Exercises for Section 4.5: Computing the Class Group

Math 582e, Winter 2009, University of Washington

Due Wednesday February 11, 2009

1. Explain as much as you can about what "goes on" when using the class group algorithm from class to compute the class groups of the following field:

   (a) $x^4 + 6$
   (b) $x^4 - 3x + 5$
   (c) $x^4 - 3x - 5$
   (d) $x^3 + 113$

   In particular,

   (a) What bound B do you use?
   (b) What is $\#T$?
   (c) What is the rank of $\mathcal{O}_K^*$ and $\mathcal{O}_{K,T}^*$?
   (d) Do the units of the form p and $\alpha - k$, for $f(k)$ smooth with $|k| < 100$, suffice to generate $\mathcal{O}_{K,T}^*$ (up to torsion)?
   (e) If the answer to the previous question is no, how many more units did you need to throw in using LLL?
   (f) What are explicit generators for the class group?

   You may use Sage as much as you want for this problem, along with the code handouts at `http://wiki.wstein.org/09/582e/code` (see the tex files for cut and pastable code). If you want, you can use the output of Sage/PARI's `class_number` command to find out what the true class number is; i.e., you don't have to use the class number formula to find $\mathrm{Reg}_K \cdot \#\,\mathrm{Cl}(K)$, and you don't have to compute an approximation to Reg_K explicitly... unless you want to.