Combinatorial Potlatch 2011

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Sage: Creating a Viable Open Source Alternative to Magma, Maple, Matlab, and Mathematica

Combinatorial Potlach 2011 (Seattle U)

William Stein (University of Washington, Seattle)

Testing, testing, ...

```
1 + 2 + 3
```

```
6
```

Prelude

- **Which do you use more?** Wikipedia or Encyclopædia Britannica?
- Which are you more likely to **contribute to**?
- The English Wikipedia alone has over 1 billion words, which is over 25 times as many as Encyclopædia Britannica.

Potlatch

Pot·latch — n. Chinook for the Native spirit of gift-giving (potlatchfund.org)

Sage Project Mission Statement

"Create A Viable Free Open Source Alternative to Magma, Maple, Mathematica, and Matlab"

- **Mathematical features:** Of Magma, Maple, Mathematica, and Matlab, with comparable speed
- **Graphics:** 2d and 3d
- **Notebook:** Interactive graphical user interface
- **Documentation:** Books, papers, curriculum, etc.

Sage is not a drop-in replacement: does not run programs written in the custom languages of the Ma's.

Sage is not like Octave (versus Matlab).

Sage's culture, architecture, programming language, and feel are different than the Ma's.

Why *not* Magma, Maple, Matlab, Mathematica?

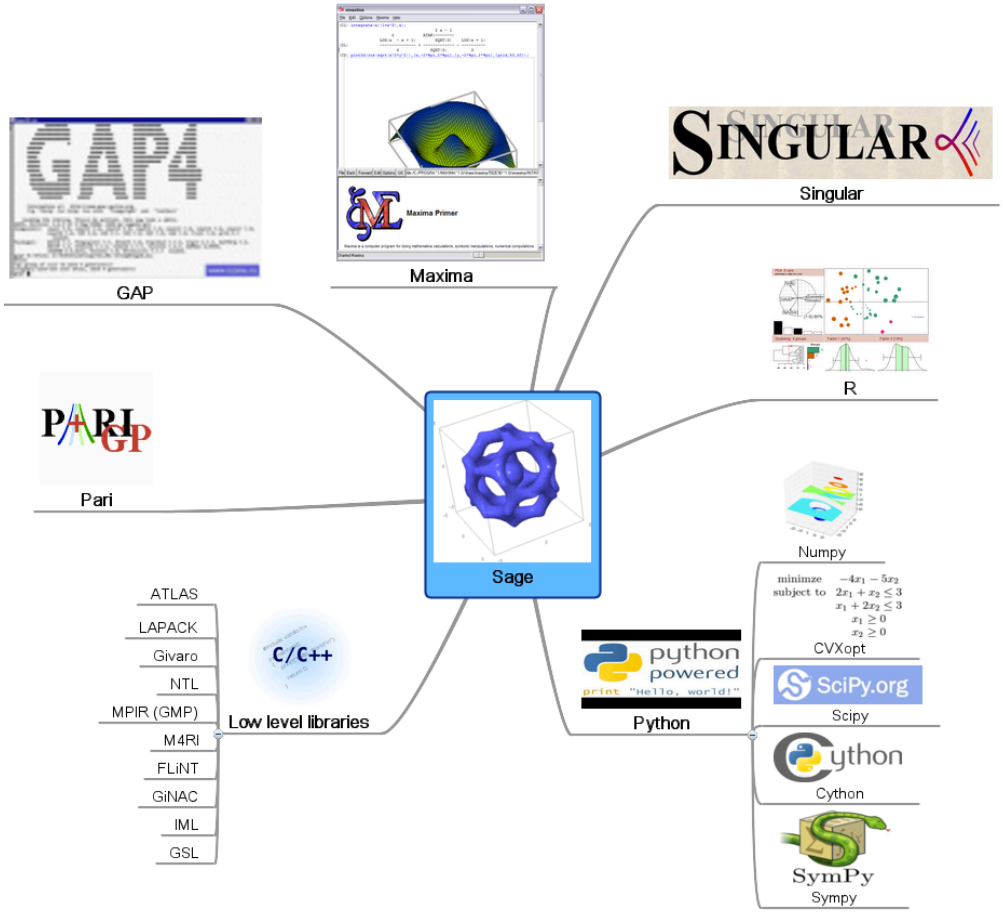
1. **Commercial:** Expensive for my collaborators and students. Not community owned.
2. **Closed:** Implementation of algorithms often secret
3. **Frustrating:** Tight control of development
4. **Copy protection:** Hard to run on supercomputer or my new laptop or after my 1-year license expires.
5. **Programming language:** All use a special math-only language
6. **Bugs:** Bug tracking is secret
7. **Compiler:** Lack of compilers for their math-only languages

Have you ever seen this? I did this morning when preparing this talk:

```
Last login: Fri Nov 18 16:16:32 on ttys008
You have mail.
deep:~ wstein$ math
Mathematica 7.0 for Mac OS X x86 (64-bit)
Copyright 1988-2009 Wolfram Research, Inc.

/Users/wstein/Library/Mathematica/Licensing/mathpass:1:
        The Mathematica license you are using has expired.
        Please contact Wolfram Research or an authorized
        Mathematica distributor to extend your license and
        obtain a new password.

You will need to get a password from your
license certificate or from Wolfram Research
(http://register.wolfram.com).
Machine name:   deep.local
MathID: 5108-53088-04270

You will need a valid license ID and password in order
to proceed. Go to http://register.wolfram.com or
consult your Getting Started documentation.

Enter your name:
```

1. **Python:** a mainstream general purpose programming language (with a compiler: Cython)
2. **Distribution:** about 100 open source packages (**written by you** and your colleagues!)
3. **Interfaces:** smoothly tie together all these libraries and packages
4. **New library:** implements novel algorithms; over a half million lines; worldwide community of several hundred people.

Distribution



Hundreds of Sage Developers

(There are at least 242 contributors in 164 different places from all around the world.)

William Stein, Tim Abbott, Michael Abshoff, Antti Ajanki, Martin Albrecht, Nick Alexander, Bill Allombert, Ethan Van Andel, Ivan Andrus, Pablo Angulo, Benjamin Antieau, André Apitzsch, Maite Aranes, Oscar Gerardo Lazo Arjona, Eviatar Bach, Jennifer Balakrishnan, Jason Bandlow, Gregory Bard, Sébastien Barthélemy, Rob Beezer, Karim Belabas, Arnaud Bergeron, Luis Berlioz, Erin Beyerstedt, Francois Bissey, Jonathan Bober, Tom Boothby, Nicolas Borie, Johan Bosman, Robert Bradshaw, Michael Brickenstein, Nils Bruin, André-Patrick Bubel, Stanislav Bulygin, Dan Bump, Iftikhar Burhanuddin, Paul Butler, Oriol Castejón, Ondrej Certik, Wilson Cheung, Dan Christensen, Craig Citro, Anders Claesson, Francis Clarke, Timothy Clemans, Alex Clemesha, Nathann Cohen, Jenny Cooley, John Cremona, Karl-Dieter Crisman, Fidel Barrera Cruz, Doug Cutrell, Alyson Deines, Vincent Delecroix, Jeroen Demeyer, Tom Denton, Maarten Derickx, Didier Deshommes, Ryan Dingman, Dan Drake, Tom Draper, Alexander Dreyer, Tim Dumol, Nathan Dunfield, Gabriel Ebner, Ben Edwards, Dana Ernst, Burcin Erocal, Ron Evans, Richard J. Fateman, Lars Fischer, Jean-Pierre Flori, Evan Fosmark, Laurent Fousse, Gary Furnish, Alex Ghitza, Andrzej Giniewicz, Alain Giorgetti, Samuele Giraudo, Amy Glen, Daniel Gordon, Chris Gorecki, Jan Groenewald, Rob Gross, Jason Grout, Ryan Grout, Mathieu Guay-Paquet, Alexey U. Gudchenko, Harold Gutch, Jonathan Gutow, Jose Guzman, Anna Haensch, Carlo Hamalainen, Marshall Hampton, Jon Hanke, David Møllerlier Hansen, Mike Hansen, Bill Hart, David Harvey, Leif Hille, Florent Hivert, Ryan Hinton, Neal Holtz, Golam Mortuza Hossain, Sean Howe, Alexander Hupfer, Wilfried Huss, Hamish Ivey-Law, Naqi Jaffery, Peter Jeremy, Peter Jipsen, Fredrik Johansson, Niles Johnson, Timo Jolivet, Benjamin Jones, David Joyner, Michael Kallweit, Josh Kantor, Kiran Kedlaya, Lloyd Kilford, Simon King, Keshav Kini, David Kirkby, Emily Kirkman, David Kohel, Ted Kosan, Ross Kyprianou, Sébastien Labbé, Yann Laigle-Chapuy, Kwankyu Lee, Julien Leroy, Richard Lindner, David Loeffler, Miguel Marco, Michael Mardaus, Robert Mařík, Jason Martin, Alexandre Blondin Massé, Peter McNamara, Gregory McWhirter, Jason Merrill, Matthias Meulien, Robert Miller, Kate Minola, Moritz Minzlaff, Joel Mohler, Thierry Monteil, Peter Mora, Bobby Moretti, Rich Morin, Guillaume Moroz, Gregg Musiker, Tobias Nagel, Brett Nakashima, Pablo De Nápoli, Johan Sebastian Rosenkilde Nielsen, Minh Van Nguyen, Andrey Novoseltsev, Christopher Olah, Johan Oudinet, Bill Page, Ronan Paixão, Willem Jan Palenstijn, John Palmieri, Dmitrii Pasechnik, Javier López Peña, Paulo César Pereira de Andrade, David Perkinson, Clement Pernet, John Perry, Pearu Peterson, David Poetzsch-Heffter, Viviane Pons, Bill Purvis, Julien Puydt, Yi Qiang, Jordi Quer, Gustavo Rama, Jens Rasch, Martin Raum, Dorian Raymer, Stefan Reiterer, R. Rishikesh, David Roe, Bjarke Hammersholt Roune, Gordon Royle, Serge A. Salamanka, Franco Saliola, Leonardo Sampaio, Kyle Schalm, Ed Scheinerman, Anne Schilling, Harald Schilly, Jack Schmidt, Michael Schneider, Christopher Schwan, Dag Sverre Seljebotn, Dan Shumow, Denis Simone, Steven Sivek, Nils-Peter Skoruppa, Jaap Spies, Jonathan Spreer, Armin Straub, Marco Streng, Kevin Stueve, Christian Stump, Blair Sutton, Chris Swierczewski, Luis Felipe Tabera Alonso, Glenn Tarbox, Philippe Theveny, Nicolas Thiery, Griffen Thoma, Emmanuel Thomé, John Thurber, Igor Tolkov, Gonzalo Tornaria, Kiminori Tsukazaki, Charlie Turner, Michel Vandenbergh, Joris Vankerschaver, Soledad Villar, John Voight, Felipe Voloch, Steve Vonn, Justin Walker, Mark Watkins, Georg S. Weber, Eric Webster, Ralf-Philipp Weinmann, Joe Wetherell, Carl Witty, Cristian Wuthrich, Soroosh Yazdani, Dal S. Yu, Gary Zablackis, Mike Zabrocki, Bin Zhang, Paul Zimmermann, Mao Ziyang

```
bernoulli??
```

History

- **2005:**SAGE-0.1 released February 1, 2005; SAGE=Software for Arithmetic Geometry Experimentation
- **2006:** (2 Sage Days workshops); Sage is not just for number theory

- **2007:** (4 Sage Days) 100% test requirements; peer review of all new code (see trac); industry funding; NSF; Trophees du Libre
- **2008:** (7 Sage Days) Release managers besides me.
- **2009:** (8 Sage Days) Better foundations; 3d graphics; more developers (e.g., sage-combinat)
- **2010:** (13 Sage Days) More devs and users; nontrivial NSF grants
- **2011:** (12 Sage Days) Much faster http://sagenb.org with >50,000 accounts; very stable releases; undergrad curriculum development
- **2012:** (5 Sage Days already planned)
- **2013:** (1 Sage Days already planned)

See this article for more details about the (pre-)history of Sage.

Question Break

????

```
factorial(3)
```

```
6
```

```
A = random_matrix(QQ, 4); b = A^(-1); b
```

```
[   0 -1/2    0 -1/2]
[-2/3   -1 -1/3 -1/2]
[   0  1/2    0    0]
[-1/3   -1  1/3 -1/2]
```

```
show(b)
```

$$\begin{pmatrix} 0 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{2}{3} & -1 & -\frac{1}{3} & -\frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 \\ -\frac{1}{3} & -1 & \frac{1}{3} & -\frac{1}{2} \end{pmatrix}$$

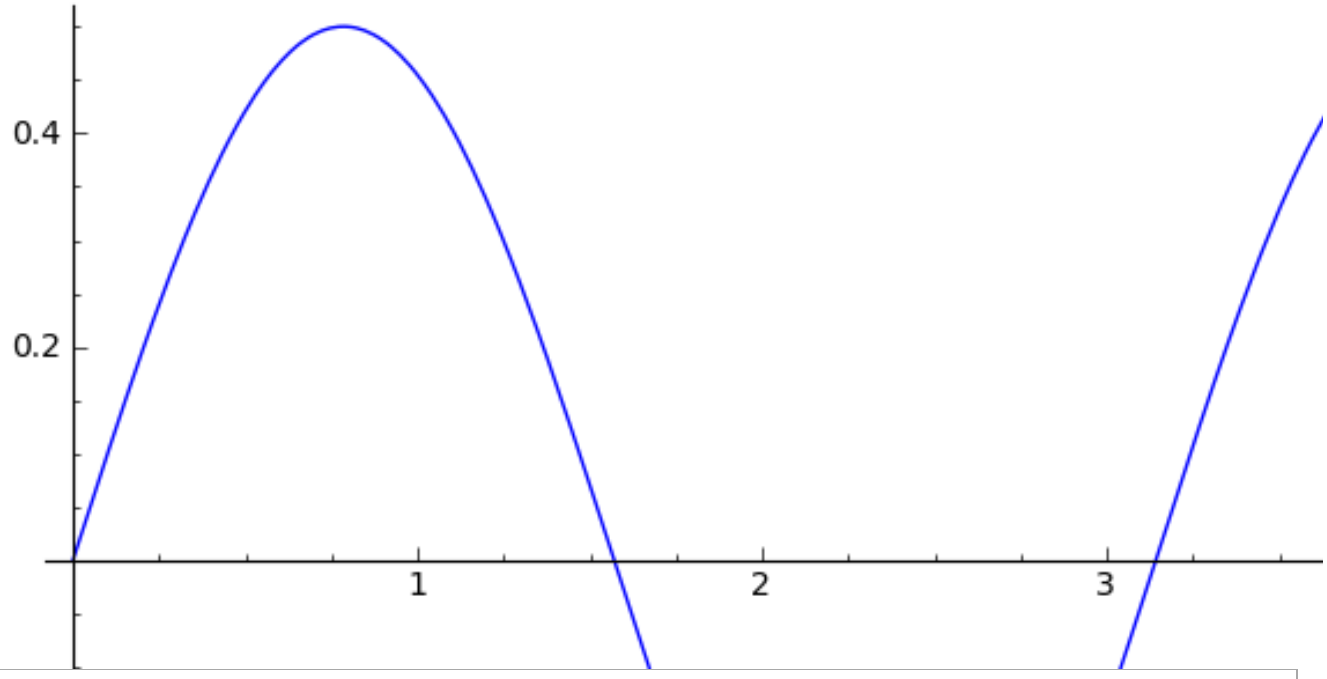
```
latex(b)
```

```
\left(\begin{array}{rrrr}
0 & -\frac{1}{2} & 0 & -\frac{1}{2} \\
-\frac{2}{3} & -1 & -\frac{1}{3} & -\frac{1}{2} \\
0 & \frac{1}{2} & 0 & 0 \\
-\frac{1}{3} & -1 & \frac{1}{3} & -\frac{1}{2}
\end{array}\right)
```

```
show(integrate(sin(x)*cos(x), x))
```

$$-\frac{1}{2}\cos\left(x\right)^{2}$$

```
G = plot(sin(x)*cos(x), (0, 4))
G
```




```
G.save('a.pdf')
```

a.pdf

```
sage.interfaces.
```

Interfaces

Use a single Mathematica session from within Sage. Here, Mathematica is running on a computer at UW, since my laptop license expired.

```
mathematica = Mathematica(server='sage.math.washington.edu',
                          server_tmpdir='/tmp/')
f = mathematica('Integrate[Sin[x],x]'); f
```

^C

```
f.Integrate(x)
```

```
    -Sin[x]
```

```
f + mathematica(sin(x)*cos(x^2))
```

```
    -Cos[x] + Cos[x^2]*Sin[x]
```

```

```

```

```

```

```

```

```

Interactive Image Compression

(using numpy)

```
import pylab, numpy

X = pylab.imread(DATA + 'wyckoff.png')
A_image = numpy.mean(X, 2)
u,s,v = numpy.linalg.svd(A_image)
S = numpy.zeros(A_image.shape)
S[:len(s),:len(s)] = numpy.diag(s)
n = A_image.shape[0]

@interact
def svd_image(i = ("Eigenvalues (quality)",
                   (20,(1..A_image.shape[0]//2)))):
```

```
    A_approx = numpy.dot(numpy.dot(u[:,:i], S[:i,:i]), v[:i,:])
    g = graphics_array([matrix_plot(A_approx),
                        matrix_plot(A_image)])
    show(g, axes=False, figsize=(6,3))
    html("%sx%s image compressed to %.1f%% of size using %s
eigenvalues."%(
            A_image.shape[0], A_image.shape[1],100*
(2.0*i*n+i)/(n*n), i))
```

Number Theory

```
factor(2009201020112012)      # uses PARI
```

```
    2^2 * 43 * 2269 * 5148259709
```

```
# Jon Bober - Rademacher's formula
time number_of_partitions(10^6)
```

```
    147168498635822339863100476060989594348403048443914212533461274
    661174189186182763301488739835975558420153741306002880959293873
    232270327849578001932784396072064228659048713020170971840761025
    986084690814282935670692978599129051989944549067221999782345287
    740222882298501367675662947818874946878790038246999881977292006
    668735996662273816798266213482417208446631027428001918132198177
    651123454259502672842445259229678119344813999466473010574256435
    949891814852853513705513994767199816914590220155991019596014174
    715430750022184895815209339012481734469448319323280150665384042
    417958775176129491624814247999880293650719525707448504757166277
    033914424951138232981952630083364898260458377122024553049963821
    028531832004519046591968302787537418118486000612016852593542741
    504626724547323732184583342751252422746539913017407694128084740
    422179992860711083363033162982891024446496968053954167918754800
    636774022023128467646919775022348562520747741843343657801534130
    197553037516970799928704028567784161934747236817177215404666430
    15630003467104673818
    Time: CPU 0.03 s, Wall: 0.03 s
```

```
@interact
def _(n=(25..10000)):
    plot(prime_pi, 0, n, gridlines='minor').show(figsize=[8,3])
```

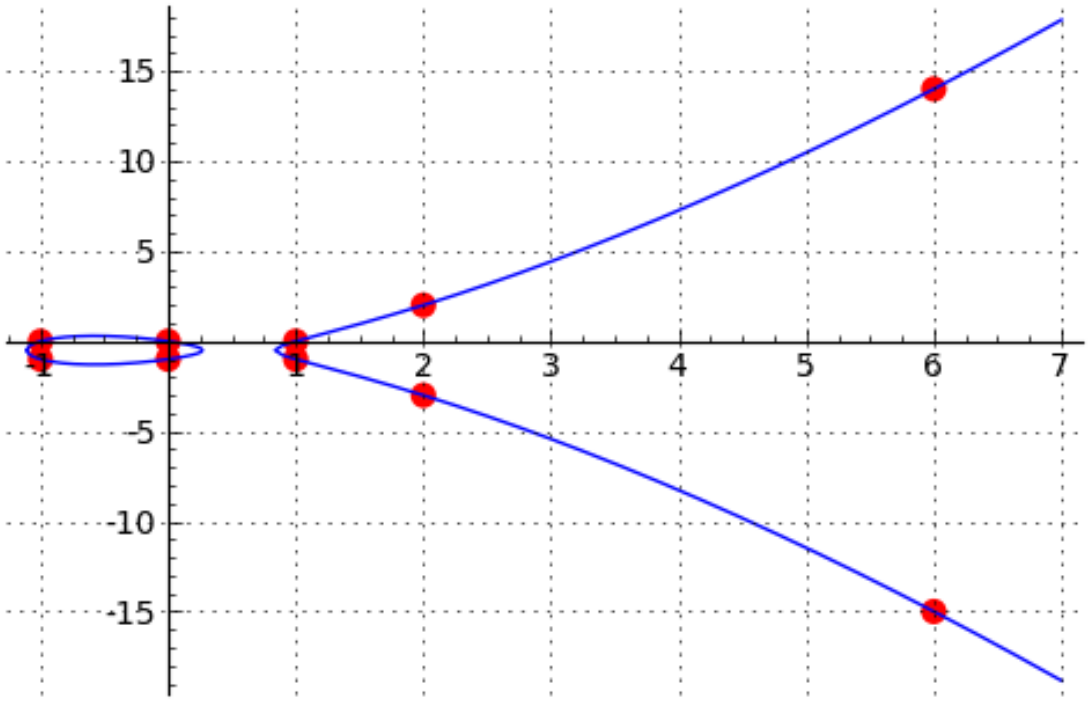
```
E = EllipticCurve('37a'); E
```

```
Elliptic Curve defined by y^2 + y = x^3 - x over Rational Field
```

```
v = E.integral_points(both_signs=True); v     # mwrank, new code
```

```
[(-1 : -1 : 1), (-1 : 0 : 1), (0 : -1 : 1), (0 : 0 : 1), (1 : -1
1), (1 : 0 : 1), (2 : -3 : 1), (2 : 2 : 1), (6 : -15 : 1), (6 :
1)]
```

```
plot(E, xmax=7, gridlines=True) + points([z[:2] for z in v],
          color='red', pointsize=50)
```

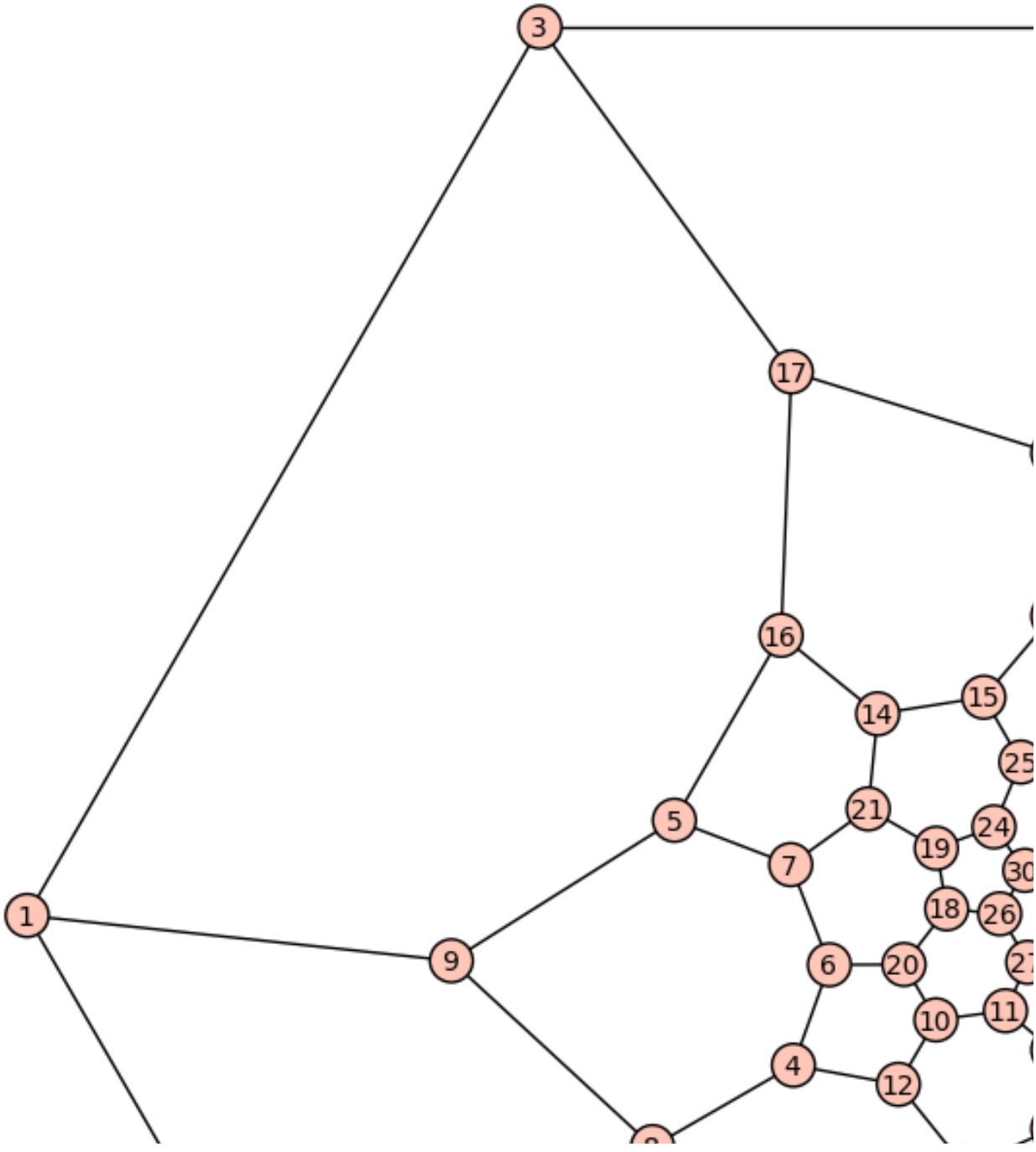


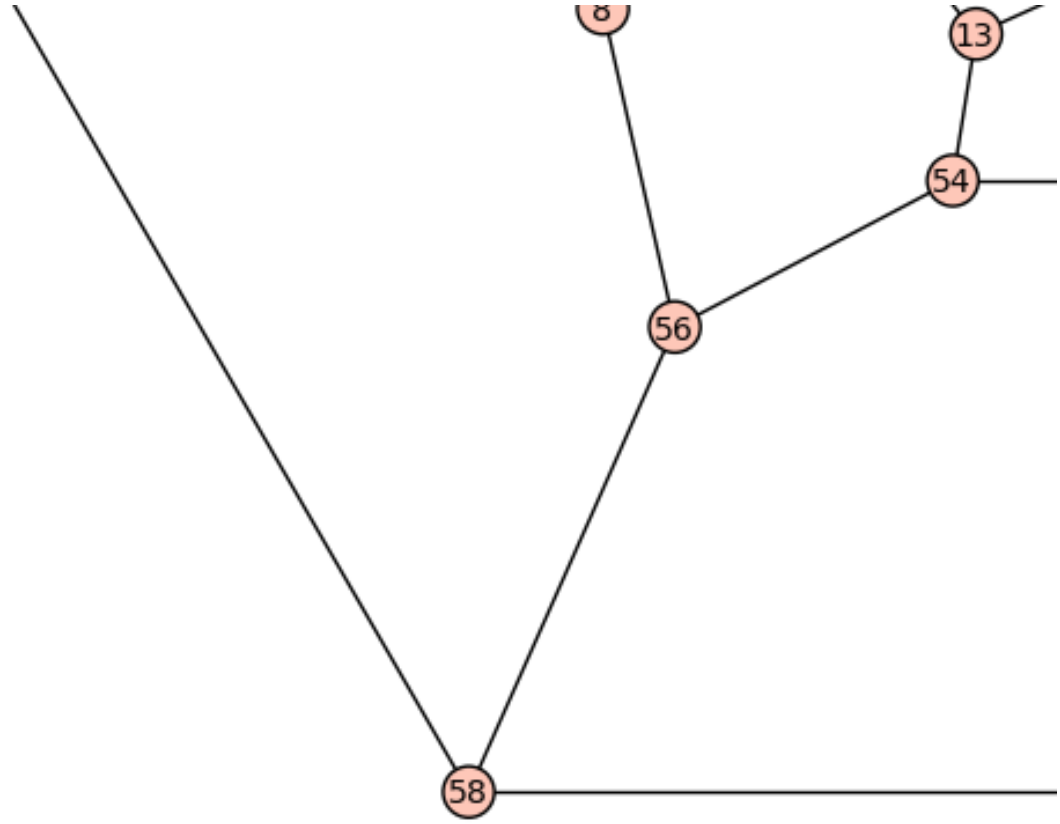
Graph Theory

"Sage's graph theory crushes anything I have met from the point of view of methods implemented. I would say: 'if you found a proprietary graph library and you are convinced that it is better for your needs than Sage, your license is on me.' But those licenses are really expensive :-D"

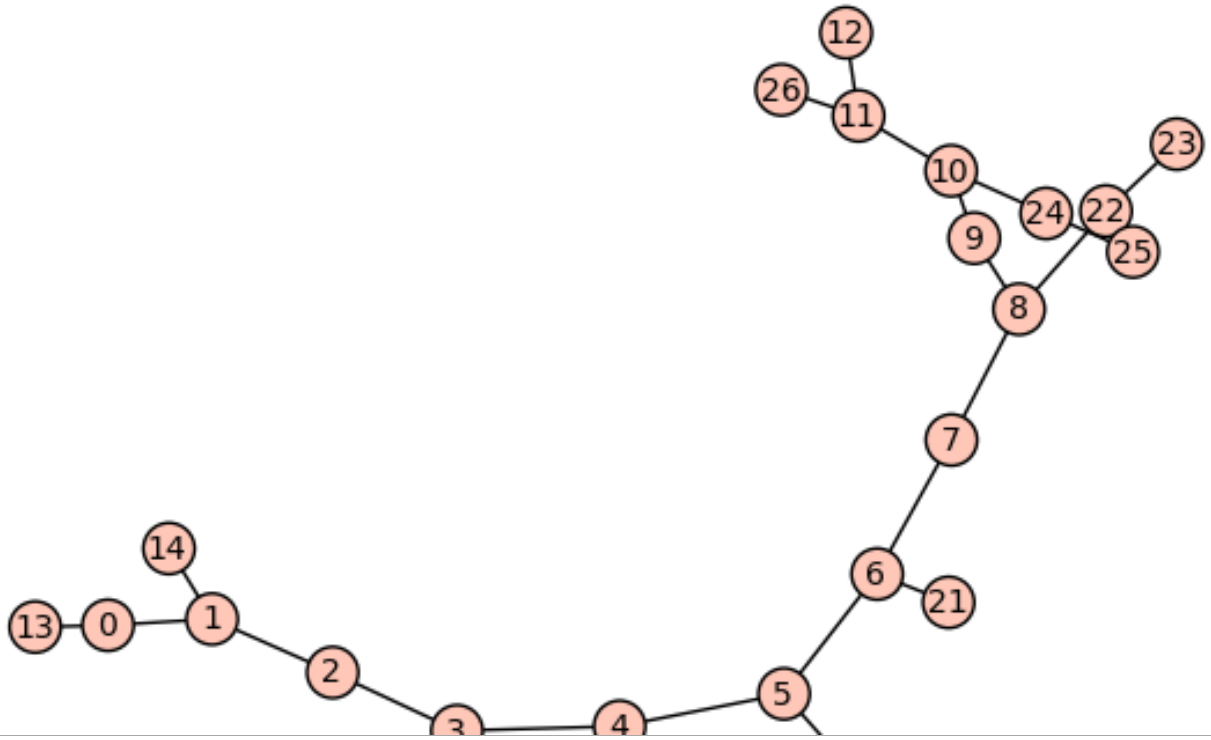
```
graphs.ClawGraph(
```

```
G = graphs.BuckyBall()
G.plot().show(figsize=8)
```






```
set_random_seed(1)
G = graphs.RandomLobster(8, .6, .3)
show(G, figsize=7)
```




```
G.automorphism_group()
```

```
    Permutation Group with generators [(12,26)]
```

```
G.chromatic_number()
```

```
    2
```

```
G.shortest_path(13,20)
```

```
    [13, 0, 1, 2, 3, 4, 5, 19, 20]
```

Algebraic Combinatorics

The sage-combinat project: "improve the open source mathematical system Sage as an extensible toolbox for computer exploration in (algebraic) combinatorics, and foster code sharing between researchers in this area."

For example, see Combinatorics Chapter of the Reference Manual

```
s = SFASchur(QQ)
h = SFAHomogeneous(QQ)
p = SFAPower(QQ)
e = SFAElementary(QQ)
m = SFAMonomial(QQ)
```

```
s
```

```
Symmetric Function Algebra over Rational Field, Schur symmetric
functions as basis
```

```
a = s([3,1]); a
```

```
s[3, 1]
```

```
a*a
```

```
s[3, 3, 1, 1] + s[3, 3, 2] + s[4, 2, 1, 1] + s[4, 2, 2] + 2*s[4,
1] + s[4, 4] + s[5, 1, 1, 1] + 2*s[5, 2, 1] + s[5, 3] + s[6, 1,
s[6, 2]
```

```
p(a)
```

```
1/8*p[1, 1, 1, 1] + 1/4*p[2, 1, 1] - 1/8*p[2, 2] - 1/4*p[4]
```

```
a.inner_plethysm(a)
```

```
s[1, 1, 1, 1] + 2*s[2, 1, 1] + s[2, 2] + 2*s[3, 1]
```

```

```

```

```

Cython

- Smooth transition between Python and compiled C code.
- Make code that involves lots of manipulation of C-level data structures optimally fast
- Heavily used in scientific computing using Python.

```
def python_sum(n):
    s = int(0)
    for i in xrange(1, n+1):
        s += i*i
    return s
```

```
python_sum(3)
```

```
14
```

```
time python_sum(2*10^6)
```

```
2666668666667000000
Time: CPU 0.16 s, Wall: 0.16 s
```

```
timeit('python_sum(2*10^6)')
```

```
5 loops, best of 3: 164 ms per loop
```

```
def python_sum2(n):
    return sum(i*i for i in xrange(1,n+1))
```

```
time python_sum2(2*10^6)
```

```
2666668666667000000
Time: CPU 0.19 s, Wall: 0.19 s
```

```
%cython
def cython_sum(long n):
    cdef long long i, s = 0
    for i in range(1, n+1):
        s += i*i
    return s
```

__Users_ws...9_code_sage81_spyx.c __Users_ws...ode_sage81_spyx

```
cython_sum(3)
```

```
14L
```

```
time cython_sum(2*10^6)
```

```
    266666866666700000L
```

```
timeit('cython_sum(2*10^6)')
```

```
    625 loops, best of 3: 671 ns per loop
```

```
165/.663
```

```
    248.868778280543
```

```
%cython
def cython_sum2(long n):
    cdef long long i
    return sum(i*i for i in range(1,n+1))
```

```
    __Users_ws...9_code_sage89_spyx.c    __Users_ws...ode_sage89_spyx
```

```
time cython_sum2(2*10^6)
```

```
    266666866666700000L
    Time: CPU 0.16 s, Wall: 0.16 s
```

```

```

Of course, it is better to choose a different algorithm:

```
var('k, n')
factor(sum(k^2, k, 1, n))
```

```
    1/6*(n + 1)*(2*n + 1)*n
```

```
def sum2(n):
    return n*(2*n+1)*(n+1)/6
```

```
sum2(2*10^6)
```

```
    2666668666667000000
```

Even then, Cython provides a speedup:

```
%cython
def c_sum2(long long n):
    return n*(2*n+1)*(n+1)/6
```

```
    __Users_ws...9_code_sage97_spyx.c    __Users_ws...ode_sage97_spyx
```

```
c_sum2(3)
```

```
    14L
```

```
c_sum2(2*10^6)
```

```
    -407786678951258603L
```

```
n = 2*10^6
```

```
timeit('sum2(n)')
```

```
625 loops, best of 3: 2.01 µs per loop
```

```
timeit('c_sum2(n)')
```

```
625 loops, best of 3: 218 ns per loop
```

```
2.01/.218
```

```
9.22018348623853
```

But at a cost!

```
c_sum2(2*10^6)        # WARNING: overflow -- it's just like C...
```

```
-4077886789512586O3L
```

```
n=2*10^6; n*(2*n+1)*(n+1) > 2^63
```

```
True
```

Solving Equations

Solve a ***cubic equation***:

```
x = var('x'); show(solve(x^3 + x - 1==0, x)[0])
```

$$x = -\frac{1}{2}\left(i\sqrt{3}+1\right)\left(\frac{1}{18}\sqrt{3}\sqrt{31}+\frac{1}{2}\right)^{\left(\frac{1}{3}\right)} + \frac{-i\sqrt{3}+1}{6\left(\frac{1}{18}\sqrt{3}\sqrt{31}+\frac{1}{2}\right)^{\left(\frac{1}{3}\right)}}$$

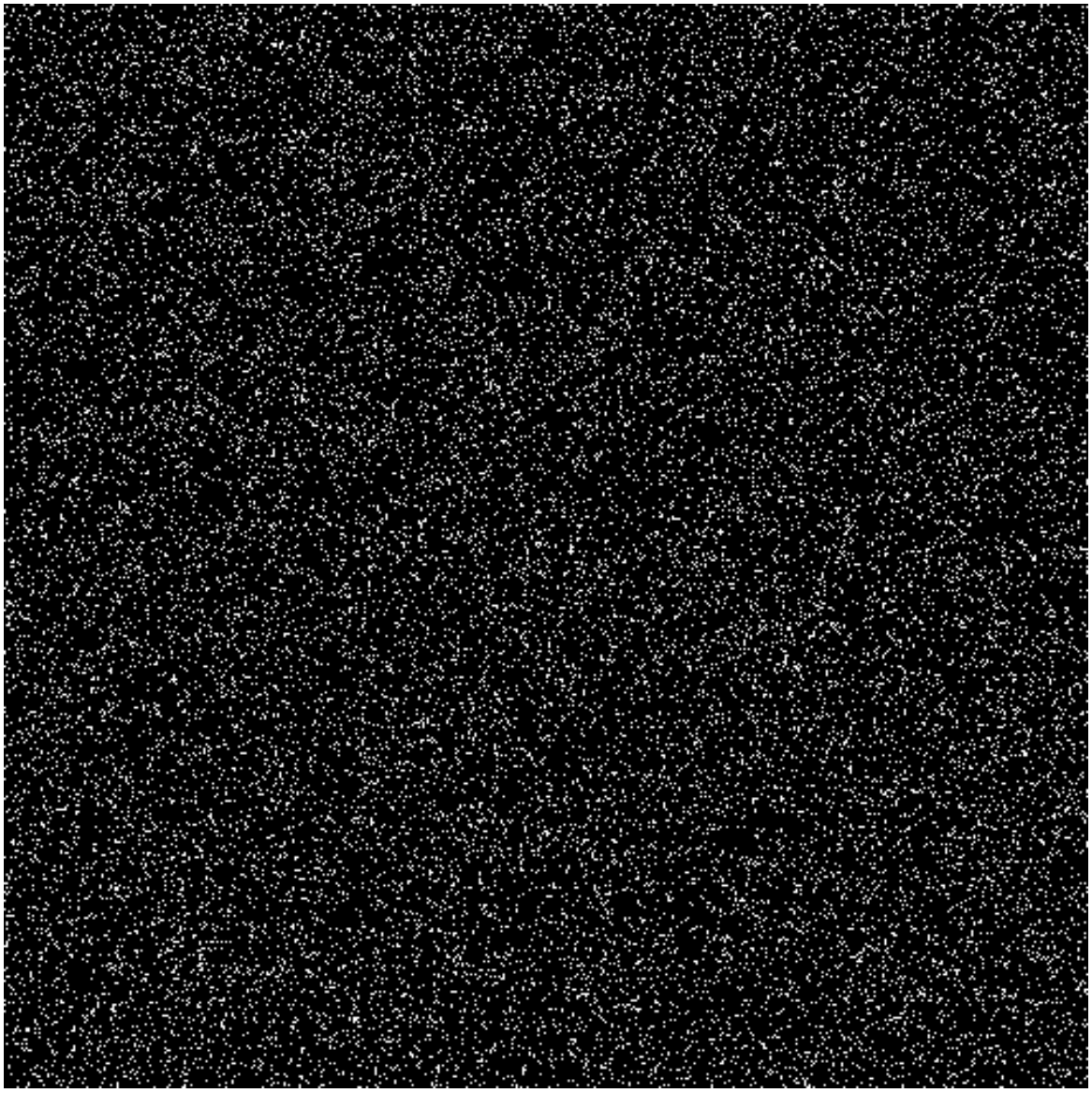
Solve a system of ***two linear equations*** with one unknown coefficient α:

```
var('alpha, y')
show(solve([3*x + 7*y == 2, alpha*x + 3*y == 8], x,y)[0])
```

$$\left[x = \frac{50}{7\,\alpha - 9}, y = \frac{2\,(\alpha - 12)}{7\,\alpha - 9}\right]$$

Solve a ***system of 500 linear equations*** exactly over the rational numbers:

```
n = 500
A = random_matrix(QQ,n,n,num_bound=10, den_bound=10)
v = random_matrix(QQ,n,1,num_bound=10, den_bound=10)
A.visualize_structure()
```



```
# IML is used -- http://www.cs.uwaterloo.ca/~astorjoh/iml.html
time w = A \ v
```

Time: CPU 3.08 s, Wall: 2.04 s

```
print w.str()
```

WARNING: Output truncated!
full_output.txt

[
-351702876068802906437552420686055850326076457904676962275100830
401420955261319182505472655403446871133375351213855350226815464
541678542779937822420479756487655377896847799471633977576723782
752013079903081977572402942342855631847634917334375567725200949
981494332871024651254158873412391649819482714094296169547438994
166187839273830840430242204133808650932888101185375789831810538
749302374084369069440792237427868339146115514095572829020266899
134355557855057198503380254232298641549695798842795869295699735
920100244120646553550505898020495083583179450653843216750029252
795806490140294417796752695156539524124805975616026523319266984
226942792117185857783063808851623050866691080434331157140193997
656405532372637424988128302930037221573841204291852109644817711
003582668463237232745438803646481729385618653719290569550431763
479911712917380077097901887716153891517164989985540840321675523
530360179417770975752951610003948983037774750319911117156732832
778539539244149073095868925297296767522098873014512743624494378
988592956277583816139788377093357552496933776948081224203156447
737484105107310982124866738518101800129697107516438736576977941
437927029350938265391137496664227427915828658742276488248475686
797308951586142050635510616384914735047939100560830346367036281
909909358743617446014742435439813694562291652176283751189754664
361180619892765924138532602101463250422678650464862058688845651
378935711191095339677561251599756523993225002326911920613591375
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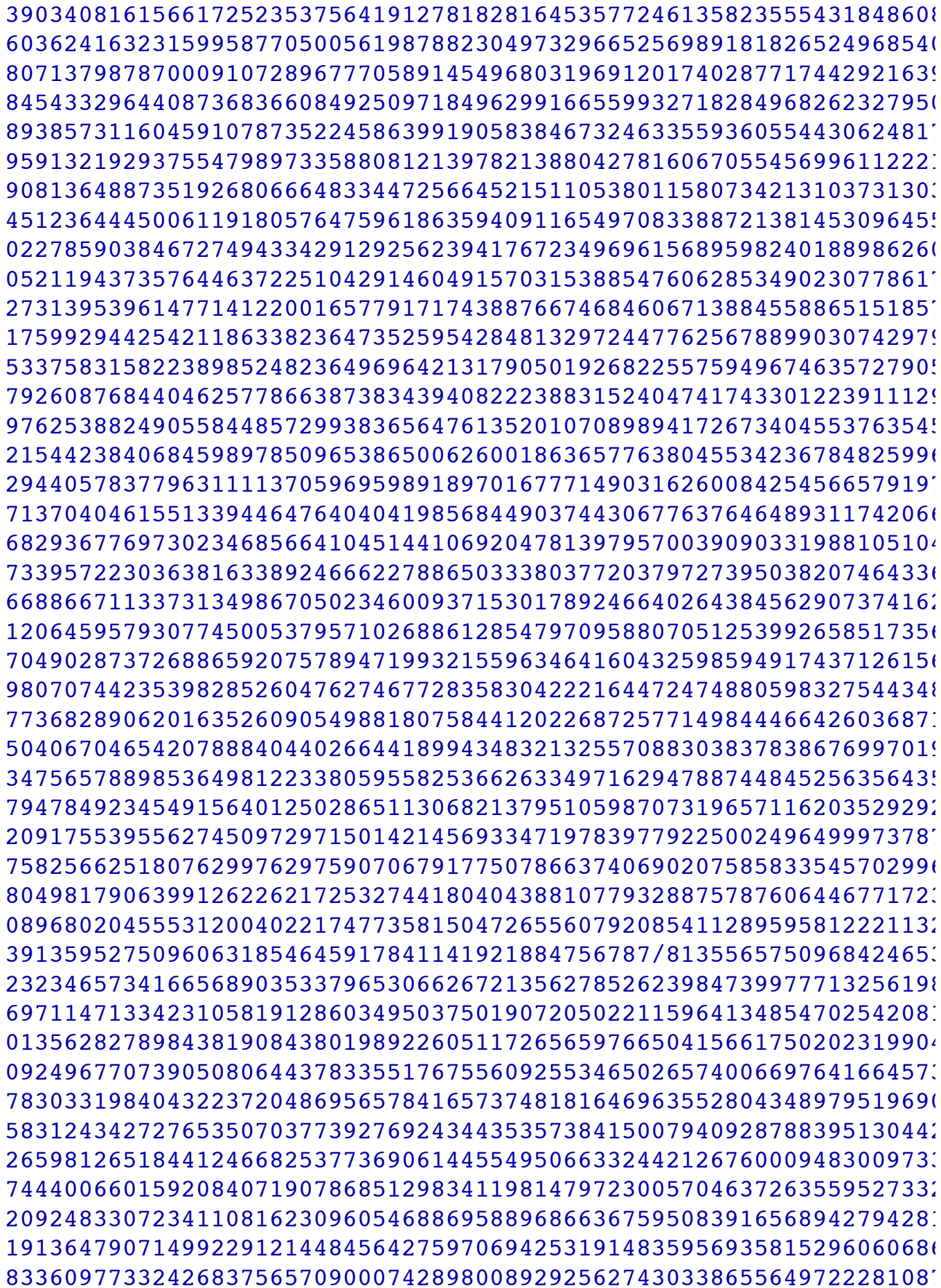

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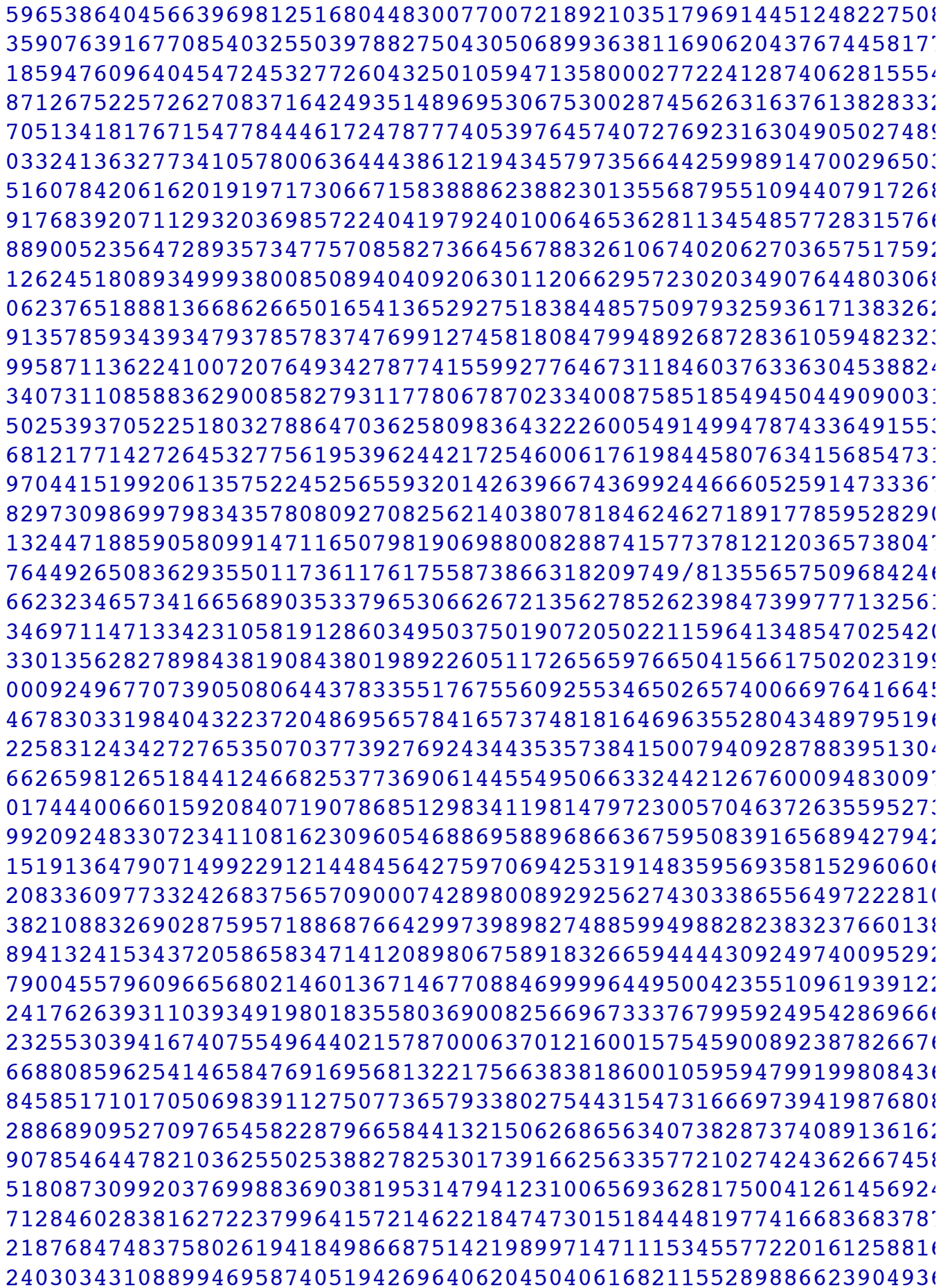

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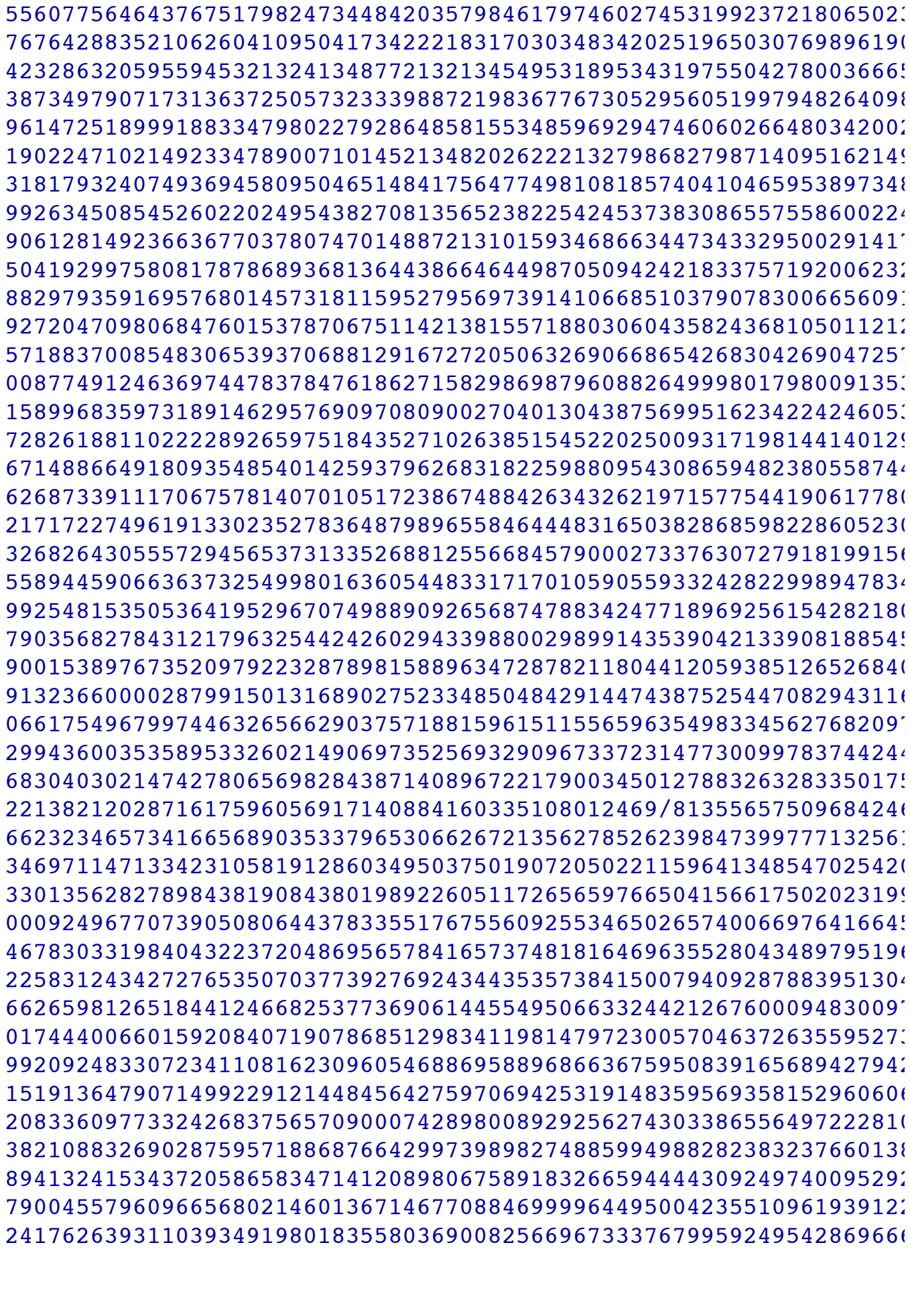

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```

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```
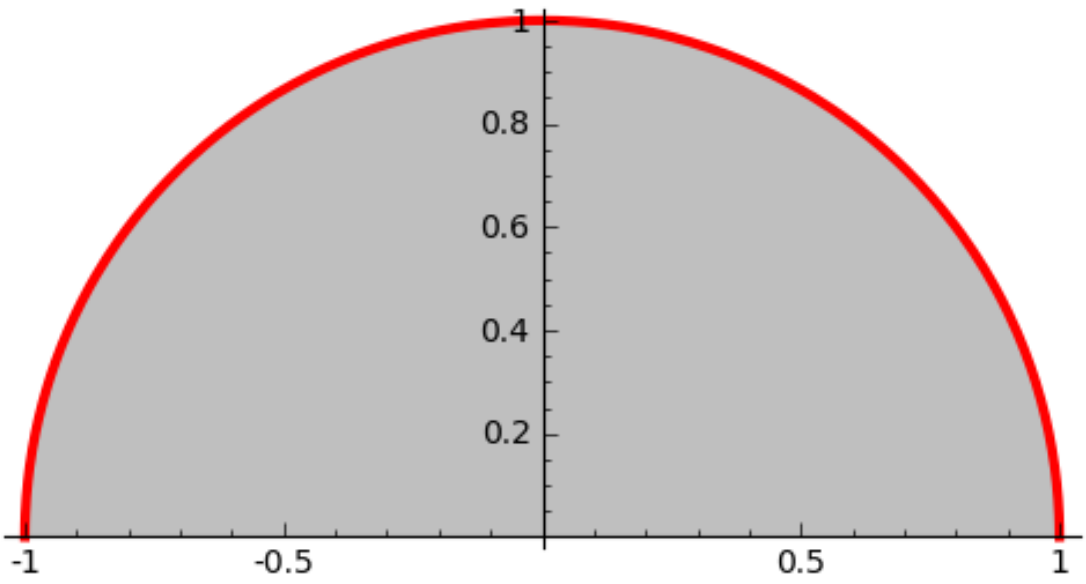

Symbolic Calculus

Symbolic Calculus makes use of **Maxima** and **Ginac** under the hood.

```
var('x')
f = sqrt(1 - x^2)
plot(f, thickness=3, color='red', aspect_ratio=1, fill=True)
```



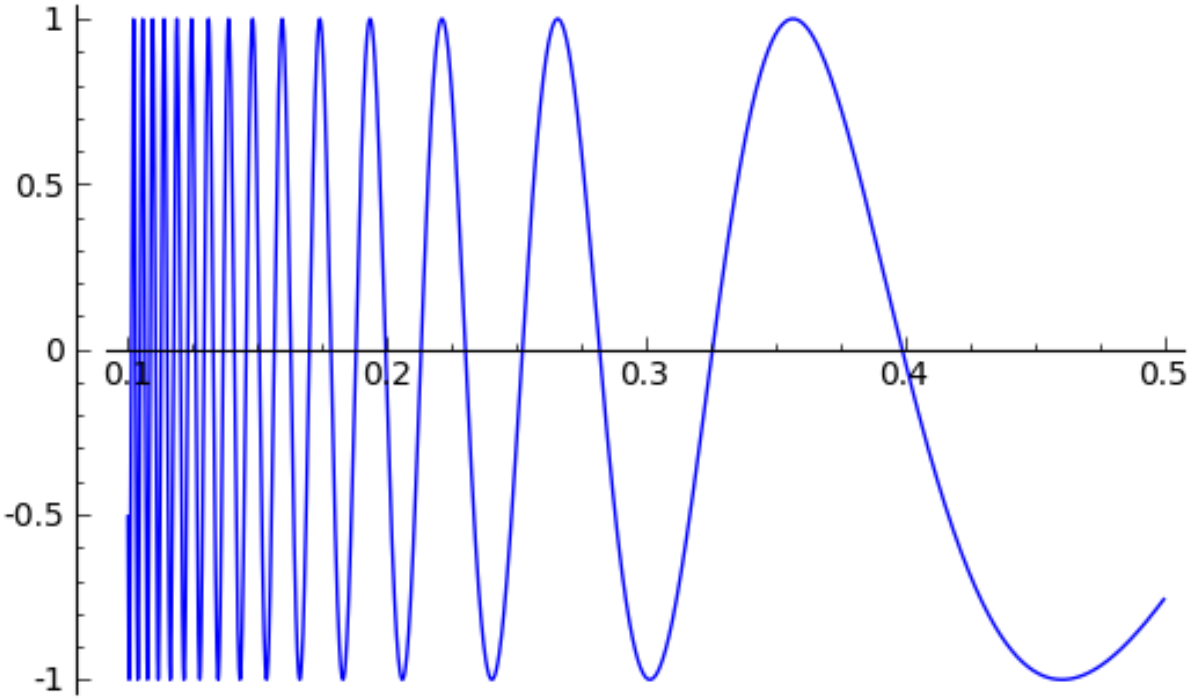

```
var('t')
assume(t+1 > 0)
g = integrate(f, (x, -1, t)); show(g)
```

```
show(g(t=1) - g(t=-1))
```

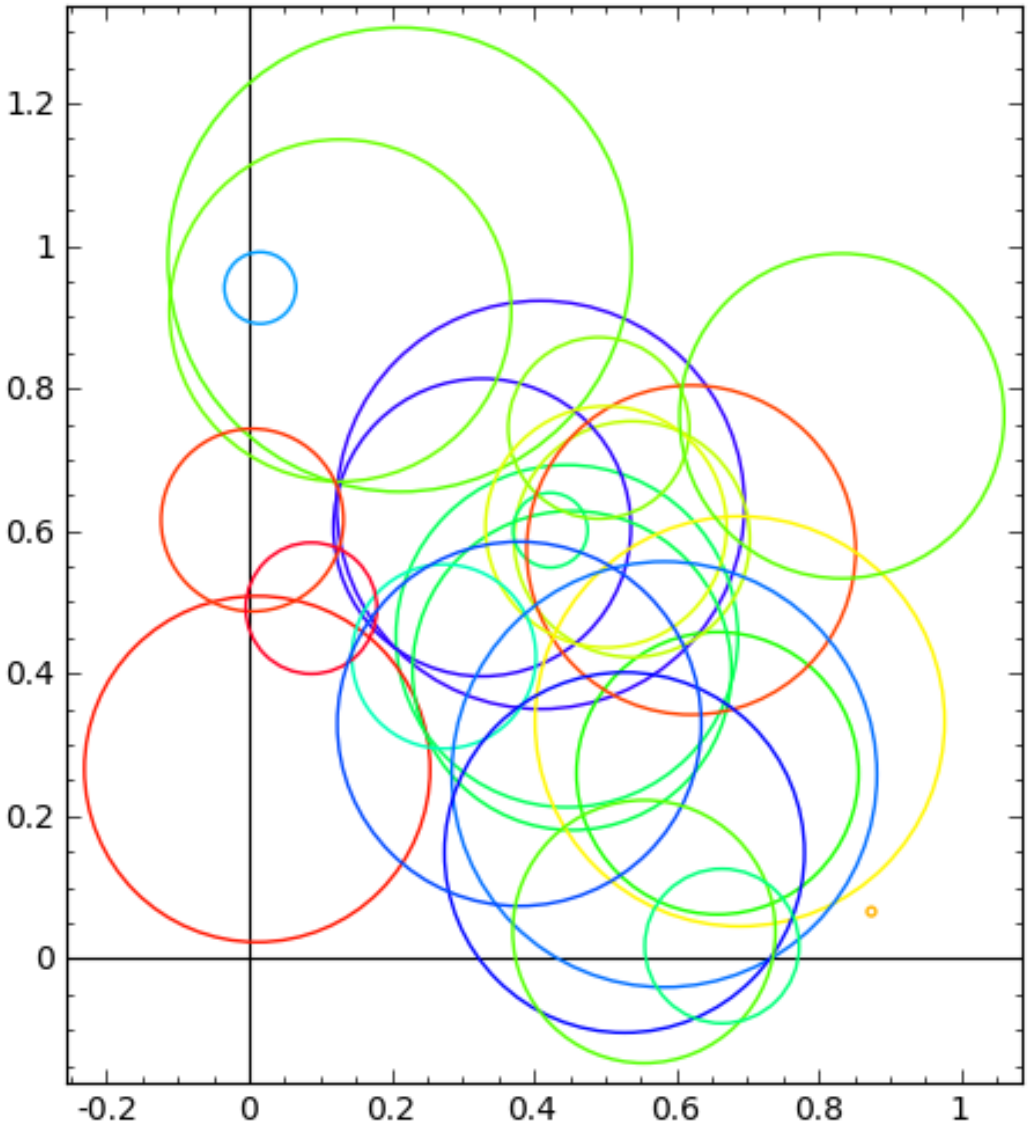
$$\frac{1}{2}\pi$$

Plotting in 2D

```
plot(sin(1/x^2), (x,.1,.5))
```



```
G = sum(circle((random(), random()), random()/3,
        color=hue(random())) for i in range(25))
G.show(aspect_ratio=1, frame=True)
```



Plotting in 3D

```
f(x,y) = sin(x - y) * y * cos(x)
plot3d(f, (x,-3,3), (y,-3,3), opacity=.9, color='red')
```

Get Image

```
G = sum(sphere((random(), random(), random()), random()/4,
        color=hue(random()), opacity=.6)
        for i in range(20))
G.show(aspect_ratio=1, frame=True)
```

Get Image

Plotting a 3D Model

See http://www.davidson.edu/math/chartier/Starwars/

```
# Yoda! (53,756 vertices)
```

```
from scipy import io
yoda = io.loadmat(DATA + 'yodapose.mat')

from sage.plot.plot3d.index_face_set import IndexFaceSet
V = yoda['V']; F3=yoda['F3']-1; F4=yoda['F4']-1
Y = IndexFaceSet(F3,V,color=Color('#444444')) +
IndexFaceSet(F4,V,color=Color('#007700'))
Y = Y.rotateX(-1)
Y.show(aspect_ratio=1, frame=False, zoom=1.2)
```

Get Image

Questions

????