SACNAS

```
1+2+3+4 # testing
```

    10

Sage: Creating a Viable Open Source Alternative to Magma, Maple, Matlab, and Mathematica

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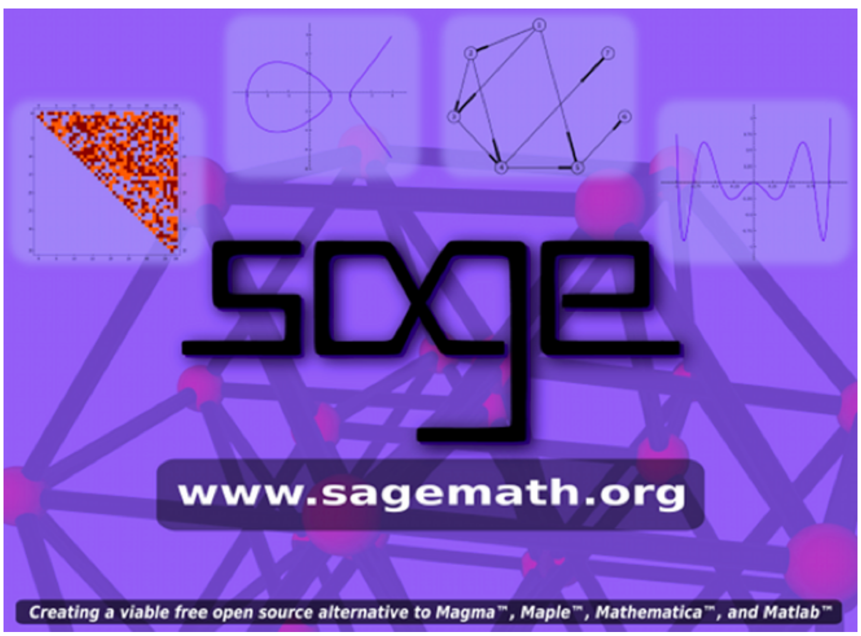
William Stein (Professor of Mathematics, University of Washington)



Sage: Mission Statement

"Create A Viable Free Open Source Alternative to Magma, Maple, Mathematica, and Matlab"

- **Mathematical features:** Of Magma, Maple, Mathematica, and Matlab, with comparable speed
- **Graphics:** 2d and 3d
- **Notebook:** Interactive graphical user interface
- **Documentation:** Books, papers, curriculum, etc.

Sage is not a drop-in replacement: does not run programs written in the custom languages of the Ma's.

Sage is not like Octave (versus Matlab).

Sage's culture, architecture, programming language, and feel are different than the Ma's.

Why *not* Magma, Maple, Matlab, Mathematica?

1. **Commercial:** Expensive for my collaborators and students. Not community owned.
2. **Closed:** Implementation of algorithms often secret
3. **Frustrating:** Tight control of development
4. **Copy protection:** Hard to run on supercomputer or my new laptop or after my 1-year license expires.
5. **Programming language:** All use a special math-only language
6. **Bugs:** Bug tracking is secret
7. **Compiler:** Lack of compilers for their math-only languages

1. **Python:** Sage uses a mainstream general purpose programming language (with a compiler: Cython)
2. **Distribution:** about 100 open source packages (**written by you** and your colleagues!)
3. **Interfaces:** smoothly tie together all these libraries and packages
4. **New library:** implements novel algorithms; over a half million lines; worldwide community of several hundred people.

```
4/6
```

    2/3

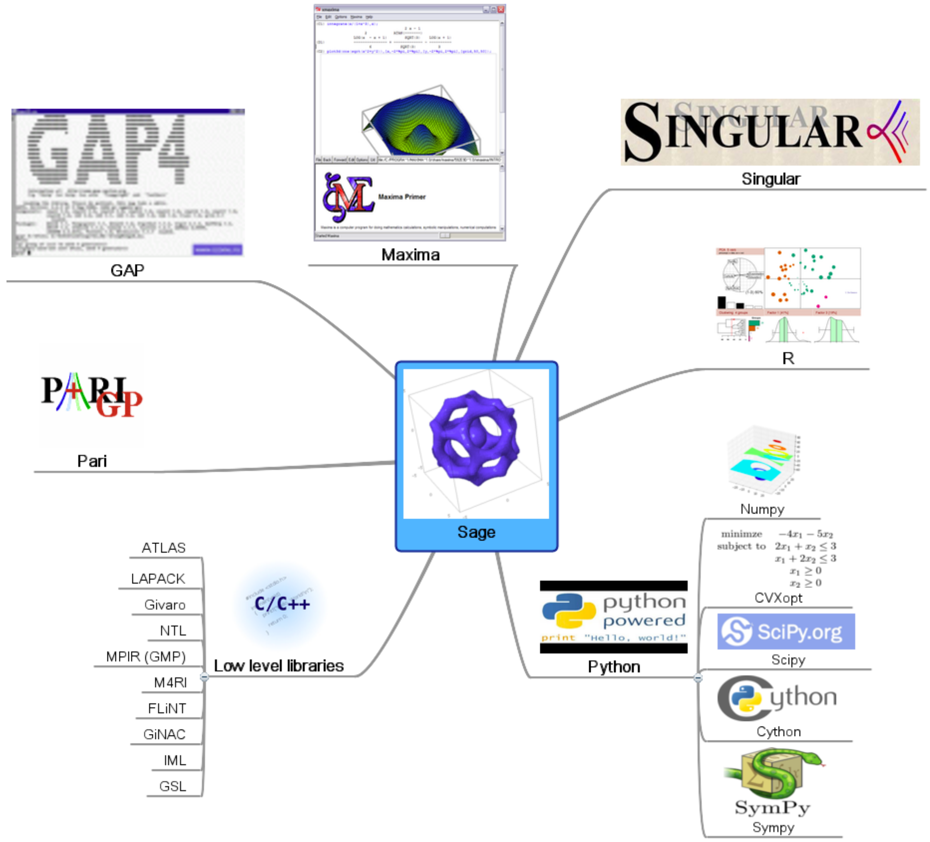
```
# Sage uses Python code

def foo(n, m):
    if n > m:
        print("%s is bigger!"%n)

foo(10, 5)
```

    10 is bigger!

```
foo('william', 'jon')
```

    william is bigger!

Distribution



... each with their own rich history!

Hundreds of Sage Developers

(There are at least 247 contributors in 167 different places from all around the world.)

William Stein, Tim Abbott, Michael Abshoff, Antti Ajanki, Martin Albrecht, Nick Alexander, Bill Allombert, Ethan Van Andel, Ivan Andrus, Pablo Angulo, Benjamin Antieau, André Apitzsch, Maite Aranes, Oscar Gerardo Lazo Arjona, Eviatar Bach, Jennifer Balakrishnan, Jason Bandlow, Gregory Bard, Sébastien Barthélemy, Rob Beezer, Karim Belabas, Arnaud Bergeron, Luis Berlioz, Erin Beyerstedt, Francois Bissey, Jonathan Bober, Tom Boothby, Nicolas Borie, Johan Bosman, Robert Bradshaw, Michael Brickenstein, Nils Bruin, André-Patrick Bubel, Stanislav Bulygin, Dan Bump, Iftikhar Burhanuddin, Paul Butler, Oriol Castejón, Ondrej Certik, Wilson Cheung, Dan Christensen, Craig Citro, Anders Claesson, Francis Clarke, Timothy Clemans, Alex Clemesha, Nathann Cohen, Jenny Cooley, John Cremona, Karl-Dieter Crisman, Fidel Barrera Cruz, Doug Cutrell, Alyson Deines, Vincent Delecroix, Jeroen Demeyer, Tom Denton, Maarten Derickx, Didier Deshommes, Ryan Dingman, Dan Drake, Tom Draper, Alexander Dreyer, Tim Dumol, Nathan Dunfield, Gabriel Ebner, Ben Edwards, Dana Ernst, Burcin Erocal, Ron Evans, Richard J. Fateman, Lars Fischer, Jean-Pierre Flori, Evan Fosmark, Laurent Fousse, Gary Furnish, Alex Ghitza, Andrzej Giniewicz, Alain Giorgetti, Samuele Giraudo, Amy Glen, Daniel Gordon, Chris Gorecki, Jan Groenewald, Rob Gross, Jason Grout, Ryan Grout, Mathieu Guay-Paquet, Alexey U. Gudchenko, Harold Gutch, Jonathan Gutow, Jose Guzman, Anna Haensch, Carlo Hamalainen, Marshall Hampton, Jon Hanke, David Møllerller Hansen, Mike Hansen, Bill Hart, David Harvey, Leif Hille, Florent Hivert, Ryan Hinton, Neal Holtz, Golam Mortuza Hossain, Sean Howe, Alexander Hupfer, Wilfried Huss, Hamish Ivey-Law, Naqi Jaffery, Peter Jeremy, Peter Jipsen, Fredrik Johansson, Niles Johnson, Timo Jolivet, Benjamin Jones, David Joyner, Michael Kallweit, Josh Kantor, Kiran Kedlaya, Lloyd Kilford, Simon King, Keshav Kini, David Kirkby, Emily Kirkman, David Kohel, Ted Kosan, Ross Kyprianou, Sébastien Labbé, Yann Laigle-Chapuy, Kwankyu Lee, Julien Leroy, Richard Lindner, David Loeffler, Miguel Marco, Michael Mardaus, Robert Mařík, Jason Martin, Alexandre Blondin Massé, Peter McNamara, Gregory McWhirter, Jason Merrill, Matthias Meulien, Robert Miller, Kate Minola, Moritz Minzlaff, Joel Mohler, Thierry Monteil, Peter Mora, Bobby Moretti, Rich Morin, Guillaume Moroz, Gregg Musiker, Tobias Nagel, Brett Nakashima, Pablo De Nápoli, Johan Sebastian Rosenkilde Nielsen, Minh Van Nguyen, Andrey Novoseltsev, Christopher Olah, Johan Oudinet, Bill Page, Ronan Paixão, Willem Jan Palenstijn, John Palmieri, Dmitrii Pasechnik, Javier López Peña, Paulo César Pereira de Andrade, David Perkinson, Clement Pernet, John Perry, Pearu Peterson, David Poetzsch-Heffter, Viviane Pons, Bill Purvis, Julien Puydt, Yi Qiang, Jordi Quer, Gustavo Rama, Jens Rasch, Martin Raum, Dorian Raymer, Stefan Reiterer, R. Rishikesh, David Roe, Bjarke Hammersholt Roune, Gordon Royle, Serge A. Salamanka, Franco Saliola, Leonardo Sampaio, Kyle Schalm, Ed Scheinerman, Anne Schilling, Harald Schilly, Jack Schmidt, Michael Schneider, Christopher Schwan, Dag Sverre Seljebotn, Dan Shumow, Denis Simone, Steven Sivek, Nils-Peter Skoruppa, Jaap Spies, Jonathan Spreer, Armin Straub, Marco Streng, Kevin Stueve, Christian Stump, Blair Sutton, Chris Swierczewski, Luis Felipe Tabera Alonso, Glenn Tarbox, Philippe Theveny, Nicolas Thiery, Griffen Thoma, Emmanuel Thomé, John Thurber, Igor Tolkov, Gonzalo Tornaria, Kiminori Tsukazaki, Charlie Turner, Michel Vandenbergh, Joris Vankerschaver, Soledad Villar, John Voight, Felipe Voloch, Steve Vonn, Justin Walker, Mark Watkins, Georg S. Weber, Eric Webster, Ralf-Philipp Weinmann, Joe Wetherell, Carl Witty, Cristian Wuthrich, Soroosh Yazdani, Dal S. Yu, Gary Zablackis, Mike Zabrocki, Bin Zhang, Paul Zimmermann, Mao Ziyang

---

History

- **2005:**SAGE-0.1 released February 1, 2005; SAGE=Software for Arithmetic Geometry Experimentation. Why did I name it "Sage"? claritalb.org
- **2006:** (2 Sage Days workshops); Sage is not just for number theory
- **2007:** (4 Sage Days) 100% test requirements; peer review of all new code (see trac); industry funding; NSF; Trophees du Libre
- **2008:** (7 Sage Days) Release managers besides me.

- **2009:** (8 Sage Days) Better foundations; 3d graphics; more developers (e.g., sage-combinat)
- **2010:** (13 Sage Days) More devs and users; nontrivial NSF grants
- **2011:** (12 Sage Days) Much faster http://sagenb.org with >50,000 accounts; very stable releases; undergrad curriculum development
- **2012:** (12 Sage Days)
- **2013:** Release Salvus...

See this article for more details about the (pre-)history of Sage.

Random Question Break

????

Some random examples involving Sage:

```
time n = factorial(10^7)
```

```
Time: CPU 11.42 s, Wall: 11.42 s
```

```
%cython
import numpy as np
cimport numpy as np

def mandelbrot_cython(float x0,float  x1,float  y0,float  y1,
                      int N=200, int L=50, float R=3):
    '''returns an array NxN to be plotted with matrix_plot
```

```
    '''
    cdef double complex c, z, I
    cdef float deltax, deltay, R2 = R*R
    cdef int h, j, k
    cdef np.ndarray[np.uint16_t, ndim=2] m
    m = np.zeros((N,N), dtype=np.uint16)
    I = complex(0,1)
    deltax = (x1-x0)/N
    deltay = (y1-y0)/N
    for j in range(N):
        for k in range(N):
            c = (x0+j*deltax)+ I*(y0+k*deltay)
            z=0
            h=0
            while (h<L and
                   z.real**2 + z.imag**2 < R2):
                z=z*z+c
                h+=1
            m[j,k]=h
    return m
```

__mnt_sage...1_code_sage12_spyx.c __mnt_sage...ode_sage12_spy:

```
import pylab
x0_default = -2
y0_default = -1.5
side_default = 3.0
side = side_default
x0 = x0_default
y0 = y0_default
options = ['Reset','Upper Left', 'Upper Right', 'Stay', 'Lower
Left', 'Lower Right']

@interact
def show_mandelbrot(option = selector(options, nrows = 2,
width=8),
                    N = slider(100, 1000,100, 300),
                    L = slider(20, 300, 20, 60),
                    plot_size = slider(2,10,1,6),
                    auto_update = False):
    global x0, y0, side
    if option == 'Lower Right':
        x0 += side/2
        y0 += side/2
    elif option == 'Upper Right':
        y0 += side/2
    elif option == 'Lower Left':
        x0 += side/2
    if option=='Reset':
```

```
        side = side_default
        x0 = x0_default
        y0 = y0_default
    elif option != 'Stay':
        side = side/2

    time m=mandelbrot_cython(x0 ,x0 + side ,y0 ,y0 + side , N, L
)
#    p = (matrix_plot(m) +
#             line2d([(N/2,0),(N/2,N)], color='red', zorder=2) +
#             line2d([(0,N/2),(N,N/2)], color='red', zorder=2))
#    time show(p, figsize = (plot_size, plot_size))
    pylab.clf()
    pylab.imshow(m, cmap = pylab.cm.gray)
    time pylab.savefig('mandelbrot.png')
```

```
random_matrix?
```

```
matrix?
```

```
a = matrix(ZZ, 10, 10, range(100), sparse=True); a
```

```
[ 0  1  2  3  4  5  6  7  8  9]
[10 11 12 13 14 15 16 17 18 19]
[20 21 22 23 24 25 26 27 28 29]
[30 31 32 33 34 35 36 37 38 39]
[40 41 42 43 44 45 46 47 48 49]
[50 51 52 53 54 55 56 57 58 59]
[60 61 62 63 64 65 66 67 68 69]
[70 71 72 73 74 75 76 77 78 79]
[80 81 82 83 84 85 86 87 88 89]
[90 91 92 93 94 95 96 97 98 99]
```

```
dir(a)
```

```
['C', 'H', 'I', 'LU', 'N', 'QR', 'T', '__abs__', '__add__',
'__array__', '__call__', '__class__', '__cmp__', '__copy__',
'__delattr__', '__delitem__', '__dict__', '__dir__', '__div__',
'__doc__', '__eq__', '__format__', '__ge__', '__getattribute__'
'__getitem__', '__getstate__', '__gt__', '__hash__', '__iadd__'
'__idiv__', '__imul__', '__init__', '__invert__', '__isub__',
'__iter__', '__le__', '__lt__', '__mod__', '__module__', '__mul_
'__ne__', '__neg__', '__new__', '__nonzero__', '__pos__', '__pow
'__pyx_vtable__', '__radd__', '__rdiv__', '__reduce__',
'__reduce_ex__', '__repr__', '__rmod__', '__rmul__', '__rpow__'
'__rsub__', '__rtruediv__', '__rxor__', '__setattr__',
'__setitem__', '__setstate__', '__sizeof__', '__str__', '__sub_
```

```
'__subclasshook__', '__truediv__', '__weakref__', '__xor__',
'_act_on_', '_acted_upon_', '_add_', '_add_parent', '_adjoint',
'_axiom_', '_axiom_init_', '_backslash_', '_base_ring', '_cache
'_charpoly_df', '_charpoly_hessenberg',
'_charpoly_over_number_field', '_check_symmetrizability',
'_cholesky_decomposition_', '_clear_cache', '_cmp_', '_coeff_re
'_column_ambient_module', '_decomposition_spin_generic',
'_decomposition_using_kernels', '_derivative', '_dict', '_div_'
'_dummy_attribute', '_echelon_classical', '_echelon_form_PID',
'_echelon_in_place_classical', '_echelon_strassen',
'_echelonize_ring', '_eigenspace_format', '_elementwise_product
'_fricas_', '_fricas_init_', '_gap_', '_gap_init_', '_get_cache
'_giac_', '_giac_init_', '_gp_', '_gp_init_',
'_gram_schmidt_noscale', '_iadd_', '_idiv_', '_ilmul_', '_im_ge
'_imul_', '_indefinite_factorization', '_interface_',
'_interface_init_', '_interface_is_cached_', '_is_atomic', '_is
'_kash_', '_kash_init_', '_latex_', '_latex_coeff_repr',
'_linbox_sparse', '_list', '_lmul_', '_macaulay2_',
'_macaulay2_init_', '_magma_init_', '_make_new_with_parent_c',
'_maple_', '_maple_init_', '_mathematica_', '_mathematica_init_
'_matrix_', '_maxima_', '_maxima_init_', '_maxima_lib_',
'_maxima_lib_init_', '_mod_int', '_mpmath_', '_mul_', '_mul_par
'_multiply_classical', '_multiply_classical_with_cache',
'_multiply_strassen', '_neg_', '_nonzero_positions_by_column',
'_nonzero_positions_by_row', '_numerical_approx', '_octave_',
'_octave_init_', '_pari_', '_pari_init_', '_pickle', '_pow_',
'_pow_naive', '_r_init_', '_reduction', '_repr_', '_richcmp_',
'_right_kernel_matrix', '_right_kernel_matrix_over_domain',
'_right_kernel_matrix_over_field',
'_right_kernel_matrix_over_number_field', '_rmul_',
'_row_ambient_module', '_sage_', '_sage_input_', '_sage_src_lin
'_scilab_', '_scilab_init_', '_set_parent',
'_set_row_to_negative_of_row_of_A_using_subset_of_columns',
'_singular_', '_singular_init_', '_solve_right_general',
'_solve_right_nonsingular_square', '_sub_', '_subdivide_on_augm
'_subdivide_on_stack', '_subdivisions', '_test_category',
'_test_change_ring', '_test_eq', '_test_not_implemented_methods
'_test_pickling', '_test_reduce', '_tester', '_travel_column',
'_unpickle_generic', '_zigzag_form', 'abs', 'act_on_polynomial'
'add_multiple_of_column', 'add_multiple_of_row', 'additive_orde
'adjoint', 'antitranspose', 'apply_map', 'apply_morphism',
'as_sum_of_permutations', 'augment', 'base_extend', 'base_ring'
'block_sum', 'cartesian_product', 'category', 'change_ring',
'characteristic_polynomial', 'charpoly', 'cholesky',
'cholesky_decomposition', 'column', 'column_module', 'column_sp
'columns', 'commutator', 'conjugate', 'conjugate_transpose', 'c
'db', 'decomposition', 'decomposition_of_subspace',
'delete_columns', 'delete_rows', 'denominator', 'dense_columns'
'dense_matrix', 'dense_rows', 'density', 'derivative', 'det',
'determinant', 'diagonal', 'dict', 'dimensions', 'dump', 'dumps
'echelon_form', 'echelonize', 'eigenmatrix_left',
```

```
'eigenmatrix_right', 'eigenspaces_left', 'eigenspaces_right',
'eigenvalues', 'eigenvectors_left', 'eigenvectors_right',
'elementary_divisors', 'elementwise_product', 'exp',
'extended_echelon_form', 'fcp', 'find', 'get_subdivisions',
'gram_schmidt', 'hadamard_bound', 'hermite_form', 'hessenberg_fo
'hessenbergize', 'image', 'indefinite_factorization',
'integer_kernel', 'inverse', 'is_bistochastic', 'is_dense',
'is_diagonalizable', 'is_hermitian', 'is_idempotent',
'is_immutable', 'is_invertible', 'is_mutable', 'is_nilpotent',
'is_normal', 'is_one', 'is_positive_definite', 'is_scalar',
'is_similar', 'is_singular', 'is_skew_symmetric',
'is_skew_symmetrizable', 'is_sparse', 'is_square', 'is_symmetric
'is_symmetrizable', 'is_unit', 'is_unitary', 'is_zero', 'iterate
'jordan_form', 'kernel', 'kernel_on', 'left_eigenmatrix',
'left_eigenspaces', 'left_eigenvectors', 'left_kernel',
'left_nullity', 'lift', 'linear_combination_of_columns',
'linear_combination_of_rows', 'list', 'matrix_from_columns',
'matrix_from_rows', 'matrix_from_rows_and_columns',
'matrix_over_field', 'matrix_space', 'matrix_window', 'maxspin',
'minimal_polynomial', 'minors', 'minpoly', 'mod',
'multiplicative_order', 'mutate', 'n', 'ncols', 'new_matrix',
'nonpivots', 'nonzero_positions', 'nonzero_positions_in_column',
'nonzero_positions_in_row', 'norm', 'nrows', 'nullity',
'numerical_approx', 'numpy', 'order', 'parent', 'permanent',
'permanental_minor', 'pivot_rows', 'pivots', 'plot',
'prod_of_row_sums', 'randomize', 'rank', 'rational_form',
'rational_reconstruction', 'rename', 'rescale_col', 'rescale_row
'reset_name', 'restrict', 'restrict_codomain', 'restrict_domain
'right_eigenmatrix', 'right_eigenspaces', 'right_eigenvectors',
'right_kernel', 'right_kernel_matrix', 'right_nullity',
'rook_vector', 'row', 'row_module', 'row_space', 'rows', 'rref',
'save', 'set_block', 'set_col_to_multiple_of_col', 'set_column',
'set_immutable', 'set_row', 'set_row_to_multiple_of_row',
'smith_form', 'solve_left', 'solve_right', 'sparse_columns',
'sparse_matrix', 'sparse_rows', 'stack', 'str', 'subdivide',
'subdivision', 'subdivision_entry', 'subdivisions', 'submatrix',
'subs', 'substitute', 'swap_columns', 'swap_rows',
'symplectic_form', 'tensor_product', 'trace', 'trace_of_product
'transpose', 'version', 'visualize_structure', 'weak_popov_form
'wiedemann', 'with_added_multiple_of_column',
'with_added_multiple_of_row', 'with_col_set_to_multiple_of_col',
'with_rescaled_col', 'with_rescaled_row',
'with_row_set_to_multiple_of_row', 'with_swapped_columns',
'with_swapped_rows', 'zigzag_form']
```

```
a.rank()
```

2

```
a.rank?
```

File: /home/salvus/sage-5.3/devel/sage/sage/matrix/matrix0.pyx

Type: <type 'builtin_function_or_method'>

Definition: a.rank()

Docstring:

TESTS:

We should be able to compute the rank of a matrix whose entries are polynomials over a finite field (trac #5014):

```
sage: P.<x> = PolynomialRing(GF(17))
sage: m = matrix(P, [ [ 6*x^2 + 8*x + 12, 10*x^2 + 4*x + 11],
...                  [8*x^2 + 12*x + 15,  8*x^2 + 9*x + 16] ])
sage: m.rank()
2
```

```
a.rank(
```

```
matrix(
```

```
A = random_matrix(QQ, 4); b = A^(-1); b
```

```
[   -1  1/4  1/4 -3/4]
[-1/2  1/8  1/8  1/8]
[   -1  3/4 -1/4 -1/4]
[-1/2 -1/8 -1/8 -1/8]
```

```
show(b)
```

$$\left(\begin{array}{rrrr}
1 & 0 & 0 & 0 \\
-\frac{27}{34} & \frac{5}{17} & -\frac{8}{17} & \frac{6}{17} \\
\frac{8}{17} & -\frac{8}{17} & \frac{6}{17} & \frac{4}{17} \\
\frac{11}{34} & \frac{3}{17} & \frac{2}{17} & \frac{7}{17}
\end{array}\right)$$

```
latex(b)
```

```
\left(\begin{array}{rrrr}
1 & 0 & 0 & 0 \\
-\frac{27}{34} & \frac{5}{17} & -\frac{8}{17} &
\frac{6}{17} \\
\frac{8}{17} & -\frac{8}{17} & \frac{6}{17} &
\frac{4}{17} \\
\frac{11}{34} & \frac{3}{17} & \frac{2}{17} &
\frac{7}{17}
```

```
\end{array}\right)
```

```
show(integrate(sin(x)*cos(x), x))
```

$$-\frac{1}{2}\,\cos\left(x\right)^{2}$$

```
f(x) = sin(x^2) * cos(sin(x)) + tan(x) - log(x); f
```

```
x |--> sin(x^2)*cos(sin(x)) - log(x) + tan(x)
```

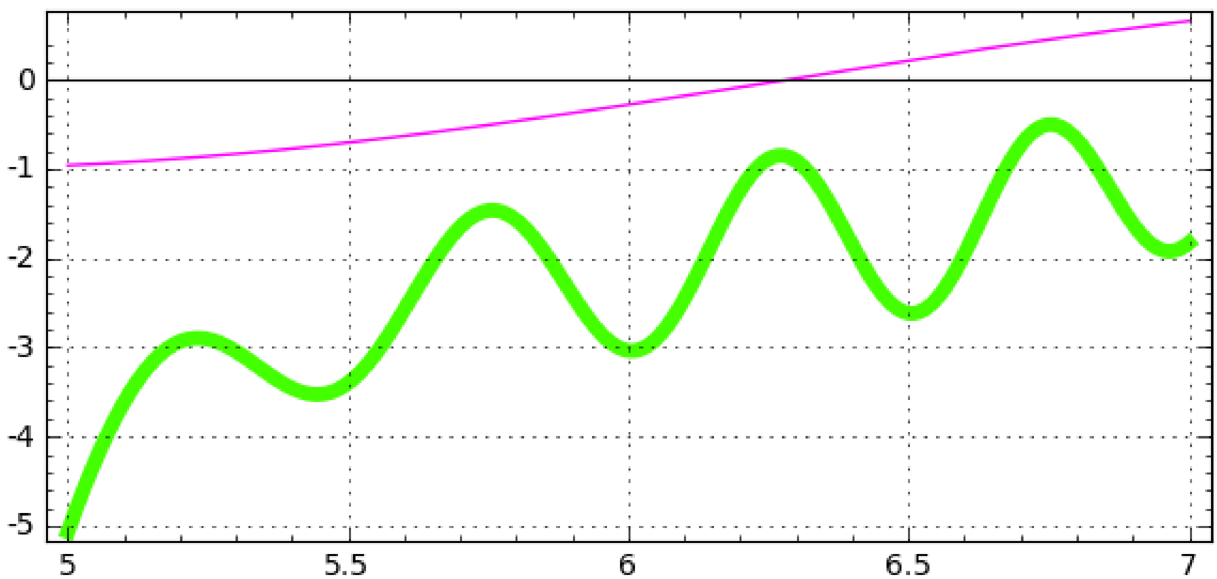
```
show(f)
```

$$x \mapsto \sin\left(x^{2}\right)\cos\left(\sin\left(x\right)\right) - \log\left(x\right) + \tan\left(x\right)$$

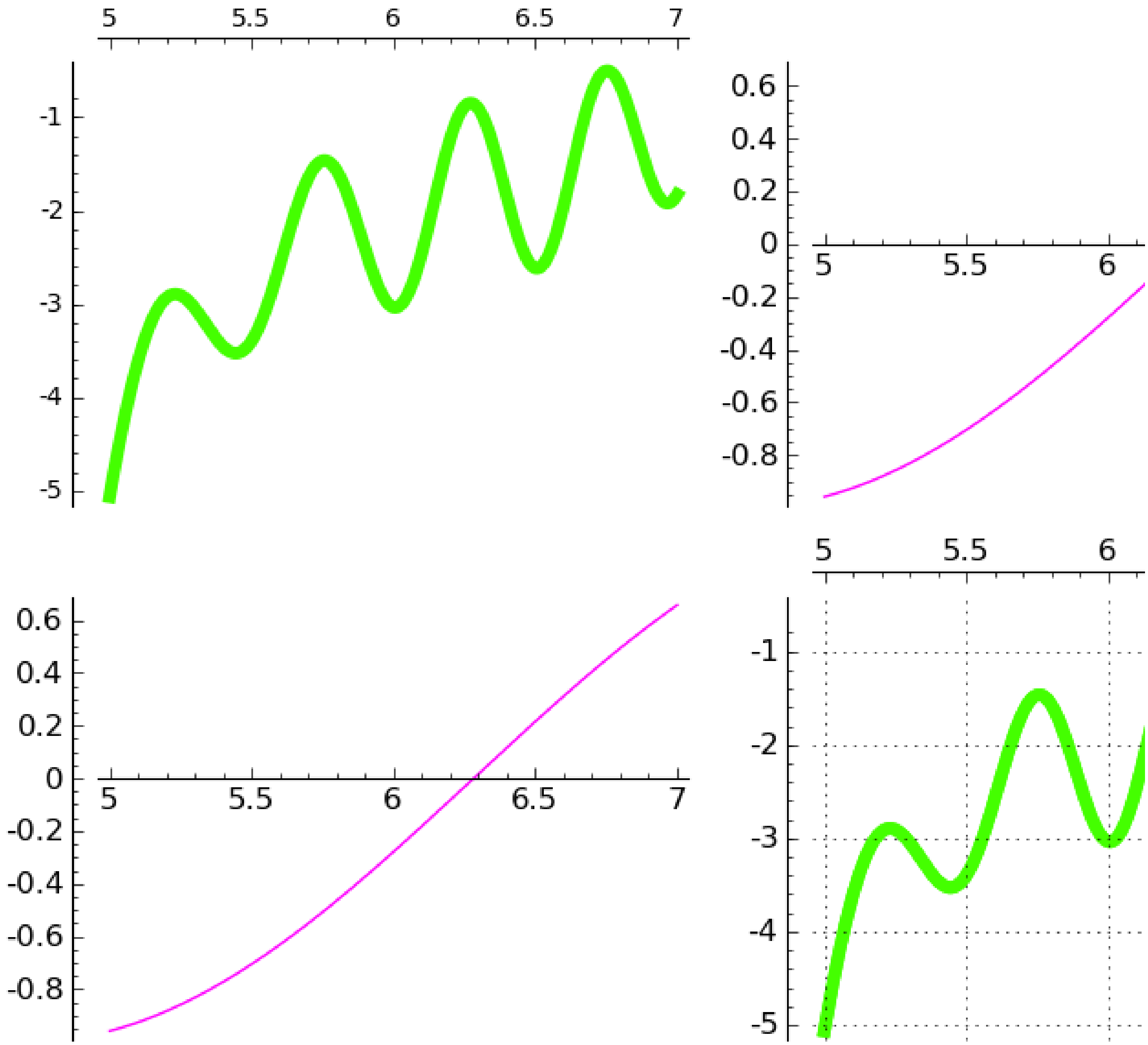
```
latex(f)
```

```
x \ {\mapsto}\ \sin\left(x^{2}\right)
\cos\left(\sin\left(x\right)\right) - \log\left(x\right) +
\tan\left(x\right)
```

```
G = plot(f, (5, 7), color='#44ff00', thickness=5,
    figsize=[6,3], gridlines=True, frame=True)
H = plot(sin(x), 5,7, color='magenta')
G + H
```




```
graphics_array([[G, H], [H, G]])
```

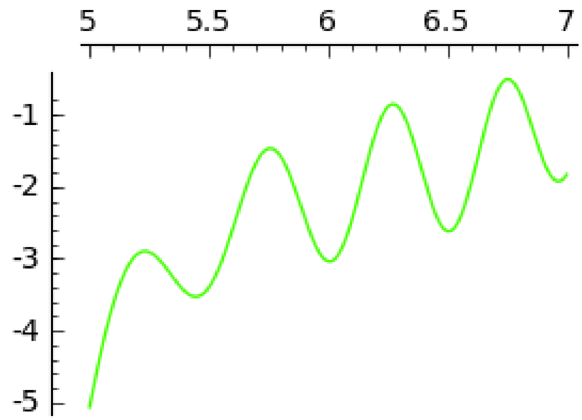


```
help(interact)
```

[Click to open help window]

```
@interact
def g(xmax=(6, 20), color=Color('red')):
    G = plot(f, (5, xmax), figsize=[6,3], color=color)
    G.show(ymin=-10, ymax=10)
```

```
g(7)
```




```
G = plot(sin, 0, 10)
G.save('a.pdf')
```

a.pdf

```
sage.interfaces.
```

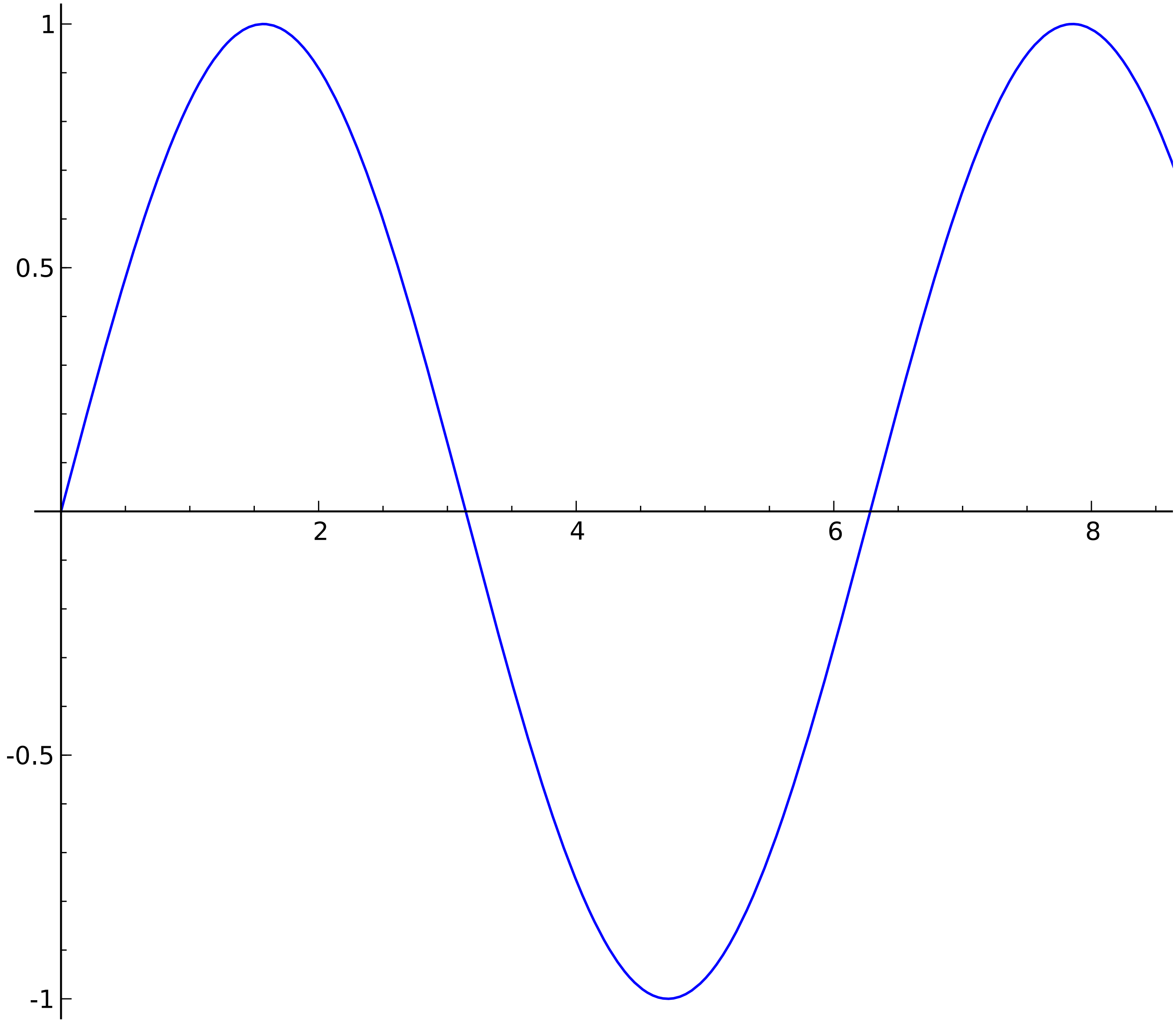
```
G.save('a.ps')
```

```
/home/salvus/sage-5.3/local/lib/python2.7/site-packages/matplot:
ight_bbox.py:55: UserWarning: bbox_inches option for ps backend
not implemented yet.
  warnings.warn("bbox_inches option for %s backend is not
implemented yet." % (format))
```

a.ps

```
G.save(
```

```
G.save('a.svg')
```




```
a = octave('rand(5)'); a
```

```
0.549036 0.512252 0.604208 0.0767369 0.747327
0.266268 0.756133 0.682624 0.645004 0.31979
0.914937 0.88774 0.630625 0.973085 0.791813
0.9205 0.500435 0.365157 0.392919 0.994673
0.651773 0.531602 0.127743 0.911378 0.685484
```

```
a.eig()
```

```
(3.02114,0)
(-0.192864,0.125433)
(-0.192864,-0.125433)
(0.377322,0)
(0.00146815,0)
```

```
%octave

x = rand(5)
eig(x)
```

```
x =
```

```
 0.343843 0.589597 0.408991 0.989844 0.0934817
 0.626751 0.00267151 0.797916 0.978089 0.465727
 0.20643 0.0241775 0.966753 0.660207 0.475048
 0.0305844 0.97238 0.381771 0.431595 0.556873
 0.758349 0.885778 0.676631 0.431916 0.627955


ans =

 (2.64135,0)
 (-0.644357,0)
 (-0.0211313,0.46375)
 (-0.0211313,-0.46375)
 (0.418085,0)
```

How to Use Sage

- Type Python code into the boxes and press shift-enter or click "evaluate". More about the notebook... (read the bottom of the page that appears)
- There is extensive documentation, including a massive reference manual.
- Get help at: live chat, ask.sagemath.org, and the sage-support google group.

Interactive Image Compression

(using numpy)

```
import pylab, numpy

X = pylab.imread(DATA + 'sacnas.png')
A_image = numpy.mean(X, 2)
u,s,v = numpy.linalg.svd(A_image)
S = numpy.zeros(A_image.shape)
S[:len(s),:len(s)] = numpy.diag(s)
n = A_image.shape[0]

@interact
def svd_image(i = ("Eigenvalues (quality)",
                   (20,(1..A_image.shape[0]//2)))):
    A_approx = numpy.dot(numpy.dot(u[:,:i], S[:i,:i]), v[:i,:])
    g = graphics_array([matrix_plot(A_approx),
                        matrix_plot(A_image)])
    show(g, axes=False, figsize=8)
    html("%sx%s image compressed to %.1f%% of size using %s
eigenvalues."%(
            A_image.shape[0], A_image.shape[1],100*
(2.0*i*n+i)/(n*n), i))
```

Number Theory

```
factor(2009201020112012)      # makes use of PARI
```

```
2^2 * 43 * 2269 * 5148259709
```

```
# Jon Bober - Rademacher's formula
time number_of_partitions(10^6)
```

```
1471684986358223398631004760609895943484030484439142125334612747
661174189186182763301488739835975558420153741306002880959293873
232270327849578001932784396072064228659048713020170971840761025
986084690814282935670692978599129051989944549067221999782345287
740222882298501367675662947818874946878790038246999881977292006
668735996662273816798266213482417208446631027428001918132198177
651123454259502672842445259229678119344813999466473010574256435
949891814852853513705513994767199816914590220155991019596014174
715430750022184895815209339012481734469448319323280150665384042
417958775176129491624814247999880293650719525707448504757166277
033914424951138232981952630083364898260458377122024553049963821
028531832004519046591968302787537418118486000612016852593542741
504626724547323732184583342751252422746539913017407694128084740
422179992860711083363033162982891024446496968053954167918754800
636774022023128467646919775022348562520747741843343657801534130
197553037516970799928704028567784161934747236817177215404666430
15630003467104673818
Time: CPU 0.03 s, Wall: 0.03 s
```

```
@interact
def _(n=(25..10000)):
    plot(prime_pi, 0, n, gridlines='minor').show(figsize=[8,3])
```

Graph Theory

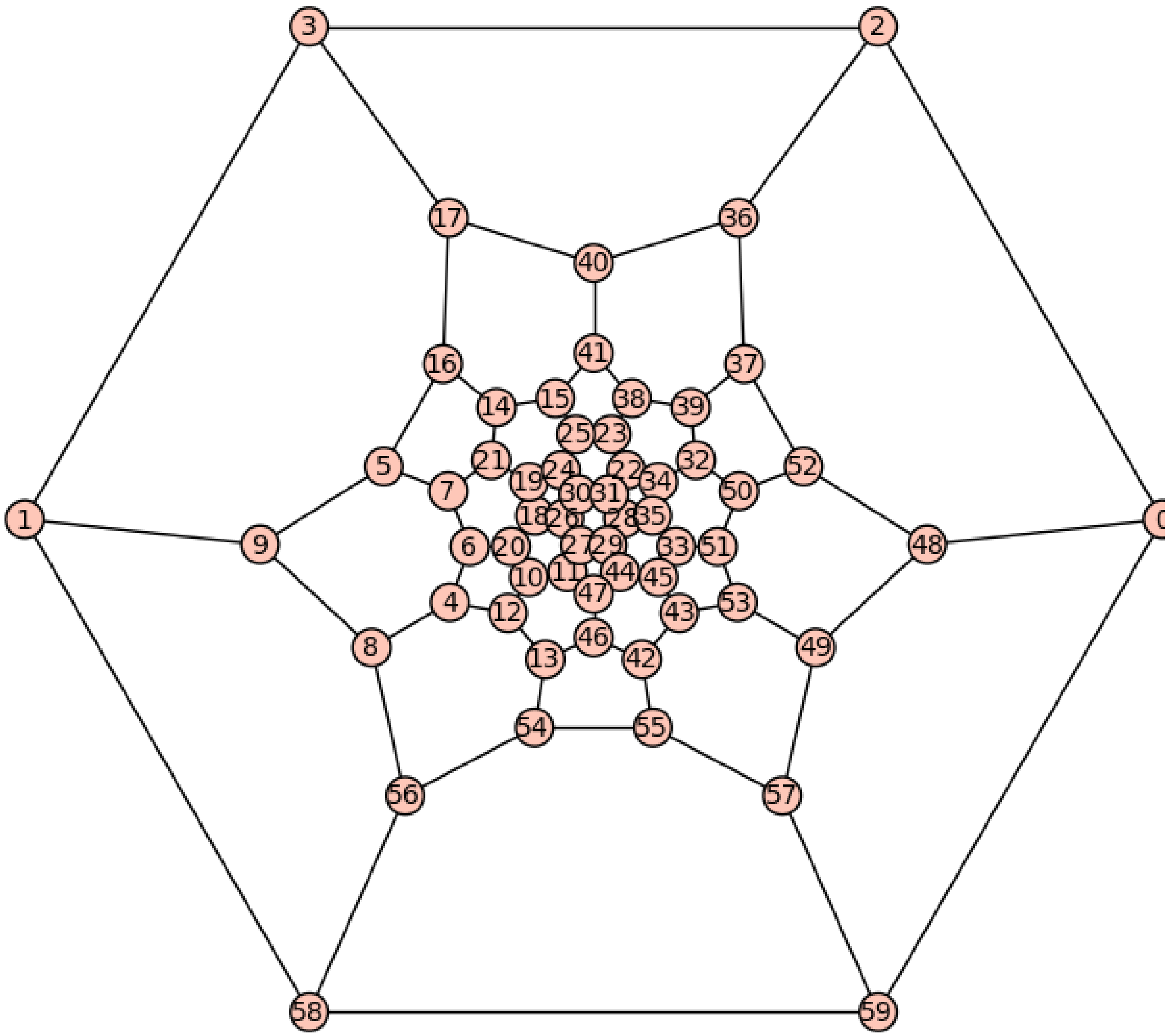
"Sage's graph theory crushes anything I have met from the point of view of methods implemented. I would say: 'if you found a proprietary graph library and you are convinced that it is better for your needs than Sage, your license is on me.' But those licenses are really expensive :-D"

```
graphs.
```

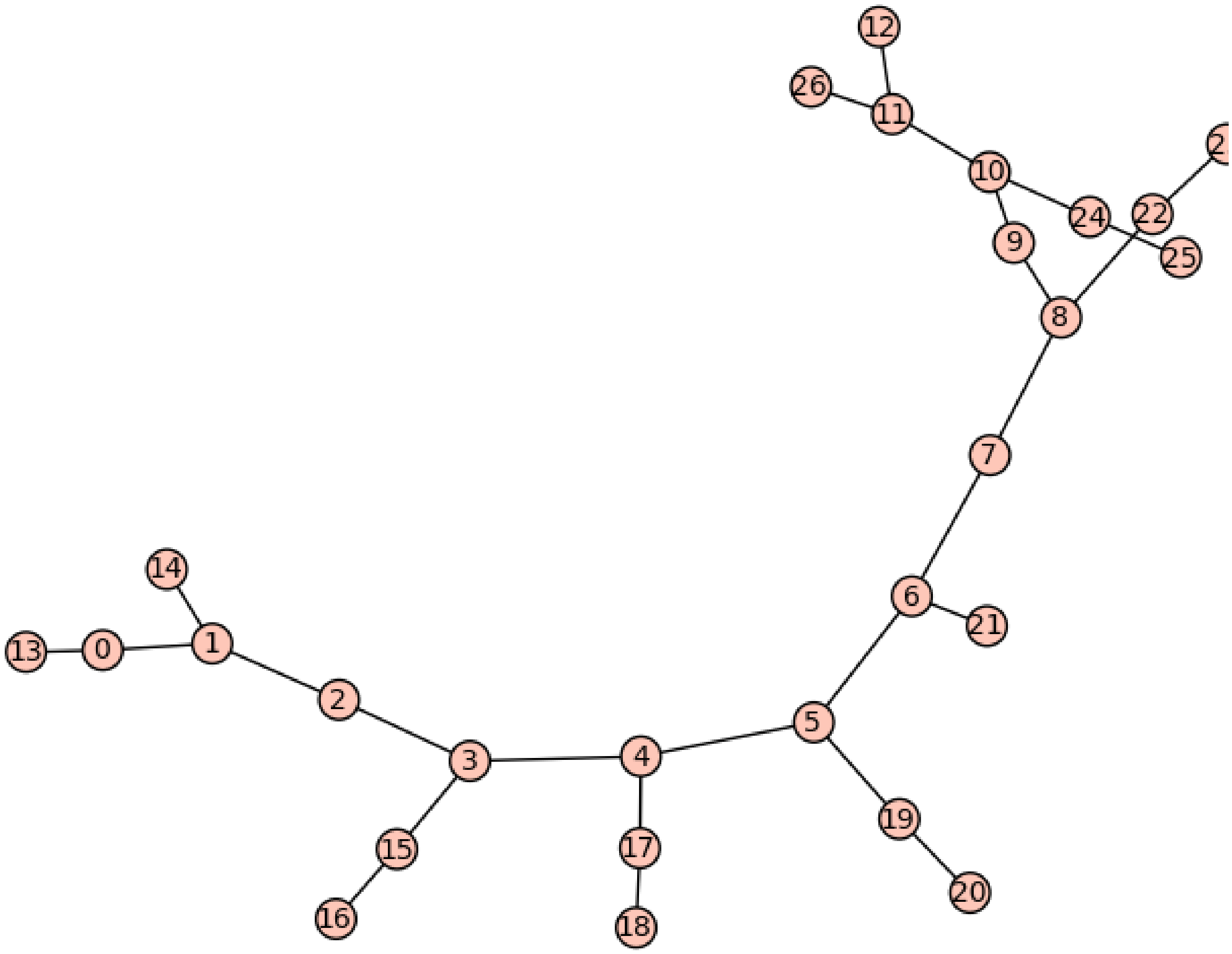
```
Traceback (click to the left of this block for traceback)
...
```

SyntaxError: invalid syntax

```
G = graphs.BuckyBall()
G.plot().show(figsize=8)
```



```
set_random_seed(1)
G = graphs.RandomLobster(8, .6, .3)
show(G, figsize=7)
```




```
G.automorphism_group()
```

    Permutation Group with generators [(12,26)]

```
G.chromatic_number()
```

    2

```
G.shortest_path(13,20)
```

    [13, 0, 1, 2, 3, 4, 5, 19, 20]

Cython

Cython C-Extensions for Python

- Smooth transition between Python and compiled C code.
- Make code that involves lots of manipulation of C-level data structures optimally fast
- Heavily used in scientific computing using Python.

```
def python_sum(n):
    s = int(0)
    for i in xrange(1, n+1):
        s += i*i
    return s
```

```
python_sum(3)
```

```
14
```

```
time python_sum(2*10^6)
```

```
2666668666667000000
Time: CPU 0.18 s, Wall: 0.18 s
```

```
timeit('python_sum(2*10^6)')
```

```
5 loops, best of 3: 173 ms per loop
```

```
def python_sum2(n):
    return sum(i*i for i in xrange(1,n+1))
```

```
time python_sum2(2*10^6)
```

```
2666668666667000000
Time: CPU 0.21 s, Wall: 0.21 s
```

```
%cython
def cython_sum(long n):
    cdef long long i, s = 0
    for i in range(1, n+1):
        s += i*i
    return s
```

__mnt_sage...1_code_sage91_spyx.c __mnt_sage...ode_sage91_spy

```
cython_sum(3)
```

```
14L
```

```
time cython_sum(2*10^6)
```

```
2666668666667000000L
Time: CPU 0.00 s, Wall: 0.00 s
```

```
timeit('cython_sum(2*10^6)')
```

```
125 loops, best of 3: 2.79 ms per loop
```

```
165/.663
```

```
248.868778280543
```

```
%cython
def cython_sum2(long n):
    cdef long long i
    return sum(i*i for i in range(1,n+1))
```

__mnt_sage...1_code_sage97_spyx.c __mnt_sage...ode_sage97_spy:

```
time cython_sum2(2*10^6)
```

```
2666668666667000000
Time: CPU 0.22 s, Wall: 0.22 s
```

Of course, it is better to choose a different algorithm:

```
var('k, n')
factor(sum(k^2, k, 1, n))
```

```
1/6*(n + 1)*(2*n + 1)*n
```

```
def sum2(n):
    return n*(2*n+1)*(n+1)/6
```

```
sum2(2*10^6)
```

```
2666668666667000000
```

Even then, Cython provides a speedup:

```
%cython
def c_sum2(long long n):
    return n*(2*n+1)*(n+1)/6
```

__mnt_sage..._code_sage104_spyx.c __mnt_sage...de_sage104_spy:

```
c_sum2(3)
```

```
14L
```

```
c_sum2(2*10^6)
```

```
-4077886789512586603L
```

```
n = 2*10^6
timeit('sum2(n)')
```

```
625 loops, best of 3: 2.29 µs per loop
```

```
timeit('c_sum2(n)')
```

    625 loops, best of 3: 145 ns per loop

```
2.01/.218
```

    9.22018348623853

But at a cost!

```
c_sum2(2*10^6)          # WARNING: overflow -- it's just like C...
```

    -4077886789512586O3L

```
n=2*10^6; n*(2*n+1)*(n+1) > 2^63
```

    True

Solving Equations

Solve a ***cubic equation***:

```
x = var('x'); show(solve(x^3 + x - 1==0, x)[0])
```

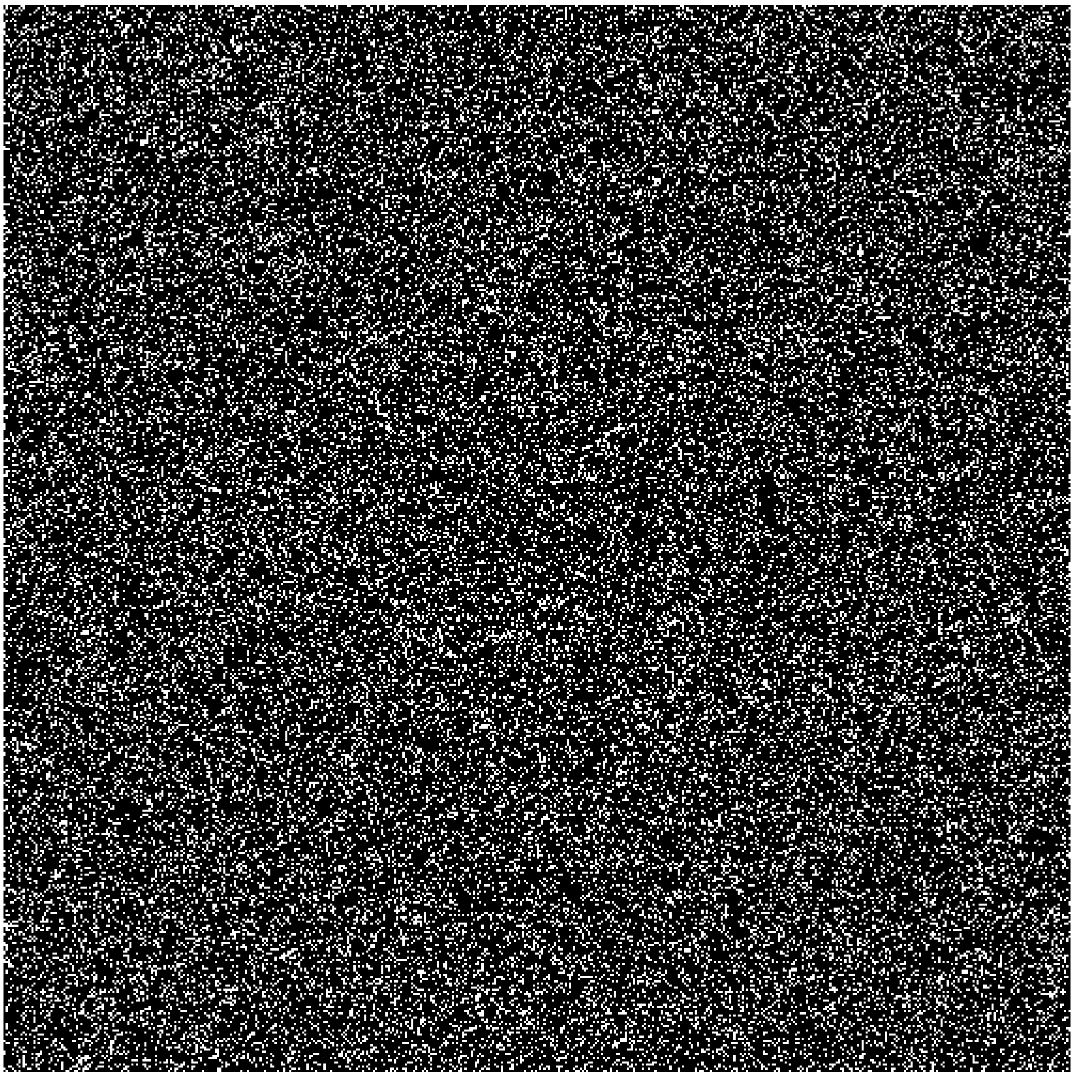
$$x = -\frac{1}{2}\left(i\sqrt{3}+1\right)\left(\frac{1}{18}\sqrt{3}\sqrt{31}+\frac{1}{2}\right)^{\left(\frac{1}{3}\right)} + \frac{-i\sqrt{3}+1}{6\left(\frac{1}{18}\sqrt{3}\sqrt{31}+\frac{1}{2}\right)^{\left(\frac{1}{3}\right)}}$$

Solve a system of ***two linear equations*** with one unknown coefficient α:

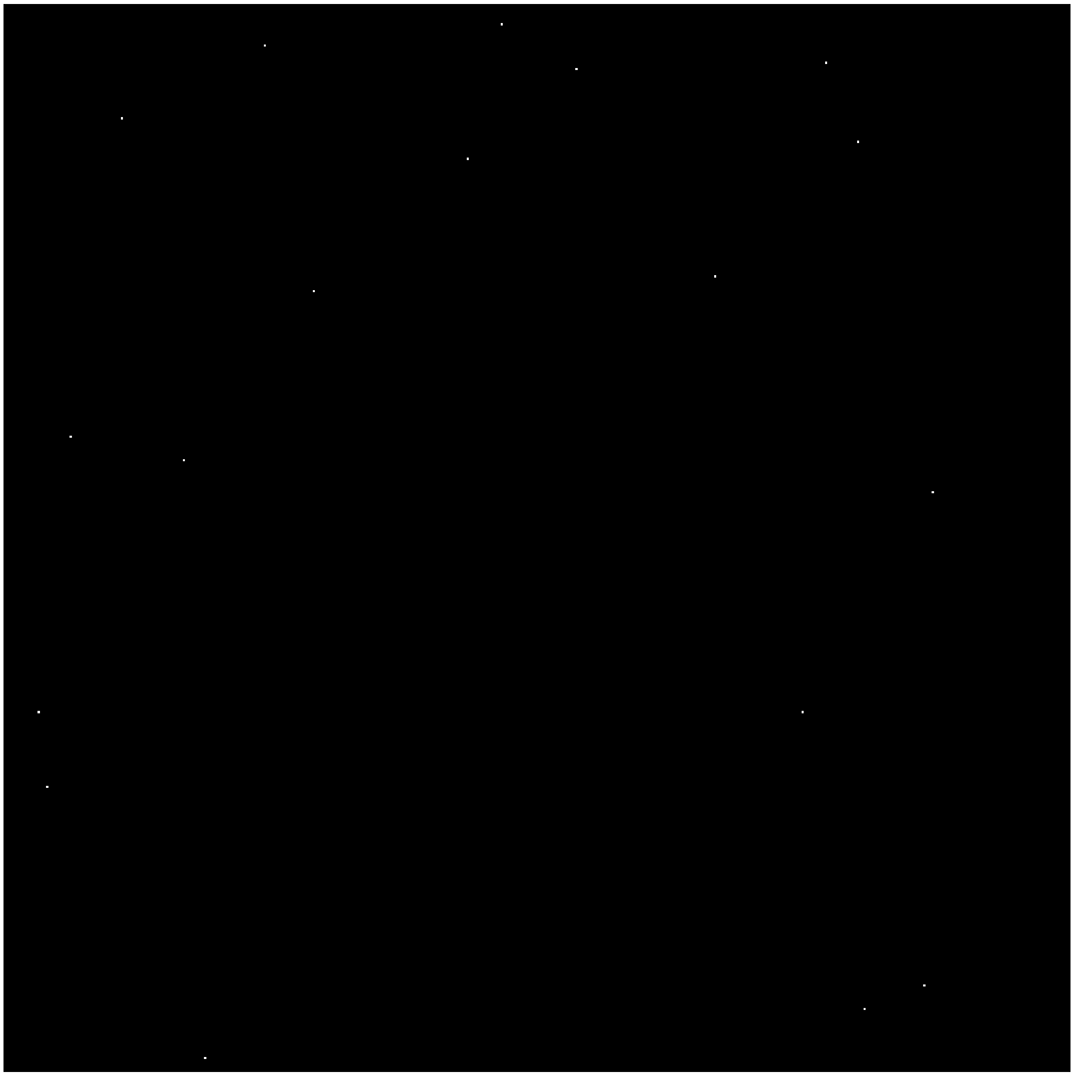
```
var('alpha, y')
show(solve([3*x + 7*y == 2, alpha*x + 3*y == 8], x,y)[0])
```

Solve a ***system of 500 linear equations*** exactly over the rational numbers:

```
n = 500
A = random_matrix(ZZ, n)
A.visualize_structure()
```



```
(A*A).visualize_structure()
```



```
import scipy.optimize
```

```
scipy.optimize.bisect(
```

```
# IML is used -- http://www.cs.uwaterloo.ca/~astorjoh/iml.html
time w = A \ v
```

```
Time: CPU 1.84 s, Wall: 1.85 s
```

```
print w.str()
```

```
WARNING: Output truncated!
```

full_output.txt

```
[
```

-351702876068802906437552420686055850326076457904676962275100083
401420955261319182505472655403446871133375351213855350226815464
541678542779937822420479756487655377896847799471633977576723782
752013079903081977572402942342855631847634917334375567725200949
981494332871024651254158873412391649819482714094296169547438994
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920100244120646553550505898020495083583179450653843216750029252
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```
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```

```
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```

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105441634514379785943438321498699491374429974941411916188300692

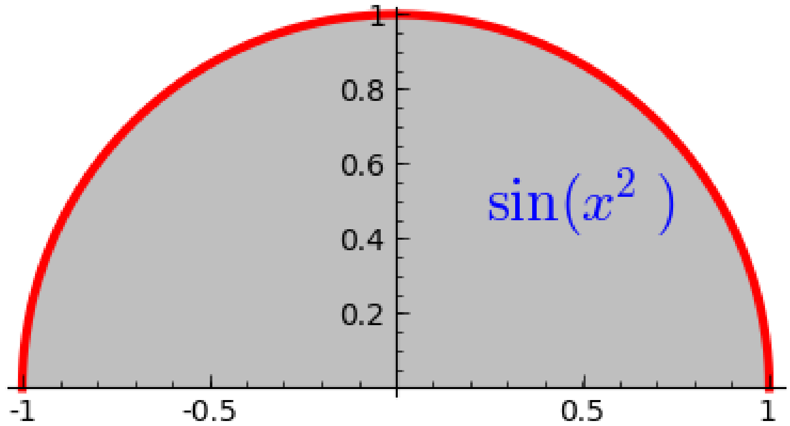
```
706620767186029329173570604490337945916332972221546248700476460
502278980483284010730068357338544234999822475021177554809695612
088131965551967459900917790184504128348366688399796247714348331
627651970837037774822010789350031850608000787729504461939133380
440429812707329238458478406610878319190930005297973995999042181
292585508525349195563753868289669013772157736583348697099384043
434454763548827291143983292206607531343281703691436870445680812
392732239105181275126941391265086958312816788605137121813337291
904365496018849941845190976573970615503284681408750206307284620
642301419081361189982078607311092373650759222409887083418418935
938423741879013097092493343757109949857355576727886100806294081
015171554449734793702597134820310225203084105776449433119524680
386618196964704137014248022985935487769788321086562349102641095
059115933183860757493359204400959331871604592993396432099609960
440125644540573643065630406530215220384502737018981682502335606
423315944686411791581384282773497317037372199630184176323632993
943867832767950499011442952233637773101716823994314895126742837
221666388715006803961777072989016656984486829254799709389454885
152072863733457370630217061791801859393643999498131050846806375
098267577867063578291309807040813378428531000012772498467389977
785823852018104843245188926381179282263086482208903119981671229
664970723542982390019017138746484439169044116493639168705177438
271394960123062314017832363588783497786530171331707763932291900
628990310160502280814134853096788081965867629862390376323081486
1447209515623489102340]
```

full_output.txt

Symbolic Calculus

Symbolic Calculus makes use of **Maxima** and **Ginac** under the hood.

```
var('x')
f = sqrt(1 - x^2)
plot(f, thickness=3, color='red', aspect_ratio=1, fill=True,
figsize=4) + text(r"$\sin(x^2)$", (.5,.5), fontsize=20)
```




```
var('t')
assume(t+1 > 0)
g = integrate(f, (x, -1, t)); show(g)
```

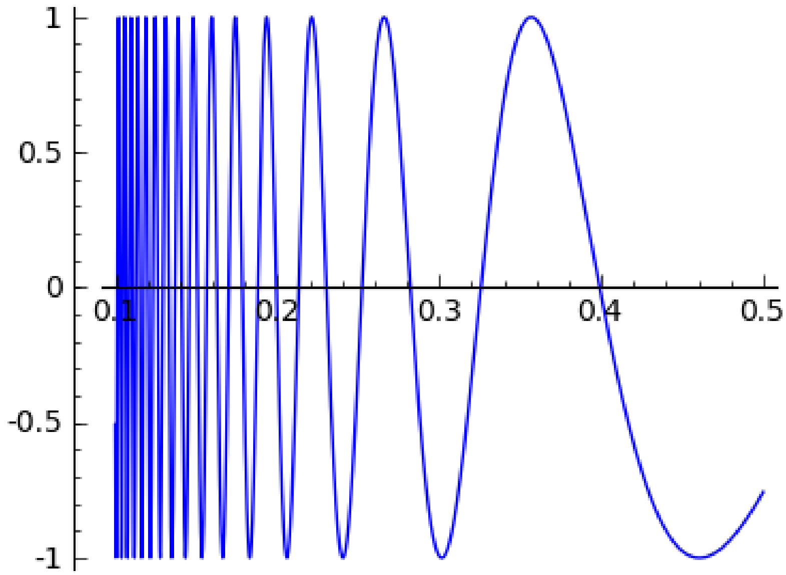
$$\frac{1}{4}\,\pi + \frac{1}{2}\,\sqrt{-t^{2} + 1}t + \frac{1}{2}\,\arcsin\left(t\right)$$

```
show(g(t=1) - g(t=-1))
```

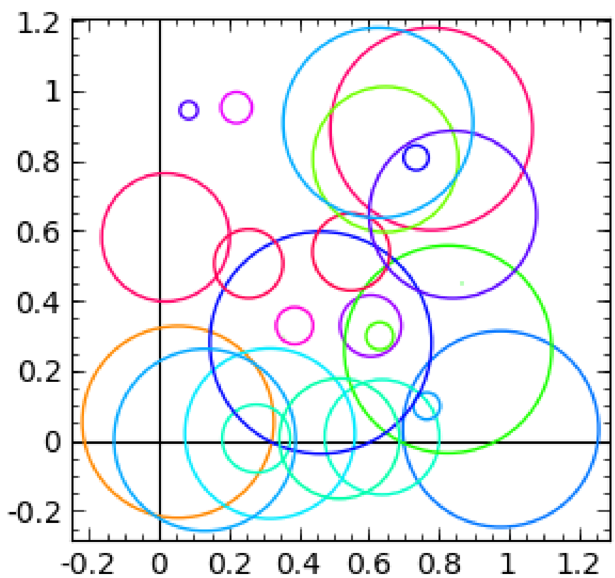
$$\frac{1}{2}\,\pi$$

Plotting in 2D

```
plot(sin(1/x^2), (x,.1,.5), figsize=4)
```




```
G = sum(circle((random(), random()), random()/3,
        color=hue(random())) for i in range(25))
G.show(aspect_ratio=1, frame=True, figsize=4)
```



Plotting in 3D

```
f(x,y) = sin(x - y) * y * cos(x)
time g = plot3d(f, (x,-3,3), (y,-3,3), opacity=.9, color='red',
figsize=3)
```

Time: CPU 0.00 s, Wall: 0.00 s

```
g
```

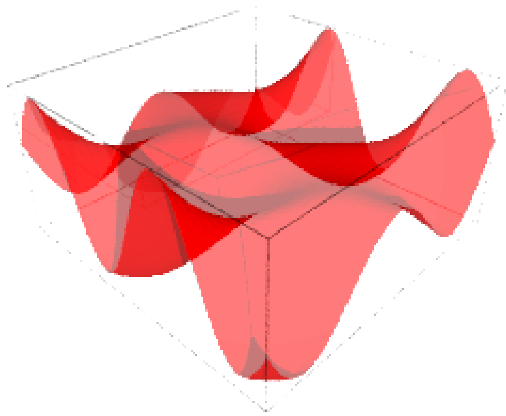
Get Image

```
icosahedron(opacity=.7, color='red') +
cube(opacity=.4,color='green')
```

Get Image

```
# user viewer='tachyon' on the ipad...
```

```
f(x,y) = sin(x - y) * y * cos(x)
plot3d(f, (x,-3,3), (y,-3,3), opacity=.9, color='red', figsize=3,
viewer='tachyon')
```



```
G = sum(sphere((random(), random(), random()), random()/4,
        color=hue(random()), opacity=.6)
        for i in range(20))
G.show(aspect_ratio=1, frame=True, figsize=3)
```

Get Image

Plotting a 3D Model

See http://www.davidson.edu/math/chartier/Starwars/

```
# Yoda! (53,756 vertices)

from scipy import io
yoda = io.loadmat(DATA + 'yodapose.mat')

from sage.plot.plot3d.index_face_set import IndexFaceSet
V = yoda['V']; F3=yoda['F3']-1; F4=yoda['F4']-1
Y = IndexFaceSet(F3,V,color=Color('#444444')) +
IndexFaceSet(F4,V,color=Color('#007700'))
Y = Y.rotateX(-1)
Y.show(aspect_ratio=1, frame=False, zoom=1.2)
```

Get Image

Questions

????

```
search_doc('ode_solver')
```

Search Documentation: "ode_solver"

1. reference/genindex-I.html
2. reference/genindex-O.html
3. reference/genindex-P.html
4. reference/genindex-all.html
5. reference/sage/calculus/desolvers.html
6. reference/sage/gsl/ode.html

If you made it this far, next try the Sage Tutorial.