743.
61. J. H. Silverman, *Advanced Topics in the Arithmetic of Elliptic Curves*, GTM 151, Springer-Verlag, 1994.
62. J. H. Silverman, *Computing canonical heights with little (or no) factorization*, Math. Comp. (1996) (to appear).
63. N. P. Smart, S. Siksek and J. R. Merriman, *Explicit 4-descents on an elliptic curve*, Acta Arith. **LXXVII.4** (1996), 385–404.
64. H. P. F. Swinnerton Dyer and B. J. Birch, *Elliptic curves and modular functions*, Modular Functions of One Variable IV, Lecture Notes in Mathematics 476, Springer-Verlag, 1975.
65. J. Tate, *Algorithm for determining the singular fiber in an elliptic pencil*, Modular Functions of One Variable IV, Lecture Notes in Mathematics 476, Springer-Verlag, 1975.
66. J. Tate, Letter to J.-P. Serre, Oct. 1, 1979.
67. D. J. Tingley, *Elliptic curves uniformized by modular functions*, DPhil thesis, University of Oxford, 1975.
68. J. Vêlu, *Isogénies entre courbes elliptiques*, C. R. Acad. Sc. Paris **273** (1971), 238–241.
69. D. Zagier, *Modular Parametrizations of Elliptic Curves*, Canadian Math. Bull. **28** (1985), 372–384.
70. J. Gebel and H. G. Zimmer, *Computing the Mordell-Weil Group of an Elliptic Curve over \mathbb{Q}* , Elliptic Curves and Related Topics (H. Kisilevsky and M. Ram Murty, eds.), CRM Proceedings and Lecture Notes, vol. 4, 1994, pp. 61–83.