Let $r_0, \ldots, r_p$ be coset representatives for $\Gamma_0(p)$ in $\mathrm{SL}_2(\mathbf{Z})$. So

$$\mathrm{SL}_2(\mathbf{Z}) = r_0\Gamma_0(p) \cup r_1\Gamma_0(p) \cup \cdots \cup r_p\Gamma_p(p).$$

E.g.,

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}, \ldots, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Let V be the $p+1$-dimensional vector space on symbols

$$x_i = r_i\{0, \infty\} = \{r_i(0), r_i(\infty)\}.$$

The space of modular symbols is

$$H_1(X_0(p), \mathbf{Q}; \text{cusps}) = V/R.$$

The subspace R of relations is the subspace generated by all

$$x_i + Sx_i$$

$$x_i + Tx_i + T^2 x_i,$$

where

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \text{ and } \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}.$$