William A. Stein ***Project Description***

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CSUMS: SAGE – Software for Algebra and Geometry Experimentation

1 Major Highlights

- A group of **40 undergraduate students** (8 each year) will do **research and development** on SAGE, which is software for algebra, geometry, number theory, cryptography, numerical analysis, statistics, and other areas.
- Students will **write proposals** and give at least **15 presentations**.
- Students will speak at **national conferences** and **workshops**.
- Students will **work together**, with the PI, with graduate students, and become part of a worldwide vibrant research and development community.
- A group of 3–6 **graduate students** will receive 1-2 quarters support each year to work with undergraduates on their projects.
- SAGE is open source software that uses GAP, Maxima, Singular, PARI, Python and more specialized software, to build an **environment for rigorous mathematics computation**, and provides **uniform interfaces** to Mathematica, Maple, Magma, Macaulay2, and other systems.
- This is a **continuing project** that has already been very successful.

2 Introduction

The proposed project is for a group of **40 undergraduate students** (8 each academic year for 5 years) at the University of Washington (UW) to do research and development on high quality mathematics software for algebra, geometry, number theory, cryptography, numerical analysis, statistics, and other areas. By joining the SAGE project, the PI hopes that participants will become involved in mathematical research, gain extensive knowledge about mathematical software, make long-term connections with a vibrant research and development community, and contribute tools that will be used by teachers and expert researchers in numerous fields. They will also learn teaching and writing skills, and really understand some part of mathematics deeply, which will prepare them to apply computational mathematics in graduate school and industry.

This proposal is not merely theoretical; it is an enhancement of a project the PI is currently directing using an NSF grant and startup funds. His experience collaborating with undergraduates in research on theoretical and computational projects has convinced him that undergraduates can do work that is esteemed by the mathematical research and education communities.

2.1 SAGE

The PI is the main author and director of SAGE—Software for Algebra and Geometry Experimentation [SAG], a project that he started in January 2005. Both the development model of SAGE and the technology itself is distinguished by a strong emphasis on openness, community, cooperation, and collaboration: *SAGE is about building the car, not reinventing the wheel.* SAGE uses the open source math software GAP [GAP], Maxima [Max], Singular [Sin], PARI [PAR], Python, and more specialized software, to build an environment for rigorous mathematics computation. SAGE also provides interfaces to Mathematica, Maple, Magma [Mag], Macaulay2 [GS], and other systems.

The overall goal of SAGE is to create an optimal computational environment for mathematical research and education. This involves the creation of software, databases, and web sites. Undergraduate participants will develop and implement advanced mathematical algorithms, design user interfaces, run computations, and give presentations in seminars and at national conferences.

> "I think the SAGE developers were very bold—maybe even audacious—to actually attempt this. And they are doing it in a largely pragmatic way without attempting to incorporate the more formal and theoretical ideas developed by the OpenMath community [Ope].
>
> One might have been tempted to predict an early failure to this effort but on the contrary SAGE seems to be growing more rapidly than any other computer algebra research and development effort."
>
> –Bill Page (one of the main current Axiom developers [Axi])

2.2 Prior Support

The PI was awarded NSF grant DMS-0555776 (and DMS-0400386) from the ANTC program in the amount of $177,917 for the period 2004–2007. Funds from this grant were used to:

1. Run a **workshop** on SAGE at UCSD [JS06a] that had one undergraduate speaker (Steven Sivek, MIT) and that several undergraduates participated in (David Roe, MIT; Alex Clemesha, UCSD; and Naqi Jaffery, UCSD).
2. Purchase a **16-processor compute server** with 64GB RAM that is the central SAGE development machine, which hosts the SAGE website, several dozen SAGE developer web sites, the online SAGE notebook, etc.
3. Run a **workshop** on SAGE at UW [JS06b] that had several undergraduate **speakers** and participants.

The PI received startup money at UCSD and UW, some of which was used to fund work on SAGE by undergraduates Alex Clemesha, Naqi Jaffery, Tom Boothby, Yi Qiang, Emily Kirkman, Jennifer Balakrishnan, and Bobby Moretti.

3 Overview

> "Students at UW did not have any easy way to get started doing mathematics research (no washing petri dishes, etc.). This is something that I have experienced personally and know that many of my math major friends are frustrated about. SAGE is opening the door to advanced mathematics research to many students that wouldn't have this chance otherwise." — Yi Qiang, UW Sophomore

3.1 Target Student Participants

The intended student participants in this project will be majors in the mathematical sciences with a strong interest in using computation to improve mathematics research and education, and being part of an exciting software project.

Students must have a strong interest in mathematics so they can understand the algorithms and theoretical background that underlies SAGE. They must also have exceptional computer skills, because work on SAGE involves significant software engineering. Perhaps because of the programming culture in the Seattle area, there are several undergraduates at UW that are talented at programming and have a strong interest in mathematics, but have not found a creative way to combine together their passions. This project targets them.

Since SAGE is designed as a tool for attacking problems at the forefront of research, students will become involved with advanced mathematics. For example, after Tom Boothby (undergraduate, UW) began working on SAGE, he carried out a project to enumerate and draw all possible isogeny diagrams of elliptic curves, in collaboration with the world leader in elliptic curve enumeration (John Cremona). Another student, Jennifer Balakrishnan (undergraduate, Harvard) implemented in SAGE much of a program for computing p-adic heights on elliptic curves, and gave three talks on the underlying theory and implementation. Alex Clemesha (undergraduate, UCSD) actively participated in [Ste06b] on a project that drew plots of modular forms and fundamental domains for congruence subgroups.

3.2 Intellectual Focus and Innovative Strategies

Like undergraduates who work in a Biology lab, students in this project will be involved with the SAGE developer community at UW and participate in an important national project. Student work will focus on algorithm research and implementation, proposal writing, and giving presentations at local seminars and national conferences. Students will develop, extend, document, test and implement algorithms for computing with mathematical structures. Their work will be reviewed, commented on, and used by the growing worldwide community of SAGE users, who may use the student's work for their research or teaching.

3.3 Organizational Structure and Timetable

The time table for the *3-quarter academic year* is given below. Every student will give at least one 30-minute presentation on their work every other week. Thus over the course of the year **each student will give over 15 presentations**.

1. **First Quarter:** Students will become familiar with mathematical software, potential projects, determine the state of current knowledge and implementation in a given area, create a proposal that will be assessed by the SAGE development community, and design a strategy for implementing it. Though each student will design their own project, they will give numerous talks, which strongly encourages collaboration.

   (a) **Basic familiarity:** (Week 1) Basic background in mathematical software. Overview of what is available and what it does.

   (b) **Examples for the SAGE documentation:** (Weeks 2–3) Creating examples for the documentation is a good way to learn SAGE and SAGE's distributed revision control system [Mer], and it improves the overall quality of SAGE.

   (c) **Choose a project:** (Week 4) Students will choose a project based on presentations by the PI, student participants from previous years, and UW graduate students. They will also be encouraged to find projects related to current research activities at UW (especially in Mathematics, Biology, Computer Science, and Physics). The undergraduates may form collaborations with graduates students at this point, as several graduate students at UW are involved in SAGE development.

   (d) **Write a proposal:** (Weeks 5–6) The students will write a proposal that contains the following:

      i. **A description** of the mathematical objects, problems, and algorithms that the project will address.

      ii. **A survey** of what existing software (both free and commercial) does toward computing with the mathematical objects described in (i). This will include lists of functions, descriptions of user interfaces, and implementation details (when available). Students will be encouraged to create benchmarks.

      iii. **A list of functionality** the student would like to implement. Students should explain why certain functionality will be omitted, e.g., some algorithms (e.g., four descents on elliptic curves) have only been implemented in closed-source non-free software, and the implementation took the world's top experts several years—it would be unreasonable for an undergraduate to recreate this (nonetheless, they should make that functionality available via SAGE interfaces to people who have the nonfree software). Whenever possible, in keeping with SAGE's strategy, students will be encouraged to implement functionality by building on other high-quality free software, e.g., students implementing numerical computation in SAGE might use [GSL], which is a mature library included with SAGE.

iv. **A timeline** describing how long students believe it will take to implement what is described above. (Such estimates are difficult, so students will get substantial feedback on their timelines.)

This proposal will be circulated for feedback not just to the PI and other student participants, but to the community of SAGE developers. All quarter, the PI will encourage the students to create prototypes and give demos in order to understand their projects more deeply.

(e) **Solicit feedback:** (Weeks 7–8) Get further feedback from the SAGE development community and mathematicians in the target area. In particular, students will make connections with experts in the target area, graduate students, and explore possibilities for collaboration.

(f) **Create final plan:** (Weeks 9–10) Revise and polish the proposal and its implementation plan, and circulate for feedback

2. **Second Quarter:** During the second quarter students will focus on implementing their projects. They will give frequent presentations on the status of their work and post regular snapshots of code. This will develop the student's communication skills and ability to work together on a large project.

3. **Third Quarter:** The focus of the third quarter will be to ensure that the student projects are of professional quality, are ready to be used by the academic research community, and will be maintainable long into the future.

   (a) **Quality control**: Students will write extra documentation, include more comments in source code, and increase the number of tests and examples. Each student's code and documentation will be refereed by other SAGE developers and other students in the program. In some cases, students will be encouraged to write research papers based on their work, e.g., because of the structured nature of SAGE, a project on numerical computation might involve creating new algorithms to convert between approximate numerical and exact algebraic objects.

   (b) **Advertise and Respond to Feedback:** The quality of student work is enhanced when people use it and provide feedback, and students put in more effort when they know they will be presenting their work at a conference. The PI receives frequent invitations to speak at conferences and workshops, and can often have some of the students give presentations or present demos. This is already a successful strategy, e.g., Alex Clemesha (UCSD, undergraduate) gave a talk at Caltech during SCIPY 2006 [SCI06], and Jennifer Balakrishnan (Harvard, undergraduate) spoke at MSRI 2006 [Ste06b]. The PI also organizes workshops and conferences, some which provide an ideal opportunity for student presentations. (The PI is organizing or co-organizing *eight* workshops during the period February 2006 – July 2007 [JS06a, SIM06, Ste06b, JS06b, BDG+07, PSS+07, CDR+07, KRR+07].) The PI also gets students involved with workshop organization, e.g., Jennifer Balakrishnan helped create the schedule for [Ste06b]. These activities train the next generation of students in the profession of being mathematicians.

4 Nature of Student Activities

The PI intends to teach a year-long course on mathematical software that the 8 students involved in this project will take. Other students who take the course would not receive stipends from this grant. There will be three meetings per week. Each week, during the Wednesday meeting the PI will give a lecture, and during the Monday and Friday meetings students will give presentations (2 per meeting).

The PI is experienced at organizing student seminars and presentations; he ran a SAGE seminar at UW in 2006 with numerous undergraduate talks, he ran an MSRI graduate student summer school [Ste06b], and led two freshman seminars at Harvard (one on elliptic curves and one on Fermat's Last Theorem) which consisted of almost 3 hours of student presentations per week.

The PI expects that students in the project will continue to contribute after their year. This would be supported by other funding sources that the PI might secure, or using support the students would obtain, e.g., the UW Mary Gates Research Scholarship provides major support for about 100 undergraduate research projects each year. The PI is involved with UW's summer REU, and some students from the SAGE project may participate in that REU. Students who go to graduate school will be likely to contribute to SAGE as part of their thesis research.

4.1 The Crucial Role of Graduate Students

Graduate students will be funded on an **opportunistic basis**. Graduate students who through the SAGE seminars, etc., form a close collaboration with an undergraduate will be offered 1–2 quarters support during which they will work very closely with that undergraduate on their project.

4.2 A Sample of Specific Projects

The PI is one of the more sought after people by mathematicians, for computational confirmation of conjectures, for algorithms, for data, and for ways of formulating problems so as to make them more accessible to algorithms. He has also done a range of consulting work for DoD, and has connections with researchers in algebraic geometry, numerical analysis, combinatorics, and other areas. His position directing SAGE also brings him into contact with a wide range of computational work. The PI is aware of numerous research problems that could become student projects. The rest of this section contains a sample of such projects.

4.2.1 The SAGE Notebook

SAGE has an innovative web-browser based interface, which was designed and implemented by the PI, Tom Boothby (undergraduate, UW) and Alex Clemesha (undergraduate, UCSD). The PI and Barry Mazur are using the notebook to bring alive a popular book they are writing on the Riemann Hypothesis, and to illustrate an article in Nature [Maz06]. Many possibilities for future work remain:

1. **Integrate** the SAGE notebook with the Moinmoin Wiki [Moi], the Trac bug and feature tracker [Tra], and Mercurial [Mer] to create a complete web-based development environment.
2. **Create typeset output** for notebook sessions.
3. Extend the notebook interface with support for **interactive parallel computation**, e.g., special parallel syntax extensions and graphical views of what dozens of processes are doing (either locally or over a network).

4.2.2 Mathematical Visualization Using SAGE

Each of the following would be usable from the web-based SAGE notebook.

1. Ray tracing – develop a 2D and 3D plotting systems that uses the Tachyon 3D [Sto] ray tracer for fast beautiful output.
2. 3D graphics – build a package that uses ray tracing, VRML (virtual reality browser plugin), and matplotlib.
3. Systematic plotting functionality for a wide range of objects in SAGE including elliptic curves, modular forms and matrices.

4.2.3 p-adic Numbers

The students would research how other people have designed software for computing with p-adic numbers, both their implementations and interfaces. This includes studying Magma, PARI, and local fields libraries. It would also include a literature search, both in mathematics and computer science, and analyses of theoretical and actual performance (see, e.g., [GG03]). Next, in collaboration with the SAGE community, students would design an ideal interface. The interface should support an implementation that takes into account (at least) the following issues:

1. **Fast arithmetic**, for small p, for large p, for $p = 2$, for both large precision, small precision, etc.; all situations must be covered.
2. **Fast conversions** between p-adic integers, p-adic rationals, plain integers and rationals, and elements of $\mathbf{Z}/p^n\mathbf{Z}$, when these conversions make sense.
3. **Finite extensions** of p-adic fields, in both the ramified and unramified cases. End users should be able to take a general number field and construct the local field corresponding to a selected prime ideal.
4. **Fast polynomial arithmetic** with p-adic coefficients. This is much trickier than it sounds at first, since many of the usual algorithms fail if the precision of the coefficients varies greatly.

This carefully researched approach is crucial, especially for this project. Otherwise much time will be wasted, e.g., Magma has already been through three implementations (even interfaces!) for p-adic numbers.

4.2.4 Graph Theory

Provide a unified interface to existing state-of-the-art libraries for graph theory, then design and implement connections with existing mathematical structures in SAGE, e.g., plotting pictures of graphs, associating graphs to matrices, Cayley graphs associated to groups, call trees when profiling code, isogeny structures of elliptic curves, subgroup lattices, etc.

4.2.5 Databases

Steven Sivek (MIT, undergraduate) created a SAGE version of Sloane's tables of integer sequences [Slo] and gave a talk about his work at [JS06a]. There are numerous similar possible student projects that involve systematically organizing the results of years of mathematical computations. Possible databases include:

1. Modular forms. The PI is an organizer of a forthcoming workshop [KRR$^+$07] on creating new databases of modular forms.
2. Elliptic curves. Extend and reorganize tables of Cremona and Stein-Watkins.
3. Modular abelian varieties. Much of the PI's work is on computing these.
4. Modular curves.
5. Graphs that are optimal with respect to some property.
6. Algebraic curves with special properties.
7. Special prime numbers, e.g., all known Mersenne primes.
8. Number theory databases and tables listed at [Mat].
9. Hooks into NIST's database of functions [NIS].
10. Dan Gordon's databases of coverings and cyclic difference sets [Gor].

This project would also involve creating a way to build web-based interfaces to SAGE databases, so that mathematicians can browse the databases without having to know any SAGE commands.

4.2.6 The Birch and Swinnerton-Dyer Conjecture

The PI has used SAGE to do substantial work with undergraduates on explicit verification of the Birch and Swinnerton-Dyer conjecture for elliptic curves [GJP+05], but much remains to be done. In particular, create packages for doing 3-descents on elliptic curves, for computing Heegner points, for computing with L-functions over number fields, and for computing p-adic L-functions. The PI submitted a proposal entitled *Explicit Approaches to the Birch and Swinnerton-Dyer Conjecture* for his research on such problems to the NSF ANTC program (Oct. 2006).

5 Connection to Regular Academic Studies

5.1 Local Impact

Undergraduate participants in the project will be encouraged to give talks in the UW undergraduate mathematics seminar to show other undergraduates the exciting work they are doing. There is an undergraduate research project day at UW in April, during which participants could give demos using a laptop. The PI has connections with Microsoft Research and has had two undergraduates working on SAGE visit there with him and give a talk. The UW mathematics department has connections with local high schools, and students will be encouraged to speak there about SAGE. The PI introduced students in UW's regular REU program to SAGE who made crucial use of SAGE in their 2006 summer research.

5.2 National Impact

Talks the students and PI give will be recorded as audio files and made easily available online. Such "podcasts" are currently popular among SAGE developers.

Some student projects will be of interest to mathematics teachers and students at all levels. Instead of them having to pay large sums of money to buy software for classroom instruction, they will have the option to use SAGE for free. Many teachers have expressed concern that currently they teach their students how to use tools that the students (or their employers) will not be able to afford after they graduate. At many institutions *and especially US high schools*, purchasing computer software is a significant burden.[1] The PI has had conversations with high school teachers who are excited about using SAGE for teaching, since they cannot afford to use commercial systems in their classrooms and much free mathematical software and its documentation does not satisfy their needs. SAGE also has a potential to impact mathematics students in third world countries; the PI has

[1] Commercial math software is expensive. The list price for one personal non-academic non-government copy of Mathematica is $**1880** (and $3135 for certain architectures), for Maple is $**1995**, for MATLAB is $**1900** (plus hundreds to thousands of dollars for each optional package), and Magma is $**1150** (educational rate—the non-educational rate is not available online).

student and faculty collaborators on SAGE development in Bogota, Colombia and in Uruguay, and regularly receives appreciative emails from third world countries for the work he has already done.

6 Research Environment and Mentoring Activities

Structure of mentoring:

- Create a **flexible lab environment**: Having a common space for SAGE developers to come together and work is vital to encouraging collaboration. Fortunately, for SAGE work dedicated lab space is not needed because of powerful laptops and ubiquitous wifi. By purchasing one laptop to be used by each participant, and making a schedule of lab meeting places, it will be possible to create a flexible and pleasant collaborative environment.
- Bring in 4 **mentors for month-long periods**. Mentor visitors would do work on SAGE while also leading and mentoring student projects. They would be chosen with regard to diversity issues and strong mentoring and computational abilities. The PI has developed professional relationships with the following potential mentor visitors: Lassina Dembele, Edray Goins, Jon Hanke, David Joyner, David Kohel, David Savitt and Helena Verrill. These are all professional mathematicians who the PI has done joint work with. Also one is a woman, one is African American (and very experienced at mentoring undergrads), and one is African Canadian.
- A **weekly SAGE seminar** attended by faculty and graduate students.
- Six quarters of **graduate student support** so that between 3–6 graduate students who develop mentoring relationships with students will be able to do focused work with the students 1–2 quarters each year.

The participants will have experiences that they likely would not have until graduate school—not only will they cut their teeth doing significant research, but they will prepare and give nontrivial talks on a regular basis.

The PI has a strong **track record of mentoring undergraduates** in theoretical and computational research. During the last five years he has directed over 20 projects with a nontrivial research component at Harvard, UCSD, and UW, many of which are available at [Ste06a]. For example, he directed the Harvard senior theses of Jayce Getz, John Gregg, Dimitar Jetchev, Andre Jorza, Seth Kleinerman, Daniellie Li, Chris Mihelich and David Speyer. He ran summer research programs on the Birch and Swinnerton-Dyer conjecture at Harvard during 2003 and 2004 with five students (Jennifer Balakrishnan, Andrei Jorza, Stefan Patrikas, Jennifer Sinnott, Tseno Tselkov). When Baur Bektemirov was a freshman at Harvard, he did a year-long project with the PI in which he computed surprising statistics about elliptic curves that led to a joint paper in the Bulletins of the AMS ([BMSW06]). The PI also worked for a year with Kevin Grosvenor, another Harvard freshman, on drawing pictures of L-functions. At UCSD the PI worked on SAGE development with Naqi Jaffery and Alex Clemesha, and since April 2006 at UW he has worked with UW undergraduates Tom Boothby, Emily Kirkman, Bobby Moretti, and Yi Qiang and MIT undergraduates Steven Sivek and David Roe on research related to SAGE. The PI is also currently working with six graduate students.

The PI has been at UW since April 2006, and has been pleased with the four undergraduate students he has worked with there; their computational skills and ability to network with other students are extraordinary. The mathematics undergraduate program at UW attracts some of the best UW students and these students are equal to the best undergraduates the PI has encountered anywhere: the mathematics department at UW has had five Goldwater Scholars in the past three years, three of the past four College of Arts and Sciences Deans Medalists in Science, five teams of Outstanding Winners in the CO-MAP Mathematical Contest in Modeling during the past five years, and most recently a Rhodes Scholar and an Astronaut Foundation Scholar. Also, the Seattle area has substantial programming talent due to the proximity of Microsoft and Digipen (the PI has given talks at both places).

7 Student Recruitment and Selection

The goal of this project is to bring together a group of 40 undergraduate students (over 5 years) to create mathematical software in collaborate with other students and faculty, and give numerous presentations. Finding and building good professional relationships with a range of students is a fascinating problem, and the PI has succeeded so far by a combination of methods. He gives talks and gets involved with student activities at UW whenever possible, e.g., he led a 2 week SIMUW [SIM06] high school workshop that used SAGE, he uses SAGE in teaching courses, visits other classes, and attends events for minorities in science[2]. He intends to continue these activities, always on the lookout for exceptional students. Also, students working on SAGE enjoy their experiences, which brings in new students.

The PI intends to invite a diverse group of mentors for up to 1 month each during the academic year to participate in the program while working with students. For example, the PI has worked on elliptic curve computations with Edray Goins (Assistant Professor of Mathematics, Purdue University), who is African American, and well situated to contribute to SAGE (he is an organizer of [BDG$^+$07]).

[2] The PI has close contacts with the Native American community in Seattle and regularly attends local events.

8 Project Management

The PI, William Stein, is a tenured associate professor at the host institution. He will bear responsibility for fulfilling the technical requirements of the project, including submission of annual reports. Because of his work on SAGE and workshop experience, the PI has become skilled at organizing projects and meetings. The PI will organize the seminar and give weekly talks. He will choose students for participation in the project in consultation with graduate students and faculty.

The following faculty have agreed to serve as an advisory board for this project:

- Chuck Doran: UW (differential geometry, number theory, string theory)
- Ioana Dumitriu: UW (probability, combinatorics, numerical analysis)
- Jim Morrow: UW (combinatorics, inverse problems)
- David Savitt: University of Arizona (number theory)
- Rekha Thomas: UW, (combinatorics, discrete geometry)

Morrow, in particular, has an incredible record working on research with undergraduates (e.g., he has run the UW REU for many years).

The advisory board will help with assessment of this project, and will be consulted on decisions regarding the program. In particular, they will be encouraged to comment on student proposals. Since their research spans a wide range of mathematics, they may be in a position to provide better feedback on potential student projects (e.g., suggest people for students to meet, conferences for them to attend, problems for them to investigate, etc.). Moreover, those faculty who are at UW might run the weekly seminar course if the PI is not available.

9 Project Evaluation and Reporting

9.1 Documentation and Dissemination

Audio and slides from talks, student proposals, code from projects, and research papers the students write, will all be freely made available online under a license compatible with the GNU Public License. Papers that make new contributions will be submitted to journals for publication. Journals will be chosen to have a copyright policy that allows papers to be distributed as part of the SAGE documentation, and to be updated by other SAGE developers later.

9.2 Quarterly Evaluation

The quarterly evaluation will involve forms filled out by the student participants. These will be both the standard university course evaluation forms, and custom forms designed by the PI, in consultation with the advisory board, which specifically address the project.

9.3 Annual Evaluation

The annual evaluation will summarize the quarterly evaluations. Near the end of the academic year all students involved in the project will give final presentations on their work at two 3-hour workshops that the UW members of the advisory board will be invited to attend. Each student will give a 45-minute presentation about their work that includes:

1. A general overview of the project and survey of relevant mathematics.
2. A report on how the student's work is used by mathematicians and students.
3. Difficulties the student had to overcome.

The PI will synthesize feedback from the advisory board and the student projects. The **ultimate measure of success** for a student project will be a combination of the following criterion, which the PI will explain to the students on day 1:

1. The extent to which the project contributes a high quality well-documented component of SAGE that is **valued by members of the mathematical community** and addresses actual needs, as attested to by letters from users.
2. How well students **respond to feedback** from refereeing (by the PI, by other students, and by outside users) during the third quarter.
3. General impression of the **advisory board**.

9.4 Tracking Beyond Project

Alumni of the program will likely have contributed a substantial component to SAGE. It will thus be *essential* to keep in touch with them for help with maintenance, for advice on how to move their work forward and modify it, and to benefit from what they have learned. Alumni will be asked, when possible, to speak to the current group of students. In some cases alumni will participate in workshops that are relevant to the software they wrote (the PI intends to run at least two SAGE-related workshops per year, indefinitely). These activities provide many opportunities for feedback about how involvement with SAGE has affected student career paths. The PI will also contact all students who were involved in the program once per year for an update on what they are currently doing, and will record the results. This will inform the five-year evaluation.

9.5 Five-Year Evaluation

The five-year evaluation will summarize the previous annual evaluations. It will also assess the long-term worldwide broader impact of the SAGE project on education and research: research papers resulting from student work, the extent to which SAGE is used in undergraduate and graduate education, and placement of students from this program in jobs and graduate schools.

References

[Axi] Axiom, *A general purpose system for doing mathematics by computer*, http://wiki.axiom-developer.org/.

[BDG+07] I. Burhanuddin, J. Demmel, E. Goins, E. Kaltofen, F. Perez, W. Stein, H. Verrill, and J. Weening, *Workshop: Interactive Parallel Computation in Support of Research in Algebra, Geometry and Number Theory*, http://modular.math.washington.edu/msri07/.

[BMSW06] B. Bektemirov, B. Mazur, W. Stein, and M. Watkins, *Average Ranks of Elliptic Curves: Tension Between Data and Conjecture*, Bulletins (2006), http://sage.math.washington.edu/papers/bmsw-rank/.

[CDR+07] J. Cremona, H. Darmon, K. Ribet, R. Sharifi, and W. Stein, *Workshop on Modular Forms*, http://www.pims.math.ca/birs/birspages.php?task=displayevent&event_id=0%7w5065.

[GAP] GAP, *Groups, algorithms, programming—a system for computational discrete algebra*, http://www-gap.mcs.st-and.ac.uk/.

[GG03] Joachim von zur Gathen and Jrgen Gerhard, *Modern Computer Algebra*, Cambridge University Press, 2003.

[GJP+05] G. Grigorov, A. Jorza, S. Patrikis, C. Patrascu, and W. Stein, *Verification of the Birch and Swinnerton-Dyer Conjecture for Specific Elliptic Curves*, To appear in Mathematics of Computation (2005).

[Gor] D. Gordon, *Databases of coverings and cyclic difference sets*, http://www.ccrwest.org/gordon/dan.html.

[GS] D. Grayson and M. Stillman, *Macaulay2: a software system devoted to supporting research in algebraic geometry and commutative algebra*, http://www.math.uiuc.edu/Macaulay2/.

[GSL] GSL, *GNU Scientific Library*, http://www.gnu.org/software/gsl/.

[JS06a] D. Joyner and W. Stein, *Workshop: SAGE Days 1*, UC San Diego (February 2006), http://sage.math.washington.edu/sage/days1/.

[JS06b] ———, *Workshop: SAGE Days 2*, University of Washington (October 2006), http://sage.math.washington.edu/sage/days2/.

[KRR+07] K. Kedlaya, M. Rubinstein, N. Ryan, N.P. Skoruppa, and W. Stein, *Workshop: L-functions and modular forms*, http://aimath.org/ARCC/workshops/lfunctionsandmf.html.

[Mag] Magma, *High performance software for Algebra, Number Theory, and Geometry*, http://magma.maths.usyd.edu.au/.

[Mat] K. Matthews, *Number Theory Web: ftp sites, calculator programs,* *archives*, http://www.numbertheory.org/ntw/N1.htm.

[Max] Maxima, *A GPL CAS based on DOE-MACSYMA*, http://maxima.sourceforge.net/.

[Maz06] B. Mazur, *Controlling our errors*, Nature **443** (7 September 2006).

[Mer] Mercurial, *A fast, lightweight source control management system designed for efficient handling of very large distributed projects,* http://www.selenic.com/mercurial/wiki/index.cgi.

[Moi] MoinMoin, *The MoinMoin Wiki Engine,* http://moinmoin.wikiwikiweb.de/.

[NIS] NIST, *DigitalMathLib: NIST Digital Library of Mathematical Functions*, http://dlmf.nist.gov.

[Ope] OpenMath, *An extensible standard for representing the semantics of mathematical objects*, http://www.openmath.org/.

[PAR] PARI, *A computer algebra system designed for fast computations in number theory*, http://pari.math.u-bordeaux.fr/.

[PSS+07] M. Papanikolas, D. Savitt, W. Stein, D. Thakur, and F. Rodriguez-Villegas, *Workshop: Arizona Winter school: p-adic Geometry*, http://modular.math.washington.edu/swc/.

[SAG] SAGE, *Software for Algebra and Geometry Experimentation*, http://sage.math.washington.edu/sage.

[SCI06] SCIPY, *Conference on Scientific Computation using Python*, http://www.scipy.org/SciPy2006/Schedule.

[SIM06] SIMUW, *Workshop: Summer Institute of Mathematics at University of Washington (the PI led 1 of 6 high-school student workshops)*, http://sage.math.washington.edu/simuw/.

[Sin] Singular, *A computer algebra system for polynomial computations*, http://www.singular.uni-kl.de/.

[Slo] N. J. Sloane, *The Online Encyclopedia of Integer Sequences*, http://www.research.att.com/~njas/sequences/index.html.

[Ste06a] W. Stein, *William Stein's Student Projects*, http://sage.math.washington.edu/projects.

[Ste06b] W Stein, *Workshop: Computing with modular forms*, http://modular.math.washington.edu/msri06.

[Sto] J. E. Stone, *The Tachyon Multiprocessor Ray Tracer,* http://jedi.ks.uiuc.edu/~johns/raytracer/.

[Tra] Trac, *An enhanced wiki and issue tracking system for software development projects,* http://trac.edgewall.org/.