Writing a Research Paper using SageMathCloud

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"A Databases of Elliptic Curves Ordered by Height and Distributions of Selmer Groups and Ranks"

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What's this about?

- Past tables are usually ordered by conductor or discriminant.
- Bhargava–Shankar: new upper bounds on the average algebraic rank ordered by height; got by studying the average sizes of n-Selmer groups.
- Make database ordered by height: compute rank and 2-Selmer size.

Background motivation

Order by conductor

- Mazur-Stein-Watkins-Bektemerov - 2006 paper: $136,832,795$ curves ordered by conductor; average rank keeps getting bigger!?

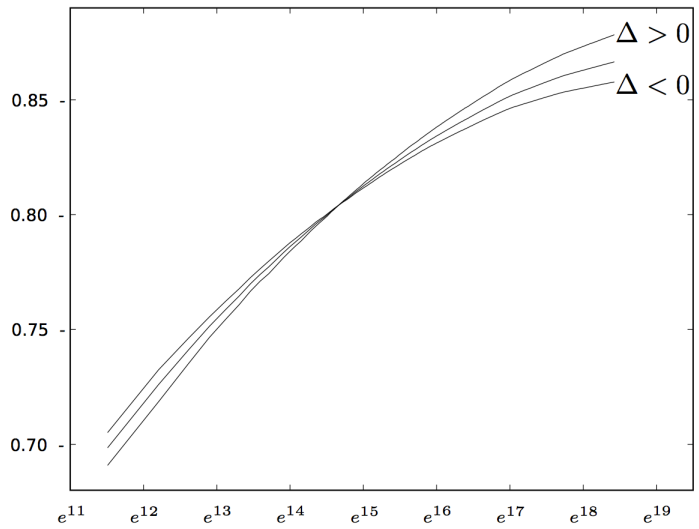


FIGURE 3. Average Rank of Stein-Watkins Curves of Conductor $\leq 10^8$

- Conjecture: average rank is $1/2$
- Manjul Bharghava: average rank is definitely bounded!

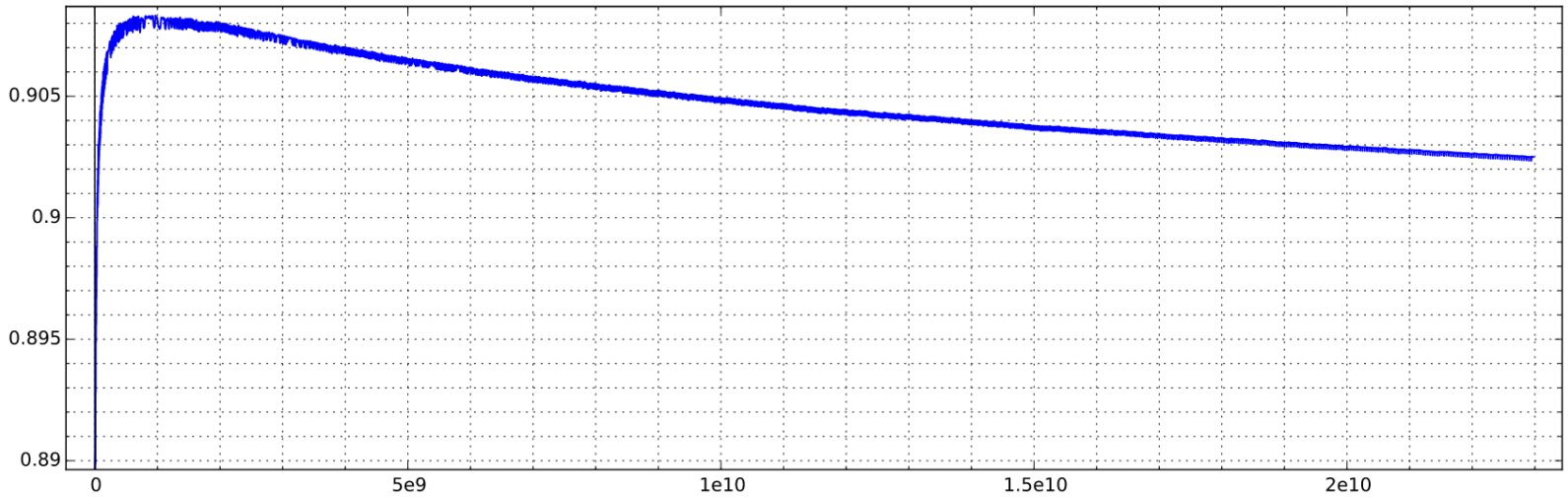
Challenge!

- Bharghava: order curves by height!
- Challenge: systematically compute the ranks of **all** elliptic curves $y^2 = x^3 + a_4x + a_6$ of height $H = \max\{4|a_4|^3, 27a_6^2\}$ far enough that we can finally clearly see the rank going down (and how).
- We did this.

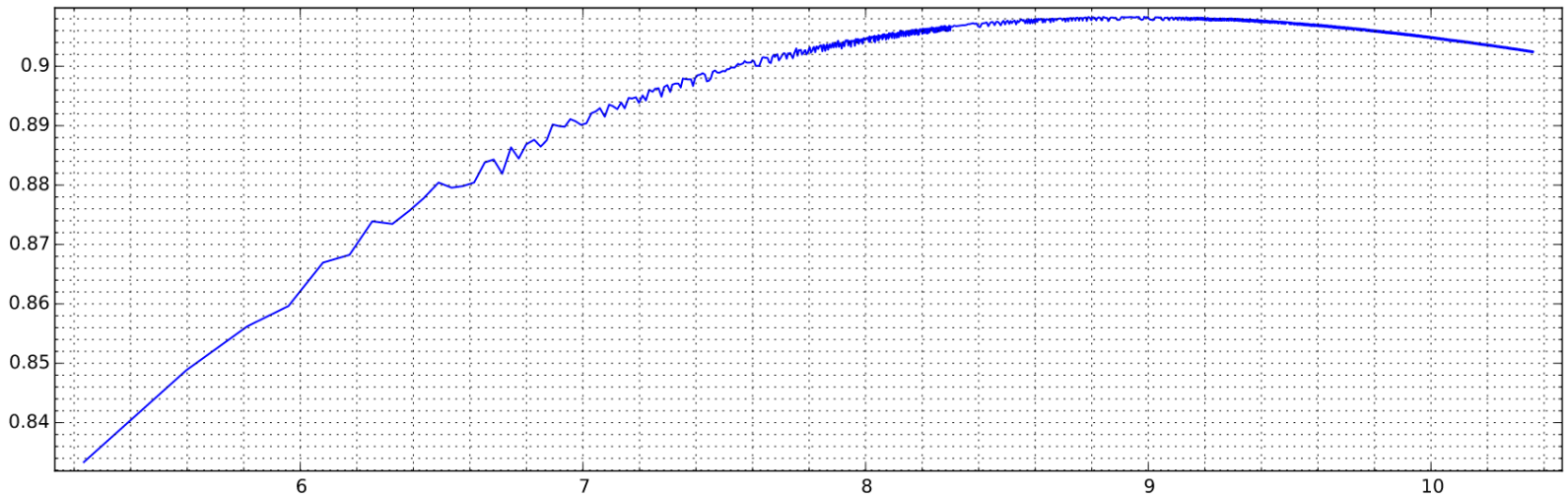
Avg. Rank: 238764310 Curves of Height $\leq 2.7 \cdot 10^{10}$

```
print v['desc'], v['count'], v['bound']
show(line(plot.derez(v['data'],5000)), figsize=[12,4], gridlines='minor', frame=True, ymin=.89)
```

```
Rank of the Mordell-Weil group. 239000520 27000000000
```



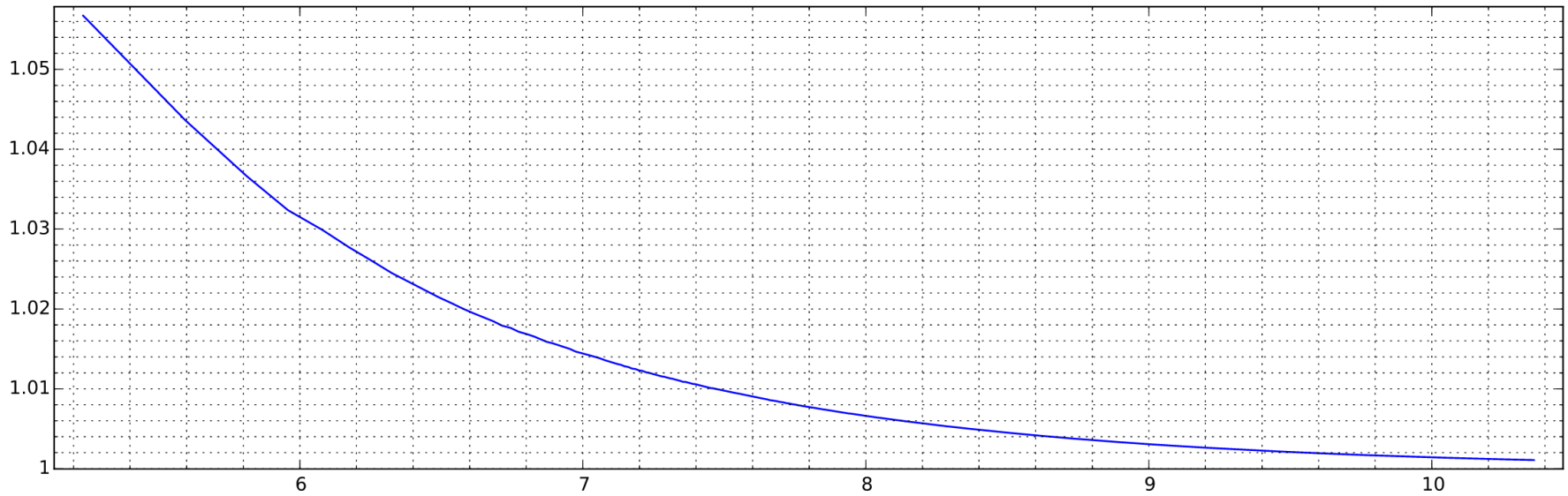

```
vlog = [(math.log(x[0],10),x[1]) for x in plot.derez(v['data'],5000)][1:]
show(line(vlog), figsize=[12,4], gridlines='minor', frame=True)
```



Order of the torsion subgroup

```
import plot; p = plot.Plots()
v = p.plot('tor')
print v['desc']
vlog = [(math.log(x[0],10),x[1]) for x in plot.derez(v['data'],5000)][1:]
show(line(vlog), figsize=[12,4], gridlines='minor', frame=True)
```

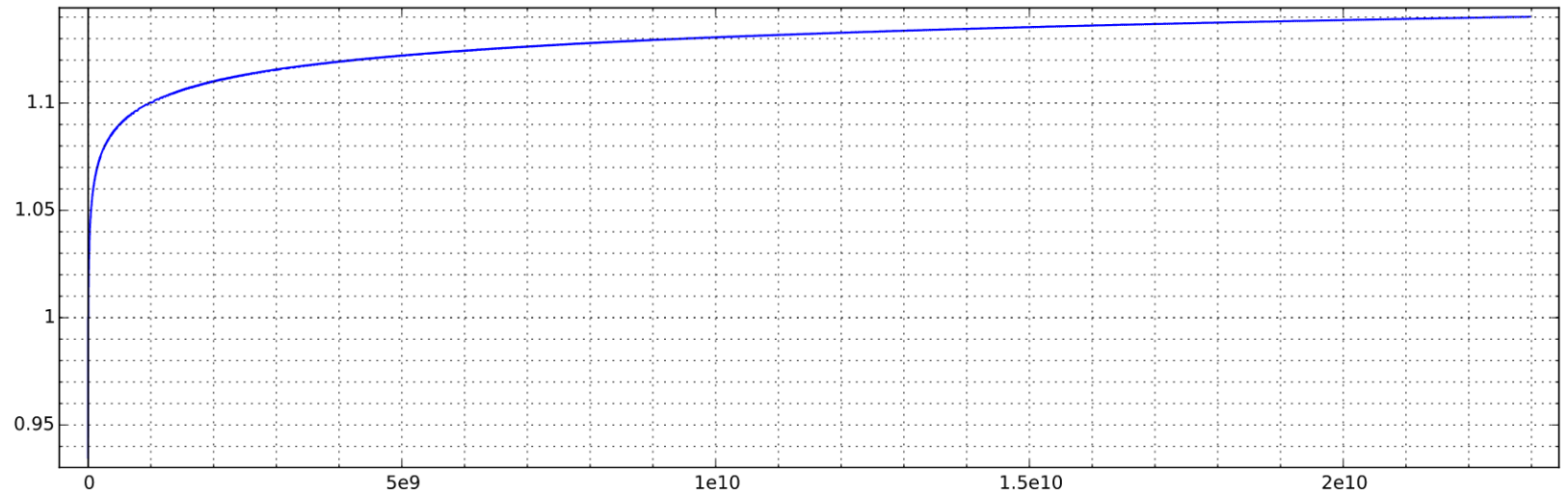
```
2015-12-20 18:09:51: getting data from dbs with index up to i=28...
Order of the torsion subgroup
```

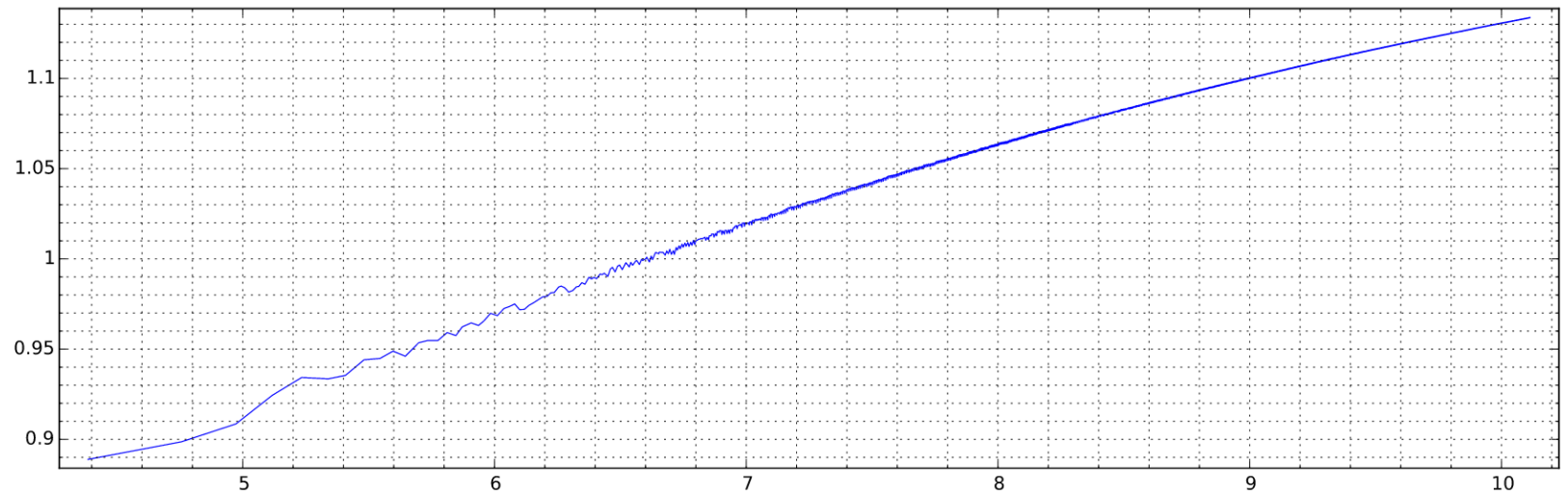


Rank of the 2-Selmer Group

```
print v['desc'], v['count'], v['bound']
show(line(plot.derez(v['data'],5000)), figsize=[12,4], gridlines='minor', frame=True)
vlog = [(math.log(x[0],10),x[1]) for x in plot.derez(v['data'],15000)][1:]
show(line(vlog, thickness=.6), figsize=[12,4], gridlines='minor', frame=True)
```

```
Rank of the 2-Selmer group. 239000520 27000000000
```

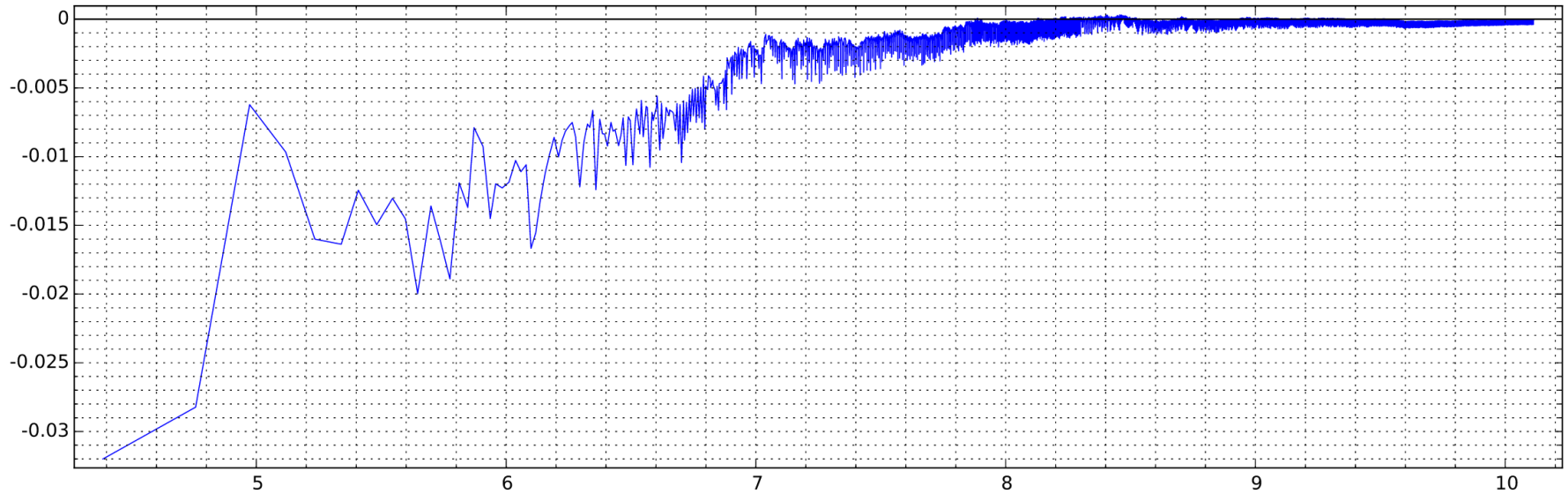




Avg. Sign of the Root Number

```
import plot; p = plot.Plots()
v = p.plot('w')
print v['desc']
vlog = [(math.log(x[0],10),x[1]) for x in plot.derez(v['data'],15000)][1:]
show(line(vlog, thickness=.6), figsize=[12,4], gridlines='minor', frame=True)
```

```
2015-12-20 18:11:54: getting data from dbs with index up to i=28...
Sign of the Atkin-Lehner involution
```



Features of SageMathCloud for writing this paper

We wrote the paper in `https://cloud.sagemath.com`

Use

- Collaborative editing of LaTeX documents
- Collaborative persistent terminals (e.g., ssh to cluster somewhere)
- Edit Python code; run from Sage worksheets (or terminals)
- Use SQLite easily
- Run project on a 32-core big-memory VM at Google
- Chat: post comments on the side of any file being edited
- Following the log

Features of Sage for writing this paper

Key things we use

- Simon Spicer's new L-function analytic rank bounding code (fully included in Sage by default, very flexible, and well documented!). Very fast (even for large conductor) code for bounding elliptic curve ranks.
- Simon Spicer's Elliptic curve enumeration (coming to Sage soon)
- MWRANK: the workhorse
- Magma interface – helped with a few hard curves at the end.